Fundamentals of Bioimaging (Prof. Gruetter)
Solutions to Problem Set No. 13

31.5.2019

Solution 1: Diffusion MRI Quiz

a) As you have learned in week 12, the spin echo sequence refocuses ("rewinds") also phase differences which are due to (external) magnetic field inhomogeneities. Since the diffusion encoding results in dephasing, additional phases due to other effects are undesirable. This is why gradient echoes (in which the phases due to inhomogeneity are not refocused) are rarely used for diffusion imaging.

b) The diffusion encoding inherently implies a very high sensitivity to motion; not only to molecular motion, but also to bulk motion, i.e. when a patient moves his head. Consequently, a single image volume (in which one diffusion direction was encoded) has to be acquired very rapidly. This is why, most of the times, the spin-echo version of one of the quickest MR imaging sequences (called echo planar imaging) is used for diffusion imaging.

c) As seen in slide 13-6, the diffusion coefficient is related to the measured signal and applied b-value as follows $S = S_0 e^{-bD}$ Thus, the diffusion coefficient yields

$$D = \frac{ln(0.4)}{-500} = 1.8 \cdot 10^{-3} \frac{mm^2}{s}$$

Solution 2: Relaxation and Diffusion Imaging

We are interested in characterizing the magnetic resonance properties of two compounds.

a) T_1 is the spin-lattice characteristic relaxation time. It characterizes the processes with which the magnetization relaxes to its equilibrium. To do that, energy has to be transferred to the environment (lattice) as heat. The energy is exchanged by quantum of $\hbar\omega_L$.
T_2 is the spin-spin characteristic relaxation time. It characterizes the processes with which the components of the macroscopic magnetization lose coherence and dephase. This dephasing is due to the little fluctuations of B_0 around each nucleus caused by Brownian motion of the molecules and the resulting dipolar couplings either between the nuclei or with the solvent.

b) T_2^* is the experimentally measured decay of the transverse magnetization. It is shorter than T_2, due to additional B_0 inhomogeneities due to the non-perfectly homogeneous magnet. This two characteristic times are related by
$\frac{1}{T_2} = \frac{1}{T2} + \gamma\Delta B_0$

c) An inversion recovery is a good way to measure T_1. You apply a 180° pulse and wait for a certain time TI. Then you apply a 90° pulse and measure the signal. TI is varied from one measurement to the next but the echo time of the measurement is kept constant to obtain a similar T_2 contribution from one measurement to the other. Interpolating the measures with different TI gives an exponential growing function with characteristic time T_1. To measure T_2, a spin-echo sequence can be applied, since it refocuses the decay due to experimental inhomogeneities (which are constant in time) (see lecture 12).
T_2^* can be measured with a gradient echo sequence (see lecture 11).

d) Decaying curves will be representative of T_2 and T_2^* decays, while rising curves are measure of the recovery of the z magnetization (characterized by T1).

For T_2 and T_2^* decays, the signal is after T_2 (or T_2^*) at about 37% of what it was at time zero ($e^{\frac{-T_2}{T_2}} \cong 0.37$).

Looking at the curves, we see that the compound A has a characteristic time of 60ms in graph 1 and 28 ms in graph 3A. We know that $T_2^* < T_2$. So, graph 3 is a plot of T_2^* decay.

For T_1, we saw in series 9 ex.3a that the signal is zero at $t = T_1 \cdot \ln(2)$. From this we can extract T_1 from the graph 2.

We get for compound A: T_1= 900ms T_2=60ms T_2^*=28ms

Similarly, for compound B: T_1= 1200ms T_2=100ms T_2^*=40ms

e) We saw (slide 12-12) that the measured signal is related to the relaxation constants by the following equation:

$$S \rho e^{\frac{-TE}{T_2}} \left(1 - 2e^{\frac{-TR}{T_1}}\right) \text{ or } S \rho e^{\frac{-TE}{T_2}} \left(1 - 2e^{\frac{-TR}{T_1}}\right) \text{ depending on if use a gradient or spin echo sequence.}$$

Measuring T_1 with very short TE gives a signal difference for $TR = \frac{1}{2}\left(T_{1_A} + T_{1_B}\right)$ (where the signal difference is optimal, see series 12 ex.1c):

$$\Delta S \left|\left(1 - 2e^{\frac{-TR}{T_{1A}}}\right) - \left(1 - 2e^{\frac{-TR}{T_{1B}}}\right)\right| = 0.21$$

Measuring T_2 with very long TR gives a signal difference for $TE = \frac{1}{2}\left(T_{2_A} + T_{2_B}\right)$:

$$\Delta S \left|e^{\frac{-TE}{T_{2A}}} - e^{\frac{-TE}{T_{2B}}}\right| = 0.19$$

Measuring T_2^* with very long TR gives a signal difference for $TE = \frac{1}{2}\left(T_{2\ A} + T_{2\ B}\right)$:

$$\Delta S \left|e^{\frac{-TE}{T_{2\ A}}} - e^{\frac{-TE}{T_{2\ B}}}\right| = 0.13 \text{The } T_1 \text{ contrast gives the best signal difference.}$$

f) Another type of contrast can be achieved with diffusion experiments.
   I. Like in point d), we know that the signal is at 37% of its value at t=0 when D=1/b. So, we find for the two compounds a diffusion coefficient of D_A=0.001 and D_B=0.002 [mm^2/s].

g) The signal each of the two compounds will be proportional to e^{-bD}. So, the signal difference can be written as:
$\Delta S \left|e^{-bD_A} - e^{-bD_B}\right|$
We search the maximum of ΔS :

$$\frac{d\Delta S}{db} = 0 \left|-D_A e^{-bD_A} + D_B e^{-bD_B}\right| = 0 \quad b = \frac{ln(D_B/D_A)}{D_B - D_A} = 693 [s/mm^2]$$

For this value, we have $\Delta S \left|e^{-bD_A} - e^{-bD_B}\right| = 0.25$

Thus, the diffusion gives even a better contrast on these two compounds than T_1.

Solution 3: Combining Your Knowledge

Question n°	Ultrasounds	CT	SPECT	PET	MRI
1	No, because of the total reflection on the skull.	No, very little soft tissue contrast.	Theoretically yes, but since MRI is working well here, one would rather choose MRI since it is non-invasive.		Yes, using a contrast agent injection (which enters the lesion because of the blood-brain barrier breakdown)
2	No, total reflection of ultrasound waves on bones.	Yes, good bone-tissue contrast	No, they need tracer injection		No, no MR signal from bones
3	No, adapted for located investigation areas and not sensitive enough.	No, very little soft tissue contrast.	Yes, with an adapted tracer linked to an energetic molecule(metastasis have a high energetic metabolism)	Yes, because high FDG concentration in metastasis after tracer injection	No, would take too much to image the whole body and analyze the images
4	No, no sensitive enough	No, no contrast	Theoretically yes, but since MRI is working well here, one would rather choose MRI since it is non-invasive.		Yes, well adapted for soft tissue contrast.
5	No, adapted for soft tissue contrast	Yes, good contrast between soft tissues and solid material.	No, they need tracer injection		No, adapted for proton (water) imaging
6	No, to deep area surrounded by bones	No, no information on blood flow	Yes, measure of the heart signal after a tracer injection allow to detect low concentration areas as low blood flow areas	Yes, for the same reason as for SPECT	No, less sensitive technique
7	Yes, non-invasive and good contrast	No, ionization an no contrast for	No, invasive and no adapted for soft tissue imaging		No, because of the foetus
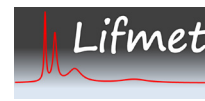
	for soft tissues	soft tissues		movements