Measurement Systems

Problem set n° 1

Sensors and signal conditioning circuits

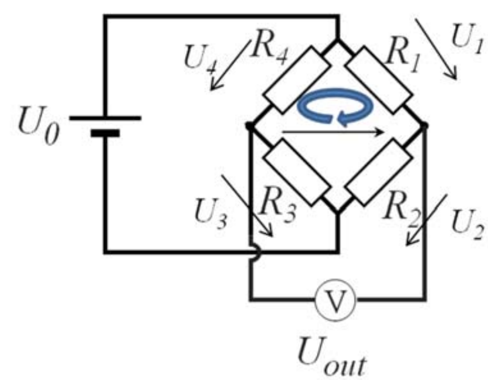
Exercise 1 (Strain gauge conditioning circuit)

a) Let U_1, U_2, U_3 and U_4 denote the voltage drops across the resistances R_1, R_2, R_3 and R_4 in the bridge.

Both branches of the Wheatstone bridge (R_1, R_2 and R_3, R_4) are connected in parallel to the voltage source:

$$U_1 + U_2 = U_3 + U_4 = U_0$$

Within each branch the resistances are connected in series:



$$\frac{U_2}{U_1} = \frac{R_2}{R_1} \text{ and } \frac{U_3}{U_4} = \frac{R_3}{R_4}$$

After adding 1 to both sides of each equation we obtain:

$$\frac{U_1 + U_2}{U_1} = \frac{R_1 + R_2}{R_1} \text{ and } \frac{U_3 + U_4}{U_4} = \frac{R_3 + R_4}{R_4}$$

$$\text{or } \frac{U_1}{U_0} = \frac{R_1}{R_1 + R_2} \text{ and } \frac{U_4}{U_0} = \frac{R_4}{R_3 + R_4}$$

Applying the mesh rule in the upper part of the Bridge leads to:

$$U_1 - U_{out} - U_4 = 0$$

$$U_{out} = U_1 - U_4 = \left(\frac{R_1}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right) U_0 = \frac{R_1R_3 - R_2R_4}{(R_1+R_2)(R_3+R_4)} U_0$$

b) By substituting $R_1 = R + \Delta R$ and $R_2 = R_3 = R_4 = R$ in the final result from a) we obtain:

$$U_{out} = \left(\frac{R+\Delta R}{R+\Delta R+R} - \frac{1}{2}\right) U_0 = \left(\frac{R+\Delta R}{2R+\Delta R} - \frac{1}{2}\right) U_0 = \frac{2R+2\Delta R-2R-\Delta R}{4R+2\Delta R} U_0 = \frac{\Delta R}{4R} \frac{1}{1+\underbrace{\frac{\Delta R}{2R}}_{\approx 0}} U_0 \approx \frac{\Delta R}{4R} U_0$$

As $\Delta R \ll R$ the ratio $\frac{\Delta R}{2R}$ is a very small number close to 0, so that $\frac{1}{1+\frac{\Delta R}{2R}} \approx 1$

c) The resistance of the unstrained wire is:

$$R = \rho\frac{l}{S} = \rho\frac{4l}{\pi d^2}$$

We obtain the change in resistance under strain as a sum of the partial derivatives of R with respect to the variables in the formula (ρ, l and d) multiplied by the absolute change of each:

$$\Delta R = \frac{\partial R}{\partial \rho}\Delta\rho + \frac{\partial R}{\partial l}\Delta l + \frac{\partial R}{\partial d}\Delta d = \frac{4l}{\pi d^2}\Delta\rho + \frac{4\rho}{\pi d^2}\Delta l - 2\frac{4\rho l}{\pi d^3}\Delta d = R\frac{\Delta\rho}{\rho} + R\frac{\Delta l}{l} - 2R\frac{\Delta d}{d}$$

$$\frac{\Delta R}{R} = \frac{\Delta\rho}{\rho} + \frac{\Delta l}{l} - 2\frac{\Delta d}{d} = \left(1 - 2\frac{\frac{\Delta d}{d}}{\frac{\Delta l}{l}}\right)\frac{\Delta l}{l} + \frac{\Delta\rho}{\rho} = (1 + 2\nu)\frac{\Delta l}{l} + \frac{\Delta\rho}{\rho}$$

Exercise 2 (Linear variable differential transformer)

a) The linear variable differential transformer (LVDT) consists of a primary coil and two secondary coils (connected in series). The primary coil is excited with a sinusoidal voltage of few volts (e.g. 5 V) with a frequency of few kHz (e.g. 3 kHz). The electromagnetic coupling between the primary and secondary coils change as a function of the displacement of the mobile core (steel) of high magnetic permeability. When the magnetic core lies in the middle (*x = 0*), the two secondary voltages have the same magnitude and the output voltage is zero. When the core is displaced, the difference between the two secondary voltages is proportional to the displacement *x*. The absolute amplitude (or RMS) of the output signal corresponds to the displacement.

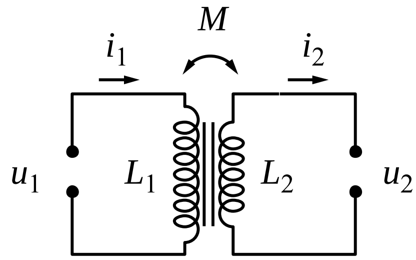
Refer to the following links:

http://en.wikipedia.org/wiki/Linear_variable_differential_transformer
http://www.a-tech.ca/doc_technote/LVDT_Principle_ATI.pdf
http://yourinstrumentation.blogspot.ch/2011/10/lvdt-basic-principle-theory-working.html
http://www.lvdt.co.uk/how-lvdts-work/
http://meas-spec.com/downloads/Principles_of_the_LVDT.pdf

(b) Self-inductance: inductive effect in each inductor due to its own current. Voltage drop u over an inductor carrying current i is:

$$u = L\frac{di}{dt}$$

where L is the inductance. The mutual inductance has a similar effect but it describes the coupling between two inductors, each in its own circuit shown on the figure for the simplest situation:



For the circuit on the left, we would then have:

$$u_1 = L_1\frac{di_1}{dt} - M\,\frac{di_2}{dt}$$

note i_2 in the second term! In the same time for the circuit on the right, we would have:

$$u_2 = L_2\frac{di_2}{dt} - M\,\frac{di_1}{dt}$$

Applied to our situation (Fig 5 from the exercise problem sheet) and together with Kirchoff's Voltage Law (KVL) on the primary side, we get:

$$u_1 = R_1 i_1 + L_1\left(\frac{di_1}{dt}\right) + (M'' - M')(\frac{di_2}{dt})$$

KVL on the secondary side:

$$u_2 = -(R_2' + R_2'')i_2 - (L_2' + L_2'')\frac{di_2}{dt} + (M'' - M')\frac{di_1}{dt}$$

Since $u_1 = U_1\sin(\omega t)$, i_1, i_2 and u_2 will be in the form of:

$$i_1 = I_1\sin(\omega t)$$
$$i_2 = I_2\sin(\omega t)$$
$$u_2 = U_2\sin(\omega t)$$

by replacing these expressions in the KVL equations:

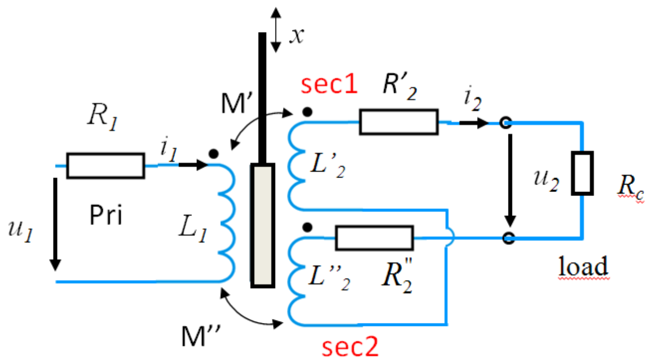
$$\underline{U}_1 = (R_1 + j\omega L_1)\underline{I}_1 + j\omega(M'' - M')\underline{I}_2$$

For Rc → ∞ : $I_2 \approx 0$

This results in:

$$\underline{U}_2 = \frac{j\omega[M''(x) - M'(x)]}{R_1 + j\omega L_1}\underline{U}_1$$

c) As the ferrite coil moves, the electromagnetic coupling with the inductor towards which the coil moves will increase, while the electromagnetic coupling with the other secondary inductor will decreased.



To clarify, let's say that the object moves towards $+x$ direction. Since the object is connected to the ferrite coil, the coil will also move in the same direction which means that it will move towards L'_2. As a result the electromagnetic coupling between the primary coil (L_1) and L'_2 will increase. The increase in the electromagnetic coupling means increase in mutual inductance, so $M'(x)$ will increase. On the other hand as the coil is moving towards L'_2, its overlap with L''_2 will be smaller. This results in less electromagnetic coupling between primary inductor L_1 and L''_2, which means that the mutual inductance $M''(x)$ will decrease.

One can give the same explanation for the situation in which the object (and thus the coil) is moving towards L''_2 and $M''(x)$ increases while $M'(x)$ decreases.

So for positive x, $M'(x)$ will increase with x increasing while $M''(x)$ will decrease. Similarly, for negative x, $M''(x)$ will increase while $M'(x)$ will decrease. Knowing this one may write the linear approximation for $M'(x)$and $M''(x)$:

$$\underline{U}_2 = \frac{j\omega\left[M''(x) - M'(x)\right]}{R_1 + j\omega L_1}\underline{U}_1$$

$$M'(x) = M(0) + ax + \dots \quad \text{for } x > 0$$

$$M''(x) = M(0) - ax + \dots \quad \text{for } x < 0$$

To find the linear relation we will take the approximation up to first order, so:

$$M''(x) - M'(x) = -2ax$$

By replacing in the above equation:

$$\underline{U}_2 = \frac{-2j\omega \cdot a\underline{U}_1}{R_1 + j\omega L_1}\underline{X}$$