Exercise Seven

Review Section 3.1, Page 27 - 36 of the LectureFour (download from Moodle).

1. Verify the statement in Example 3.25 (5) about immersed manifold.

2. Prove Lemma 3.28.

3. Show that the definition of inner product of k-tensors is independent of the choice of coordinates. Furthermore, if $F\colon M \to M$ is a diffeomorphism, then it preserves the inner product in the following sense:

$$F^*\big(\langle u, v\rangle_g\big) = \langle F^*u, F^*v\rangle_{F^*g}.$$

4. Prove Lemma 3.31 about the existence of local orthonormal frames.

5. Prove Lemma 3.32 about the existence of volume form.