Problem Set 2 (solution)

Exercise 1

(i)

Demand (D) = Average revenue (AR): $P = 2{,}34 - 1{,}34Q$

$TR = P \cdot Q = 2{,}34Q - 1{,}34Q^2 \quad \Rightarrow \quad MR = 2{,}34 - 2{,}68Q$

$TC = UTC(Q) \cdot Q = 0{,}85Q^2 - 0{,}83Q \Rightarrow \quad MC = 1{,}7Q - 0{,}83$

$\pi = TR - TC = 2{,}34Q - 1{,}34Q^2 - (0{,}85Q^2 - 0{,}83Q) = -2{,}19Q^2 + 3{,}17Q$

$\frac{\partial \pi}{\partial Q} = -4{,}38Q + 3{,}17 = 0 \quad \Leftrightarrow \quad 3{,}17 = 4{,}38Q \quad \Rightarrow \quad Q^* = 0{,}72$

(Remark: By equalizing MR and MC we have the same result.)

$P^*_M = 2{,}34 - 1{,}34Q^* = 2{,}34 - 1{,}34 \cdot 0{,}72 = 1{,}375$

$\pi^*_M = -2{,}19(Q^*)^2 + 3{,}17Q^* = -2{,}19(0{,}72)^2 + 3{,}17 \cdot 0{,}72 = 1{,}15$

(ii)

$$\varepsilon = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = -\frac{1}{1{,}34} \cdot \frac{1{,}375}{0{,}72} = -1{,}425$$

(iii)

Under perfect competition: $MC = P$

$MC = 1{,}7Q - 0{,}83 = P \quad \Rightarrow \quad Q_S = (P + 0{,}83)/1{,}7$

Equilibrium on this market:

$Q_S = (P^* + 0{,}83)/1{,}7 = 2{,}34/1{,}34 - P^*/1{,}34 = Q_D$

$1{,}34 \cdot (P^* + 0{,}83) = (2{,}34 - P^*) \cdot 1{,}7$

$3{,}04P^* = 2{,}8658 \quad \Rightarrow \quad P^*_{PC} = 0{,}94 \ (< P^*_M) \quad \Rightarrow \quad Q^*_{PC} = 1{,}04$

(iv)

Consumers' surplus in the monopoly situation:

$P_{max} = 2{,}34$

$CS_M = (P_{max} - P^*) \cdot (Q^*/2) = (2{,}34 - 1{,}38) \cdot (0{,}72/2) = 0{,}34$

Consumers' surplus under perfect competition:

$CS_{PC} = (P_{max} - P^*_{PC}) \cdot (Q^*_{PC}/2) = (2{,}34 - 0{,}94) \cdot (1{,}04/2) = 0{,}73$

$\Rightarrow$ Change of the consumers' surplus: $0{,}73 - 0{,}34 = 0{,}39$

Exercise 2

(i)

At the optimum: $MR_1(q_1) = MR_2(q_2) = MC(q_1 + q_2)$

$q_1 = -P_1/8 + 4 \quad \Leftrightarrow \quad P_1 = 32 - 8q_1 \quad \Leftrightarrow \quad TR_1 : 32q_1 - 8q_1^2$

$q_2 = -P_2/10 + 2 \quad \Leftrightarrow \quad P_2 = 20 - 10q_2 \quad \Leftrightarrow \quad TR_2 : 20q_2 - 10q_2^2$

$MR_1 = 32 - 16q_1$ and $MR_2 = 20 - 20q_2$

The two marginal revenues must be equal:
$MR_1 = 32 - 16q_1 = 20 - 20q_2 = MR_2 \quad \Leftrightarrow \quad 12 + 20q_2 = 16q_1 \quad \Rightarrow \quad q_1 = 0{,}75 + 1{,}25q_2$

$MR_2 = 20 - 20q_2 = 3Q^2 - 12Q + 15 = MC$ where $Q = q_1 + q_2 = 0{,}75 + 2{,}25q_2$

$$20 - 20q_2 = 3(0{,}75 + 2{,}25q_2)^2 - 12(0{,}75 + 2{,}25q_2) + 15$$
$$20 - 20q_2 = 1{,}6875 + 10{,}125q_2 + 15{,}1875\, q_2^2 - 9 - 27q_2 + 15$$
$$15{,}1875\, q_2^2 + 3{,}125q_2 - 12{,}3125 = 0$$

$$q_2 = \frac{-3.125 \pm \sqrt{3.125^2 - 4 \cdot 15.1875 \cdot (-12.3125)}}{2 \cdot 15.1875} = \frac{-3.125 \pm \sqrt{757.75}}{30.375} \cong \begin{matrix} 0.8034 \\ -1.009 \end{matrix}$$

We choose the positive quantity ($q_2 = 0.8034$) from which we find q_1:

$q_1 = 0{,}75 + 1{,}25q_2 \Leftrightarrow \quad q_1 = 0{,}75 + 1{,}25 \cdot 0.8034 \cong 1.7542$

$q_1 = 1.7542$	$q_2 = 0.8034$	$Q = q_1 + q_2 \cong 2.5576$

The prices fixed on the markets are the following:

$P_1 = 32 - 8q_1 = 32 - 8 \cdot 1.7542 \cong 17.966$	$P_2 = 20 - 10q_2 = 20 - 10 \cdot 0.8034 \cong 11.966$

The total cost of the firm is:
$TC(Q) = Q^3 - 6Q^2 + 15Q = (2.5576)^3 - 6(2.5576)^2 + 15(2.5576) = 15.846$

The total profit of the firm is:
$\pi_{tot} = TR_1(q_1) + TR_2(q_2) - TC(Q) = 17.966 \cdot 1.7542 + 11.966 \cdot 0.8034 - 15.846 \cong 25.28$

(ii)

$$\varepsilon_1 = \frac{dq_1}{dP_1} \cdot \frac{P_1}{q_1} = -\frac{1}{8} \cdot \frac{17{,}966}{1{,}7542} = -1{,}28$$
$$\varepsilon_2 = \frac{dq_2}{dP_2} \cdot \frac{P_2}{q_2} = -\frac{1}{10} \cdot \frac{11{,}966}{0{,}8034} = -1{,}49 \qquad |\varepsilon_1| < |\varepsilon_2| \qquad \Rightarrow \qquad P_1 > P_2$$

The demand in the second market is more elastic so that the price in this market is lower than that of market 1.