Measurement Systems

Problem set n° 4

Modeling a measurement system

Exercise 1 (*System order a: IR sensor*)

a) We analysed the conversion of the constant radiation flux emitted Φ_o in temperature $T(t)$:

- The following electrical correspondence can be found concerning the different variables :

Power consumption :	$e \cdot S \cdot \Phi_o$	$\Longrightarrow$	Current Source:	I_Φ
Temperature difference:	$T(t) - T(t_o)$	$\Longrightarrow$	Potential :	U_T(t)
Thermal Conductance:	G_T	$\Longrightarrow$	Electrical Conductance:	$G_{el} = \frac{1}{R_{el}}$
Thermal capacity:	C_T	$\Longrightarrow$	Electrical capacity :	C_{el}

The analog electrical diagram corresponding to Figure 1

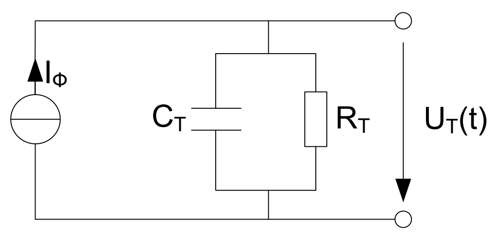

Figure 1: Analogy for electrical diagram.

- The time response of the temperature $T(t\ )$ can be easily deduced by the model provided (with: $\Delta T(t) = T(t) - T(t_o)$)

$$\tau \cdot \frac{dy(t)}{dt} + y(t) = k \cdot x_o \qquad \Longrightarrow \qquad \frac{e \cdot S}{G_{th}} \cdot \Phi_o = \frac{C_{th}}{G_{th}} \cdot \frac{d\Delta T(t)}{dt} + \Delta T(t)$$

We found for $T(t)$, τ and T_{stable} :

$$T(t) = \frac{e \cdot S}{G_T} \cdot \left(1 - \exp(\frac{-t}{C_T/G_T})\right) \cdot \Phi_o + T(t_o)$$ with :

$$\tau = C_T/G_T$$

$$T_{stable} = \frac{e \cdot S}{G_T} \cdot \Phi_o + T(t_o)$$

b) We find for the conversion of the temperature $T(t)$ with load variation $Q_p(t)$:

- The tension $U_o(t)$ which is :

$$U_o(t) = R \cdot I_p(t) = R \cdot \frac{dQ_p(t)}{dt} = R \cdot \gamma_p \cdot S \cdot \frac{dT(t)}{dt}$$

- By introducing the temperature response time $T(t)$ (see point a)), we find for the voltage $U_o(t)$:

$$U_o(t) = R \cdot \gamma_p \cdot S \cdot \frac{dT(t)}{dt} = \frac{R \cdot \gamma_p \cdot S^2 \cdot e}{C_T} \cdot \exp(\frac{-t}{C_T/G_T}) \cdot \Phi_o \qquad \text{with : } U_{o,stable} = 0$$

c) For the frequency response of the IR sensor, we obtain by applying a sinusoidal flux $\underline{\Phi}(j\omega)$:

- The frequency response of the temperature change $\underline{\Delta T}(j\omega)$:

$$e \cdot S \cdot \underline{\Phi}(j\omega) = C_T \cdot j\omega \cdot \underline{\Delta T}(j\omega) + \underline{\Delta T}(j\omega) \cdot G_T$$

$$\underline{\Delta T}(j\omega) = \frac{e \cdot S}{G_T \cdot (1 + j \cdot \frac{\omega}{G_T/C_T})} \cdot \underline{\Phi}(j\omega) = \frac{e \cdot S}{G_T \cdot (1 + j \cdot \frac{\omega}{\omega_c})} \cdot \underline{\Phi}(j\omega)$$

The conversion of the radiation-temperature is **low-pass** type and we find the cutoff frequency f_c :

$$f_c = \frac{\omega_c}{2 \cdot \pi} = \frac{1}{2 \cdot \pi \cdot \tau} = \frac{G_T/C_T}{2 \cdot \pi}$$

- The frequency response of the voltage $\underline{U_o}(j\omega)$:

$$\underline{U_o}(j\omega) = R \cdot \gamma_p \cdot S \cdot j\omega \cdot \underline{\Delta T}(j\omega)$$

$$\underline{U_o}(j\omega) = \frac{R \cdot \gamma_p \cdot e \cdot S^2}{G_T} \cdot \frac{j\omega}{(1 + j \cdot \frac{\omega}{G_T/C_T})} \cdot \underline{\Phi}(j\omega)$$

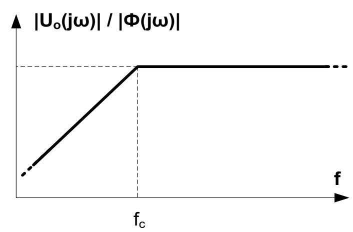

Figure 2: Frequency Response IR sensor

The IR sensor is **high-pass** type with cutoff frequency f_c. For example, it can't measure a DC component. The output voltage is then, if the frequency is above the cutoff frequency:

$$\underline{U_o}(j\omega) = R \cdot \gamma_p \cdot e \cdot S \cdot C_T \cdot \underline{\Phi}(j\omega)$$

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Exercise 2 (*2nd order system: accelerometer*)

a) For the time-dependent response, we obtain :

- The sensitivity S obtained by static calibration is:

$$S = \frac{y_{stat}}{g} = 2.04 \cdot 10^{-5} s^2$$

- From the step response we can deduce the damping coefficient ξ:

$$\xi = \left(\sqrt{1 + \left(\frac{\pi}{ln\left(\frac{y_1}{S \cdot x_o} - 1\right)}\right)^2}\right)^{-1} = 6.9 \cdot 10^{-1}$$

The natural frequency of the system is:

$$f_o = \frac{f_T}{\sqrt{1-\xi^2}} = 1.38\ kHz$$

b) For the frequency response we find:

- The response $\underline{y}(j\omega)$:

$$\left(\frac{(j\omega)^2}{{\omega_o}^2} + \frac{2 \cdot \xi \cdot j\omega}{\omega_o} + 1\right) \cdot \underline{y}(j\omega) = S \cdot \underline{a}(j\omega) \qquad \Rightarrow \qquad \underline{y}(j\omega) = \frac{S}{\left(1 - \left(\frac{\omega}{\omega_o}\right)^2 + \frac{2 \cdot \xi \cdot j\omega}{\omega_o}\right)} \cdot \underline{a}(j\omega)$$

- The absolute value and the phase:

$$\frac{|\underline{y}(j\omega)|}{S \cdot |\underline{a}(j\omega)|} = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_o}\right)^2\right)^2 + 4 \cdot \xi^2 \cdot \left(\frac{\omega}{\omega_o}\right)^2}} \quad \text{and} \quad arg\left(\frac{\underline{y}(j\omega)}{S \cdot \underline{a}(j\omega)}\right) = atan\left(\frac{-2 \cdot \xi \cdot \frac{\omega}{\omega_o}}{1 - \left(\frac{\omega}{\omega_o}\right)^2}\right) = atan\left(\frac{2 \cdot \xi}{\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}}\right)$$

- The system is of the **low-pass** type and the cutoff frequency $f_c$is:

$$f_c = f_o \cdot \sqrt{1 - 2 \cdot \xi^2} \cong 300\ Hz$$