Measurement Systems

Problem set n° 2

Sensors and signal conditioning circuits

Exercise 1 (Temperature compensation)

a) We know that the voltage U_o can be expressed for the case of four gauges:

$$U_o = -\frac{\Delta R}{R} \cdot U_i = -\frac{\Delta R}{R_O \cdot (1+\alpha_T \cdot \Delta T)} \cdot U_i$$

The system is considered temperature compensated if $\Delta T \ll {\alpha_T}^{-1}$. However, if the temperature variations become too large, the bridge will start being sensitive to these changes. Variations ΔT exceeding a certain limit will thus induce a variation of the measurand.

b) In this new system composed of a temperature-sensitive gauge, the bridge voltage U_o and sensitivity S are :

$$U_o = -\frac{1}{4} \cdot K_T \cdot \epsilon \cdot U_i \qquad \text{and} \qquad S = -\frac{1}{4} \cdot K_T \cdot U_i$$

The change in sensitivity $\Delta S/S$ is :

$$\frac{\Delta S}{\Delta T} = -\frac{U_i}{4} \cdot \frac{\Delta K_T}{\Delta T} = \frac{U_i}{4} \cdot \beta_T \cdot K_T \quad \xrightarrow{\Delta S/S} \quad \frac{\Delta S}{S} = \frac{\Delta K_T}{K_T} = -\beta_T \cdot \Delta T = -5\ \%$$

We add in parallel the thermistor in order to compensate the system's temperature dependence. The voltage drop U_p at the bridge's terminals, knowing that R_p is the equivalent resistor of the bridge, is:

$$U_p = \frac{R_p}{R_p + R_T} \cdot U_i \qquad \text{with : } R_p = \frac{(2 \cdot R) \cdot (2 \cdot R)}{(2 \cdot R) + (2 \cdot R)} = R$$

The change in sensitivity $\Delta S/S$ of the bridge depends on the gauge factor K_T and on the bridge voltage U_p :

$$\frac{\Delta S}{\Delta T} = -\frac{U_p}{4} \cdot \frac{\Delta K_T}{\Delta T} - \frac{K_T}{4} \cdot \frac{\Delta U_p}{\Delta T} \quad \xrightarrow{\Delta S/S} \quad \frac{\Delta S}{S} = \frac{\Delta K_T}{K_T} + \frac{\Delta U_p}{U_p}$$

The change in voltage on the bridge's terminals due to temperature is:

$$\frac{\Delta U_p}{U_p} = -\frac{\Delta(R_T + R)}{R_T + R} = -\frac{\Delta R_T}{R_T + R} = -\frac{\gamma_T \cdot R_T \cdot \Delta T}{R_T + R} \qquad \text{with : } \gamma_T = \frac{1}{R_T} \cdot \frac{\Delta R_T}{\Delta T}$$

c) The temperature compensation implies $\Delta S/S = 0$ and we find the temperature coefficient of the thermistor γ_T to be :

$$\frac{\Delta K_T}{K_T} = -\frac{\Delta U_p}{U_p} = -\beta_T \cdot \Delta T = \frac{\gamma_T \cdot R_T \cdot \Delta T}{R_T + R} \quad \Longrightarrow \quad \gamma_T = -\beta_T \cdot \left(1 + \frac{R}{R_T}\right)$$

Exercise 2 (***Desired quantity, modifying and interfering inputs***)

The wires of resistance $R_{c,2}$ connecting the measuring area of the bridge to the gauge carry almost no current ($i_{c,2} \approx 0$) and therefore cause no voltage drop across the bridge. This is because the voltmeter connected to measure the voltage drop across the input has a very large input resistance, a common characteristic of voltmeters. The voltage U_o is then :

$$U_o = \left(\frac{1}{2} - \frac{R+R_{c,3}}{\Delta R+2\cdot R+R_{c,1}+R_{c,3}}\right)\cdot U_i$$

In order to apply the concepts of interfering and modifying inputs, we will now linearize this expression.

$$U_o = \left(\frac{1}{2} - \frac{R+R_{c,3}}{\Delta R+2\cdot R+R_{c,1}+R_{c,3}}\right)\cdot U_i = \left(\frac{\Delta R+2R+R_{c,1}+R_{c,3}-2\left(R+R_{c,3}\right)}{2(\Delta R+2R+R_{c,1}+R_{c,3})}\right)U_i =$$

$$= \left(\frac{\Delta R+R_{c,1}-R_{c,3}}{2(\Delta R+2R+R_{c,1}+R_{c,3})}\right)U_i$$

If we look at the different values of R, we can see that ΔR , $R_{c,1}, R_{c,2}, R_{c,3}$ are all relatively small compared to *R*. This allows us to neglect ΔR in the denominator (we can't do it elsewhere, because it would make the result insensitive to strain. Note that $\frac{\Delta R}{R} = K\epsilon$):

$$U_o \approx \left(\frac{\Delta R+R_{c,1}-R_{c,3}}{2\left(2R+R_{c,1}+R_{c,3}\right)}\right)U_i = \left(\frac{RK}{2\left(2R+R_{c,1}+R_{c,3}\right)}\varepsilon + \frac{R_{c,1}-R_{c,3}}{2\left(2R+R_{c,1}+R_{c,3}\right)}\right)U_i$$

1. The resistors $R_{c,1}$ and $R_{c,3}$ of the wires alter the sensitivity of the voltage U_o and its offset. These resistances of the wires are both modifying inputs x_m and interfering inputs x_i when measuring the deformation ϵ.
2. Due to the symmetry of the bridge, we must have resistances $R_{c,1}$ = $R_{c,3}$ to minimize their effect. According to the following equation, we see that it is sufficient to choose the wires of the same length $L_{c,1}$ and $L_{c,3}$:

   $R_{c,1} = \rho \cdot \frac{L_{c,1}}{S} = \rho \cdot \frac{L_{c,3}}{S} = R_{c,3} \quad \Longrightarrow \quad L_{c,1} = L_{c,3}$ (shortest possible)

   However, the resistors $R_{c,1}$ and $R_{c,3}$ remain modifying inputs x_m that decrease the sensitivity of the deformation measurement ϵ.