Master in Financial Engineering
Financial Econometrics

Principal Components Analysis

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Factor Models

1. Factors as observed portfolios of traded assets :
   - CAPM as one-factor model
   - Fama-French three-factor model
   - Illustration : see Econometrics course, 1st semester
2. Factors as observed economic variables
   - Macroeconomic factors : GNP, inflation rate, unemployment rate, etc.
3. Factors as latent (estimated) variables
   - Principal Component Analysis
   - Factor Analysis
   - Illustration : **Our goal for today**

Road map

1. Part I : Swap rates or first-difference swap rates ?

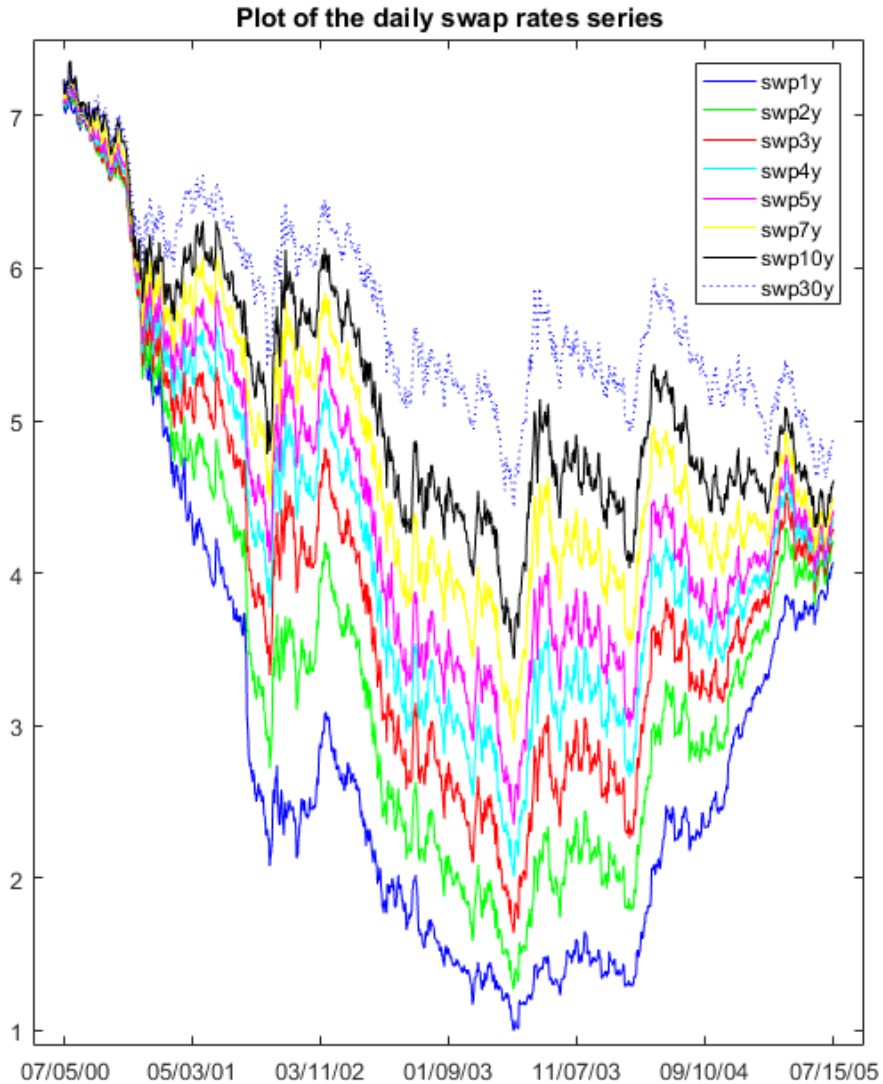
2. Part II : Principal Component Analysis

Part I : Swap rates or first-difference swap rates ?

- Data :
  - Daily swap rates from 7/3/2000 to 7/15/2005.
  - Different maturities : 1-year, 2-year,..., 30-year.
- Remark :
  - Let $\{x_t\}$ denote a (financial) time series.
  - The first-differenced series, $\{y_t\}$ is defined as :

$$y_t = x_t - x_{t-1}$$

- Tasks :
  1. Plot the daily swap rates series and the first-difference of each series.
  2. Determine the covariance (resp. correlation) matrix for the level series and the first-differenced series.
  3. Comment on the results.


Plot of the daily swap rates series
swp1y
swp2y
swp3y
swp4y
swp5y
swp7y
swp10y
swp30y
7
6
5
4
3
2
1
07/05/00
05/03/01
03/11/02
01/09/03
11/07/03
09/10/04
07/15/05

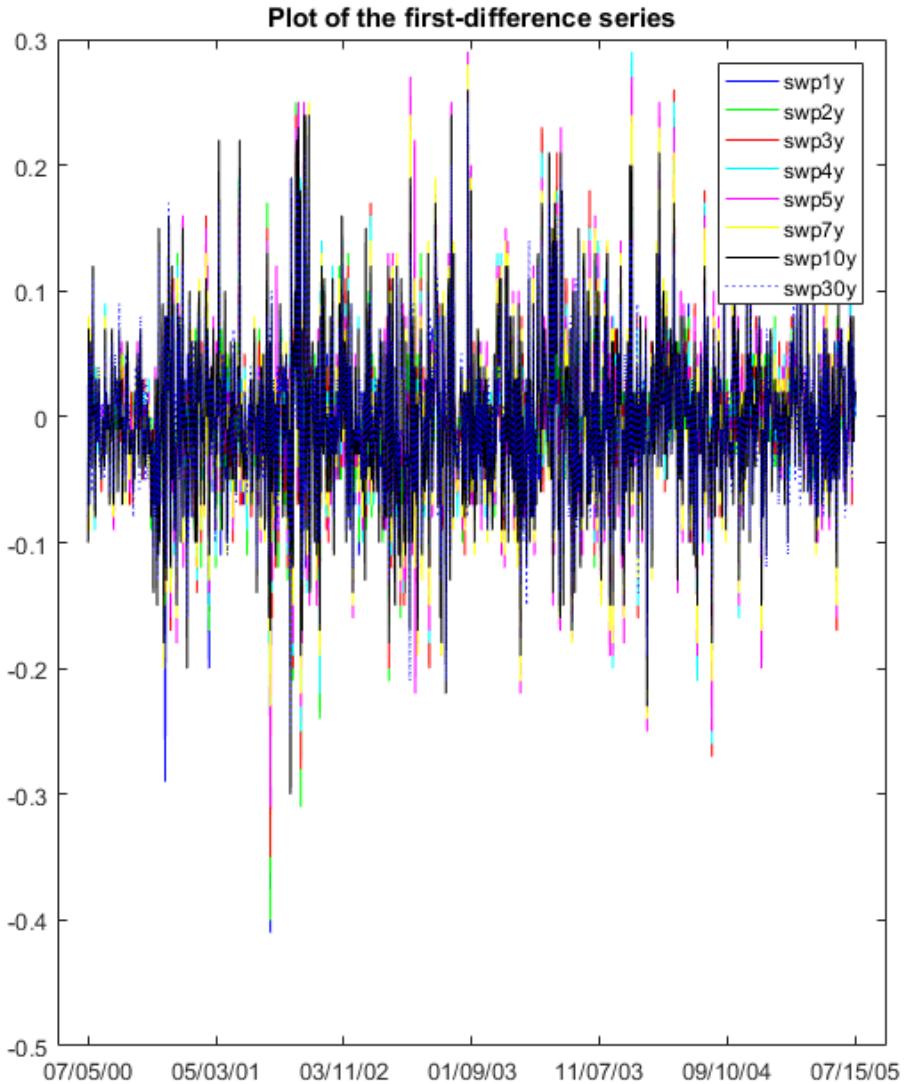

Plot of the first-difference series
swp1y
swp2y
swp3y
swp4y
swp5y
swp7y
swp10y
swp30y
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
07/05/00
05/03/01
03/11/02
01/09/03
11/07/03
09/10/04
07/15/05

FIGURE – Covariance Matrix : Daily Swap Series

2.700196							
2.371115	2.133177						
2.079241	1.901130	1.716569					
1.848071	1.708523	1.557459	1.423516				
1.662560	1.549692	1.423232	1.308504	1.208691			
1.400411	1.320653	1.226002	1.136883	1.057617	0.935497		
1.161642	1.107518	1.038847	0.971460	0.910067	0.813701	0.715597	
0.763452	0.742098	0.710213	0.675025	0.640964	0.585198	0.525433	0.402382

FIGURE – Covariance Matrix : First-Difference Series

0.002329							
0.003004	0.004378						
0.003032	0.004544	0.004883					
0.002966	0.004532	0.004921	0.00508				
0.002917	0.004507	0.004939	0.00514	0.005427			
0.002682	0.004207	0.004659	0.004897	0.005128	0.004983		
0.002404	0.003843	0.004306	0.004583	0.004774	0.004721	0.004673	
0.001691	0.002783	0.003182	0.003442	0.003618	0.003651	0.003686	0.003216

FIGURE – Correlation Matrix : Daily Swap Series

1.000000							
0.987965	1.000000						
0.965776	0.993500	1.000000					
0.942627	0.980452	0.996333	1.000000				
0.920284	0.965105	0.988069	0.997554	1.000000			
0.881123	0.934876	0.967474	0.985175	0.994602	1.000000		
0.835681	0.896402	0.937318	0.962519	0.978546	0.994510	1.000000	
0.732428	0.800993	0.854553	0.891907	0.919088	0.953811	0.979184	1.000000

FIGURE – Correlation Matrix : First-Difference Series

1							
0.940889	1						
0.898996	0.982646	1					
0.862339	0.960871	0.98802	1				
0.820606	0.924492	0.959481	0.978967	1			
0.787311	0.900777	0.944556	0.973258	0.986042	1		
0.728656	0.849569	0.901473	0.940633	0.947953	0.978388	1	
0.61786	0.741574	0.8029	0.851534	0.865974	0.912197	0.95087	1

Part II : Principal Component Analysis

- Data :
  - See Part I.
  - Use all curves.
- Tasks :
  1. Run a principal component analysis.
  2. Provide the eigenvalue decomposition, the contribution of each component and the cumulative contribution.
  3. How many components can one choose ?
  4. Interpret the principal components.
  5. Propose an approximate factor model with 1, 2, 3 factor(s) respectively.

Figure – PCA of Covariance Matrix (raw data)

Number	Eigenvalue	Forward Difference	Proportion	Cumulative Value	Cumulative Proportion
1	10.75806	10.31334	0.9575	10.75806	0.9575
2	0.444720	0.414394	0.0396	11.20277	0.9971
3	0.030326	0.028392	0.0027	11.23310	0.9998
4	0.001933	0.001486	0.0002	11.23503	0.9999
5	0.000447	0.000357	0.0000	11.23548	1.0000
6	8.97E-05	5.47E-05	0.0000	11.23557	1.0000
7	3.49E-05	1.56E-05	0.0000	11.23560	1.0000
8	1.93E-05	---	0.0000	11.23562	1.0000

Figure – PCA of Covariance Matrix (raw data) : Eigenvectors

Variable	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8
SWP1Y	0.485128	-0.598328	0.541604	-0.312288	0.098997	-0.073487	-0.005504	-0.023751
SWP2Y	0.442053	-0.254841	-0.208135	0.573604	-0.444325	0.351794	0.044392	0.210115
SWP3Y	0.398964	-0.011642	-0.361292	0.240393	0.251219	-0.401050	-0.205044	-0.621591
SW4Y	0.362240	0.140580	-0.313529	-0.114340	0.345573	-0.345759	0.092672	0.700063
SW5Y	0.331397	0.239932	-0.206510	-0.363973	0.171819	0.535649	0.522416	-0.261008
SW7Y	0.286123	0.350159	0.006751	-0.324382	-0.172345	0.273956	-0.763334	0.053104
SW10Y	0.243156	0.417992	0.230446	-0.111990	-0.624467	-0.460485	0.302394	-0.084044
SW30Y	0.167435	0.449740	0.581992	0.502345	0.401321	0.128188	0.018122	0.028555

Figure – PCA of Correlation Matrix (raw data)

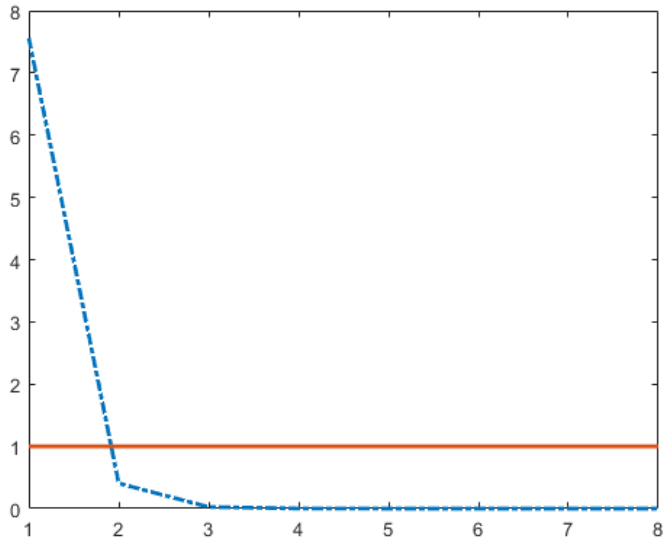
Number	Eigenvalue	Forward Difference	Proportion	Cumulative Value	Cumulative Proportion
1	7.566005	7.160278	0.9458	7.566005	0.9458
2	0.405727	0.379617	0.0507	7.971732	0.9965
3	0.026110	0.024503	0.0033	7.997842	0.9997
4	0.001607	0.001172	0.0002	7.999448	0.9999
5	0.000435	0.000364	0.0001	7.999883	1.0000
6	7.06E-05	3.69E-05	0.0000	7.999954	1.0000
7	3.37E-05	2.13E-05	0.0000	7.999988	1.0000
8	1.24E-05	---	0.0000	8.000000	1.0000

Figure – PCA of Correlation Matrix (raw data) : Eigenvectors

Variable	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8
SWP1Y	0.339760	-0.534770	0.628221	-0.399870	0.172826	-0.112592	-0.017281	-0.034371
SWP2Y	0.353490	-0.365233	0.021996	0.449559	-0.510677	0.451773	0.082432	0.259573
SWP3Y	0.360191	-0.202757	-0.237122	0.387210	0.040681	-0.346814	-0.217693	-0.673549
SW4Y	0.362667	-0.072085	-0.320119	0.108062	0.301456	-0.466059	-0.043435	0.662683
SW5Y	0.362950	0.035200	-0.319279	-0.194342	0.367517	0.358417	0.654100	-0.189585
SW7Y	0.360481	0.194684	-0.215170	-0.359039	0.056881	0.427708	-0.686078	0.008462
SW10Y	0.354464	0.347256	-0.022500	-0.383780	-0.653505	-0.362993	0.211605	-0.048640
SW30Y	0.333195	0.612039	0.545351	0.403021	0.226797	0.050722	0.017415	0.015592

How many components can one choose ?

FIGURE – PCA of Correlation Matrix (raw data) : Scree Graph



Recall :

- The **Scree Graph** plots the eigenvalues against the number of components.
- Detect whether there is a 'break', i.e. decide at which component k the slopes of the lines are steep (resp. not steep) to the left of k.

Interpretation of the principal components (1/4)

Parallel Shift (level) Component :

- The factor loadings of the first principal component are roughly constant over different maturities.
- This means that a change in the swap rate for one maturity is accompanied by roughly the same change for other maturities.

Tilt Component :

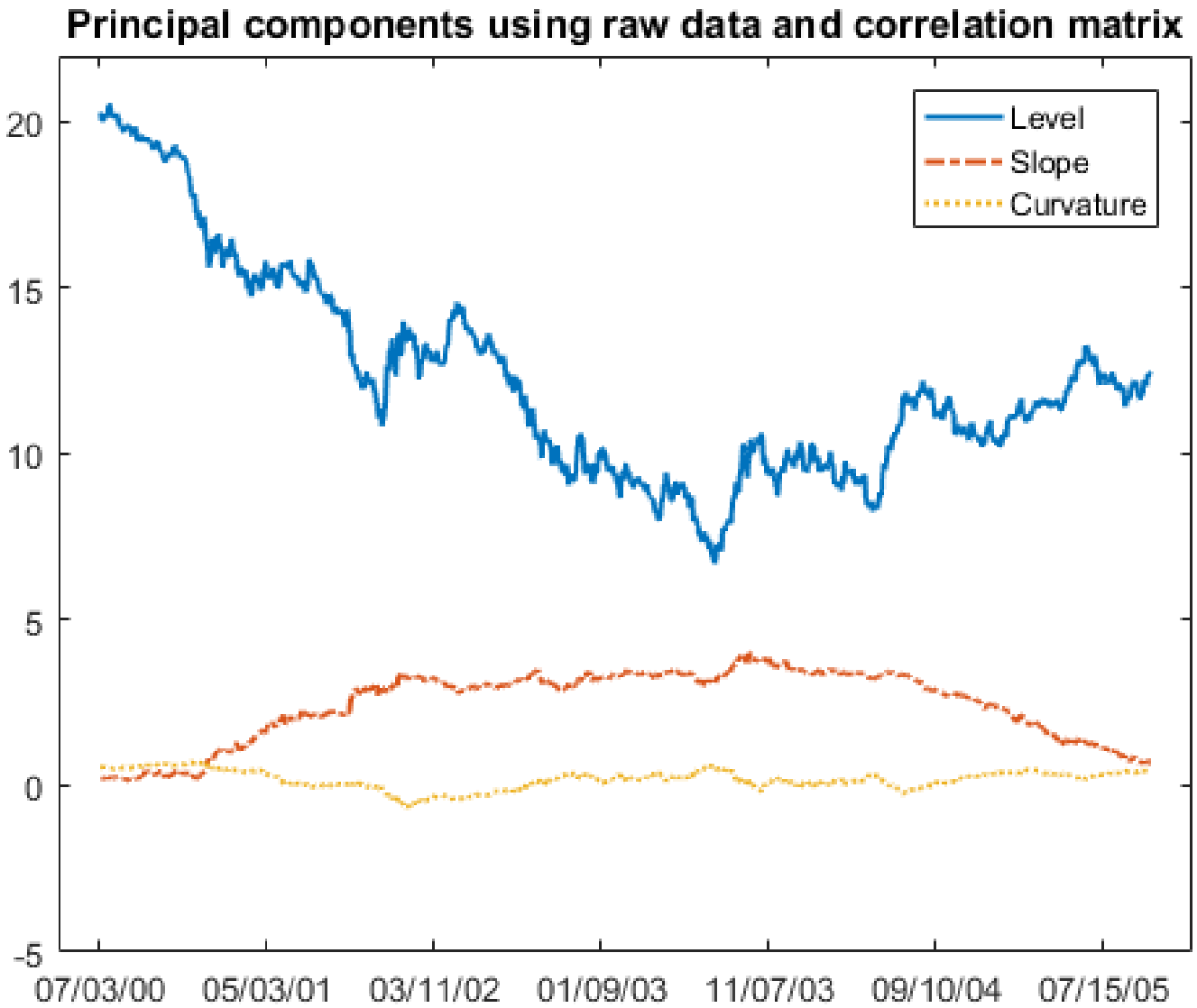
- The factor loadings of the second principal component have a monotonic change with maturities.
- Changes in short-maturity and long-maturity swap rates in this component have opposite signs.

Interpretation of the principal components (2/4)

Curvature Component :

- The factor loadings of the second principal component have a monotonic change with maturities.
- The factor loadings of the third principal component are different for the midterm rates and the average of short- and long-term rates, revealing a curvature of the graph that resembles the convex shape of the relationship between the rates and their maturities.

Interpretation of the principal components (3/4)



Interpretation of the principal components (4/4)