Exercise Five

1. Prove that $\mathbb{S}^2$ is a smooth manifold of dimension 2.

2. Prove that $\{(x,t) \in \mathbb{R}^{n+1} | t \geq f(x)\}$ for a smooth map $f : \mathbb{R}^n \to \mathbb{R}$ is a smooth manifold of dimension $n+1$, with boundary.

3. Define what is a smooth application between two manifolds and then show that the composition of two smooth application is a smooth application.

4. Prove that T_pM is a vector space of dimension n.

5. Prove that TM is a smooth manifold of dimension $2n$.