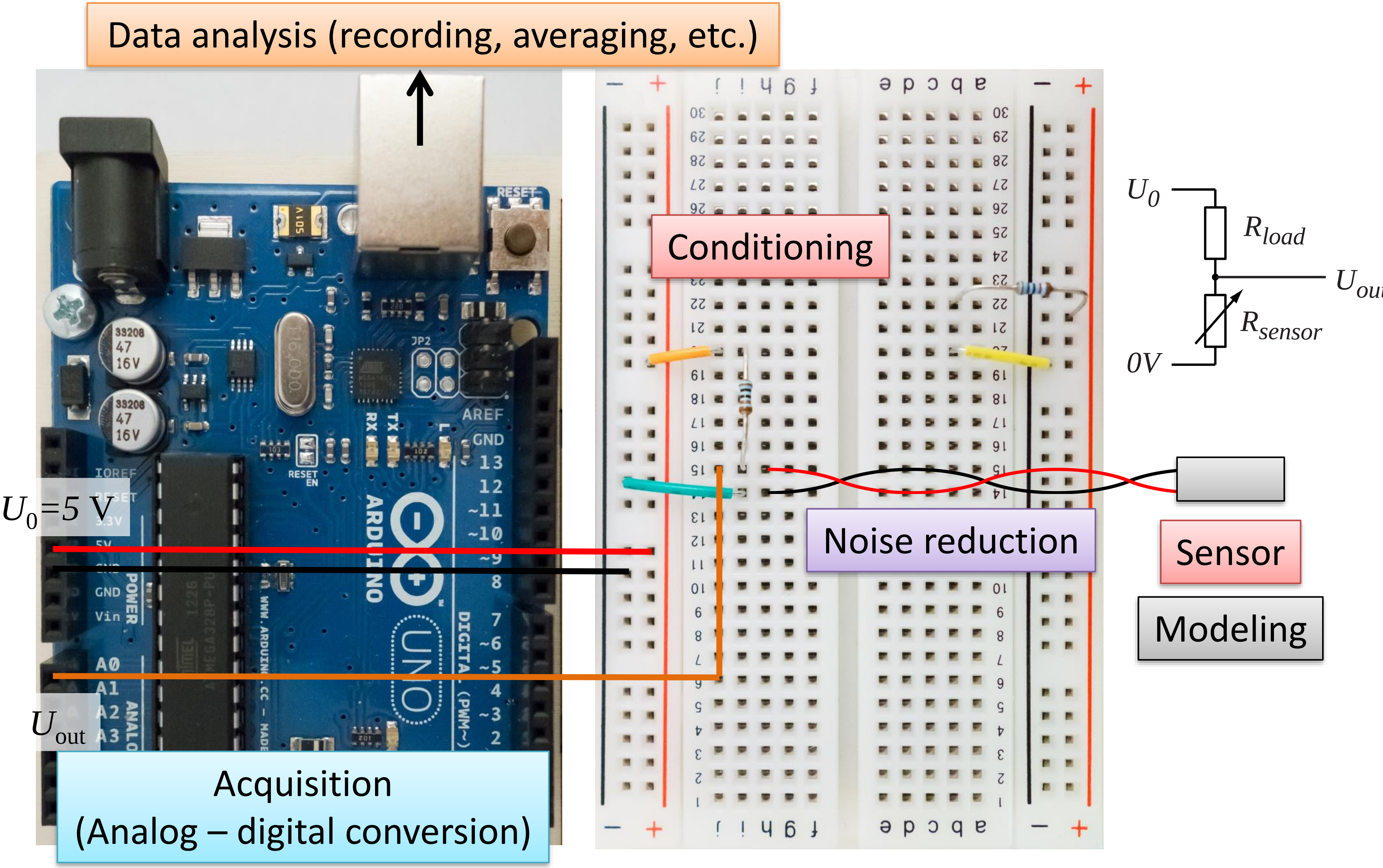
Measurement systems

Lecturer: Andras Kis

Chapter 3: Noise

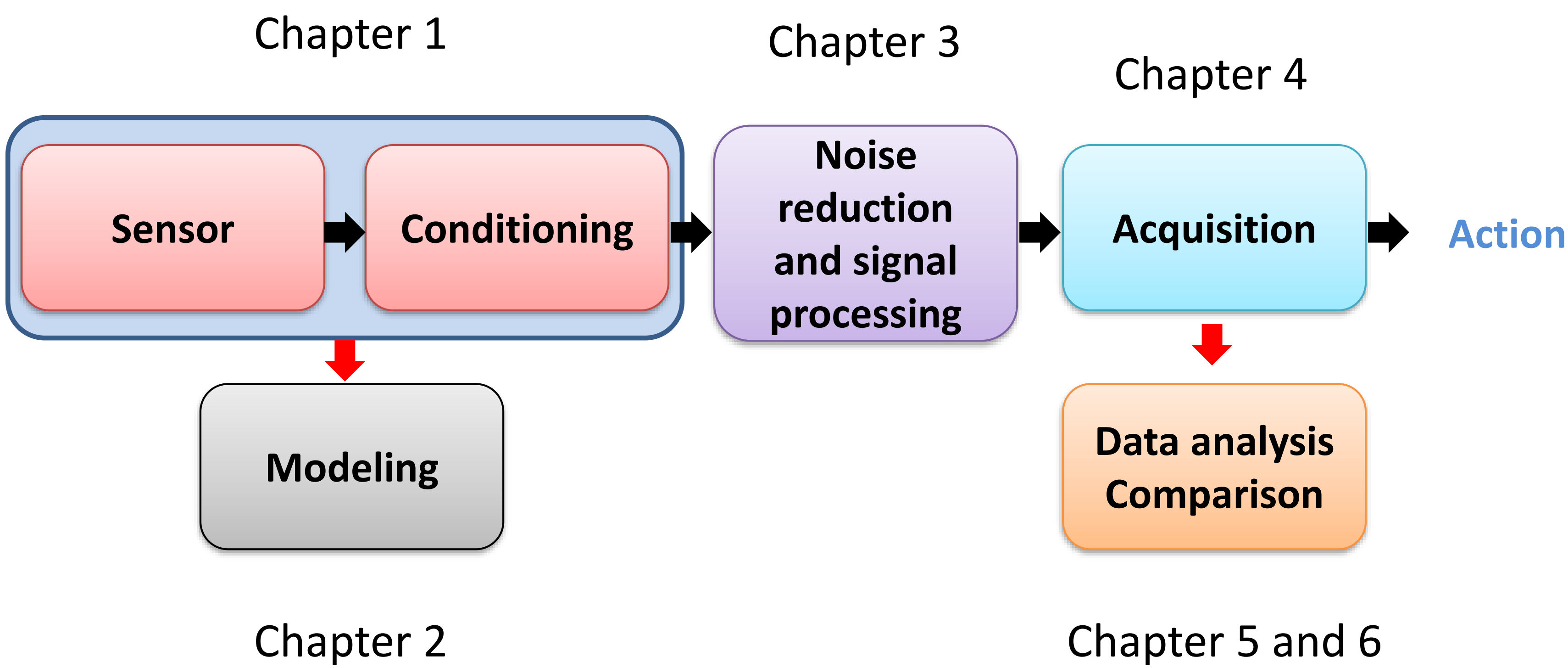
Measurement chain



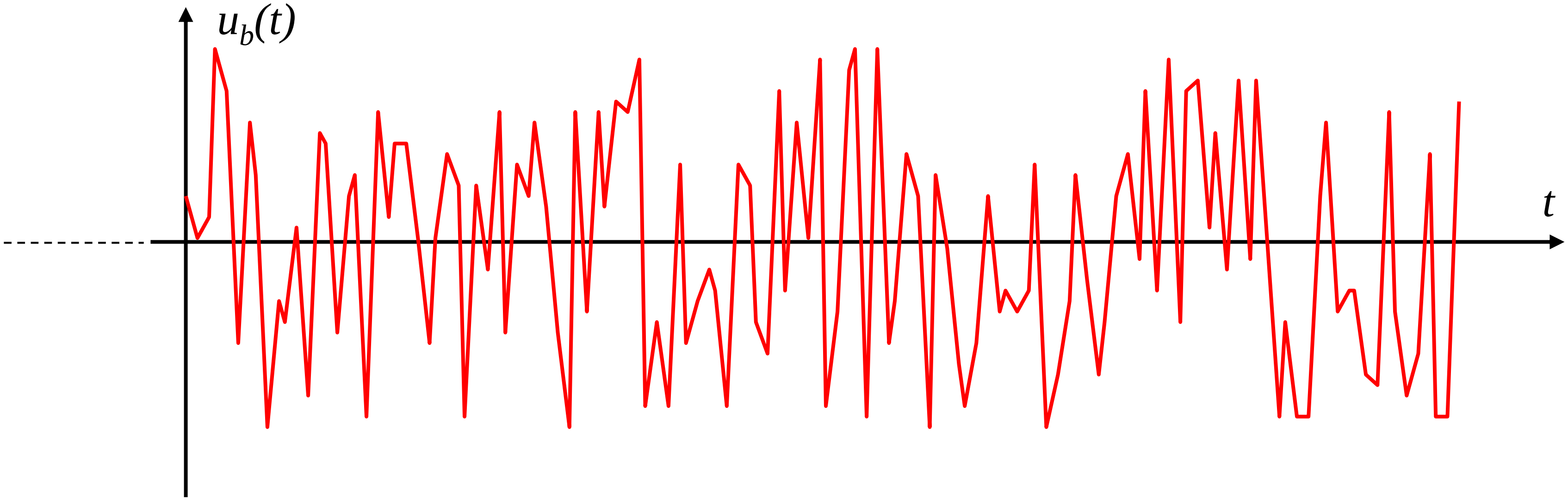
Arduino UNO board

Conditioning circuit

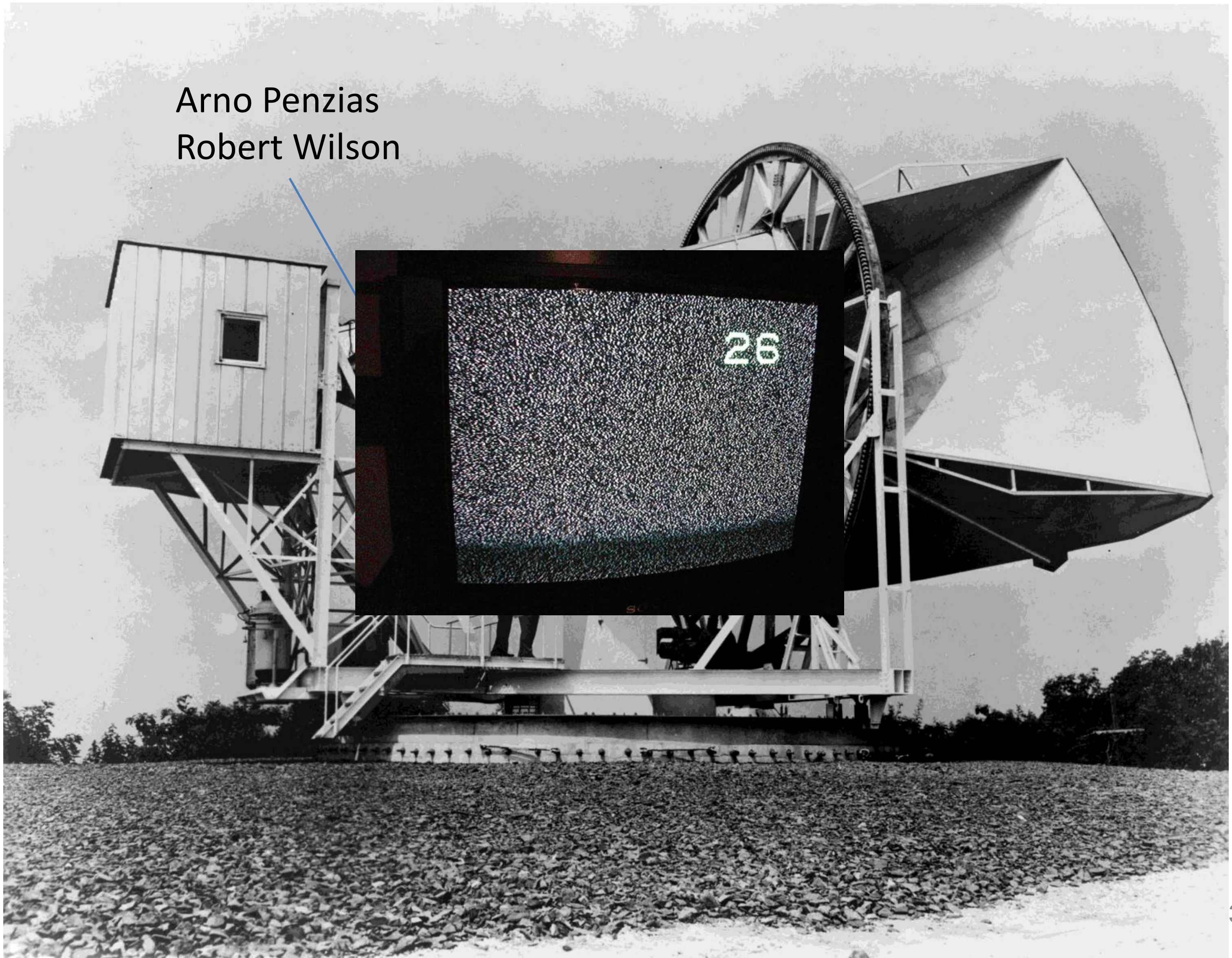
Measurement chain



Noise: Example



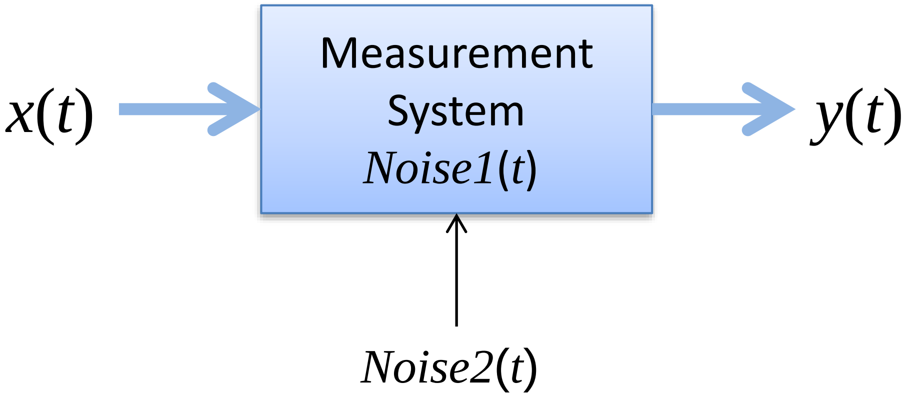
Noise is not always bad



Noise estimation and suppression

- Sources
- Extrinsic noise
  - Conductive coupling
  - Capacitive coupling
  - Magnetic coupling
- Noise suppression using differential measurements
  - Common mode voltage
  - Suppression of the common mode voltage
  - Instrumentation amplifier
- Intrinsic noise
  - Thermal noise
  - Shot noise
  - 1/f noise
  - Noise estimation

Noise sources



Extrinsic noise $Noise2(t)$

External influence
- Electrical
- Magnetic
- Electromagnetic
- Mechanical (vibration, sound)
- Thermal (temperature variation)

Intrinsic noise $Noise1(t)$

Internal to the circuit and device
- Thermal
- Shot noise
- 1/f

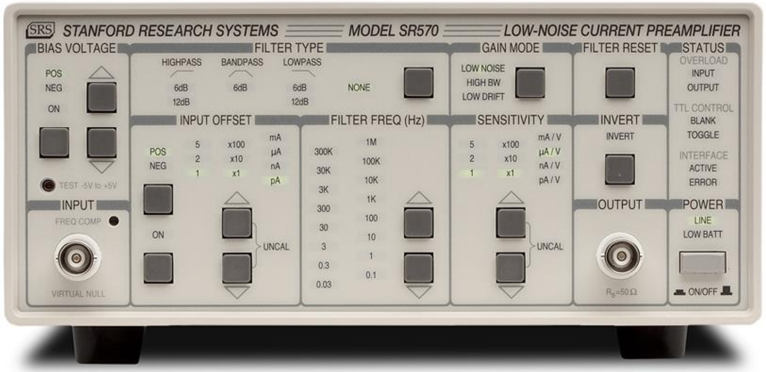
Extrinsic noise: perturbations

- Coupling mechanisms
  - Conductive (galvanic) coupling
  - Capacitive (electrostatic) coupling
  - Magnetic coupling

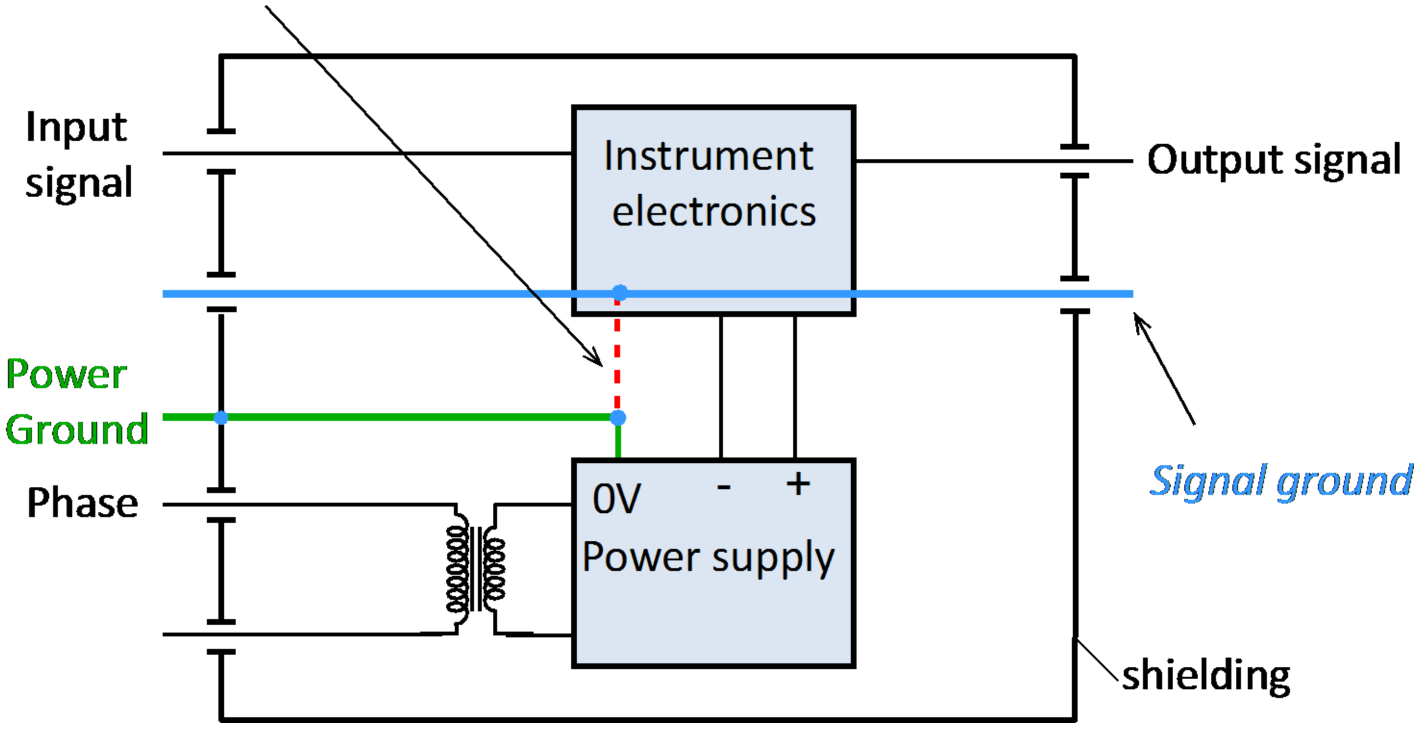
- Coupling modes
  - Common mode
  - Differential mode

Conductive coupling

- Power supplies and the ground

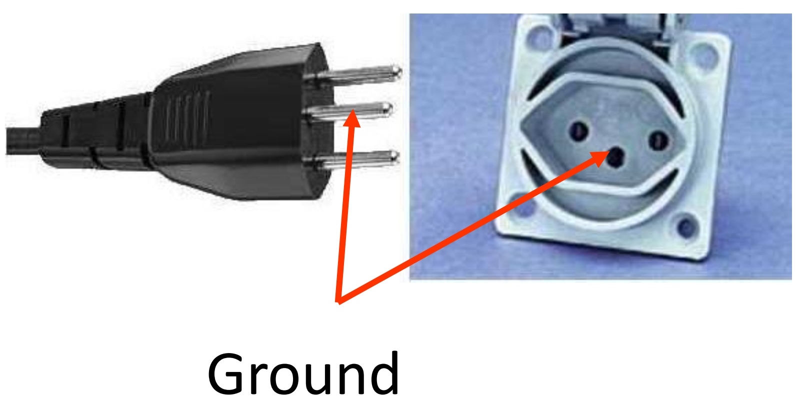


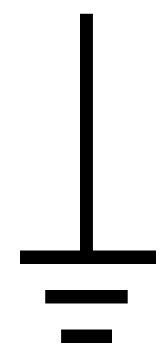
Possible connection signal ground – power ground



Reference connections of a circuit

- Power ground
  - sometimes also referred to as simply “ground”, Earth
  - the Earth is considered to be a perfect conductor and a sink for charge
  - ground potential is by reference 0V
  - connection provided by the power supply line or a dedicated line

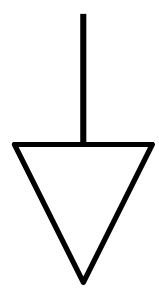




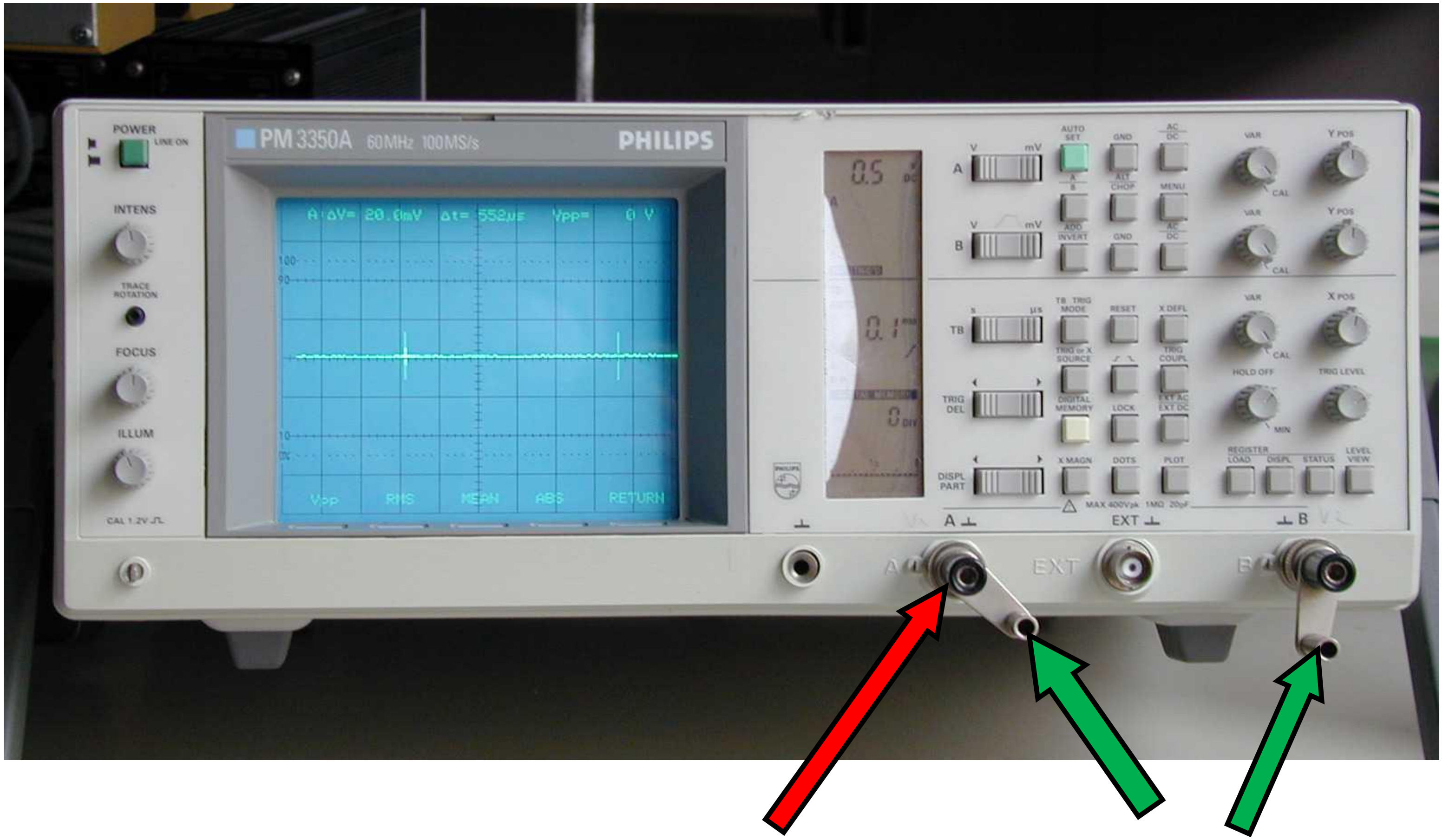
Reference connections of a circuit

- Signal ground
  - voltage reference against which all the voltages on the input/output terminals are measured against
  - may be connected to the power ground, usually through the chassis, either directly or through a ~1MOhm resistor
  - signal ground that is not connected to the power ground is called a "floating ground".

Symbol



Example: Oscilloscope



signal

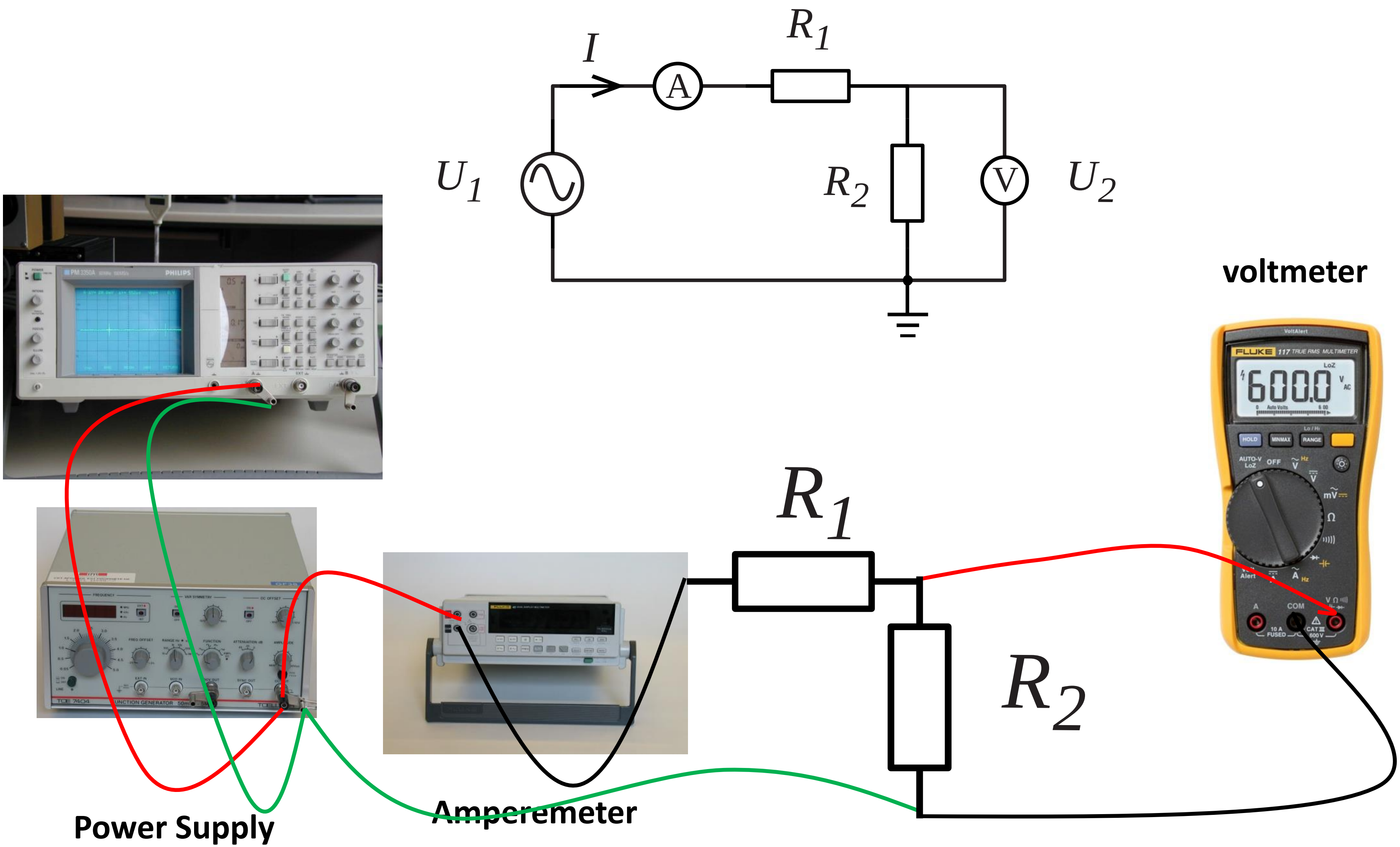
ground

Example: battery-powered voltmeter

Signal
ground

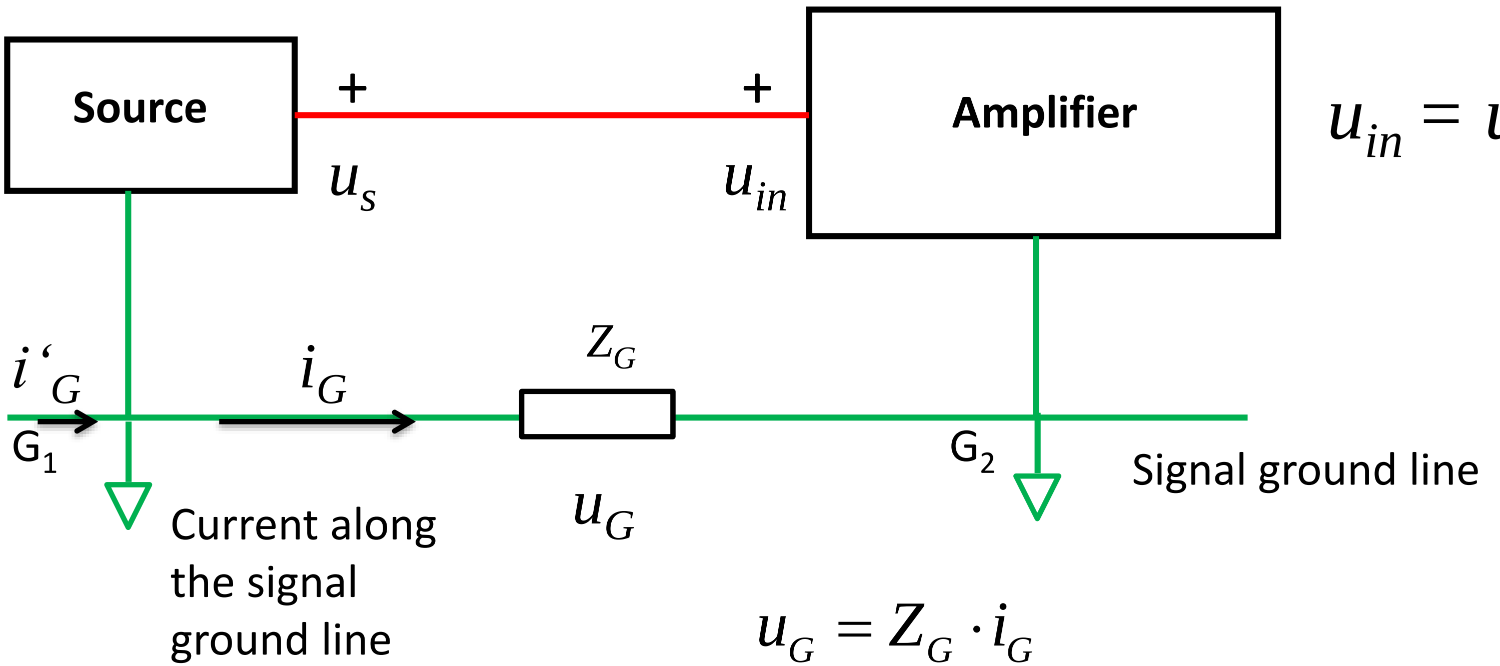
signal

Example of connections



Conductive coupling – ground loop

- Cause: finite resistance of connecting wires
- Influence of the signal ground potential difference
- Occurs also if the signal ground is connected to the ground



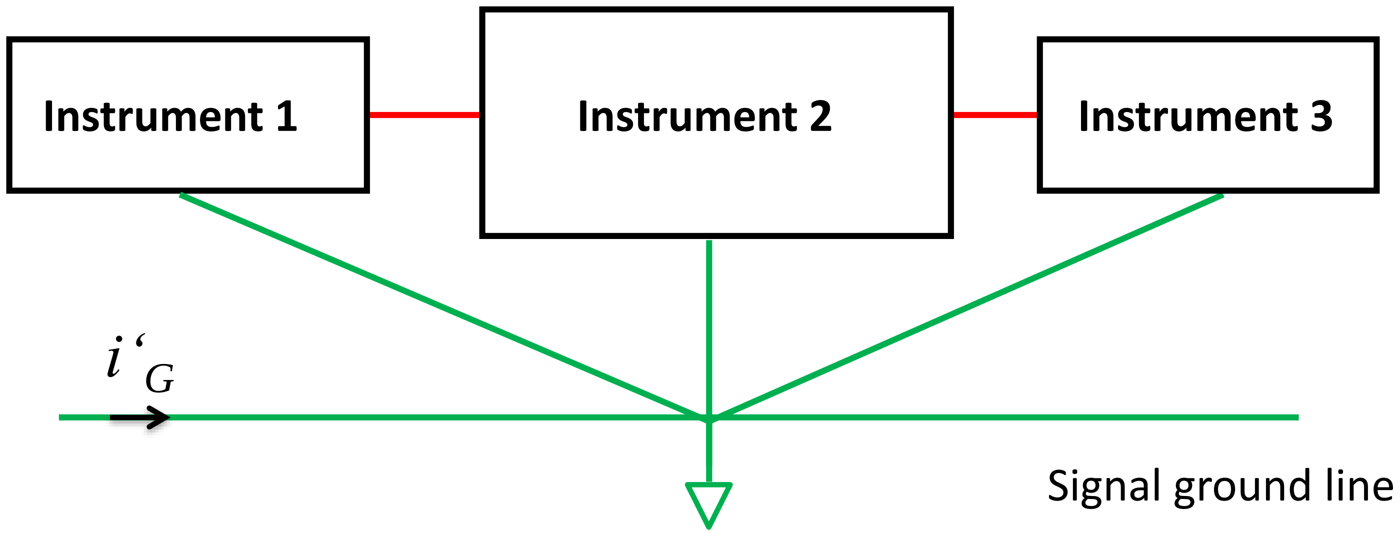
$$u_{in} = u_s + u_G$$

$$u_G = Z_G \cdot i_G$$

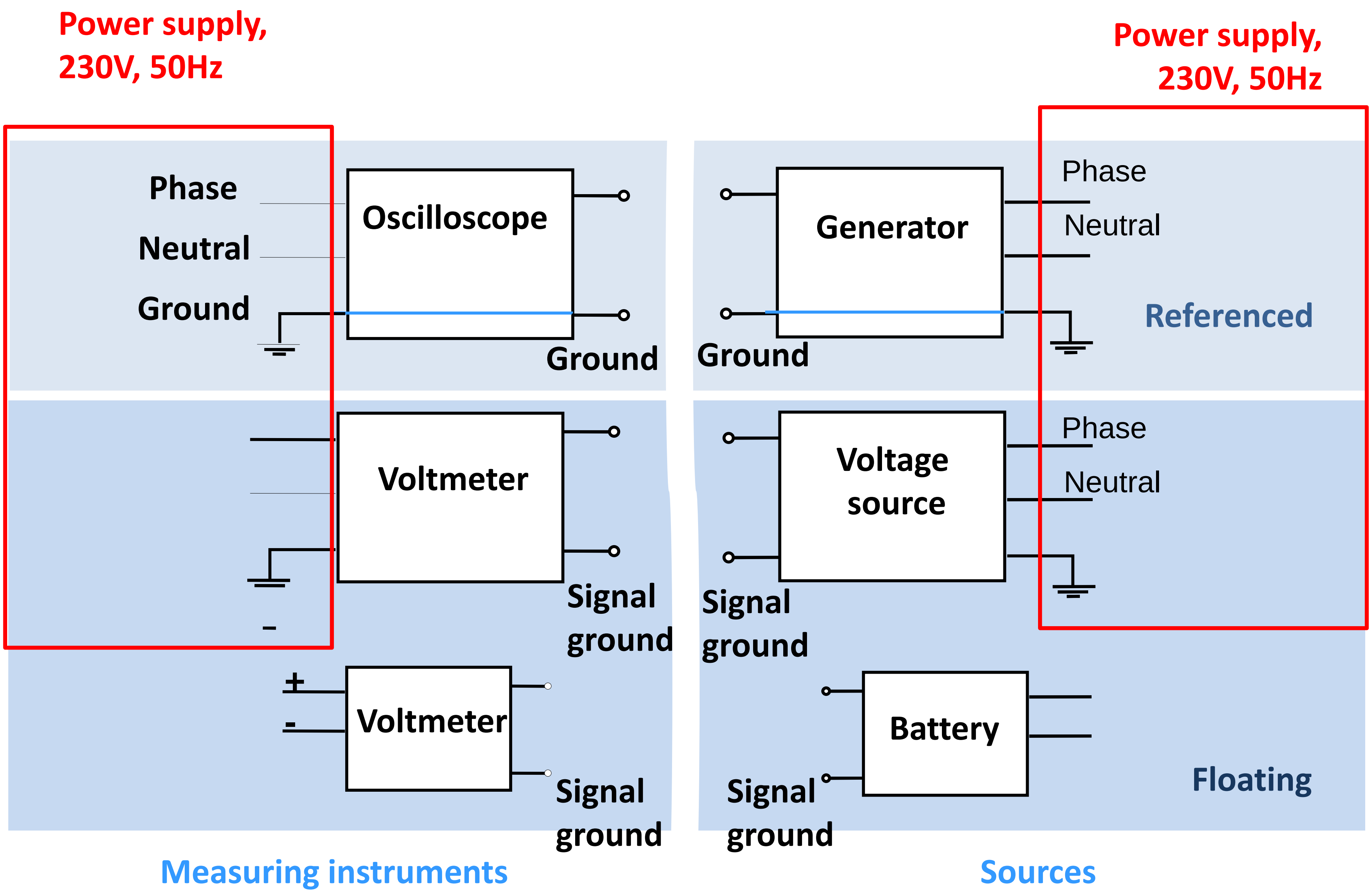
$$Z_G = R_G + jL_G\omega$$

Reducing the conductive coupling

- Reducing ground connections: star grounding



Connecting instruments

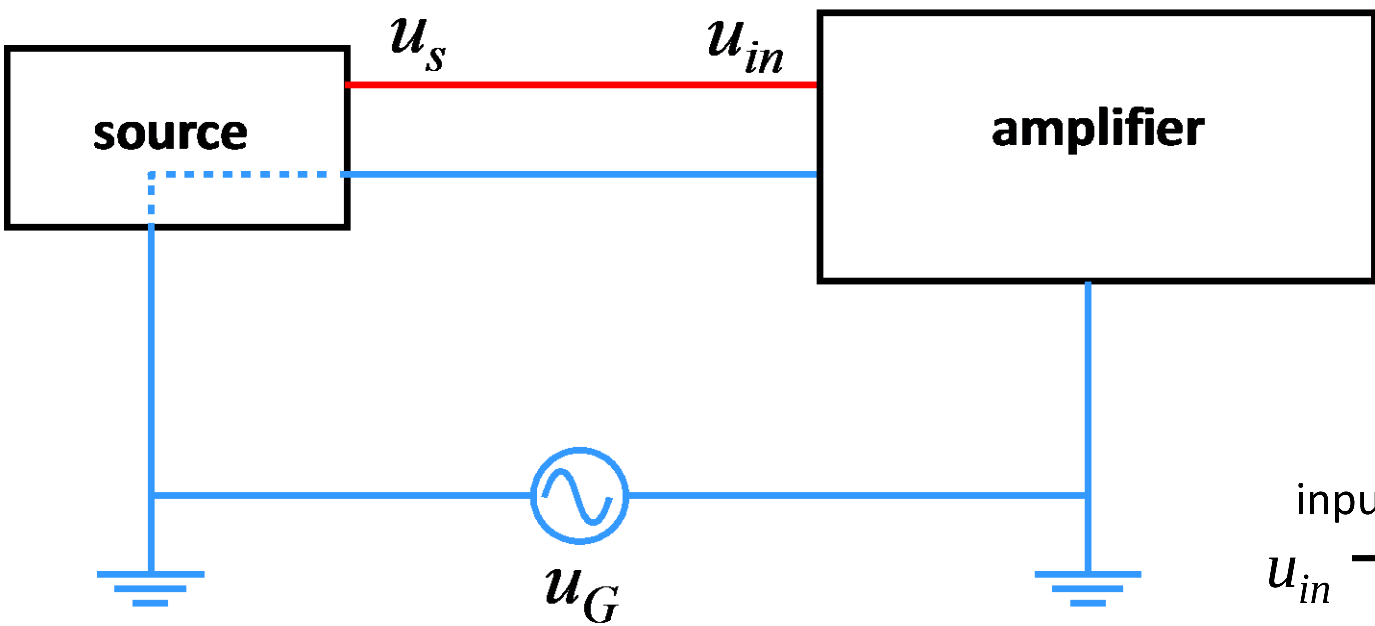


Referenced and floating sources and amplifiers

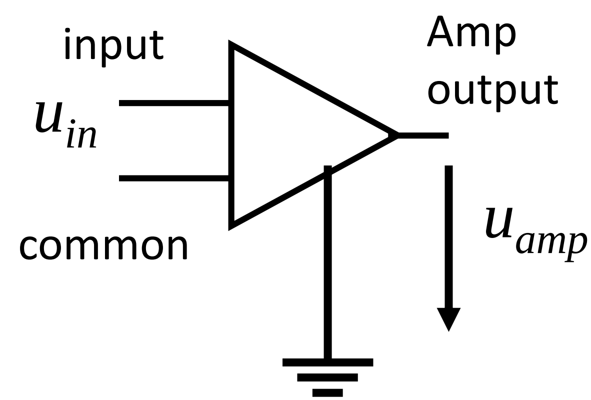
- Referenced source (with respect to the signal or power ground)
- Floating source (isolated from the signal or power ground)
- Asymmetric referenced amplifier (with respect to the power ground)
- Asymmetric floating amplifier (isolated from the power ground)

Referenced source – floating amplifier

- Amplifier with a non-referenced single-ended input (NRSE)

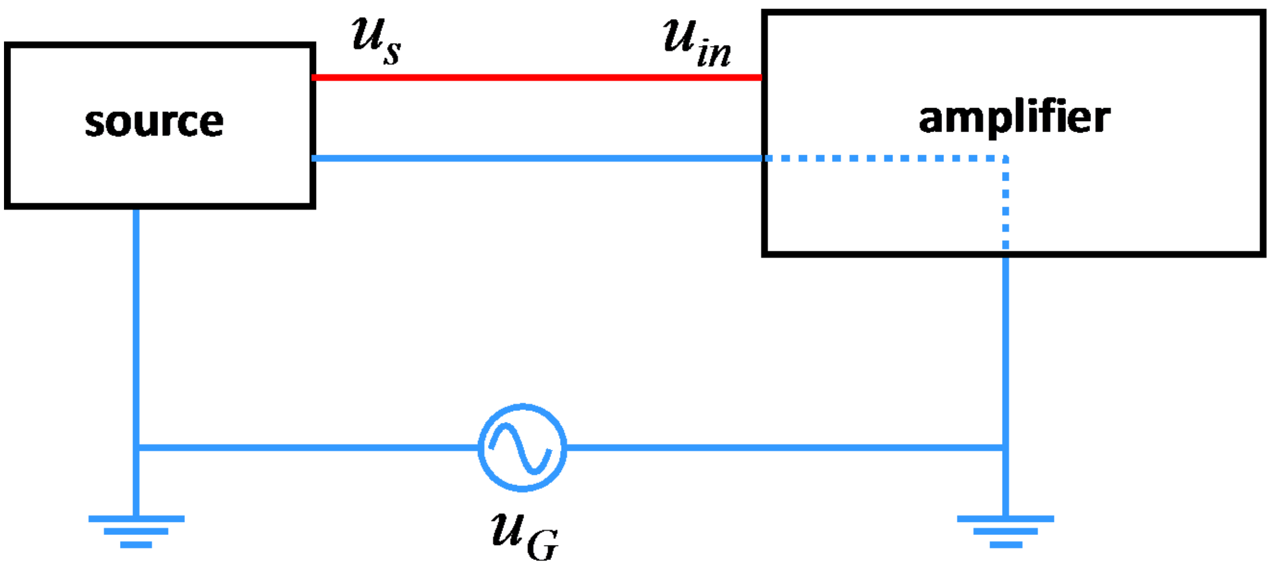


$$u_{in} = u_s$$

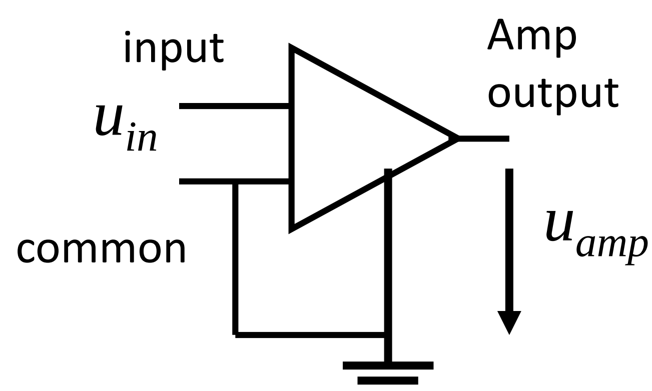


Floating source – referenced amplifier

- Amplifier with a referenced single-ended input (RSE)

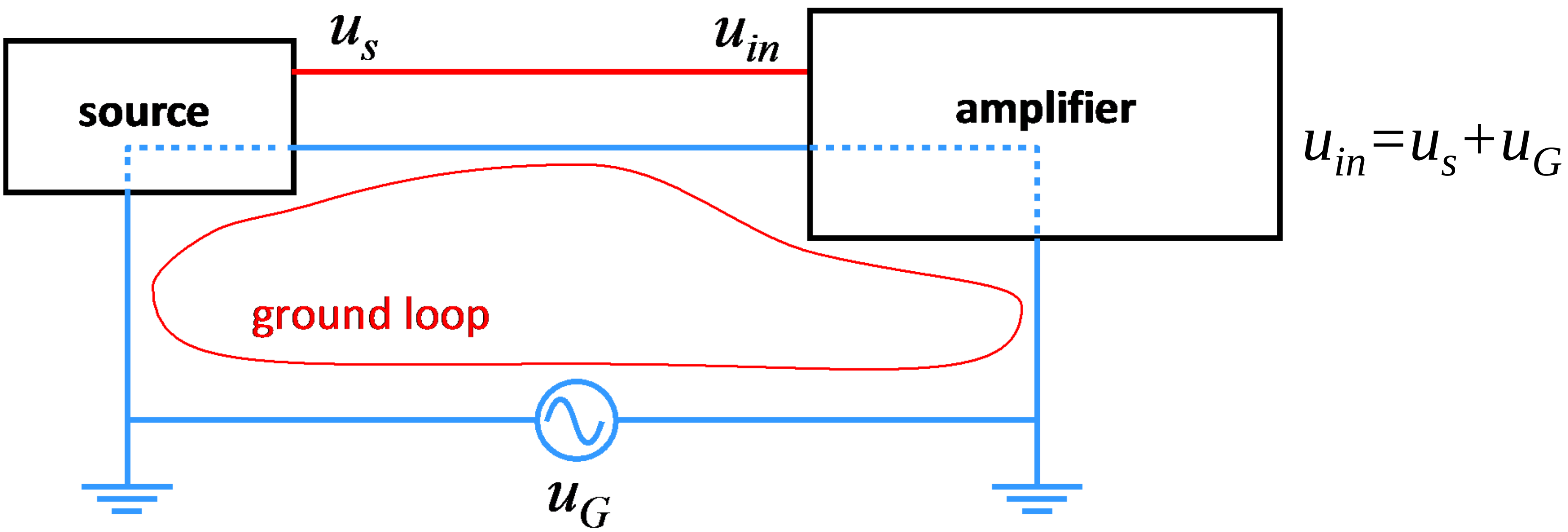


$$u_{in}=u_s$$



Referenced source – referenced amplifier

- Results in a ground loop
- Avoid



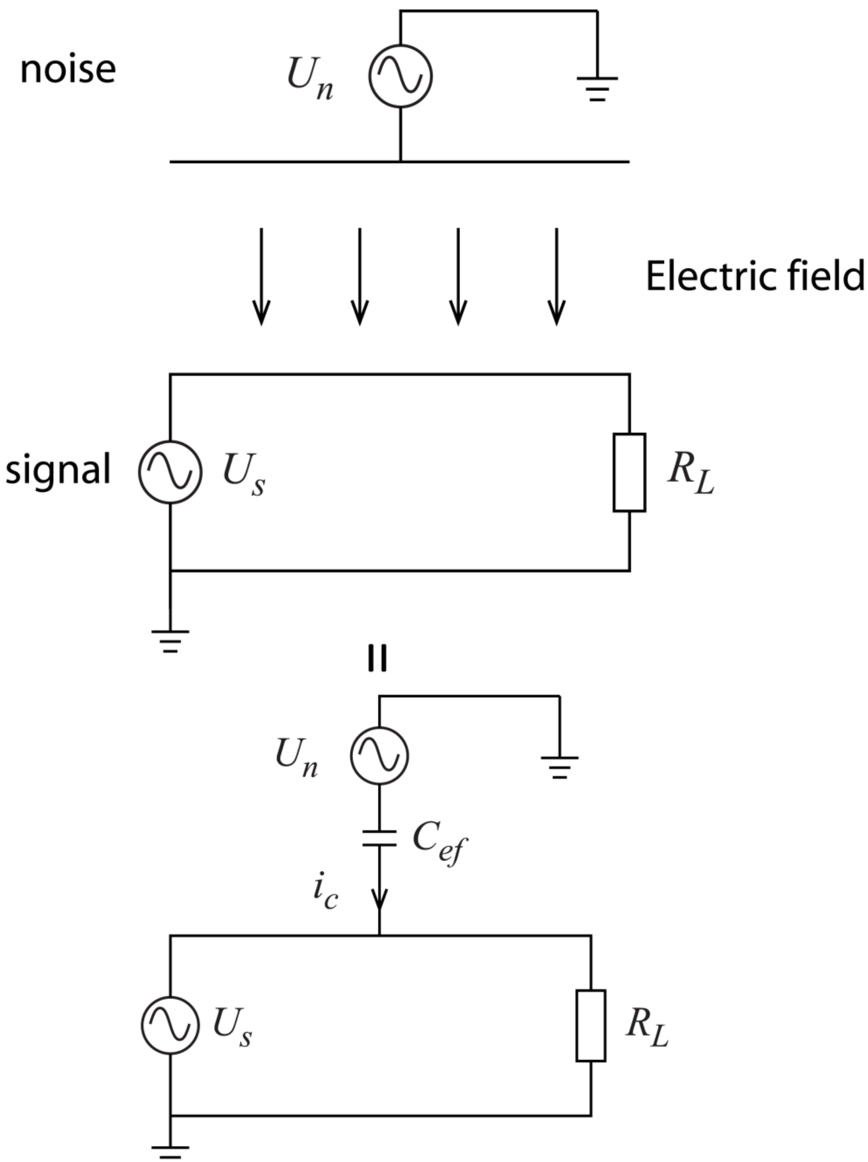
Complex impedances in an ac circuit

	Resistor	Capacitor	Inductor
Impedance Z	$Z_R = R$	$Z_C = \frac{1}{jC\omega}$	$Z_L = jL\omega$
Differential equation	$v = iR$	$i = C\frac{dv}{dt}$	$v = L\frac{di}{dt}$
Phase difference (i with respect to v)	0	+90° (i ahead of v)	-90° (i lagging behind v)

Ohm's Law: $V = I \cdot Z_{total}$ V and I are also complex numbers (amplitude and phase)

1. Apply regular Kirchhoff rules, calculate Z_{total} according to rules for parallel and serial addition of resistors
2. Keep complex numbers until the end
3. Calculate absolute values and phase (if interested in it)

Capacitive coupling

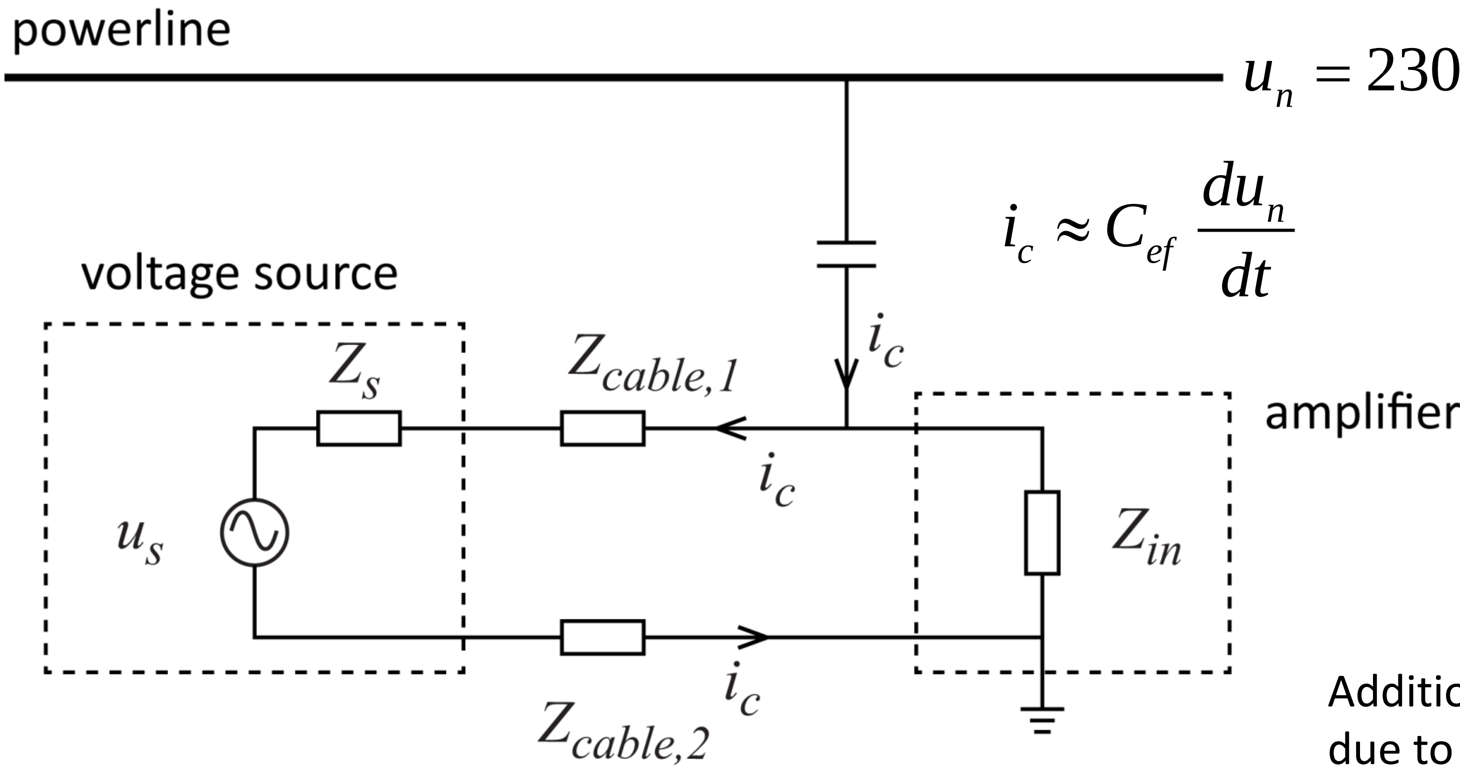


- Capacitive coupling between the measurement circuit and a noise source (for example the 220 V powerline network)

$$i_c \approx C_{ef} \frac{dU_n}{dt}$$

C_{ef} – typically 0.1-1000 pF

Capacitive coupling - example



$$u_n = 230\cos(2\pi \cdot 50 \cdot t)$$

$$i_c \approx C_{ef}\,\frac{du_n}{dt}$$

Additional voltage
due to capacitive coupling

Voltage drop
at the amplifier input

$$C_{ef} = 1.4\ pF$$

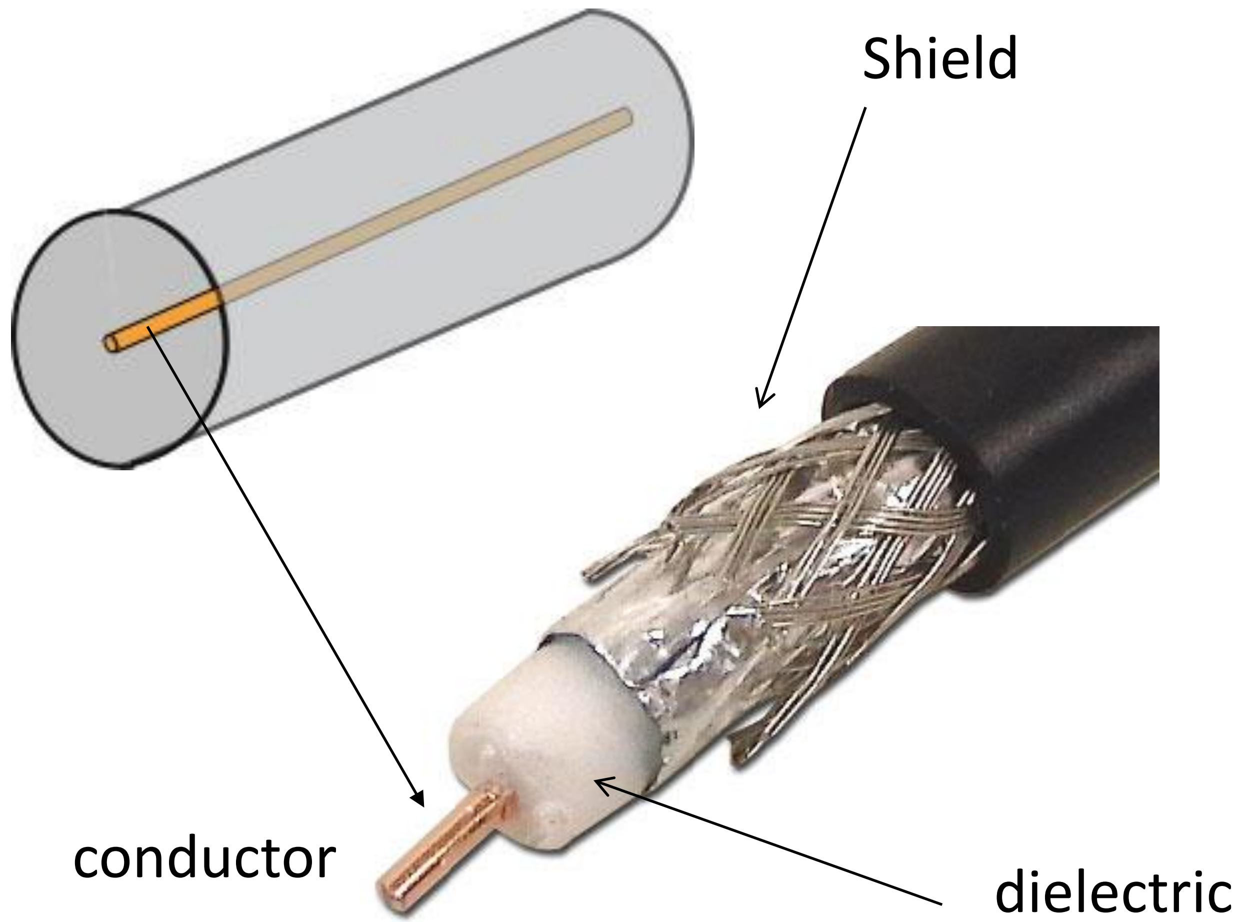
$$Z_s + Z_{cable,1} + Z_{cable,2} = 10\,\Omega$$

$$Z_{in} = 10\,MOhm$$

$$u_A = \frac{Z_{in}}{Z_{cable,1} + Z_{cable,2} + Z_{in} + Z_S}\,u_s + u_{c,A}$$

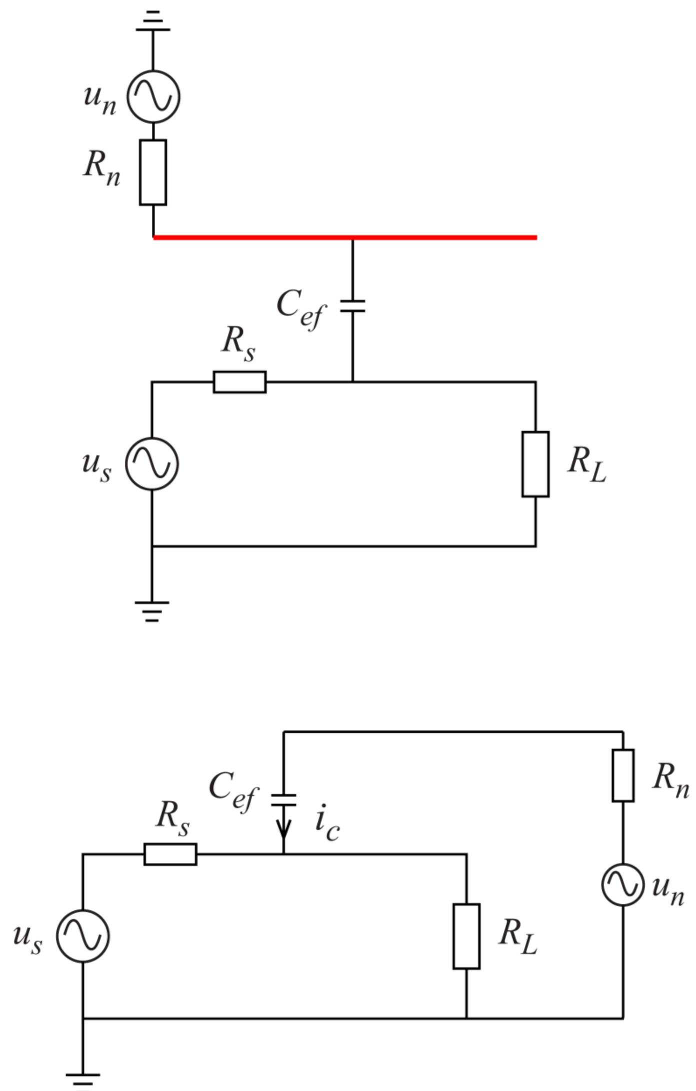
$$u_{c,A} = \left(Z_{cable,1} + Z_{cable,2} + Z_S\right) i_c \approx 1\mu V$$

Shielding

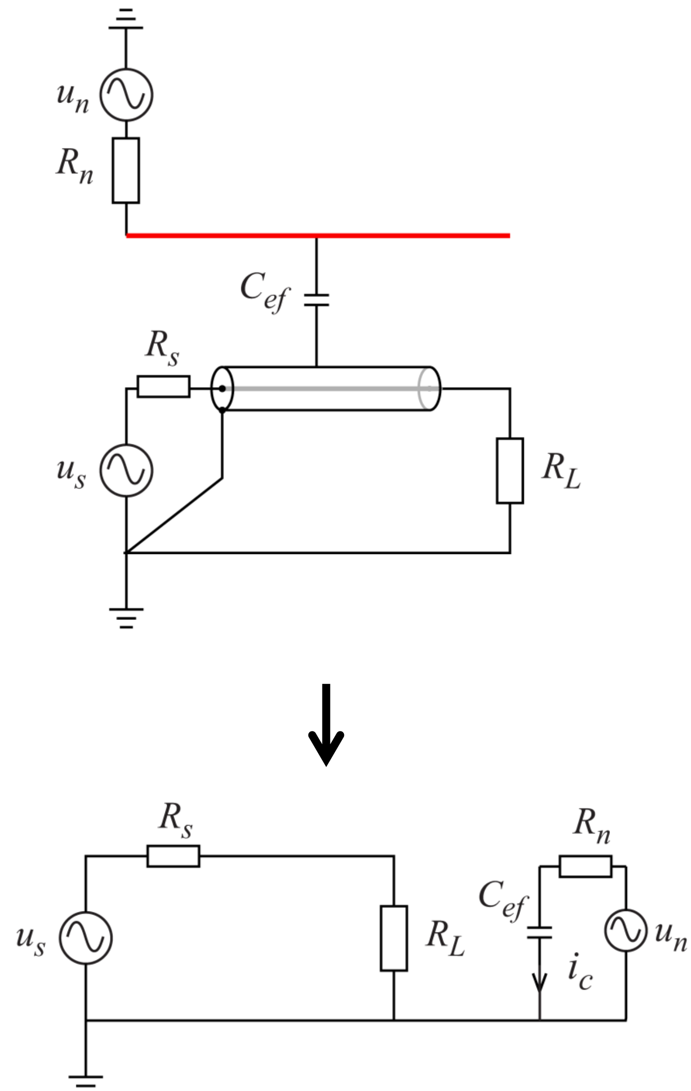


Shielding

Without



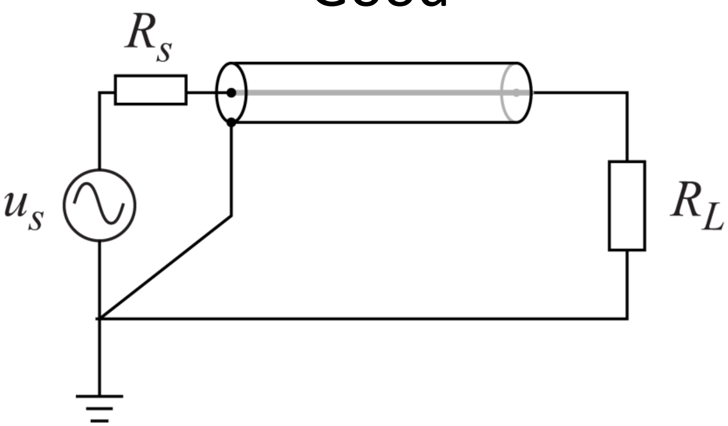
With



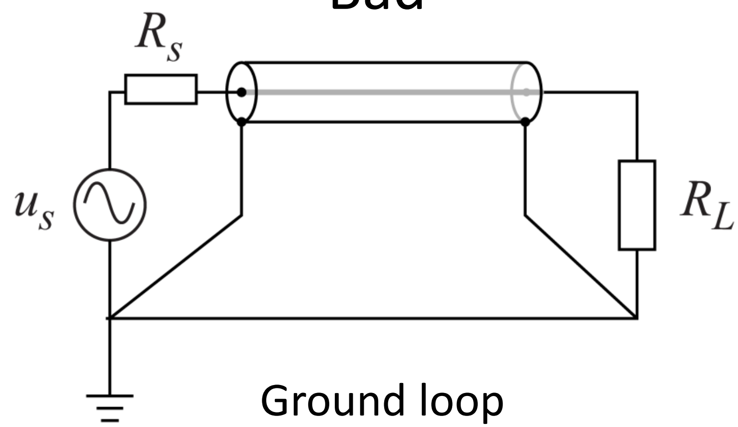
How to connect the shield

- Needs to be connected to the ground or the reference

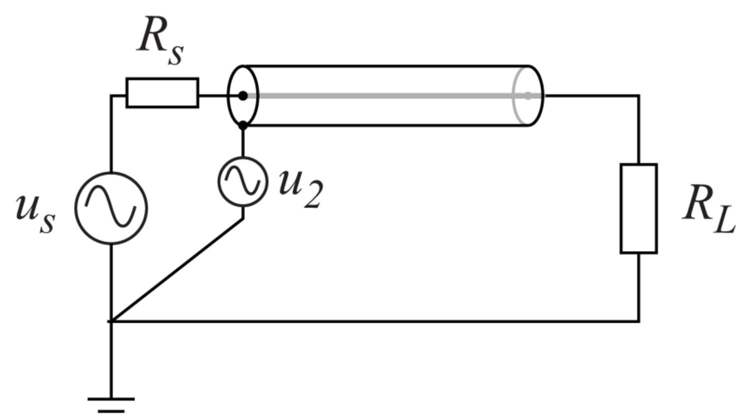
Good



Bad



Ground loop

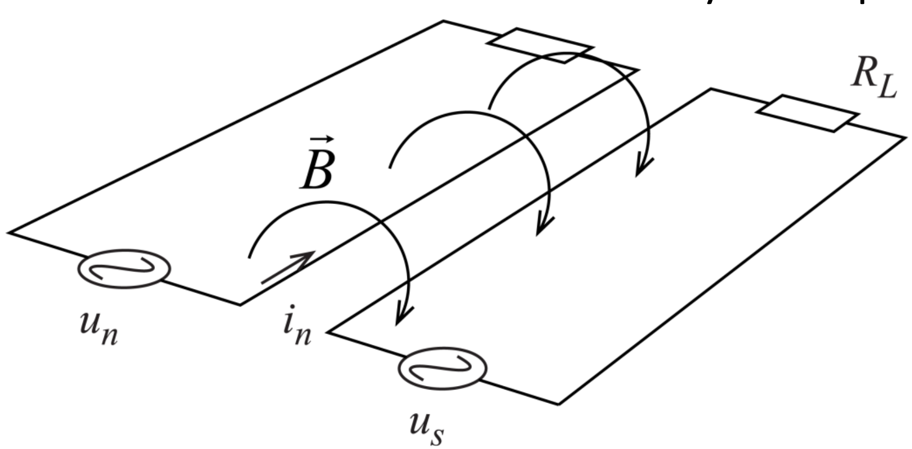


u_2 capacitively couples into the signal

Magnetic coupling

- Magnetic field generated by one circuit induces EMF in the other (measurement) circuit

Physical representation



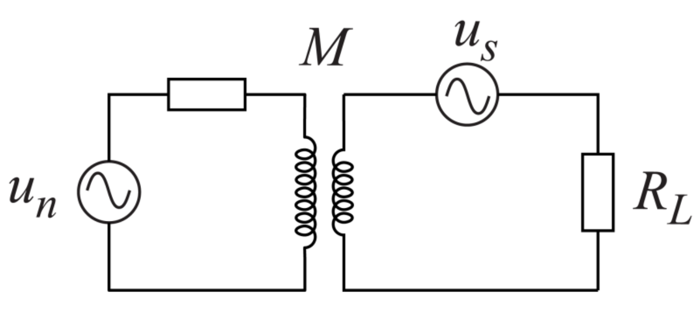
$$\phi = \int_{\substack{\textit{measurement}\\ \textit{circuit area}}} \vec{B}\,d\vec{A} = M i_n$$

A: area of the measurement (receiving) circuit

M: mutual inductance

Electromotive force $$\mathcal{E}_i = -\frac{d\phi}{dt}$$

$$u_{induced} = M\,\frac{di_n}{dt}$$

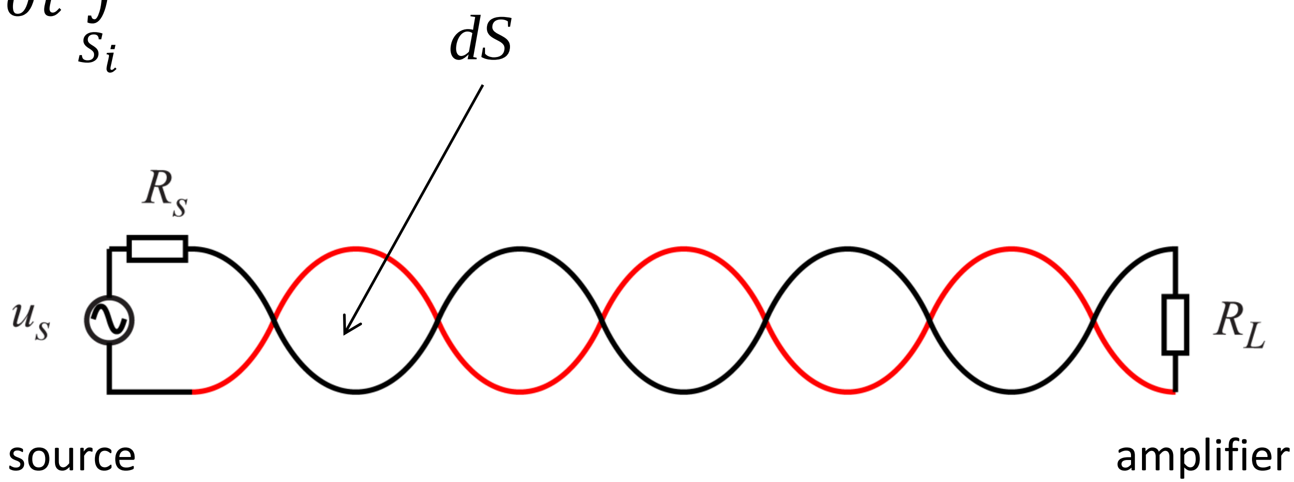


Equivalent circuit

Protection: twisted pairs

- Induced EMF in each twist

$$\mathcal{E}_i = -\frac{\partial}{\partial t}\int_{S_i} \vec{B}\cdot d\vec{S}$$



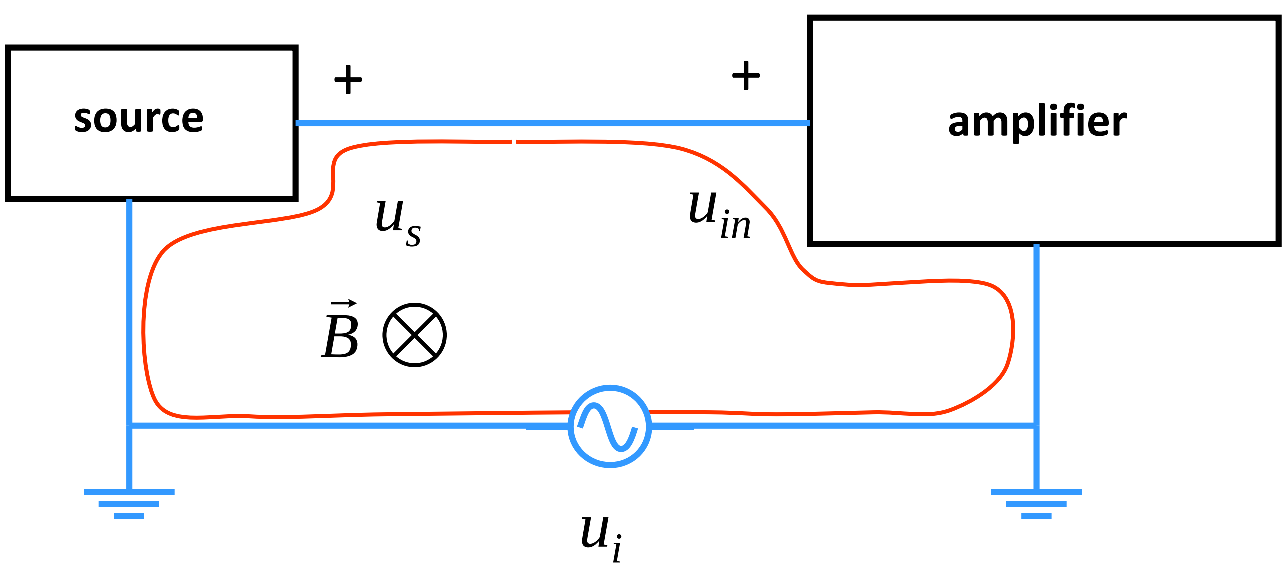
Opposite signs of EMF in neighbouring twists

Magnetic coupling to ground loops

- External time-varying magnetic field B induces a voltage u_i

$$u_i \approx A\frac{dB}{dt}$$

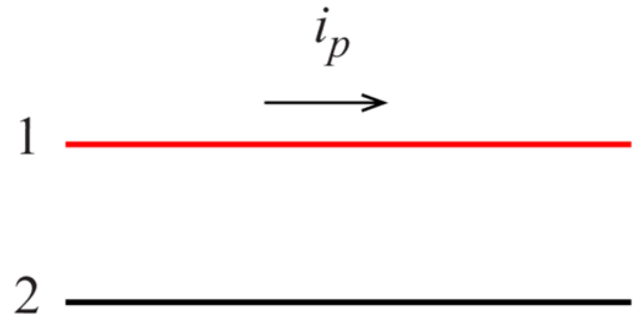
A - loop area



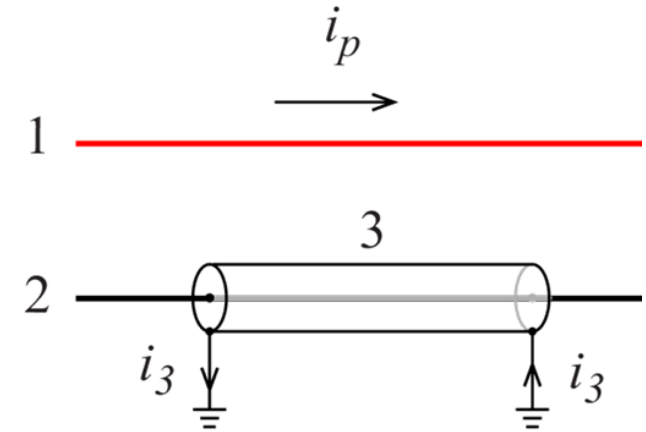
$$u_{in}=u_s+u_i$$

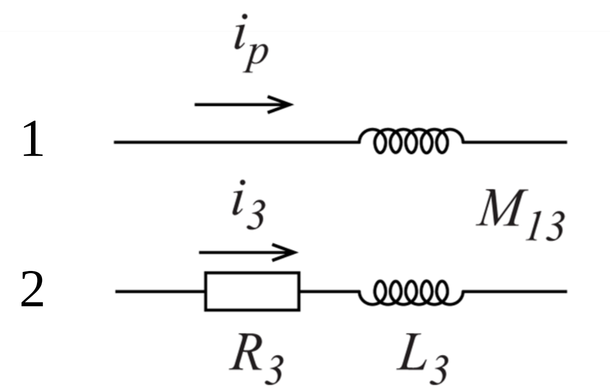
Magnetic shielding

Without shielding



Grounded shielding





Voltage induced in 2 by 1:

$$u_{2,0} = j\omega M_{12} i_p$$

Voltage drops over the shield (3):

$$i_3(R_3 + j\omega L_3) = -j\omega M_{13} i_p$$

Voltage drops over the inner conductor (2):

$$u_2 = j\omega M_{12} i_p + j\omega M_{23} i_3$$

Magnetic shielding

$$\frac{u_2}{u_{2,0}} = \frac{j\omega M_{12} i_p + j\omega M_{23} i_3}{j\omega M_{12} i_p} = 1 + \frac{M_{23} i_3}{M_{12} i_p}$$

$$i_3 = \frac{-j\omega M_{13} i_p}{R_3 + j\omega L_3}$$

$$M_{12} \equiv M_{13}$$

$$= 1 + \frac{M_{23}}{M_{12}} \cdot \frac{-j\omega M_{13}}{R_3 + j\omega L_3}$$

$$M_{23} \cong L_3$$

$$= \frac{R_3 + j\omega L_3 - j\omega M_{23}}{R_3 + j\omega L_3}$$

$$f_c = \frac{1}{2\pi} \frac{R_3}{L_3}$$

$$= \frac{R_3}{R_3 + j\omega L_3}$$

$$\omega_c = \frac{R_3}{L_3}$$

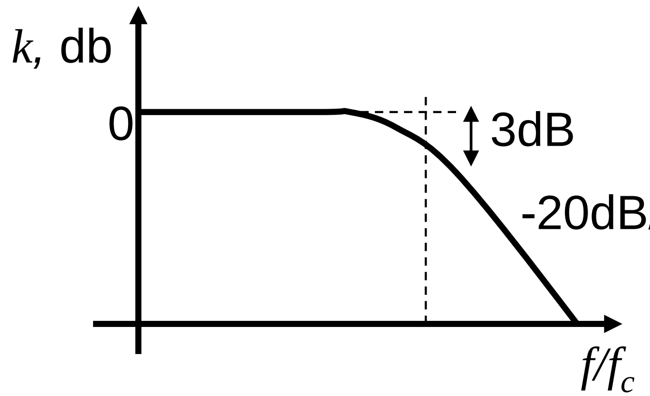
$$\frac{u_2}{u_{2,0}} = \frac{1}{1 + j\dfrac{f}{f_c}}$$

$$\left|\frac{u_2}{u_{2,0}}\right|^2 = \frac{1}{1 + \left(\dfrac{f}{f_c}\right)^2}$$

Effectiveness of shielding depends on the frequency

Magnetic shielding

$$k = 10\log\left|\frac{u_2}{u_{2,0}}\right|^2 = 10\log\frac{1}{1+\left(\frac{f}{f_c}\right)^2}$$



Induced voltage

$$u_2 = \frac{1}{1+j\left(\frac{f}{f_c}\right)}u_{2,0} = \frac{j\omega M_{12} i_p}{1+j\left(\frac{f}{f_c}\right)}$$

$$|u_2| = \frac{2\pi f M_{12}|i_p|}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}$$

For f >>

$$|u_2| \cong \omega_c M_{12}|i_p| \qquad \omega_c = \frac{R_3}{L_3}$$

- Shielding not effective for $f < f_c$, however induced voltage is also small there
- For f >>, u_2 depends on M and ω_c
- We should:
  - Decrease M_{12}
  - Decrease ω_c (decrease R_3, increase L_3)

Noise estimation and suppression

- Sources
- Extrinsic noise
  - Conductive coupling
  - Capacitive coupling
  - Magnetic coupling
- Noise suppression using differential measurements
  - Common mode voltage
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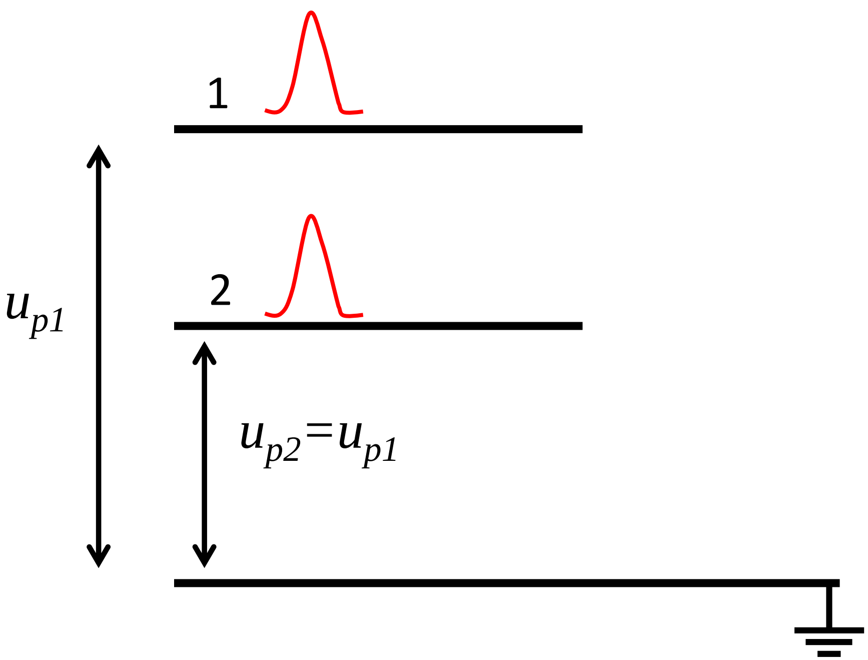
- Intrinsic noise
  - Thermal noise
  - Shot noise
  - 1/f noise
  - Noise estimation

Common and differential mode signals and perturbations

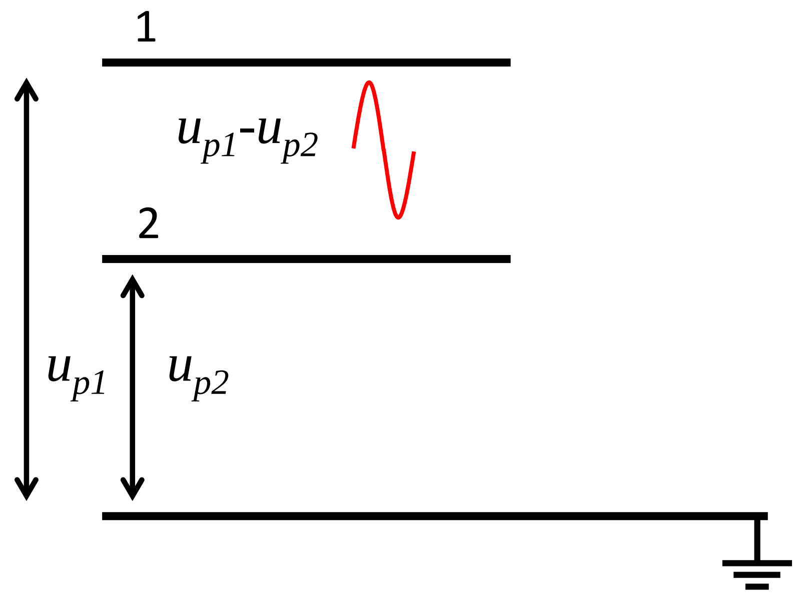
Common mode perturbation



Common mode parasitic voltage

$$u_{cm,p} = \frac{u_{p1} + u_{p2}}{2}$$

Differential mode perturbation

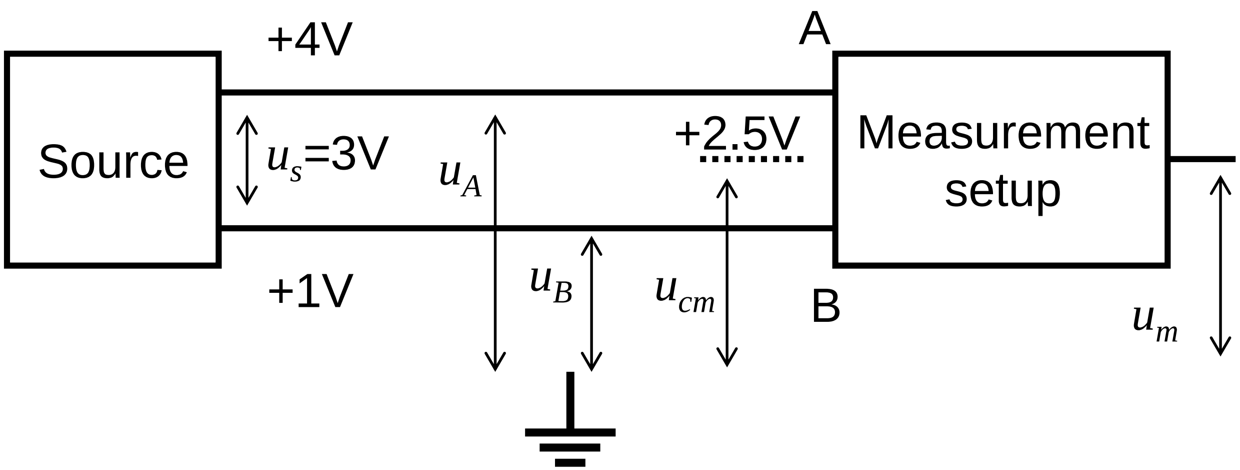


Differential mode parasitic voltage

$$u_{diff,p} = u_{p1} - u_{p2}$$

Common mode voltage - example

- Common mode voltage u_{cm}: voltage common to u_{A} and u_{B} that does not carry a useful signal



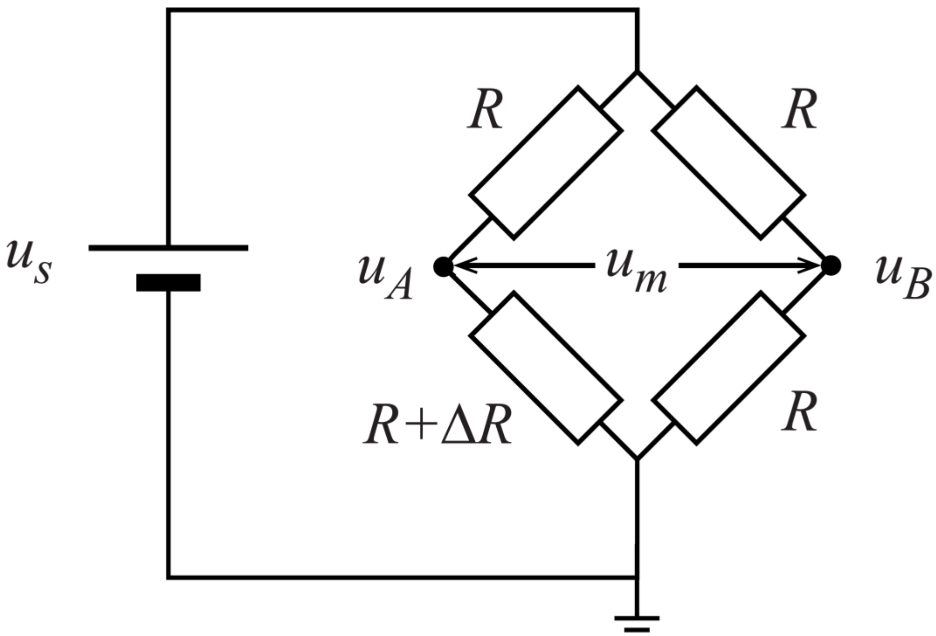
u_{m}: output voltage

$$u_A = u_{cm} + \frac{u_s}{2}$$

$$u_B = u_{cm} - \frac{u_s}{2}$$

$$u_{cm} = \frac{u_A + u_B}{2} \qquad u_s = u_A - u_B$$

Common mode voltage - example



$$u_B = \frac{u_s}{2}$$

$$u_A = \frac{u_s}{2} + \frac{\Delta R}{4R} u_s$$

$$u_m = u_A - u_B = \frac{\Delta R}{4R} u_s$$

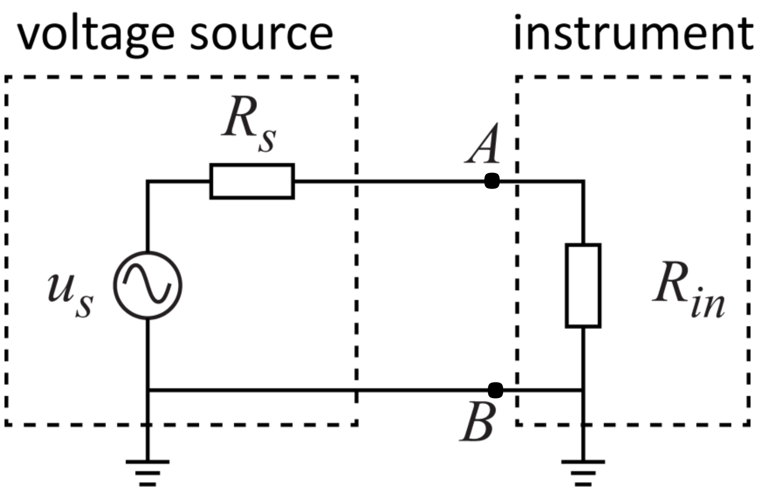
$$u_{cm} = \frac{u_A + u_B}{2} \approx \frac{u_s}{2}$$

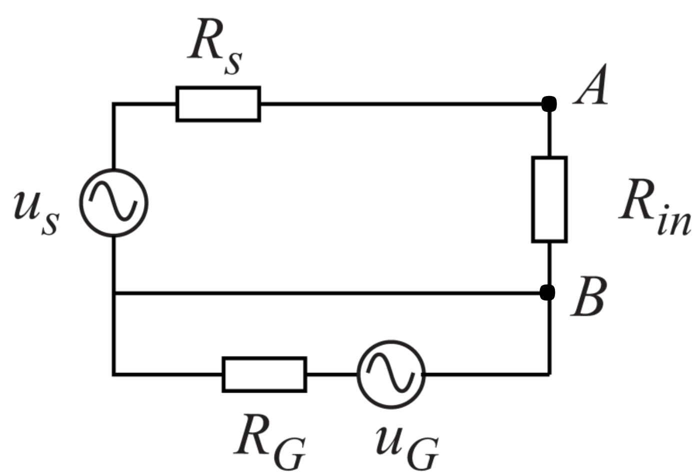
Example: u_s=10V, $\Delta R/R$=0.01
u_m= 25mV and u_{cm}=5V >>u_m

u_{cm} can be much larger than u_{m}

Common mode voltage - example

- Common mode voltage from a ground loop





R_{in} : large (open circuit, no current through R_s)

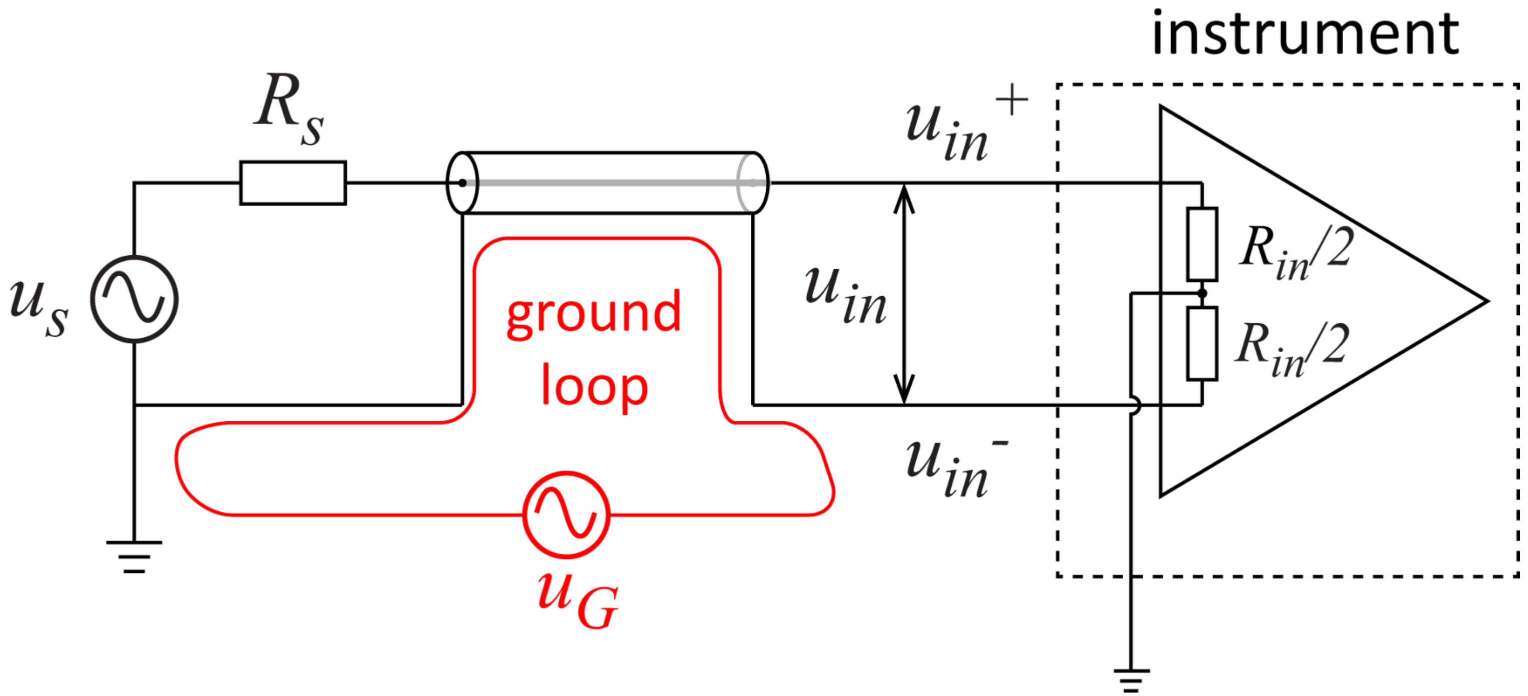
$$u_A = u_S + u_G \qquad u_B = u_G$$

$$u_{cm} \approx u_G \quad \text{for} \quad u_S << u_G$$

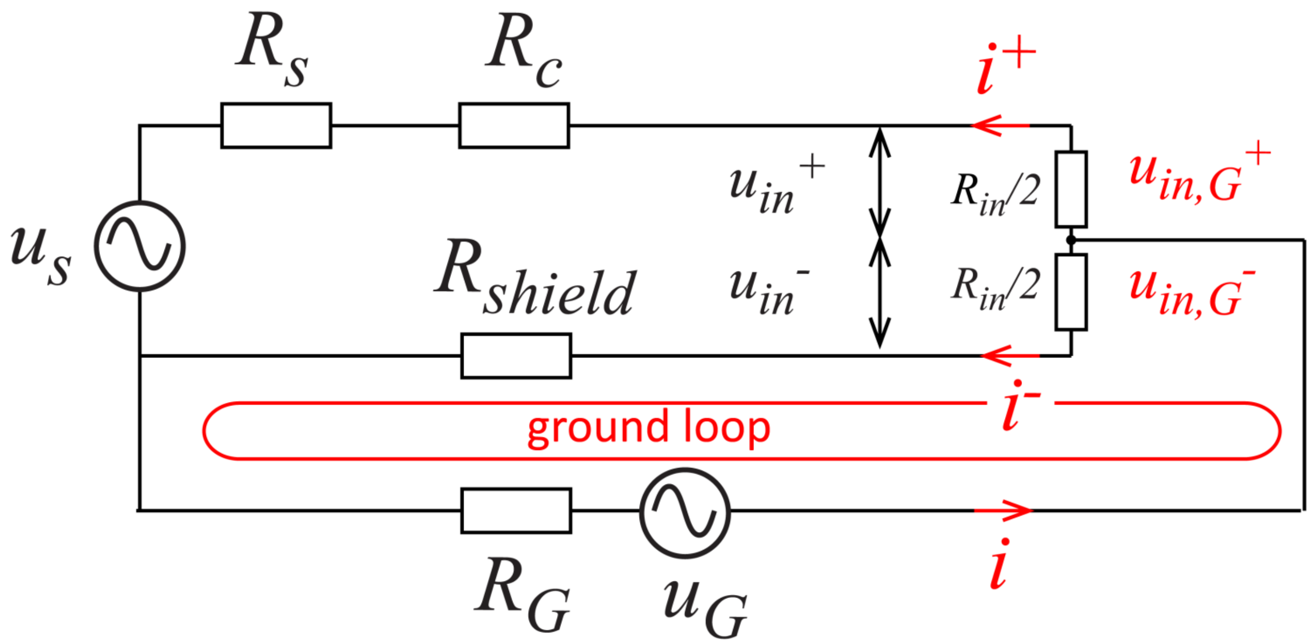
For the case of an ideal differential amplifier:

$$u_{input} = u_A - u_B = u_S$$

Differential measurements – shielded cables



Differential measurements – shielded cables



Usually:

$R_{in} >> R_s, R_c, R_{shield}, R_G$

Typical values:

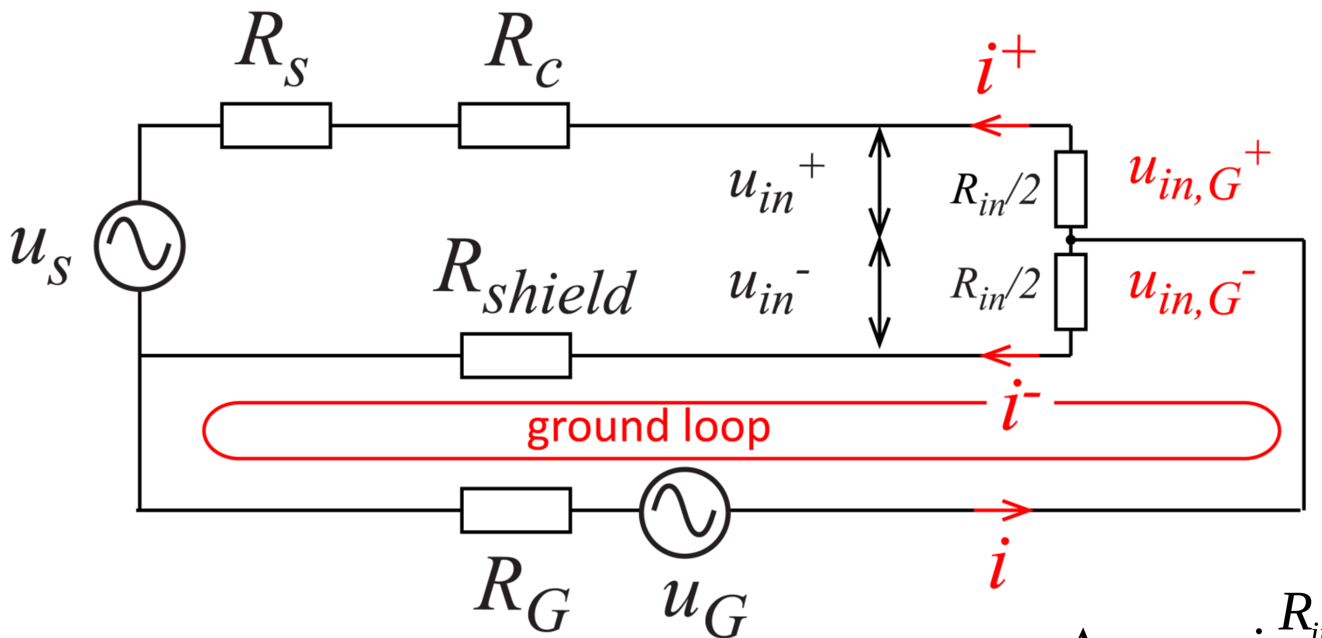
$R_{in} = 10\ \mathrm{M\Omega}$, $R_s = 1\ \mathrm{k\Omega}$,
$R_c = R_{shield} = 10\ \Omega$
$R_G = 1\ \Omega$

$$i^+ = i\,\frac{R_{in}/2 + R_{shield}}{R_{in} + R_s + R_{shield} + R_c} \approx i\,\frac{R_{in}/2}{R_{in} + R_s}$$

$$i^- = i\,\frac{R_{in}/2 + R_s + R_c}{R_{in} + R_s + R_{shield} + R_c} \approx i\,\frac{R_{in}/2 + R_s}{R_{in} + R_s}$$

$$\Delta u_{in,G} = u_{in,G}{}^+ - u_{in,G}{}^- = \frac{R_{in}}{2}\left(i^+ - i^-\right)$$

Differential measurements – shielded cables



$$\Delta u_{in,G} = i\,\frac{R_{in}}{2}\left(\frac{R_{in}/2}{R_{in}+R_s} - \frac{R_{in}/2+R_s}{R_{in}+R_s}\right)$$

Usually:

$R_{in} >> R_s,\ R_c,\ R_{shield},\ R_G$

$$i \approx -\frac{u_G}{R_{in}/4} = -\frac{4u_G}{R_{in}}$$

$$= -\frac{4u_G}{R_{in}}\,\frac{R_{in}}{2}\left(-\frac{R_s}{R_{in}+R_s}\right)$$

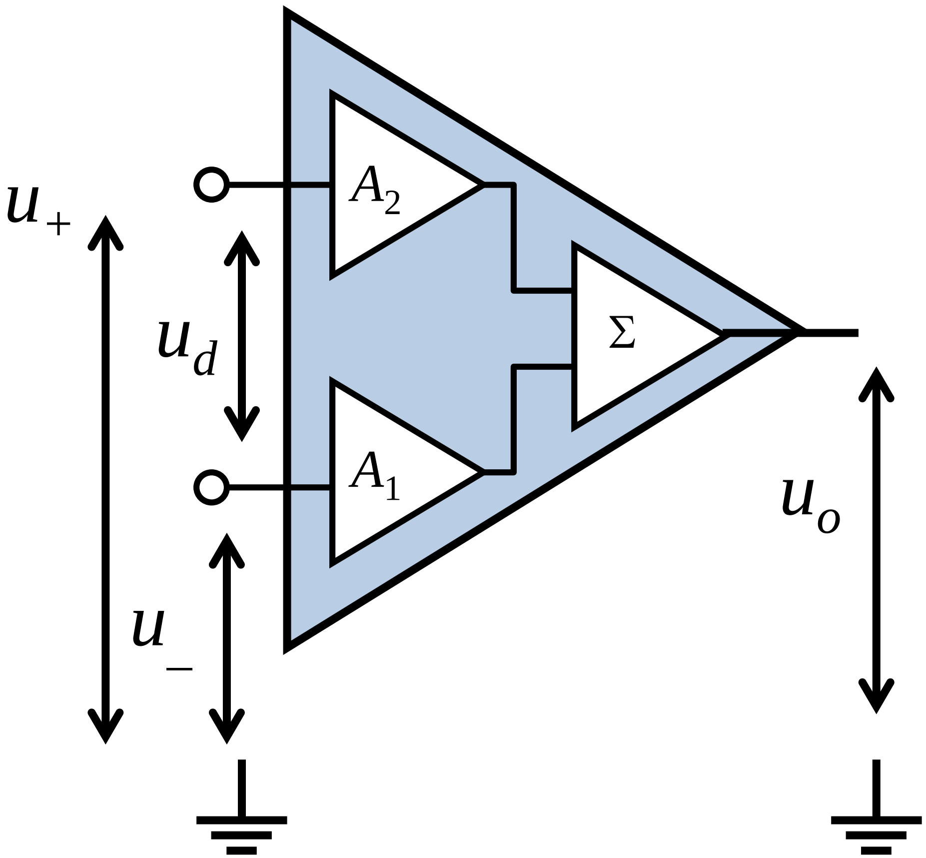
Typical values:

$R_{in} = 10\ \mathrm{M\Omega},\ \ R_s = 1\ \mathrm{k\Omega},$
$R_c = R_{shield} = 10\ \Omega$
$R_G = 1\ \Omega$

$$\boxed{\Delta u_{in,G} = \frac{2R_s}{R_{in}}u_G} \approx 0.0002u_G$$

Huge reduction in noise!

(A more realistic) Differential amplifier



$$u_o = A_2 u_+ - A_1 u_-$$

$$u_{cm} = \frac{u_+ + u_-}{2} \qquad u_d = u_+ - u_-$$

$$u_o = \underbrace{\frac{A_1 + A_2}{2}}_{A_d} \cdot u_d + \underbrace{(A_2 - A_1)}_{A_{cm}} \cdot u_{cm}$$

A_d : differential gain

A_{cm} : common mode gain

For an ideal amplifier:

$A_2 = A_1$

Common mode rejection ratio (CMRR)

$$CMRR = \frac{A_d}{A_{cm}}$$

$$CMRR\big|_{dB} = 20\log\left|\frac{A_d}{A_{cm}}\right|, dB$$

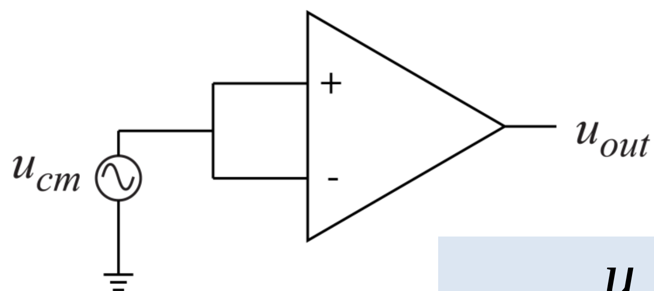
Output voltage:

$$u_{out} = A_d \cdot u_d + A_{cm} \cdot u_{cm}$$

$$u_{out} = A_d\left(u_d + \frac{1}{CMRR}u_{cm}\right)$$

Example:
Standard: cca 90dB CMRR
For u_{cm} = 10V → ± 316 μV on the output

How to measure A_{cm}:



$$A_{cm} = \frac{u_{out}}{u_{cm}}$$

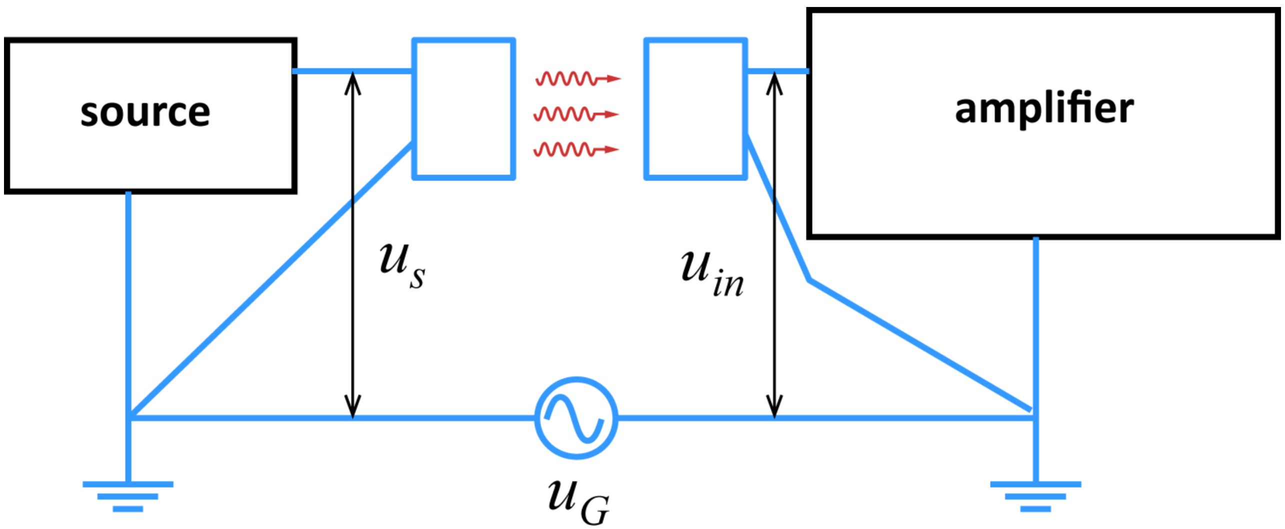
Instrumentation amplifier

- High-performance differential amplifier

  - High CMRR: 100 db (at 50 Hz)
  - Low input current (nA or pA)
  - Low output impedance: 0.1 Ω
  - Large input impedance: 10^{10} Ω
  - Large temperature stability
  - Programmable differential gain

Isolation amplifier

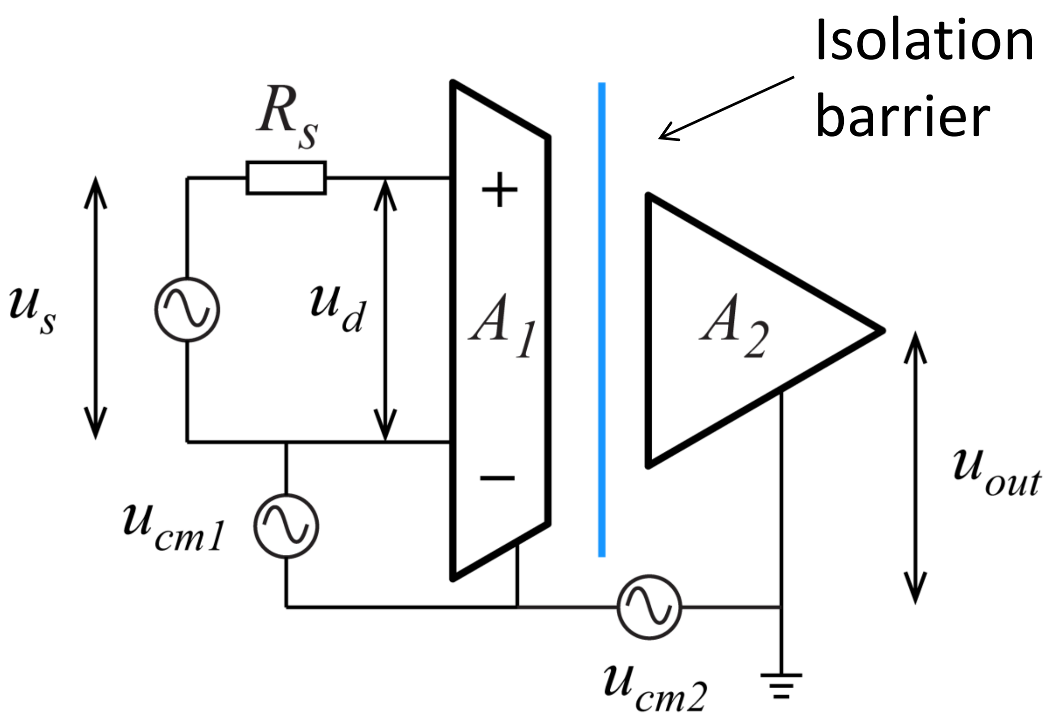
- Signal is transmitted through magnetic (transformer), optical (LED – photodiode pair) or capacitive (capacitor) coupling



- Recommended for u_{cm} > 70% of the power supply voltage

Isolation amplifier

- Decreases high levels of common mode voltage by breaking ground loops
- Protects the source from voltage surges in the instrument – important for medical applications ("source" is usually a person)



Stage A_1: instrumentation amplifier
- u_s floating source
- C_1 connected to the source ref

Stage A_2: buffer amplifier (gain=1)
- C_2 connected to the ground or instrument reference

Isolation barrier: no ohmic connections between A_1 et A_2
- Signal transmitted between A_1 et A_2 by coupling (magnetic, optical)

Isolation mode rejection ratio

- u_{cm1} : 10s of volts
- u_{cm2}: can be >1000 V

$$u_{out} = A_d \left(u_d + \frac{1}{CMRR} u_{cm1} + \frac{1}{IMRR} u_{cm2} \right)$$

- CMRR: common mode rejection ratio (>100 db – factor 10^5)
- IMRR: isolation mode rejection ratio (>140 db – factor 10^7)
- A_d : differential gain

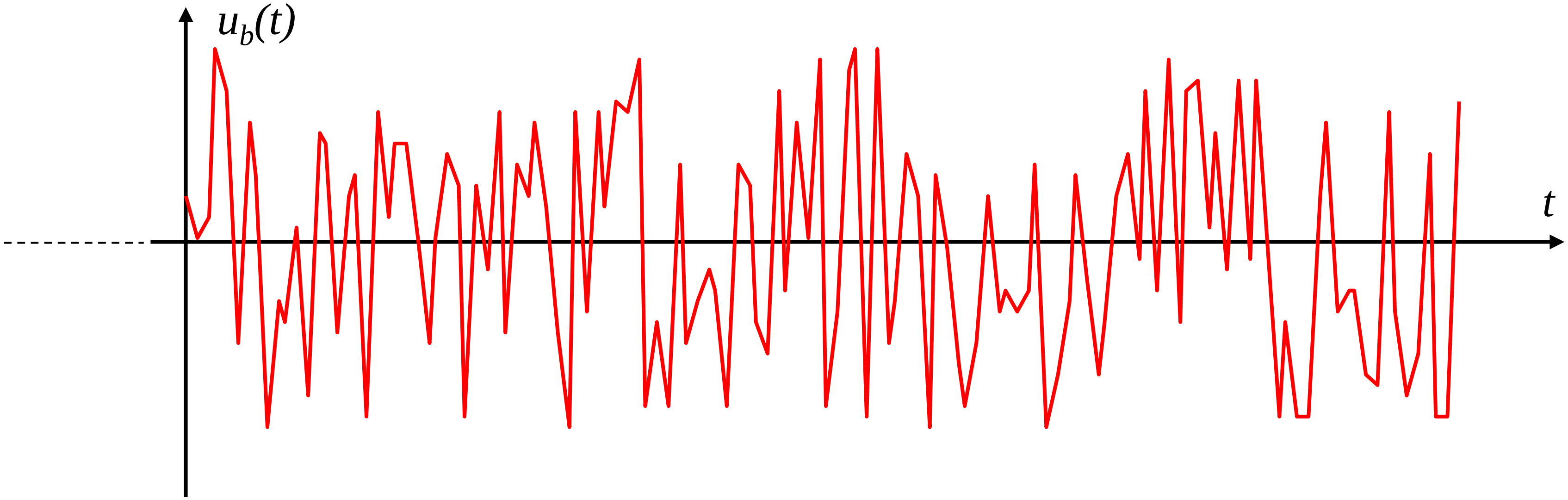
Key points

- Common mode voltage is a signal (sometimes noise, sometimes just a voltage offset) present on both inputs of the amplifier
- In case it corresponds to noise, we would of course like to eliminate it
- In case it corresponds to an offset, we would like to avoid it saturating or destroying the amplifier
- Differential, instrumentation and isolation amplifiers reduce the effect of common mode voltage

Noise estimation and suppression

- Sources
- Extrinsic noise
  - Conductive coupling
  - Capacitive coupling
  - Magnetic coupling
- Noise suppression using differential measurements
  - Common mode voltage
  - Suppression of the common mode voltage
  - Instrumentation amplifier
- Intrinsic noise
  - Thermal noise
  - Shot noise
  - 1/f noise
  - Noise estimation

Noise: Example



Intrinsic noise

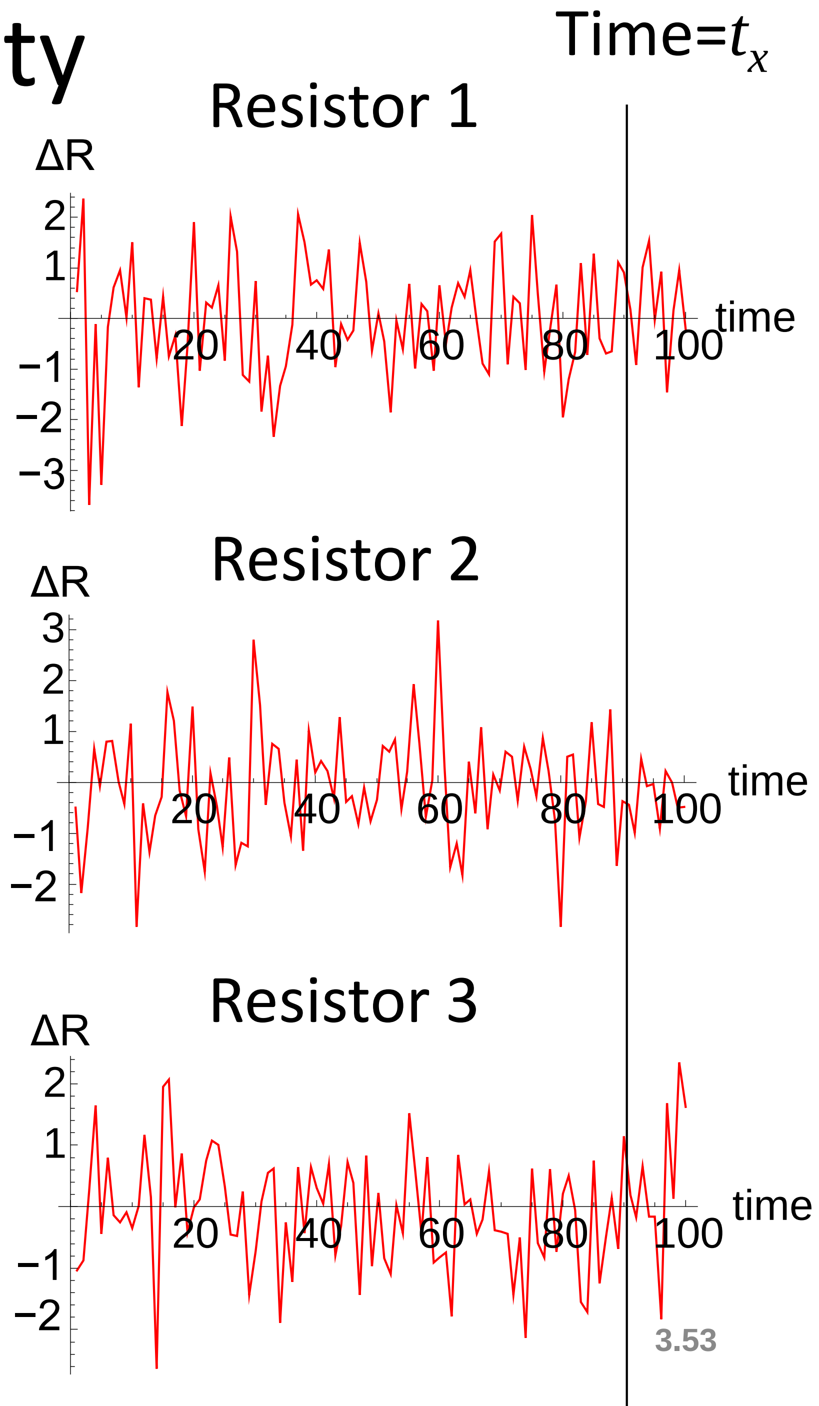
- Stationary:
  - All statistical parameters (average, standard deviation, etc) are time-independent
  - Noise estimated during one interval of time ΔT_1 is the same as in another interval ΔT_2
- Ergodic
  - Time average is the same as the ensemble average

- Stationary ergodic noise: a stationary noise for which the probability that the noise voltage lies within any given interval at any time is nearly equal to the fraction of time that the noise voltage lies within this interval if a sufficiently long observation interval is recorded

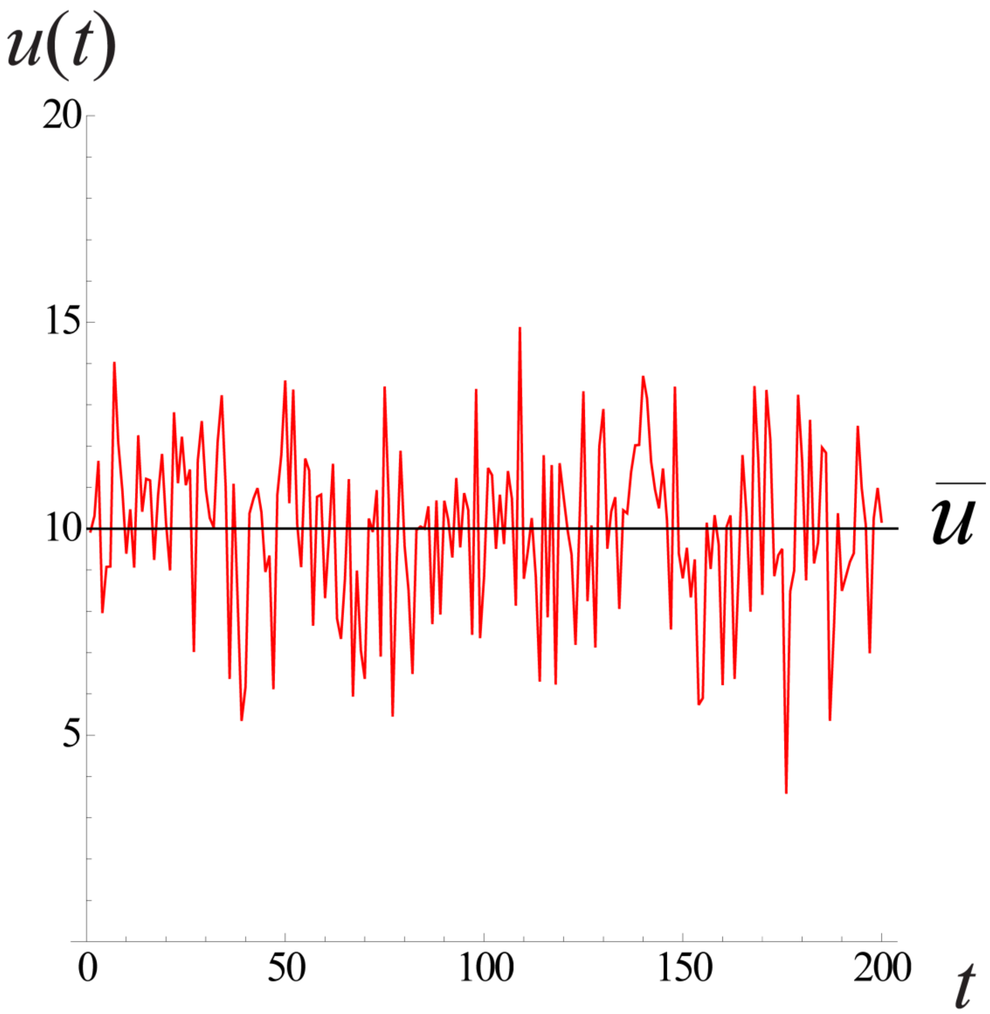
Ergodicity

1. Take N resistors
2. Measure their resistance vs. time -> N traces of R vs. time
3. Calculate average over time for the trace of any one resistor: **time average**
4. Now look at all the N traces and choose points from each trace corresponding to the same time t_x
5. Calculate the average -> **ensemble average**

Time average should have the same value as the ensemble average



Characteristics of noise



$u(t)$ – voltage measurement as a function of time

$$u_n(t) = u(t) - \overline{u}$$

$u_n(t)$ – noise

$u_n(t)$ and $i_n(t)$ – voltage or current noise, deviation from the average

Characteristics of noise

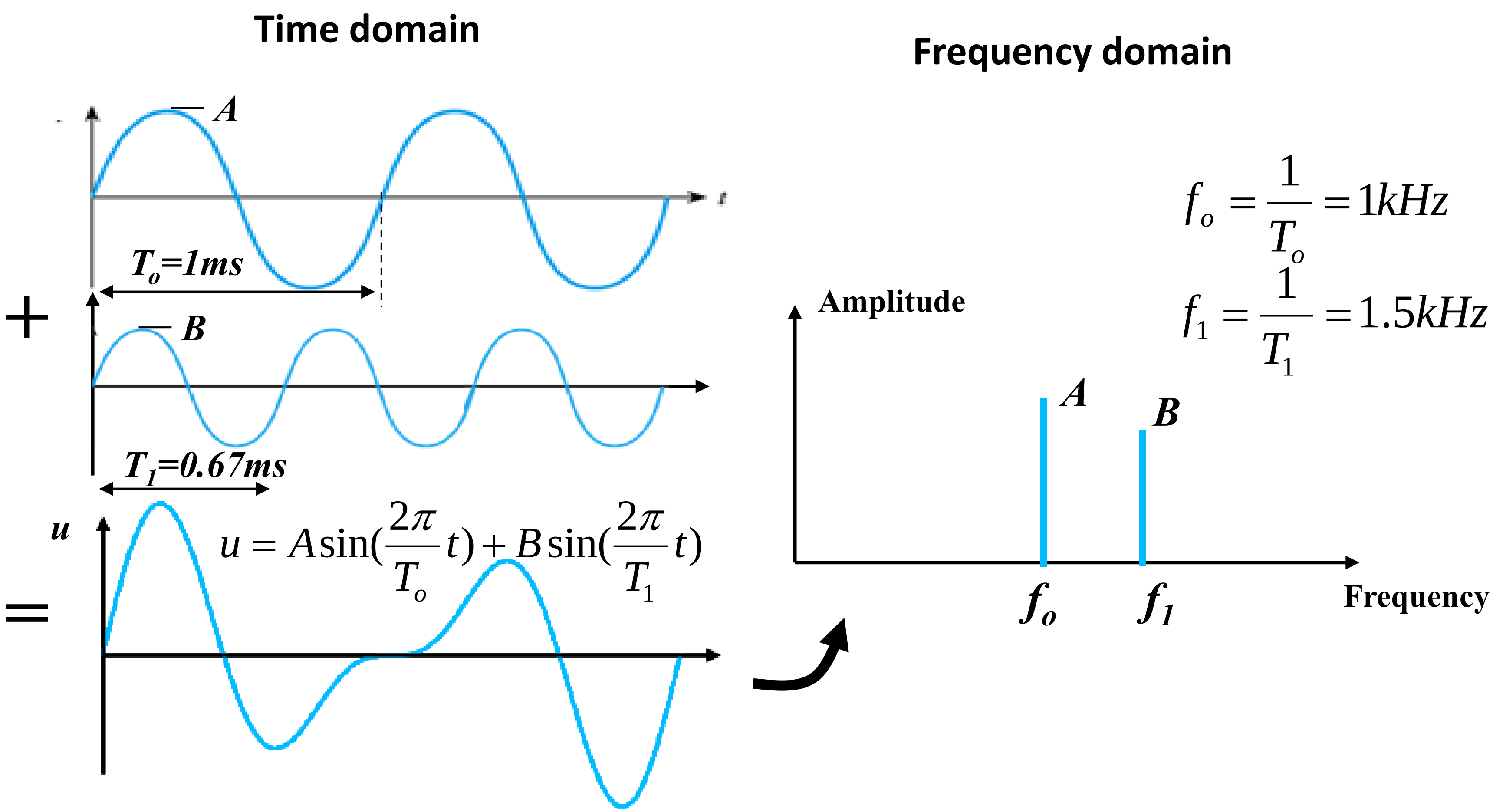
	Time-based	**Statistical**
Mean value (of the deviation)	$\overline{u_n} = \frac{1}{T}\int_0^T u_n(t)dt = 0$	$\mu_{un} = \frac{1}{N}\sum_{i=1}^{N} u_n(i) = 0$
Mean square of the deviation (variance)	$\overline{u^2{}_n} = \frac{1}{T}\int_0^T u^2{}_n(t)dt$	$\sigma_{un}^2 = \frac{1}{N}\sum_{i=1}^{N} u^2{}_n(i)$

Characteristics of noise

	Time-based	**Statistical**
Effective value (DC voltage that would give you the same power dissipation as the noise)	$U_{n,eff} = \sqrt{\frac{1}{T}\int_0^T u^2{}_n(t)dt}$ $= \sqrt{\overline{u^2{}_n}}$	$\sigma_{un} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} u^2{}_n(i)}$
Average power	$P_n = \frac{1}{T}\int_0^T \frac{u^2{}_n(t)}{R}dt = \frac{\overline{u^2{}_n}}{R}$ $= \frac{U^2_{n,eff}}{R}$	$\frac{1}{N}\sum_{i=1}^{N} \frac{u^2{}_n(i)}{R} = \frac{\sigma^2_{un}}{R}$ $\frac{1}{N}\sum_{i=1}^{N} Ri^2{}_n(i) = R\sigma^2_{in}$

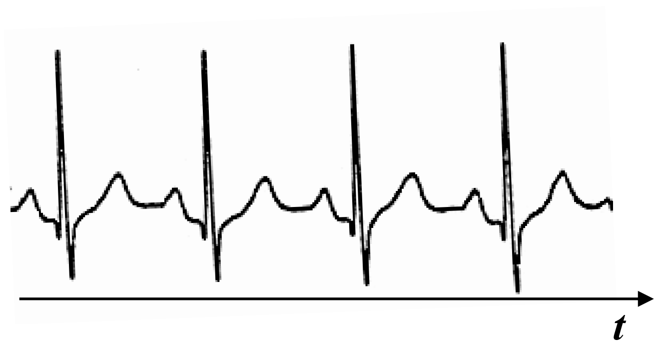
Reminder: frequency spectrum

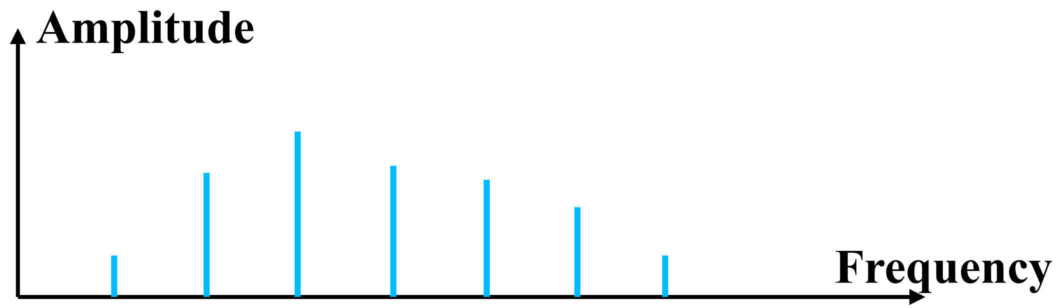
- Sinusoidal signals



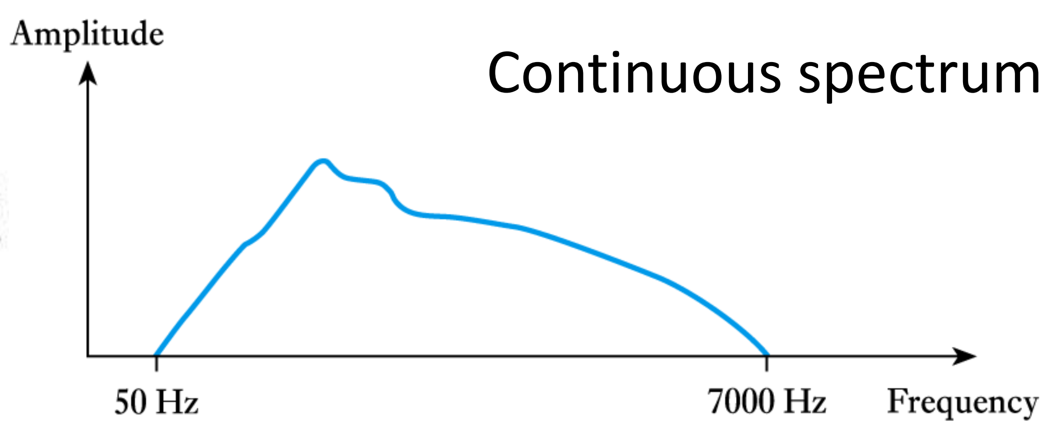
Generalisation

- Periodic signals





- Arbitrary signals



Continuous spectrum

Noise spectral density

- Voltage or current noise is a superposition of periodic noise with a spectrum ranging from 0 to infinity
- The noise amplitude depends on the frequency bandwidth:

$$f_{max} - f_{min}$$

- Noise spectral density

$$\Phi_{u,n}(f) = \frac{d(\overline{u_n^2})}{df} \quad \text{V}^2/\text{Hz} \qquad \Phi_{i,n}(f) = \frac{d(\overline{i_n^2})}{df} \quad \text{A}^2/\text{Hz}$$

- Allows us to express the electrical power due to noise in the measurement bandwidth

Voltage (Current) noise mean square

$$\overline{u^2}_n = \int_{f_{\min}}^{f_{\max}} \Phi_{u,n}(f)df; \qquad \overline{i^2}_n = \int_{f_{\min}}^{f_{\max}} \Phi_{i,n}(f)df;$$

For spectral density independent of frequency:

$$\overline{u^2}_n = \Phi_{u,n} \cdot (f_{\max} - f_{\min})$$

Summary

Instantaneous values	$u_n(t)$	$i_n(t)$
Effective values	$U_{n,eff}$	$I_{n,eff}$
Average power	$P_n = \dfrac{\overline{u^2{}_n}}{R} = \dfrac{U^2_{n,eff}}{R}$	$P_n = R\overline{i^2_{eff}} = RI^2_{n,eff}$
Spectral densities	$\Phi_{u,n}$, V^2/Hz $\sqrt{\Phi_{u,n}}$, $\mathrm{V}/\sqrt{\mathrm{Hz}}$	$\Phi_{i,n}$, A^2/Hz $\sqrt{\Phi_{i,n}}$, $\mathrm{A}/\sqrt{\mathrm{Hz}}$
Estimation of noise intensity (noise indep. of frequency)	$U_{n,eff} = \sqrt{\Phi_{u,n} \cdot (f_{\max} - f_{\min})}$	$I_{n,eff} = \sqrt{\Phi_{i,n} \cdot (f_{\max} - f_{\min})}$

Example

AC Electrical Characteristics

$T_A = T_J = 25^\circ C$, $V_S = \pm 15V$

Symbol	Parameter	Conditions	LF155/355	LF156/256/ 356B	LF156/256/356/ LF356B	LF257/357	Units
			Typ	Min	Typ	Typ	
SR	Slew Rate	LF155/6: A_V=1,	5	7.5	12		V/μs
		LF357: A_V=5				50	V/μs
GBW	Gain Bandwidth Product		2.5		5	20	MHz
t_s	Settling Time to 0.01%	(Note 7)	4		1.5	1.5	μs
e_n	Equivalent Input Noise Voltage	R_S=100Ω					
		f=100 Hz	25		15	15	$nV/\sqrt{Hz}$
		f=1000 Hz	20		12	12	$nV/\sqrt{Hz}$
i_n	Equivalent Input Current Noise	f=100 Hz	0.01		0.01	0.01	$pA/\sqrt{Hz}$
		f=1000 Hz	0.01		0.01	0.01	$pA/\sqrt{Hz}$
C_{IN}	Input Capacitance		3		3	3	pF

The noise for amplifier LF356 in the frequency band [0-40Hz]:

$$U_{n,eff} / \sqrt{\Delta f} = 15 nV / \sqrt{Hz}$$

Effective value: $$U_{n,eff} = 15 nV \times \sqrt{40} = 94 nV$$

Signal to noise ratio (SNR)

$$SNR = \frac{Signal\ power}{Noise\ power} = \frac{s}{n}$$

$$\boxed{SNR_{dB} = 10\log\frac{s}{n}}$$

- Example: signal U_{eff} = 10 mV, noise $U_{n,eff}$ = 4.9 mV

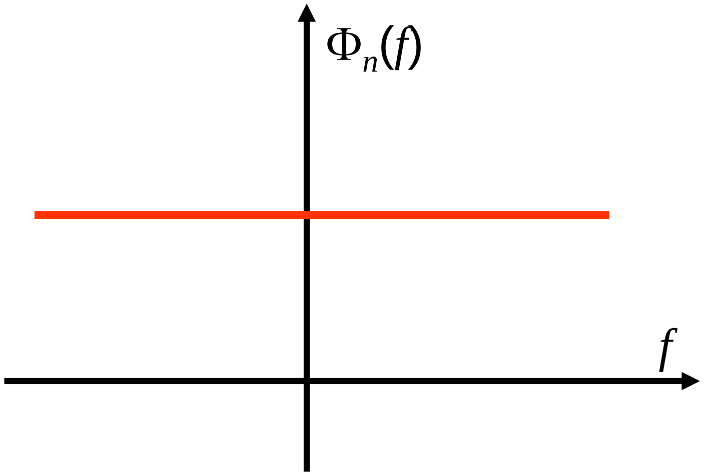
$$SNR_{dB} = 20\log\frac{U_{eff}}{U_{n,eff}} = 20\log\frac{10}{4.9} = 6.2dB$$

Types of intrinsic noise

- **Thermal noise (Johnson noise)**
  - Fluctuations in resistance due to random motion of atoms that influence conduction electrons
  - White noise (same for all frequencies)
- **Shot noise**
  - Fluctuations in current due to the fact that the current is composed of discrete charge carriers
  - White noise
- **1/f noise (flicker noise, pink noise)**
  - Fluctuations in resistance due to instabilities in contacts, atom migration, impurities in the conductive channel

White noise

- $\Phi_n(f)$ does not depend on f
- In practice, we consider the white noise in a limited frequency band



$$\Phi_n(f) = \text{const} \qquad -\infty < f < +\infty$$

Thermal noise

- Power spectral density of voltage variance (i.e. voltage variance per Hz of bandwidth) across a resistor at finite temperature T:

$$\boxed{\Phi_{u,n} = 4k_B TR}$$

white noise!

k_B : Boltzmann constant, $k_B = 1.38\times10^{-23}$ J/K
T : temperature in Kelvins
R : Resistance
Δf : Bandwidth

- Example: 50Ω resistor at room temperature (300 K), 1 Hz bandwidth

$$U_{n,eff} = \sqrt{\Phi_{u,n} \cdot \Delta f} = \sqrt{4k_B TR \cdot \Delta f} = 1\,\mathrm{nV}$$

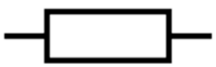
- A resistor in short circuit dissipates a noise power of:

$$\boxed{P = \frac{U_{n,eff}^2}{R} = 4k_B T\Delta f}$$

Independent of R!

Equivalent circuits

Noisy resistor at temperature T



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Noisless resistor

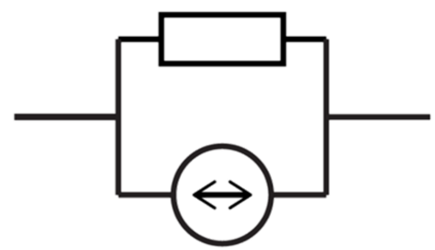
Noise voltage

$U_{n,eff}^2 / \Delta f = 4k_B TR$

Thevenin equivalent circuit

or

Noisless resistor



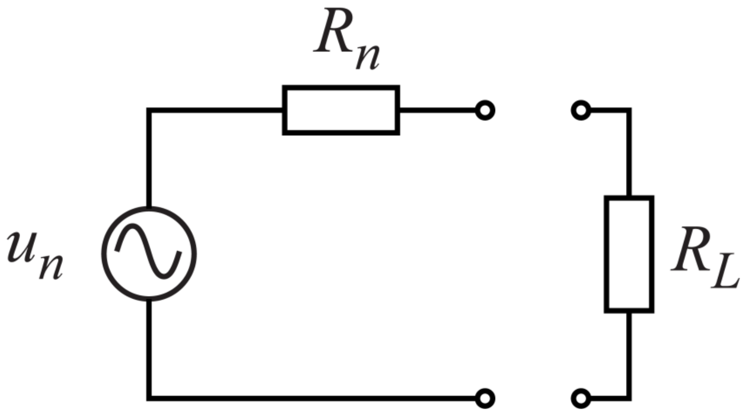
Noise current

$I_{n,eff}^2 / \Delta f = 4k_B T / R$

Norton equivalent circuit

Maximal dissipated noise

- Noise delivered to R_L from R_n : $P_{n,L}$



Source of noise

Load

$$P_{n,L} = I^2_{n,eff} R_L \qquad I_{n,eff} = \frac{U_{n,eff}}{R_n + R_L}$$

$$P_{n,L} = \frac{U^2_{n,eff} R_L}{\left(R_n + R_L\right)^2}$$

$$P_{n,L} = \max \text{ for } R_n = R_L$$

$$U^2{}_{n,eff} = 4k_B T R_n \Delta f$$

$$\boxed{P_{n,L} = \frac{U^2_{n,eff}}{4R_n} = k_B T \Delta f}$$

Independent of R!

Shot noise

- Schottky noise
- Due to fluctuations in the number of charge carriers, described by Poisson distribution

$$SNR = \frac{N}{\Delta N} = \frac{N}{\sqrt{N}} = \sqrt{N}$$

$$\boxed{I_{n,eff} = \sqrt{2ei\Delta f}}$$

i : average current flowing through the circuit

e : elementary charge (1.6×10^{-19} C)

1/f noise

- In general dominant under 500 Hz

$$\Phi_n \sim \frac{1}{f^{\alpha}} \qquad 0.8 < \alpha < 1.3 \qquad \text{For } \alpha=1: \quad \boxed{\Phi_n = \frac{K}{f}}$$

- Noise in the frequency band [$f_{\min}$, f_{max}]:

$$\int_{f_{\min}}^{f_{\max}} \Phi_n(f)\,df = K \ln \frac{f_{\max}}{f_{\min}}$$

Same dissipated power between 1-10 Hz and 0.1-1 Hz!

Total noise due to multiple sources

- Different noise sources can be considered to be independent
- The total dissipated power is the sum of dissipated powers from each source
- Squares of current or voltage are summed up

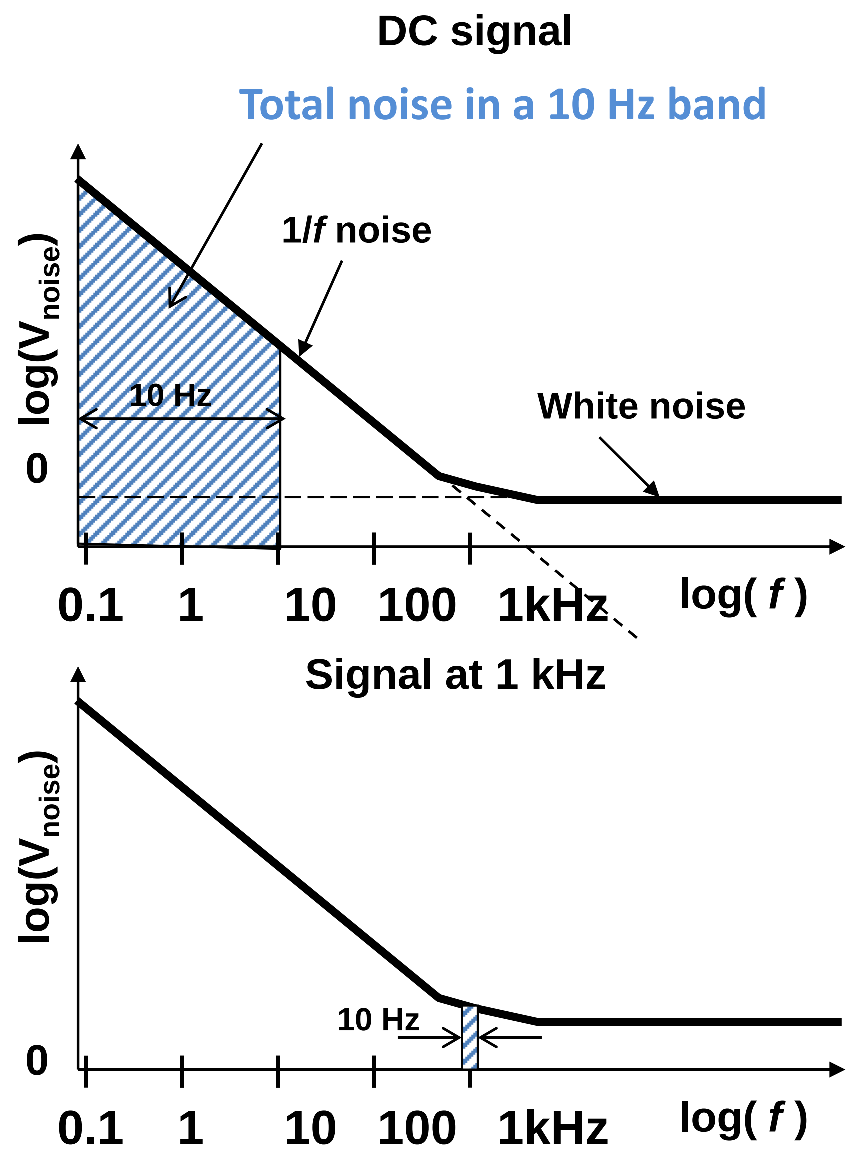
$$u_n^2 = u_{n1}^2 + u_{n2}^2 + i_{n3}^2 R^2{}_{n3} + \ldots + u_{nn}^2$$

$$i_n^2 = i_{n1}^2 + i_{n2}^2 + u_{n3}^2 / R^2{}_{n3} + \ldots + i_{nn}^2$$

Signal and noise

Noise at different frequencies

- Low frequencies: 1 / *f* dominant
- High frequencies: white noise
  - Thermal noise, shot noise
- Total noise depends on the frequency
  - High for DC measurements, better in white noise region
- **Problem: low-frequency and DC measurements**



Noise reduction

- Limiting the bandwidth
- Filtering
- Averaging

Key points

- **Extrinsic noise** is most often due to galvanic, electrostatic or magnetic coupling
- **Ground loops** induce noise in measurement circuits
- **Common mode voltage** appears on both inputs to an amplifier
- **Differential and isolation amplifiers** reduce the effect of common mode voltage
- Common mode voltage can be reduced by **shielding**
- Noise coming from the system itself is **intrinsic**: thermal, shot and 1/f
- Equivalent noise power is calculate from the **power spectral density** (V^2/Hz) by taking into account the bandwidth
- Intrinsic noise can be reduced by filtering, averaging,...