Name / first name

Exam 16.June 2009
Neural Networks and Biological Modeling

- Write your name in readable letters on top of this page
- The exam lasts 160 minutes
- **All** responses must be on these exam sheets
- Except for one paper A4 of **handwritten** notes, no documentation is allowed.
- You may use a pocket calculator, but not a programmable computer
- The total number of points that can be achieved is 44

Evaluation

Section 1: ....../7 pts

Section 2: ....../12 pts

Section 3: ....../9 pts

Section 4: ....../10 pts

Section 5: ....../6 pts

....................../44

The exam has 10 pages, the back of the pages is also used!

Question 1: Ion channel (7 points)

We consider the following model of a ion channel

$$I_{ion} = g_0 \, x^p \, (u - E)$$

where u is the membrane potential. The parameters g_0, p and $E = 0$ are constants.

(a) What is the name of the variable E? ....

Why does it have this name, what does it signify (give answer in one sentence)

.... /1 point

(b) The variable x follows the dynamics

$$\frac{dx}{dt} = -\frac{x - x_0(u)}{\tau}$$

Suppose we make a voltage step from a fixed value E to a new constant value u_0. Give the mathematical solution $x(t)$ for $t > 0$

$x(t) =$ .... /2 points

An electrophysiologist tells you that he is able to apply voltage steps as in (b) and that by measuring the current he wants to determine the parameters g_0 and $x_0(u)$ of the ion channel in (a) and (b)

(c) How should he proceed to measure the parameter p? What would be different between the case p=1 and $p = 4$? You can sketch a little figure to illustrate your answer.

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....

/2 points

(d) Under the assumption that $x_0(u)$ is bounded between zero and 1, how can he measure g_0?

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....

/1 point

(e) Given the value of g_0 and p, how can he measure the value $x_0(u_1)$ at some arbitrary value u_1? If possible, give a mathematical expression to illustrate your explanation.

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/1 point

Question 2: Integrate-and-Fire Model/Phase Plane analysis (12 points)

An integrate-and-fire neuron model with adaptation is described by the two differential equations

$$\frac{du}{dt} = F(u) - w + I \qquad (1)$$

$$\tau \frac{dw}{dt} = -w + a\,(u + 10) \qquad (2)$$

If $u \geq 10$ the variable u is reset to $u = 0$. The variable w is increased by an amount of 4 during reset.

We take

$$F(u) = -(u + 10) \quad \text{for } u \leq 0 \qquad (3)$$

$$F(u) = -10 + 5u \quad \text{for } u > 0 \qquad (4)$$

(a) Plot the nullclines in the phase plane (u, w) for $I = 0$ and $a = 0.5$ using the space here:

/2 points

(b) In the same graph, add representative arrows indicating qualitatively the flow on the nullclines and in different regions of the phase plane (you may assume $\tau = 2$). /2 points

(c) In the same graph, mark the rest state. /1 point

(d) In the same graph, indicate a trajectory in the phase plane, after a stimulus $I(t) = 11\delta(t)$ has been applied starting from rest
[δ denotes the Dirac delta function]. /1 point

(e) In the same graph, indicate a trajectory in the phase plane, after a stimulus $I(t) = 15\delta(t)$ has been applied starting from rest
[δ denotes the Dirac delta function]. /1 point

(f) Plot qualitatively the solution $u(t)$ for $t > -1$ for the cases in (d) and (e) in the space below. Pay particular attention to the moment around $t = 0$ and to the situation after a very long time.

/2 points

(g) assume that $\tau \gg 1$ (e.g. $\tau = 5$). Assume that we have applied for a long time in the past a negative (hyperpolarizing) current $I = -40$. Draw the nullclines again for this situation in the space below.

What happens if we stop the hyperpolarizing current at $t = 0$ so that $I = 0$ for $t \geq 0$? Explain in words the evolution of the voltage after the current has been stopped and draw a sketch of the trajectory $u(t)$ for $t \geq -1$

....
....
....

/3 points

Question 3: Stochastic Model: Poisson process (9 points)

We consider a linear neuron model under stochastic spike arrival. The neuron receives input at a rate r at a synapse with weight w. Each input causes a postsynaptic potential $\alpha(s) = 2 - s$ for $0 < s < 2$ and zero elsewhere. The total membrane potential is

$$u(t) = \sum_{t^f} w\alpha(t - t^f) + u_0 \tag{5}$$

The sum is over all spike times arriving at the synapse.

(a) Suppose a fixed value of u_0. What is the mean membrane potential?

$\langle u \rangle = \bar{u} =$ ....

....

/2 points

(b) What is the variance of the membrane potential?

$\langle (u - \bar{u})^2 \rangle =$ ....

....

/2 points

(c) Suppose now that because of external input the reference u_0 is periodically modulated at a period of $T = 20$ (e.g., $T = 20ms$, but you can do the calculation unit-free)

$$u_0(t) = \Delta u \sin(2\pi t/T)$$

Stochastic input is the same as before and arrives at a constant rate of $r = 2$ (e.g. $r = 2kHz$). Sketch 2 sample trajectories of the potential $u(t)$ corresponding to 2 repetitions of the experiment in the space here. Indicate in your sketch $u_0(t)$ as well as the mean and the variance. Pay attention to the relation between r and $1/T$.

/2 points

(d) Suppose that the neuron fires whenever the membrane potential $u(t)$ hits the threshold θ from below. Starting from the figure in (c), choose three different values of theta that lead to qualitatively different firing behavior, and discuss the results. Use 1 sentence for each of the three cases You can use the space below if you want to make additional sketches.

case 1: ....
....

case 2: ....
....

case 3: ....
....

/3 points

QUESTION 4: DYNAMICS OF HOPFIELD MODEL (10 points)

Consider a network of $N = 20000$ neurons that has stored 4 patterns

$\xi^1 = \{\xi^1_1, \dots \xi^1_N\}$
$\xi^2 = \{\xi^2_1, \dots \xi^2_N\}$
$\xi^3 = \{\xi^3_1 \dots \xi^3_N\}$
$\xi^4 = \{\xi^4_1 \dots \xi^4_N\}$

using the synaptic update rule $w_{ij} = (J/N) \sum_\mu \xi^\mu_i \xi^\mu_j$ where $J > 0$ is a parameter. **Each pattern has values $\xi^\mu_i = \pm 1$ so that exactly 50 percent of neurons in a pattern have $\xi^\mu_i = +1$.**

Assume stochastic dynamics: neurons receive an input $h_i(t) = \sum_j w_{ij} S_j(t)$ where $S_j(t) = \pm 1$ is the state of neuron j. Neurons update their state

$$Prob\{S_i(t+1) = +1 | h_i(t)\} = 0.5[1 + g(h_i(t))] \qquad (6)$$

where g is an **odd and monotonically increasing function**: $g(h) = 2h$ for $|h| < 0.5$ and $g(h) = 1$ for $h \geq 0.5$ and $g(h) = -1$ for $h \leq -0.5$.

(a) Rewrite the righ-hand-side of equation (6) by introducting an overlap $m^\mu(t) = (1/N) \sum_j \xi^\mu_j S_j(t)$.

....

/1 point

(b) What is the significance of the overlap? Describe its meaning in one sentence; give examples if necessary.

....
....
....

/1 point

(c) Assume that the four patterns are orthogonal, i.e., $\sum_i \xi^\mu_i \xi^\nu_i = 0$ if $\mu \neq \nu$. **Assume that the overlap with pattern 4 at $t = 0$ has a value of 0.3 and $m^\mu(0) = 0$ for all other patterns.**

Suppose that neuron i is a neuron with $\xi^4_i = -1$.

What is the probability that neuron i fires in time step 1? Give the formula for arbitrary J and evaluate then for $J = 1$.

....

What is the probability that another neuron k with $\xi^4_k = +1$ fires in time step 1? Give the formula for arbitrary J and evaluate then for $J = 1$.

....

/2 points

(d) For the same assumptions as in (c), what is the expected overlap for $< m^4(t) >$ after the first time step.

$< m^4(1) >=$..
....

/2 points

(e) For the same assumptions as in (c) and (d), write the evolution of the overlap for $m^4(t)$ for an arbitrary time step and arbitrary J. **Assume that N is large ($N \to \infty$).** Because of the orthogonality of the patterns, you may also assume that $m^\mu(t) = 0$ for $\mu \neq 4$).

$m^4(t+1) =$..
....

/1 point

(f) Can you relate the evolution of the overlap to the more general picture of mean-field analysis of coupled networks?

....
....

/2 points

Question 5: Fokker-Planck equation (6 points)

For a population of integrate-and-fire neurons the continuity equation reads

$$\tau \frac{\partial}{\partial t} p(u,t) = -\frac{\partial}{\partial u} J(u,t) \tag{7}$$

(a) What is the meaning of the term $p(u,t)$. Give the name and mathematical definition. Describe its significance of p in your own words

....
....
....

/1 point

(b) What is the meaning of the term $J(u,t)$? Give the name or definition. Describe the significance of J in your own words.

....
....
....

/1 point

(c) A professor who sometimes makes mistakes on the blackboard writes

$$A(t) = J(\vartheta, t)$$

Is this formula correct? Justify your answer.

....
....
....

/2 points

(d) How can we include the reset of integrate-and-fire models in the continuity equation above?

....
....
....

/2 points