# *Measurement Systems*

Problem set n° 1

***Sensors and signal conditioning circuits***

**Exercise 1 (Strain gauge conditioning circuit)**

Consider a strain gauge sensor incorporated in a Wheatstone bridge as shown below. The sensor corresponds to $R_1$.

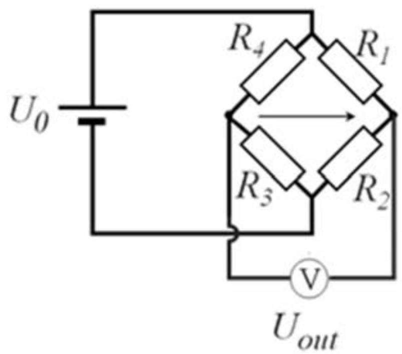


Fig. 1. General conditioning circuit based on the Wheatstone bridge

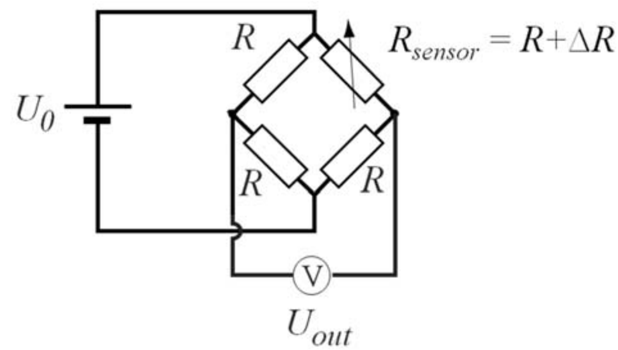


Fig. 2. Special case of the conditioning circuit

a) Derive an expression for $U_{out}$ as function of $R_1$, $R_2$, $R_3$, $R_4$ and $U_0$.
b) Determine $U_{out}$ if $R_1= R+\Delta R$ is the sensor and $R_2=R_3=R_4=R$, for $\Delta R/R<<1$ (Fig. 2.). (Attention, don't go too far in your approximation).
c) Knowing that the electrical resistance of a wire depends on its resistivity $\rho$, length $l$ and the cross-sectional area $S=\pi d^2/4$ (see figure below) where d is the wire diameter, find an expression for the ratio $\Delta R/R$ after strain as function of the relative changes $\Delta l/l$ and $\Delta\rho/\rho$. Simplify your expression using the Poisson ratio $\nu = -\frac{\Delta d/d}{\Delta l/l}$.

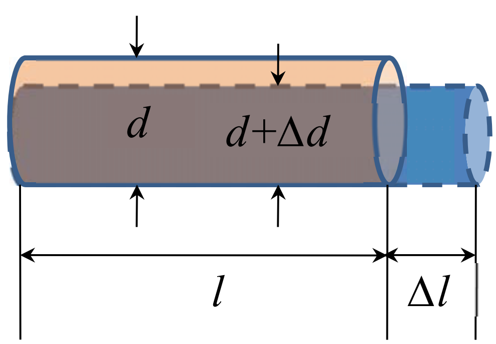


Fig. 3. Current-carrying wire under mechanical deformation.

## Exercise 2 (Linear variable differential transformer)

The linear variable differential transformer (LVDT) is a type of an electrical transformer used for measuring linear displacement (position), shown on Fig. 4. It is composed of a mobile ferrite core, placed inside a transformer with the primary and secondary coil arranged in a way shown on Fig. 5.

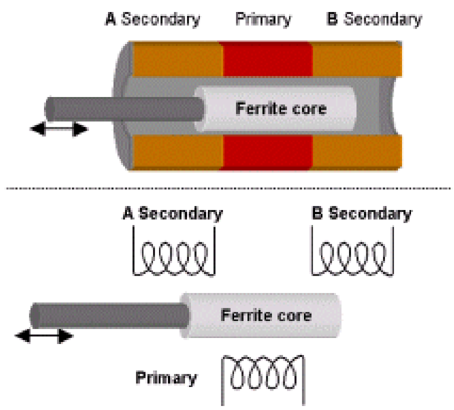


Fig. 4. Schematic representation of an LVDT system.

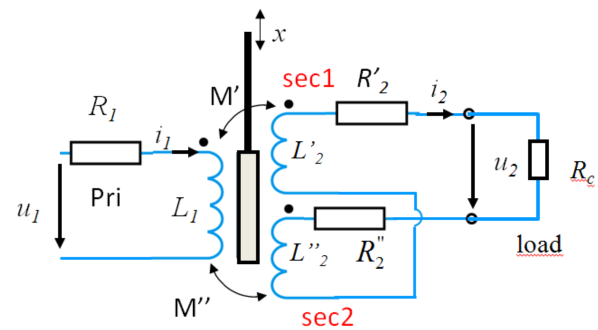


Fig. 5. Equivalent electrical circuit for an LVDT.

a) Provide a qualitative description of how this system measures the distance. Your description should cover the fundamental physics laws based on which the system is operating, the different elements in the system and their functions, how the system measures the magnitude and the direction of displacement and what could be the advantages of this system.

b) Knowing that $u_1$ is a sinusoidal voltage with angular frequency $\omega$, use the equivalent circuit in Fig. 5., the Kirchhoff's voltage law (KVL) and the Ohm's law for a resistor, inductor and a transformer (coupled inductors) to find the output voltage ($u_2$) as a function of the mutual inductances $M'(x)$ and $M''(x)$. To start, write down the equations for voltage drops on the left and right-hand sides of the circuit. The voltage drop over the inductor 1 (primary inductor on Fig 4) due to its self-inductance $L_1$ is given by $L_1 \frac{di_1}{dt}$. Because of the coupling with the secondary inductors A and B, there are additional voltage drops induced on the primary inductor due to the AC current flowing in the secondary circuit. The voltage drop induced on inductor 1 due to current $i_2$ flowing the through the inductor $L_2{}'$ is given by $M' \frac{di_2}{dt}$.

Hint 1: For a system with the input of constant frequency, it is useful to write the equations in frequency domain using the complex impedances for describing the various components.

Hint 2: Assume that the load resistance is very high

c) Using the results of part b) and a linear approximation for $M'(x) and\ M''(x)$ around the origin up to the first order, find the linear relation between input voltage $u_1$ and displacement $x$.