

Exercise Six

Review Section 3.1, Page 27 - 36 of the LectureFour (download from Moodle).

1. Prove Lemma 3.8.
2. Prove Lemma 3.16
3. Prove Lemma 3.19
4. Suppose (U, φ) is a local coordinate chart on a smooth manifold M . Let $V \neq \emptyset$ be an open subset of M with $\bar{V} \subset U$ and \bar{V} being compact. Show that there exists a smooth function $h: M \rightarrow \mathbb{R}$ such that
 - $0 \leq h \leq 1$
 - $h \equiv 1$ on V and $h \equiv 0$ outside U .

Hint: Use Exercise Two, 2 and 3.

5. Prove Lemma 3.20 (partition of unity) using the previous exercise.