Master in Financial Engineering, EPFL

Course: Financial Econometrics

Homework 1: GARCH and portfolio management—An application to cryptocurrencies

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Overview

The objectives are: (i) To compare a static asset allocation with constant expected returns and volatility and a dynamic (two-week) asset allocation with time-varying volatility (using GARCH models) and (ii) to assess whether there is a so-called "timing-volatility", i.e. some gains when forecasting correctly the volatility dynamics.

Data

The Excel file homework_1_epfl_2019.xls includes the following daily financial variables over the period January 2016-April 2019: (1) the close price of 20 of the largest cryptocurrencies by market capitalization (Bitcoin, Ethereum, Ripple,...) and (2) the daily effective federal funds rate, rf.

You consider investing in a portfolio of two cryptocurrencies and a risk-free asset (i.e., the effective federal funds rate). Among the 20 currencies provided in the Excel spreadsheet (except Tether[1]), select two currencies you would like to invest in. Explain your choice carefully. If needed, provide the appropriate analyses to support your argument.

- Compute the (daily) log-returns of the two cryptocurrencies;
- Compute the (daily) log-returns of the risk free asset:

  $$r_{f,t} = \log\left(1 + r_t/100\right)$$

  where r_t is the one-day interest rate;
- Compute the excess log-returns of the first cryptocurrency, denoted $r_{1,t}$ and of the second cryptocurrency, denoted $r_{2,t}$.
  Remark: Do not multiply log-returns by 100.

Preliminary step: Characterization of the (co-) volatility

1. For both cryptocurrencies:
   - Plot an EWMA-based estimate of the (weekly) volatility.
   - Plot a GARCH-based estimate of the (weekly) volatility. In so doing, the following constant expected (log-) return model with GARCH-based error terms ($i = 1, 2$) is considered:

     $$\begin{aligned} r_{i,t} &= m_i + \epsilon_{i,t} \\ \epsilon_{i,t} &= \sigma_{i,t} z_{i,t} \\ \sigma_{i,t}^2 &= \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \end{aligned} \qquad (1)$$

     where $\omega_i > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$, and $z_{i,t} \sim \mathtt{i.i.d}\mathcal{N}(0,1)$.
   - Compute the term structure of volatility over the next 21 days.
   - Interpret the results.
   - Does one observe "more" volatility than a standard stock (say, the S&P500)? Explain carefully.
2. Provide some measures of conditional and unconditionnal correlation(s) between the two (log-) returns.

Static allocation using the constant expected returns model

Consider the case where you, as an investor, believe returns have constant first and second moments, and are i.i.d. You would like to invest your wealth in a portfolio composed of

[1] Please do not select Tether in your portfolio.

the two cryptocurrencies and cash (risk free asset) for **two consecutive days**. Given a degree of risk aversion λ, a standard mean-variance description of your allocation problem is the following:

$$\max_{w} \quad \mu_p[2] - \frac{\lambda}{2}\sigma_p^2[2] \tag{2}$$

where

- $w = (w_1, w_2, w_f)^\top$ denote the vector of weights such that

  $$w_f = 1 - w_1 - w_2$$
  $$w_1 \text{ and } w_2 \text{ are left unconstrained}$$

  where w_1 (resp., w_1) is the weight associated to the first (resp., the second) cryptocurrency.

- The first moment of the portfolio return over two consecutive days is given by:

  $$\mu_p[2] = w^\top \mu[2] + r_f[2]$$

  where $\mu[2] = (\mu_1[2], \mu_2[2])^\top$ denotes the vector of **expected excess log-returns** and $r_f[2]$ is the average two-period risk-free rate;

- The second moment of the portfolio return over two consecutive days is given by:

  $$\sigma_p^2[2] = w^\top \Sigma[2] w$$

  where $\Sigma[2]$ is the variance-covariance matrix of the 2-period (expected) log-returns.

Suppose that the one-period (expected excess) log-returns are given by the sample means and that the 1-period variance-covariance matrix is given by the sample variance-covariance matrix.

3. Compute the 2-period moments $\mu[2]$ and $\Sigma[2]$.
   Hint: Use the fact that you believe that the first two moments are constant (for $\mu[2]$ and $\Sigma[2]$) and the `i.i.d.` assumption (for $\mu[2]$ and $\Sigma[2]$).

4. Compute the optimal weights for $\lambda = 2$ and $\lambda = 10$.
   Hint: The optimal weights can be computed by using the close form solution of the optimization problem (Eq. 1) or by using some numerical procedures.

Dynamic asset allocation with time-varying volatility

It is now assumed that the assets are allocated for two consecutive days (i.e., the portfolio is rebalanced every second period) for $t = 1, 3, 5, \cdots, T$. The dynamic mean-variance allocation problem is the following:

$$\max_{w_t} \quad \mu_{p,t}[2] - \frac{\lambda}{2}\sigma_{p,t}^2[2] \tag{3}$$

where $\mu_{p,t}[2] = w_t^\top \mu[2] + r_f[2]$ and $\sigma_{p,t}^2[2] = w_t^\top \Sigma_t[2] w_t$ and $w_t = (w_{1,t}, w_{2,t}, w_{f,t})^\top$ with $w_{1,t}$, $w_{2,t}$ left unconstrained and $w_{f,t} = 1 - w_{1,t} - w_{2,t}$. In so doing, one needs to compute $\Sigma_t[2]$ using the GARCH specification (Eq. 3) and then proceeds with the determination of the optimal weights.

5. Determination of $\Sigma_t[2]$

   – Using Eq. 3, write down the 1-step ahead and the 2-step ahead variance forecast, $\sigma^2_{i,t}(1) = \mathbb{V}[\epsilon_{i,t+1} \mid I_t]$ and $\sigma^2_{i,t}(2) = \mathbb{V}[\epsilon_{i,t+2} \mid I_t]$ with I_t the information set at time t, as a function of the parameters and the squared residuals.

   – Write down the variance forecast for the 2-period (log-) returns $\sigma^2_{i,t} = \mathbb{V}[\epsilon_{i,t+1} + \epsilon_{i,t+2} \mid I_t]$ as a function of the parameters and the squared residuals.
   Hint: Use the fact that $\mathbb{C}ov[z_{i,t+1}, z_{i,t+2}] = 0$.

   – Using the estimates of the GARCH-based specification in Question 1, estimate $\sigma^2_{i,t}[2]$ at $t = 1, 3, 5, \cdots, T$ for $i = 1, 2$. Plot the two series on the same figure and comment them.

6. Determination of optimal weights $w^\star_t$.
   Using Question 5 and assuming that the correlation between the residuals, denoted ρ_{sb}, is constant, the conditional two-period variance-covariance matrix $\Sigma_t[2]$ at time t can be written as:

$$\Sigma_t(2) = \begin{pmatrix} \sigma^2_{1,t} & \sigma_{12,t} \\ \sigma_{12,t} & \sigma^2_{2,t} \end{pmatrix}$$

   where $\sigma_{12,t}[2] = \rho_{12} \times \sigma_{1,t}[2] \times \sigma_{2,t}[2]$.

   – Compute the optimal weights of the dynamic mean-variance allocation problem (Eq. 4) for $t = 1, 3, 5, \cdots, T$ and $\lambda = 2$ and 10.

   – For $\lambda = 2$ and 10, plot the optimal weights of the stock index and the bond index for the two allocation problems (static and dynamic).

   – Comment your results for $\lambda = 2$ and 10.

7. Comparison

   – Compute the cumulative (log-) returns of the optimal portfolio for the static and dynamic allocation problem (when $\lambda = 2$ and 10) using the optimal portfolio weights and the realized returns. For instance, in the case of the dynamic allocation problem, the cumulative (log-) return is given by (for $t = 1, 3, \cdots, T$):

$$\texttt{CR}_t = \sum_{\tau=1}^{t} r_{p,\tau}[2]$$

   where

   - $r_{p,\tau}[2] = w^\star_{s,\tau} r_{s,\tau}[2] + w^\star_{b,\tau} r_{b,\tau}[2] + (1 - w^\star_{s,\tau} - w^\star_{b,\tau}) r_f[2]$
   - $r_{i,\tau}[2] = r_{i,\tau+1} + r_{i,\tau+2}$.

   – Plot the two series of cumulative returns (for $\lambda = 2$ and 10). Which asset allocation method performs better?

Pair trading

8. Pair trading using an error correction model

   – Do you think that the two currencies that you have selected would be good candidates for a pair trading strategy using an error correction model? Explain your answer in detail. If needed, provide the appropriate analyses to support your argument.

   – Why would one use an error correction model for pair trading?

   – Imagine that your two currencies are good candidates, explain carefully how you would implement such strategy.

Conclusion

9. Conclude briefly.

   – Provide a brief conclusion of your findings.

   – Was your GARCH specification useful in timing volatility? Explain.

   – Is there anything about your portfolio and the characteristics of the assets that you think is worth highlighting? Would you consider investing in such a portfolio?

Extension(s): bonus questions

10. What would happen in the case of a static or dynamic allocation over two consecutive weeks (14 days)?
    Remark: Weekly excess log-return must be computed carefully.

11. How would you improve the specification of the dynamic allocation problem? Especially, is it consistent to assume that the linear correlation coefficient is constant?

12. You were asked not to select Tether for your portfolio. Explain why Tether has a different volatility structure than other cryptocurrencies. Hint: look it up online.

Additional information

- You are free to choose your programming language (Matlab, R, or Python).

- Deadline: May 2, 2019

- Please send your report along with your code to maxime.couvert@epfl.ch by the deadline.

- Your report should be detailed. Interpreting results is a key part of the exercise.