Position Control by Examples

1 Device description

Consider the following device (Figure 1). It is a cable driven disc steered by a Brushless Maxon[1] DC motor. It is equipped with two incremental encoders for position measurement. The first one is on the motor shaft and the second one is on the output shaft.

The device parameters are considered as follows:

- M_D and J_D are respectively the Mass and the Inertia of the disc relative to its rotation center.
- r_g is the distance of the center of mass of the disc to its rotation center.
- I_m is the inertia of the motor.
- The gear ratio of the cable based transmission is 15.

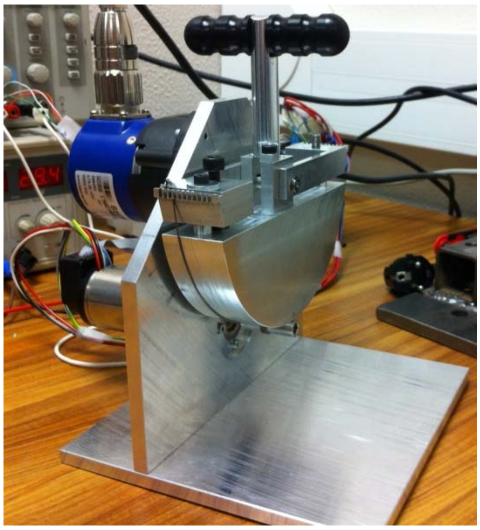

Figure 1-Haptic Device used for position control

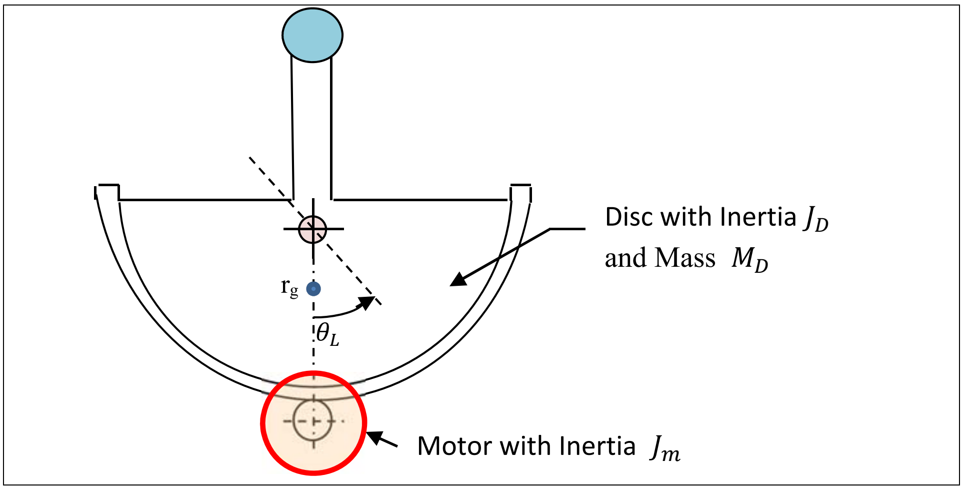

Figure 2- Parameterization of the Haptic Paddle

2 Device Dynamic Model

By considering k_{vis} and Γ_{dry}, respectively the viscous friction constant and the dry friction at the output side, the Euler dynamic equation is written as follows

$$\sum M = \mathcal{J}_{RL}\ddot{\theta}_L = \Gamma_{act} - \Gamma_{dry} - k_{vis}\dot{\theta}_L - \Gamma_g \quad \text{(eq. 1)}$$

$$\Gamma_g = M_D.g.r_g.\sin(\theta_L) \quad \text{(eq. 2)}$$

Γ_g is the gravity torque. Γ_{act} is the actuating torque reported at the output side.

[1] www.maxonmotor.ch

J_{RL} is the total inertia reported at the load side

$$J_{RL} = J_D + n^2 J_n \quad \text{(eq. 3)}$$

Combining (eq.2) and (eq.3) leads to:

$$J_{RL}\ddot{\theta}_L = \Gamma_{act} - \Gamma_{dry} - k_{vis}\dot{\theta}_L - M_D\, g\, r_g \sin(\theta_L) \quad \text{(eq. 4)}$$

3 Control implementation

3.1 Exact compensation strategy

The exact compensation control strategy consists of using the motor torque control to compensate all the resistant torques. It can be as simple as compensating the gravity, the friction or any other known effects.

Hence, the actuating control torque may be written as:

$$\Gamma_{act} = \Gamma_{act}^{*} + \Gamma_{compensation} \quad \text{(eq. 5)}$$

Where $\Gamma_{compensation}$ is the compensating torque and Γ_{act}^{*} is the additional actuating torque assuring a desired objective (position, velocity or force control). This additional torque may come from a PID regulator.

(eq. 4) may be rewritten as follows:

$$J_{RL}\ddot{\theta}_L = \Gamma_{act} - \underbrace{M_D g\, r_g \sin(\theta_L) - \Gamma_{dry} - k_{vis}\,\dot{\theta}_L}_{\text{Torques (effects) to be compensated}}$$

We need to compensate the gravity, viscosity and the dry friction. This must be done by rejecting these values. We pose:

$$\Gamma_{act} = \Gamma_{act}^{*} + M_D g\, r_g \sin(\theta_L) + \Gamma_{dry} + k_{vis}\,\dot{\theta}_L \quad \text{(eq. 6)}$$

That means that the compensating torque is given by:

$$\Gamma_{compensation} = M_D g\, r_g \sin(\theta_L) + \Gamma_{dry} + k_{vis}\,\dot{\theta}_L \quad \text{(eq. 7)}$$

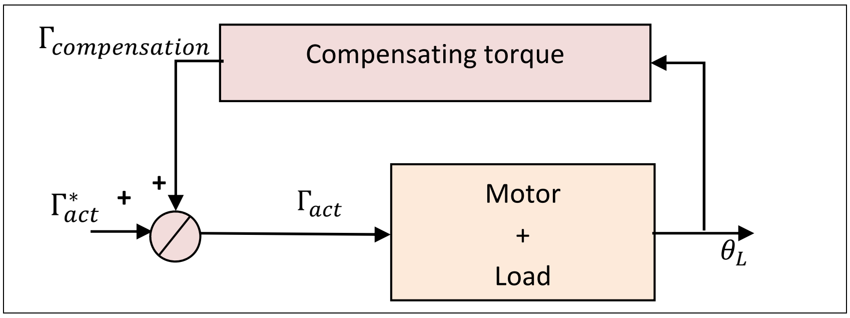

Figure 3- Compensating feedback loop

The relation between θ_L and the new control variable torque is then given by the following dynamic equation:

$$J_{RL}\,\ddot{\theta} = \Gamma^{*}_{act} \qquad \text{(eq. 8)}$$

Important

Note that this relation between θ_L and Γ_{act} (eq. 4) is nonlinear. After applying the compensating feedback, the relation between θ_L and $\Gamma^{*}{}_{act}$ becomes linear. This operation is also called exact nonlinear compensation or exact linearization. Furthermore, the obtained system (eq.8) is a double integrator that may be controller by any conventional controller.

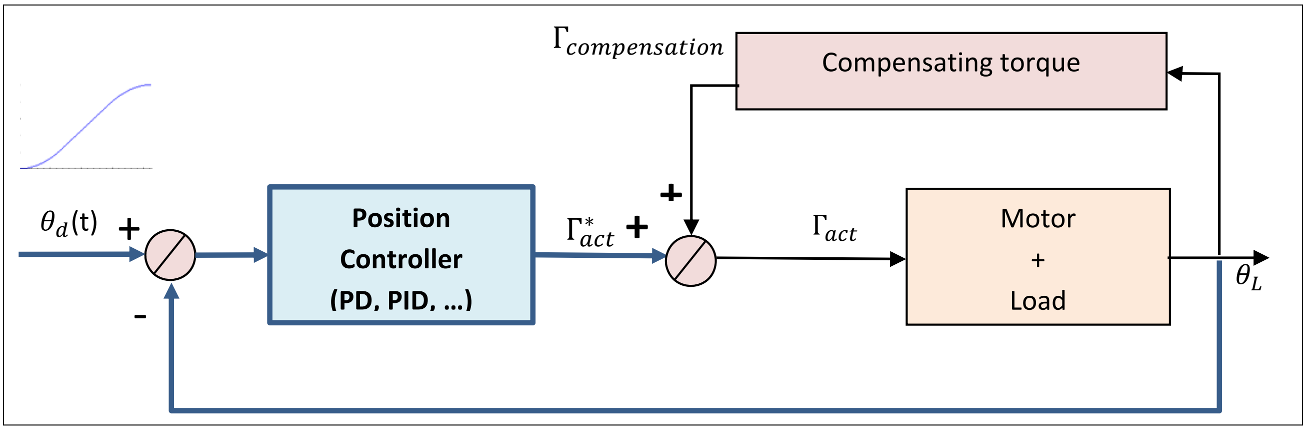

Figure 4- Position control with a an exact compensating feedback loop

Finally, the control expression of the torque applied to the motor may be given by:

$$\Gamma_{act} = \Gamma_{PID} + M_D g\, r_g\, sin(\theta_L) + \Gamma_{dry} + k_{vis}\,\dot{\theta}_L \qquad \text{(eq. 9)}$$

Where $\Gamma_{PID} = K_p \cdot \left(e + T_d \cdot \frac{de}{dt} + \frac{1}{T_i} \cdot \int e(\tau)d\,\tau\right)$ (eq. 10)

3.2 Feed forward (A priori) strategy

We can evaluate the needed torque to realize a known trajectory defined by the desired position, velocity and acceleration thanks to the dynamic inverse model. This torque is called the "a priori torque" and is defined as the motor torque for a given time-defined trajectory (eq. 11).

$$\boldsymbol{\Gamma_{ap} = \Gamma_{motor}(\theta = \theta_d(t), \dot{\theta} = \theta_d(t), \ddot{\theta} = \theta_d(t))} \qquad \text{(eq. 11)}$$

$$\Rightarrow \Gamma_{ap} = \underbrace{J_{RL}\ddot{\theta}_d}_{\text{Inertia feed forward “A priori”}} + \underbrace{M_d g\, r_g \sin(\theta_d)}_{\text{Gravity feed forward “A priori”}} + \underbrace{k_{vis}\,\dot{\theta}_d + \Gamma_{dry}}_{\text{Friction feed forward “A priori”}} \qquad \text{(eq. 12)}$$

The final control torque is the sum of both contributions: the a priori torque computed thanks to the desired position and a closed loop torque provided by a chosen control algorithm such as a PID function (eq.13).

The actuating control torque at the load side may be expressed as follows:

$$\Gamma_{act} = \Gamma_{PID} + \mathcal{J}_{RL}\ddot{\theta}_d + M_d g\, r_g \sin(\theta_d) + k_{vis}\,\dot{\theta}_d + \Gamma_{dry} \qquad \text{(eq. 13)}$$

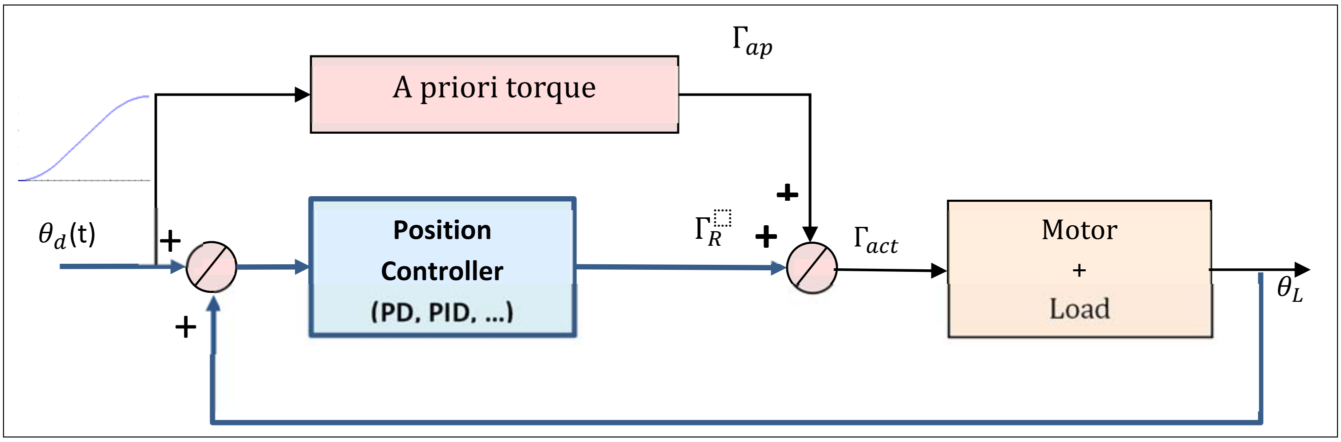

Figure 5- Position control with an a priori torque

<u>The dynamic behavior</u>

By considering only the a priori loop, the dynamic behavior is deduced and given by (eq. 14)

$$\mathcal{J}_{RL}\ddot{\theta} = \Gamma_R + \mathcal{J}_{RL}\ddot{\theta}_d + M_d g\, r_g \sin(\theta_d) + k_{vis}\,\dot{\theta}_d + \Gamma_{dry} - M_d g\, r_g \sin(\theta_L) - k_{vis}\,\dot{\theta}_L - \Gamma_{dry}$$

$$\varepsilon = \theta - \theta_d$$

$$\Rightarrow \mathcal{J}_{RL}\ddot{\varepsilon}_\theta = \Gamma_R - k_{vis}\,\dot{\varepsilon}_\theta + \left\{M_d g\, r_g \sin(\theta_d) - M_d g\, r_g \sin(\theta_L)\right\} \qquad \text{(eq. 14)}$$

<u>Important</u>

This behavior does not correspond to a linear system cause to the presence of the sinus function. To carry out the stability analysis of such dynamics after using a linear controller (PD, PID,...), one must make a linearization around an operational point. Another solution is to use nonlinear stability analysis tools such as Lyapounov stability purposes.

This means that in the presence of nonlinearities, the proof of the stability when using the a priori controller must be taken with a particular care. Even though, this scheme remains commonly used in robotic industrial control. This is due to its easiness of implementation while having the possibility to take into account coupling effects when multi-DOF robots are considered.