Measurement Systems

Problem set n° 4

Exercise 1 (First Order Systems)

We want to build an Infrared (IR) sensor using a **pyroelectric element**. The radiation flux is first converted into a temperature change ΔT through absorption (see point a); the latter is converted into a charge variation ΔQ through the pyroelectric element (see point b). The pyroelectric effect is based on the physical principle according to which, for certain materials, a temperature change generates a change in the electrical polarization (i.e. a variation of the charge).

$$\Phi \overset{(a)}{\Longrightarrow} \Delta T \overset{(b)}{\Longrightarrow} \Delta Q$$

a) For the conversion of the flux into the temperature change, we find the following balance equation, assuming a constant flux Φ_o en W/m^2 that is initiated at the time t_o (see "step response") and a temperature $T(t)$ at the time instant t :

$$e \cdot S \cdot \Phi_o = C_T \cdot \frac{dT(t)}{dt} + (T(t) - T(t_o)) \cdot G_T$$

e : Coefficient of emissivity
S : Surface area
C_T : Thermal capacitance of the sensor
G_T : Thermal conductivity

Give the expression of the time response of the temperature $T(t)$, the value of the temperature T_{stable} in the steady state and the time constant τ of the response.

b) The pyroelectric effect relates the charge $Q_p(t)$ to the temperature variation $\Delta T(t)$ according to the following equation:

$$Q_p(t) = \gamma_p \cdot S \cdot \Delta T(t)$$ (with: γ_p pyroelectric coeff. in $C/m^2/s$ and S the surface)

We model the pyroelectric element as a current source $I_p(t)$ following the electrical scheme on Figure 1. The resistor R allows conditioning of the element to measure the voltage $U_o(t)$

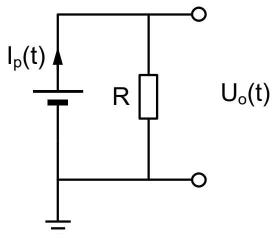

Figure 1. Modeling and conditioning of the pyroelectric element

- Give the expression for $U_o(t)$ as a function of the temperature $T(t)$.
- Insert the expression for the temperature time response $T(t)$ in the expression for the voltage $U_o(t)$ (see point **a**). Give the expression $U_{o,stable}$ in the steady state.

Exercise 2 (*2nd order system: accelerometer*)

We would like to characterize the uniaxial accelerometer shown on Figure 2.

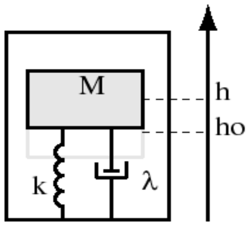

Figure 2: Model of the accelerometer

a) We first find the time response of the system:

- The sensitivity *S* is determined statically when the system is subjected to gravity alone, resulting in an output value y_{stat}. Calculate *S*.
- A step input of amplitude x_o is applied to the accelerometer to determine the natural frequency f_o and damping coefficient ξ. We obtain a response where the amplitude of the first oscillation is y_1 and a measured oscillation frequency f_T. Calculate the values of f_o and ξ.

b) We now characterize the frequency response of the system by applying a sinusoidal signal $\underline{a}(j\omega)$:

- Determine $\underline{y}(j\omega)$ as a function of $\underline{a}(j\omega)$.
- Calculate the cutoff frequency f_c of the system and determine if the accelerometer is of low-pass or high-pass type.

Numerical data :

$y_{stat} = 200\ \mu m$ $\quad$ $x_o = 49.05\ m/s^2$

$y_1 = 1050\ \mu m$ $\quad$ $f_T = 1\ kHz$