Problem 1 : spin detection

a) Indicate the nuclear spin I of the three following nuclei : ^4He, ^6Li and ^7Li. In general, can all the nuclei be detected by NMR? If not what is the condition on their nuclei to be detectable and which nuclei among the three above can be detected?
b) What is the energy difference between the two spin states of ^1H in a magnetic field of 5.87 T? And that of ^{13}C?
c) For both nuclei ^1H and ^{13}C, calculate which fraction of the spin population is in the upper state (higher energy)? What can you deduce for the magnetization of the two nuclei?
Hint: assume T=300K.
d) Explain why proton signal is used for imaging in NMR and not another nucleus.
e) One wants to measure glucose signal, using proton (H^1) and carbon (C^{13}) MR spectroscopy. Investigating total Glc signal, calculate sensitivity ratios between both techniques for Glc.

Problem 2 : B_1 field and radiofrequency

In many MR experiments it is necessary to flip a spin which is initially aligned along the z axis into the x'-y' plane by using an appropriate rotating magnetic field B_1 during a certain amount of time (pulse) (in the rotating frame, B_1 is a static field orthogonal to B_0). This is referred to as a 90^o pulse. If the desired pulse time is 1.0ms, what B_1 magnitude is required for

a) A proton spin?
b) A carbon spin?
c) At which frequency is the magnetization originating from each of the two atoms precessing ?
d) At a B_0 of 9.4T, at which frequency must B_1 rotate (in the lab frame) in resonance conditions for a proton spin?
e) What is then the wavelength and energy of a photon of this field? Compare it to what is used for X-ray imaging.

Problem 3 : Rotating frame and effective field

In a static frame, the equation of motion for a spin, *m(t)*, in a magnetic field $\boldsymbol{B_0} = B_0\boldsymbol{e_z}$ is given by:

$$\frac{d\vec{m}(t)}{dt} = \gamma\vec{m}(t) \times \overrightarrow{B_0}$$

a) Write down the equation of motion in a rotating frame with precession speed of $\vec{\omega} = \omega \cdot \vec{e_z}$
Hint: use the following theorem, for any vector $\vec{A}(t)$

$$\left[\frac{d\vec{A}(t)}{dt}\right]_{static\ frame} = \left[\frac{\partial\vec{A}(t)}{\partial t} + \vec{\omega} \times \vec{A}(t)\right]_{rotating\ frame}$$

b) Explain graphically what happens to the spin's precession speed, if we increase the precession speed of the rotating frame gradually from 0 to $\omega_0 = \gamma B_0$,.
Hint: draw $\boldsymbol{B_{eff}}$ in the rotating frame as ω increases.

c) Now we have an exciting magnetic field $\boldsymbol{B_1}$ perpendicular to $\boldsymbol{B_0}$ and precessing around $\boldsymbol{B_0}$ at angular frequency ω. Calculate the effective field, $\boldsymbol{B_{eff}}$, in the rotating frame (where $\boldsymbol{B_1}$ is fixed) and the tangent of the angle between the $\boldsymbol{B_0}$ field and the $\boldsymbol{B_{eff}}$.

Hint: Replace γB_0 by $\omega_0 = -\gamma B_0$

$$tan\theta = f(\omega, \omega_0, \omega_1)$$

d) Discuss the effect of $\boldsymbol{B_{eff}}$ on the spin motion depending on ω.
e) Draw the precession of the spin in case of resonance ($\omega = \omega_0$).