

# Environmental Bioprocesses Engineering

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based on lecture from **Sirous Ebrahimi (sirous.ebrahimi@epfl.ch)**

What do you learn in this course?

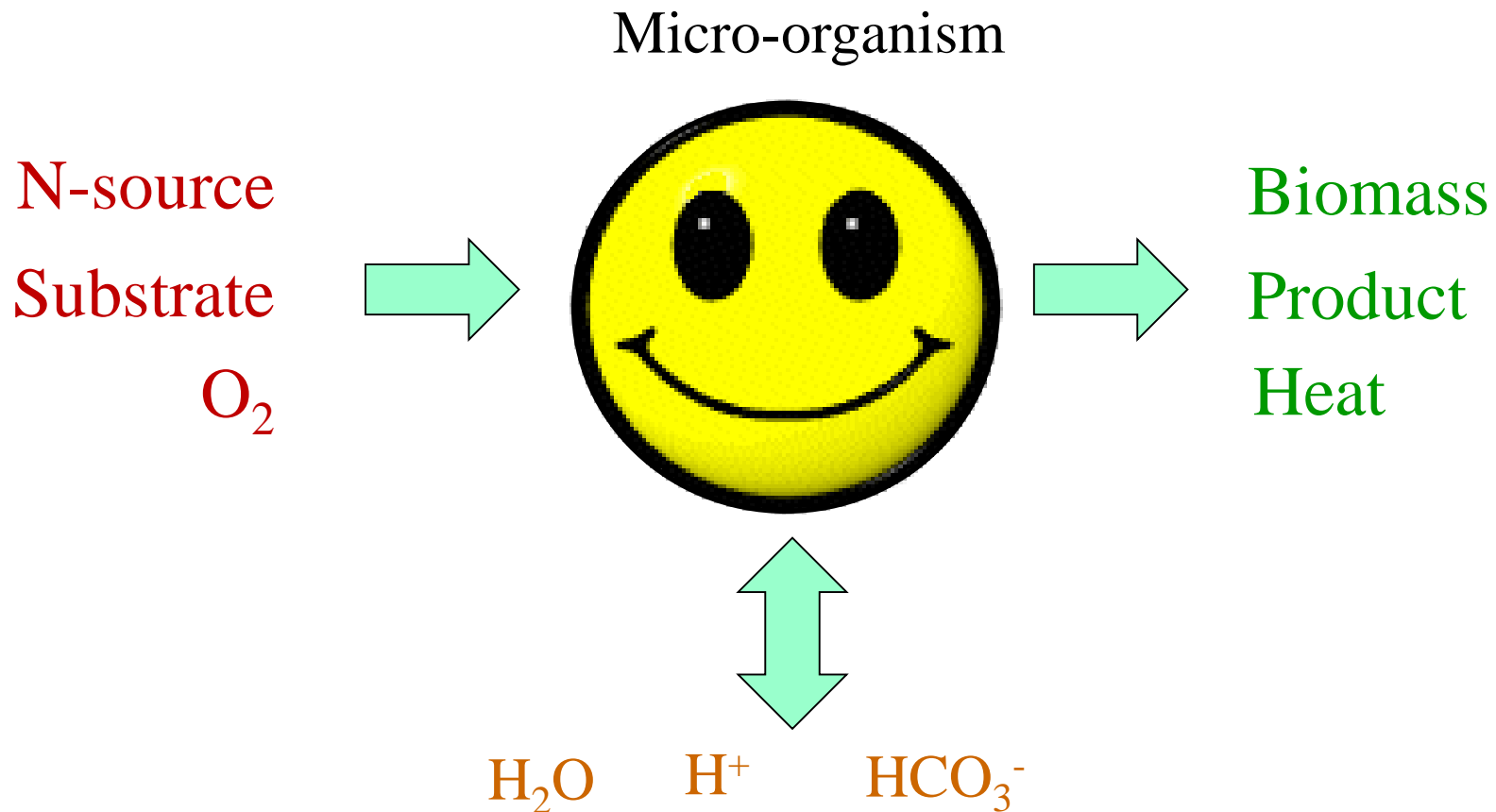
*You will learn the **engineering tools** you need as a  
**bioprocess engineer***

What are the tools?

*The ways to calculate microbial rates*

# Quantification of Microbial Rates

## Consumption or Production



# Quantification of Microbial Rates

## Consumption or Production



Rates of consumption or production are obtained from

**Mass balances**

Over reactor, for **each compound** and from measured:

1. Volumes
2. Concentrations
3. Flow rates

# Mass Balances over reactors

$$\text{Accumulation} = \text{Transports} + \text{Conversions}$$

Transport rate **IN TO** the system (+)  
or  
Transport rate **OUT OF** the system (-)

rate of **PRODUCTION** (+)  
or  
rate of **CONSUMPTION** (-)

$$\text{Accumulation [amount/h]} = \text{In [amount/h]} - \text{Out [amount/h]} + \text{Conversions [amount/h]}$$

For each compound  $C_i$  in the biosystem:

$$\text{Accumulation}_i = \sum R_i^{IN} - \sum R_i^{OUT} + \sum R_i^{\text{Conversions}}$$

Important: **At Steady State, Accumulation = 0**

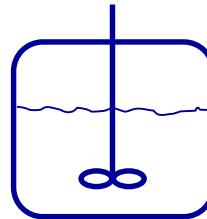
# Mass Balances over reactors

$$Accumulation_i = \sum R_i^{IN} - \sum R_i^{OUT} + \sum R_i^{Conversions}$$

Accumulation = 0, at Steady State

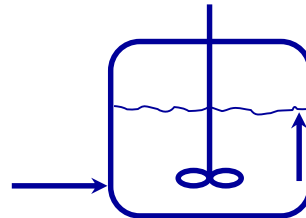
3 types of  
bioreactor

Batch



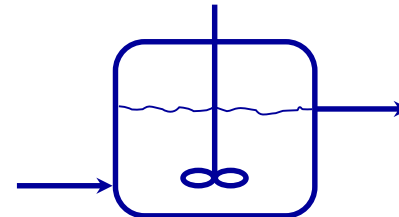
Transport IN = 0  
Transport OUT = 0  
Accumulation = Conversion

Fed batch



Transport OUT = 0  
Accumulation = In + Conversion

Chemostat



Accumulation = 0  
(at steady state)  
IN - OUT + Conversion = 0

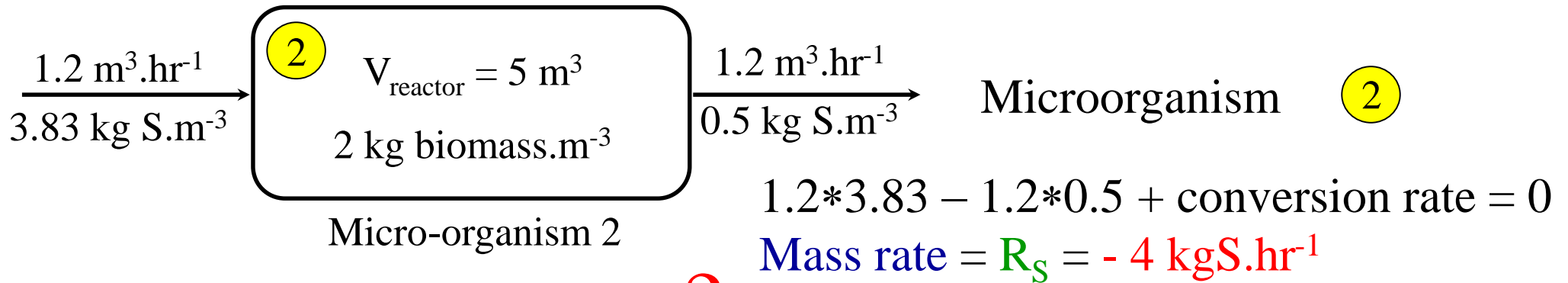
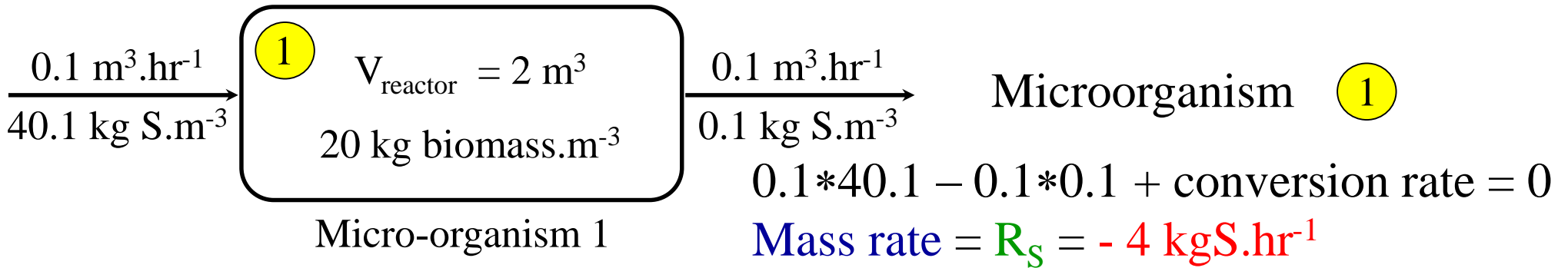
# Rates from Mass Balances

How are rates defined? (1)

Consider 2 chemostats where microorganism consumes glucose

$$Accumulation_i = \sum R_i^{IN} - \sum R_i^{OUT} + \sum R_i^{Conversions}$$

[amount.hr<sup>-1</sup>],[kg.hr<sup>-1</sup>] or [mole.hr<sup>-1</sup>]



Same rates, but not same reactors ?

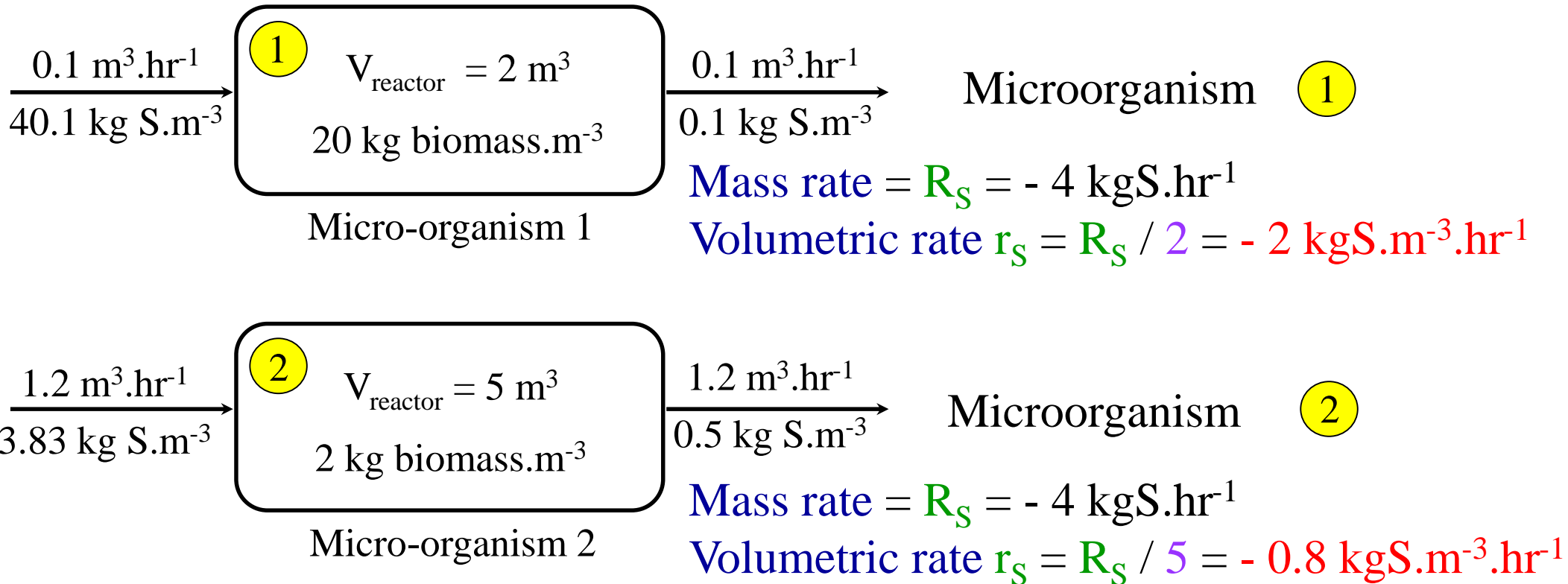
# Rates from mass balance

How are rates defined? (2)

Reactor ② is bigger than ①!  
More expensive...

$$r_i \text{ Volumetric rate} = R_i / V_{\text{reactor}}$$

[amount.m<sup>-3</sup>.hr<sup>-1</sup>], [kg.m<sup>-3</sup>.hr<sup>-1</sup>] or [mole.m<sup>-3</sup>hr<sup>-1</sup>]



Reactor ① seems the best !



# Rates from mass balance

How are rates defined? (3)

But who do the job? Microorganisms!!!

- Do both microorganisms have equal rates?
- How fast do they "work"?

$q_i$  Specific rate =  $R_i$  / Biomass amount

$q_i$  Specific rate =  $r_i$  / Biomass concentration

[amount S.amount  $X^{-1}.h^{-1}$ ],[kgS.kg $X^{-1}.h^{-1}$ ] or [moleS.mole $X^{-1}.h^{-1}$ ]

1

$$V_{\text{reactor}} = 2 \text{ m}^3$$

$$20 \text{ kg biomass.m}^{-3}$$

Microorganism 1

$$\text{Mass rate} = R_S = -4 \text{ kgS.hr}^{-1}$$

$$\text{Volumetric rate } r_S = R_S / 2 = -2 \text{ kgS.m}^{-3}\text{.hr}^{-1}$$

$$\text{Specific rate } q_S = R_S / (2 \cdot 20) = -0.1 \text{ kgS.kgX}^{-1}\text{.hr}^{-1}$$

$$\text{Specific rate } q_S = r_S / (20) = -0.1 \text{ kgS.kgX}^{-1}\text{.hr}^{-1}$$

2

$$V_{\text{reactor}} = 5 \text{ m}^3$$

$$2 \text{ kg biomass.m}^{-3}$$

Microorganism 2

$$\text{Mass rate} = R_S = -4 \text{ kg.hr}^{-1}$$

$$\text{Volumetric rate } r_S = R_S / 5 = -0.8 \text{ kg.m}^{-3}\text{.hr}^{-1}$$

$$\text{Specific rate } q_S = R_S / (5 \cdot 2) = -0.4 \text{ kgS.kgX}^{-1}\text{.hr}^{-1}$$

$$\text{Specific rate } q_S = r_S / (2) = -0.4 \text{ kgS.kgX}^{-1}\text{.hr}^{-1}$$

Microorganism 2 is 4 times more efficient than microorganism 1 !

# Three types of rates

$R_i$ : Mass Rate

amount of  $C_i$  per hour

$$R_i: \text{kg}C_i \cdot \text{hr}^{-1}$$

$r_i$ : Volumetric rate

amount of  $C_i$  per hour  
 $\frac{\text{amount of } C_i \text{ per hour}}{\text{m}^3 \text{ reactor}}$

$$r_i: \text{kg}C_i \cdot \text{m}^{-3} \cdot \text{hr}^{-1}$$

$q_i$ : Specific rate

amount of  $C_i$  per hour  
 $\frac{\text{amount of } C_i \text{ per hour}}{\text{amount of micro-org.}}$

$$q_i: \text{kg}C_i \cdot \text{kg}X^{-1} \cdot \text{hr}^{-1}$$

$$R_i = r_i \times V_r$$

$$\frac{\text{amount/hour}}{\text{m}^3 \text{ reactor}} = \frac{\text{amount / hour}}{\text{m}^3 \text{ reactor}} \times \text{m}^3 \text{ reactor}$$

$$r_i = q_i \times C_X$$

$$\frac{\text{Amount/hour}}{\text{m}^3 \text{ reactor}} = \frac{\text{amount/hour}}{\text{kg X}} \times \frac{\text{kg X}}{\text{m}^3 \text{ reactor}}$$

For each compound:

substrate	$(-r_S)$	$=$	$(-q_S) C_X$
oxygen	$(-r_{O_2})$	$=$	$(-q_{O_2}) C_X$
biomass	$r_X$	$=$	$q_X C_X = \mu C_X$
$C_{O_2}$	$r_{CO_2}$	$=$	$q_{CO_2} C_X$
product	$r_P$	$=$	$q_P C_X$

# Yield = ratio of rates

Definition of yield  $Y_{ij}$  of compound  $j$  over  $i$  :

$$Y_{ij} = \left| \frac{\text{rate}_j}{\text{rate}_i} \right| = \left| \frac{R_j}{R_i} \right| = \left| \frac{R_j/V_R}{R_i/V_R} \right| = \left| \frac{r_j}{-r_i} \right| = \left| \frac{q_j \cdot C_X}{q_i \cdot C_X} \right| = \left| \frac{q_j}{q_i} \right|$$

Often, yields are related to biomass growth rate  
i.e  $Y_{ix}$

$$Y_{sx} = \left| \frac{r_x}{-r_s} \right| = \left| \frac{\mu \cdot C_X}{-q_s \cdot C_X} \right| = \left| \frac{\mu}{-q_s} \right|$$

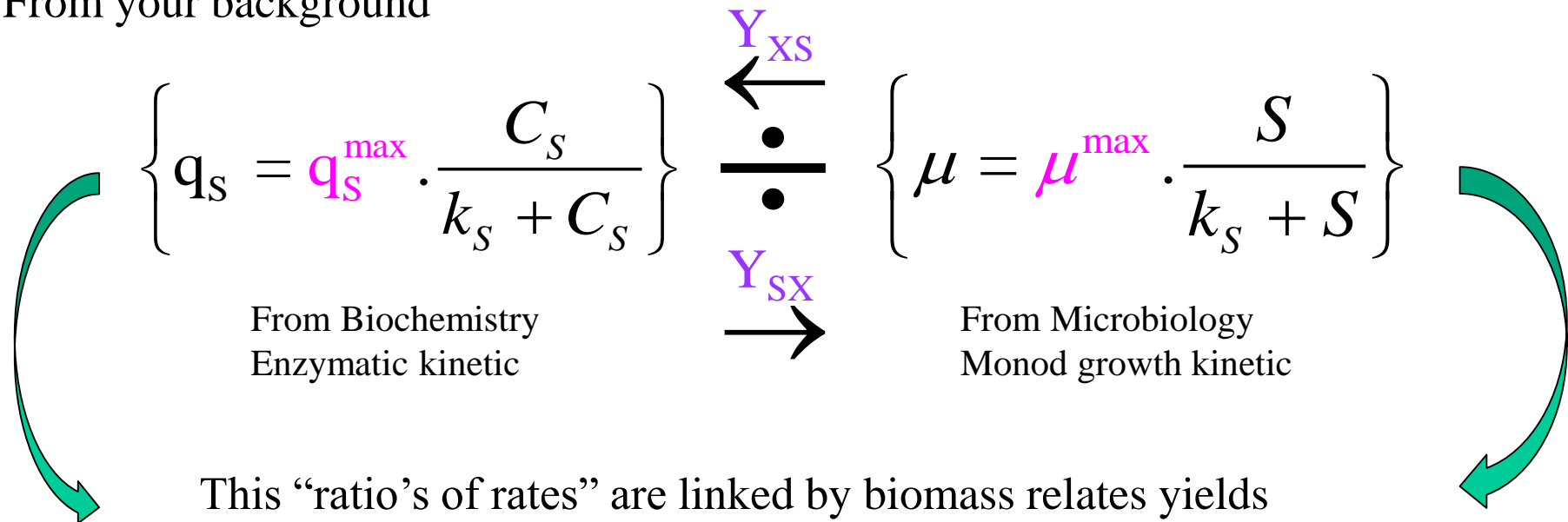
But all  $Y_{ij}$  yields can be obtained with  $Y_{ix}$  yields

$$Y_{sp} = \left| \frac{q_p}{-q_s} \right| = \left| \frac{q_p/\mu}{-q_s/\mu} \right| = \left| \frac{1/Y_{px}}{1/Y_{sx}} \right| = \left| \frac{Y_{sx}}{Y_{px}} \right|$$

# Yield = ratio of rates

$$Y_{SX} = \left| \frac{r_X}{-r_S} \right| = \left| \frac{\mu \cdot C_X}{-q_S \cdot C_X} \right| = \left| \frac{\mu}{-q_S} \right| = \frac{1}{Y_{XS}}$$

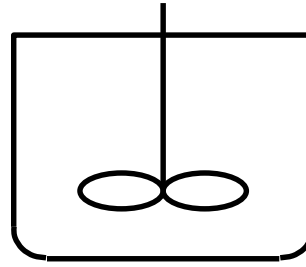
From your background



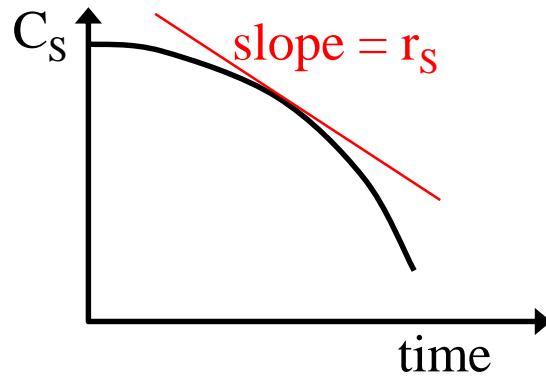
$$\frac{\mu}{q_s} = Y_{SX} = \frac{\text{Specific rate of biomass production [kgX.kgX}^{-1}\text{.hr}^{-1}]}{\text{Specific rate of substrate consumption [kgS.kgX}^{-1}\text{.hr}^{-1}]}$$

# Quantification of $r_i$ , $q_i$ , $Y_{ij}$ in BATCH reactor

BATCH reactor



constant volume



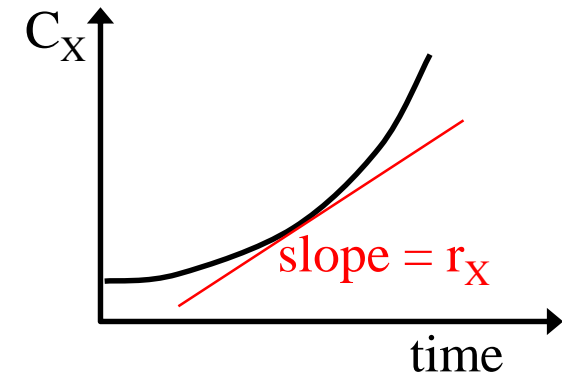
Substrate rate?

Use substrate mass balance

$$\frac{d(V C_s)}{dt} = r_s V \Rightarrow r_s = \frac{1}{V} \frac{d(V C_s)}{dt}$$

$$\text{constant volume} \Rightarrow r_s = \frac{dC_s}{dt} = \text{slope}$$

$$q_s = \frac{1}{C_x} \frac{dC_s}{dt}$$



Biomass rate ?

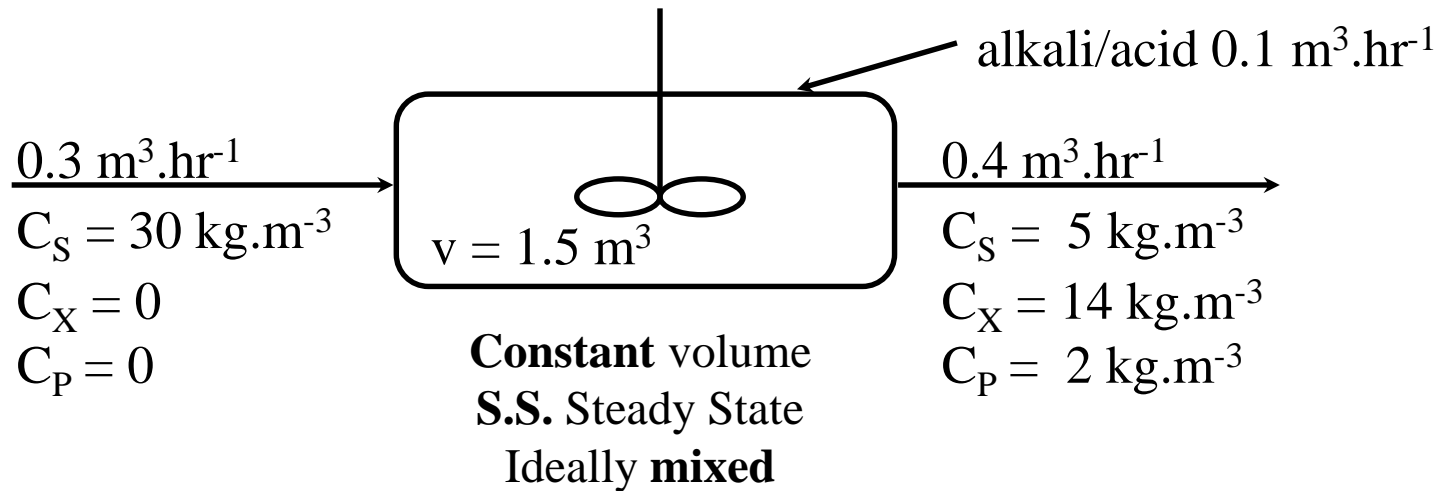
Use the biomass mass balance

$$\frac{d(V C_x)}{dt} = r_x V \Rightarrow r_x = \frac{1}{V} \frac{d(V C_x)}{dt}$$

$$\text{constant volume} \Rightarrow r_x = \frac{dC_x}{dt} = \text{slope}$$

$$q_x = \mu = \frac{r_x}{C_x} = \frac{1}{C_x} \frac{dC_x}{dt}$$

# Quantification of $r_i$ , $q_i$ , $Y_{ij}$ in CHEMOSTAT reactor



$$\text{Accumulation}_i = \sum R_i^{IN} - \sum R_i^{OUT} + \sum R_i^{\text{Conversions}}$$

Mass Balance at Steady State  
**→ No accumulation**

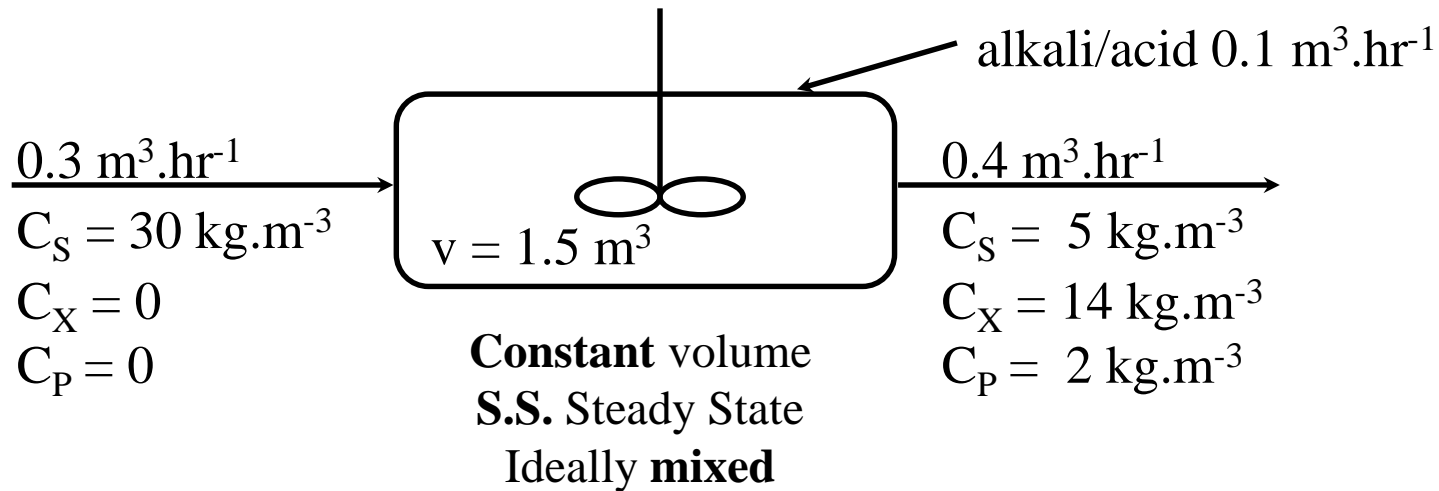
(1) Calculation of  $R_i$  and  $r_i$  rates for Biomass X, Substrate S and Product P

X: growth + in – out = 0 ;  $R_X = \text{growth} = - (\text{in} - \text{out})$  [kg X.hr<sup>-1</sup>]

S: consumption + in – out = 0 ;  $R_S = \text{consumption} = - (\text{in} - \text{out})$  [kg S.hr<sup>-1</sup>]

P: production + in – out = 0;  $R_P = \text{production} = - (\text{in} - \text{out})$  [kg P.hr<sup>-1</sup>]

# Quantification of $r_i$ , $q_i$ , $Y_{ij}$ in CHEMOSTAT reactor



Mass Balance at Steady State  $\rightarrow$  **No accumulation**

(1) Calculation of  $R_i$  and  $r_i$  rates for Biomass  $X$

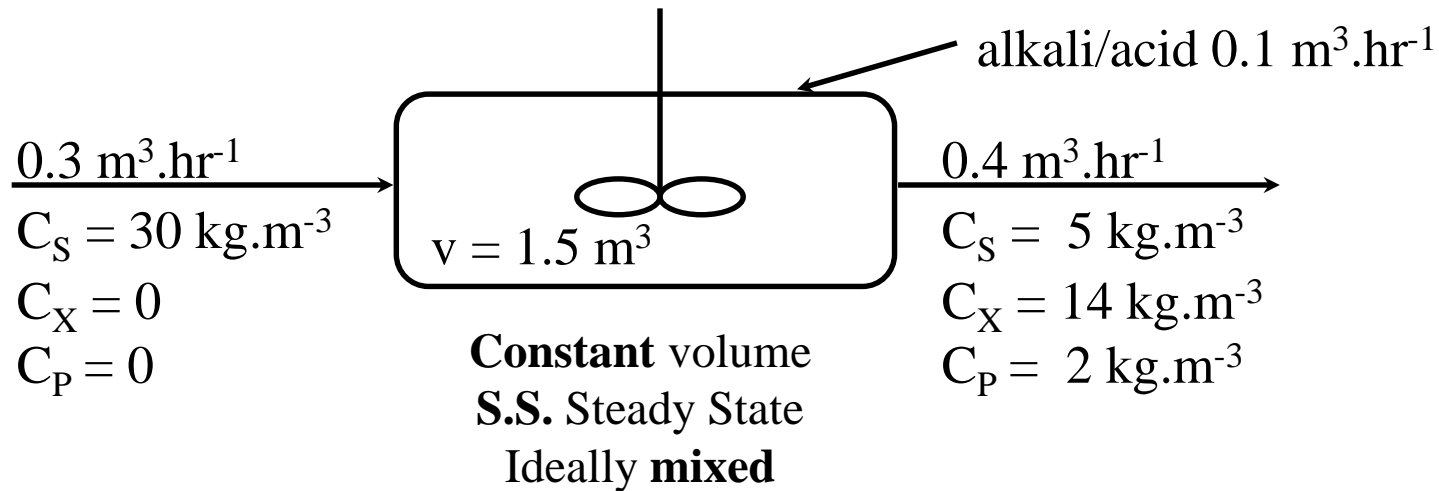
$$r_i \text{ Volumetric rate} = R_i / V_{\text{reactor}}$$

$$X: \text{growth} + \text{in} - \text{out} = 0 ; R_X + (0 \cdot 0.3) - (14 \cdot 0.4) = 0 \text{ [kgX} \cdot \text{h}^{-1}]$$

$$R_X = -[0 - (14 \cdot 0.4)] = 5.6 \text{ [kgX} \cdot \text{h}^{-1}]$$

$$r_X = R_X / V = (14 \cdot 0.4) / 1.5 = 3.73 \text{ [kgX} \cdot \text{m}^{-3} \cdot \text{h}^{-1}]$$

# Quantification of $r_i$ , $q_i$ , $Y_{ij}$ in CHEMOSTAT reactor



Mass Balance at Steady State  $\rightarrow$  **No accumulation**

(1) Calculation of  $R_i$  and  $r_i$  rates for Substrate S

$$r_i \text{ Volumetric rate} = R_i / V_{\text{reactor}}$$

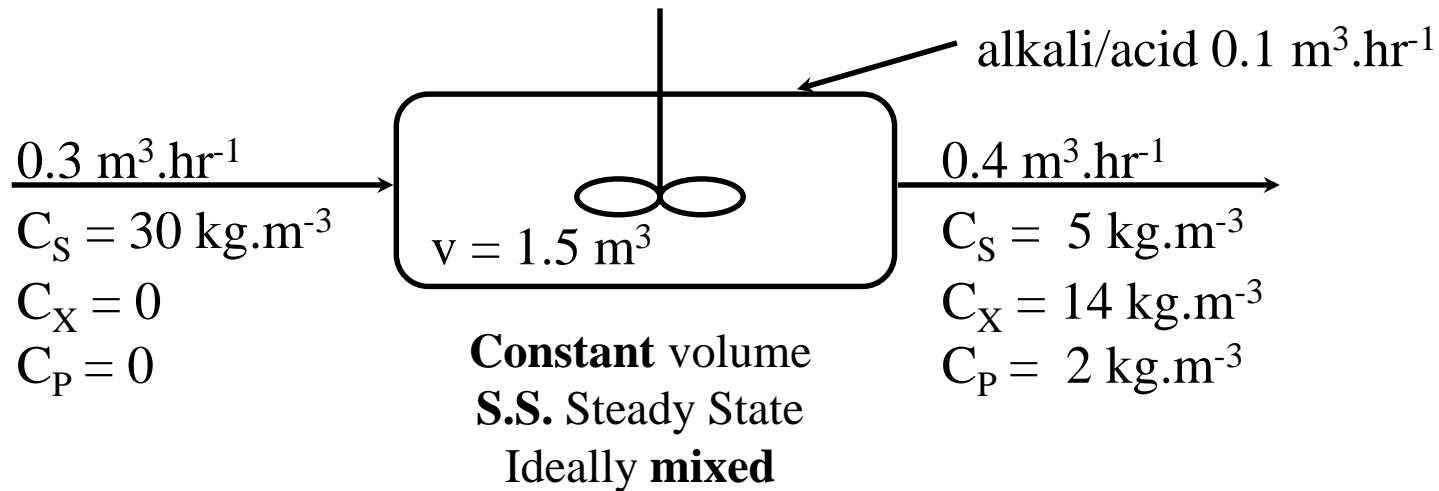
$$S: \text{consumption} + \text{in} - \text{out} = 0; R_S + (30 \cdot 0.3) - (5 \cdot 0.4) = 0 \text{ [kgS} \cdot \text{h}^{-1}\text{]}$$

$$R_S = -[(30 \cdot 0.3) - (5 \cdot 0.4)] = -7 \text{ [kgS} \cdot \text{h}^{-1}\text{]}$$

$$r_S = R_S / V = -7 / 1.5 = -4.66 \text{ [kgS} \cdot \text{m}^{-3} \cdot \text{h}^{-1}\text{]}$$



# Quantification of $r_i$ , $q_i$ , $Y_{ij}$ in CHEMOSTAT reactor



Mass Balance at Steady State  $\rightarrow$  **No accumulation**

(1) Calculation of  $R_i$  and  $r_i$  rates for Product P

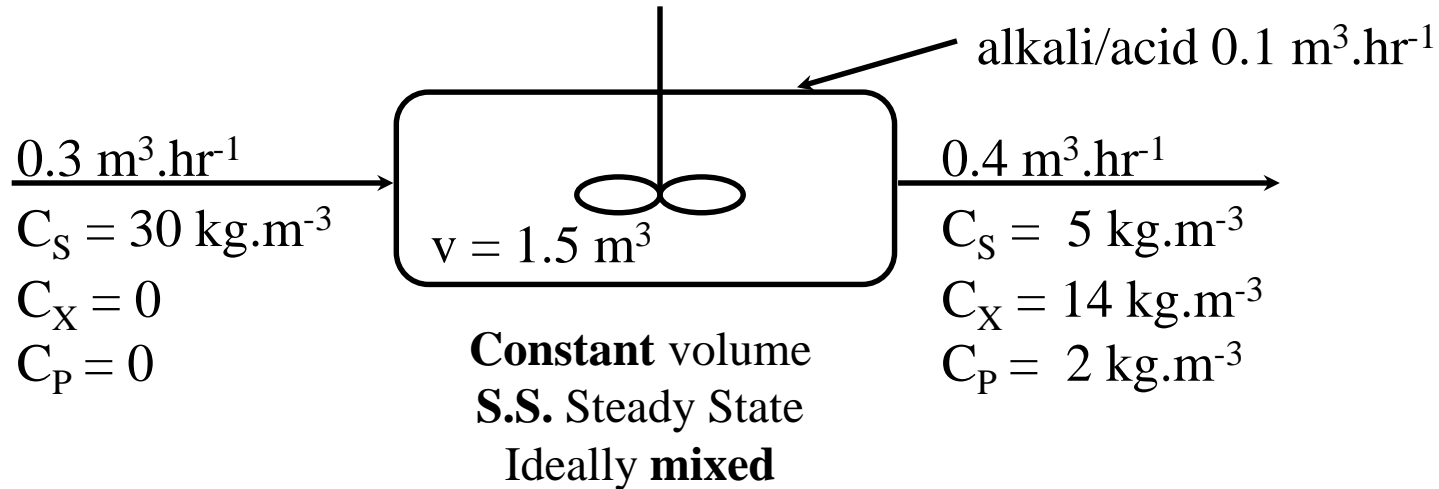
$$r_i \text{ Volumetric rate} = R_i / V_{\text{reactor}}$$

$$P: \text{production} + \text{in} - \text{out} = 0; R_P + (0 \cdot 0.3) - (2 \cdot 0.4) = 0 \text{ [kgP} \cdot \text{h}^{-1}\text{]}$$

$$R_P = -[(0 \cdot 0.3) - (2 \cdot 0.4)] = 0.8 \text{ [kgP} \cdot \text{h}^{-1}\text{]}$$

$$r_P = R_P / V = 0.8 / 1.5 = 0.53 \text{ [kgP} \cdot \text{m}^{-3} \cdot \text{h}^{-1}\text{]}$$

# Quantification of $r_i$ , $q_i$ , $Y_{ij}$ in CHEMOSTAT reactor



(2) Calculation of  $q_i$  specific rates

$$q_i \text{ Specific rate} = r_i / \text{Biomass concentration}$$

$$r_X = 3.73 \text{ [kgX} \cdot \text{m}^{-3} \cdot \text{h}^{-1}]$$

$$r_S = -4.66 \text{ [kgS} \cdot \text{m}^{-3} \cdot \text{h}^{-1}]$$

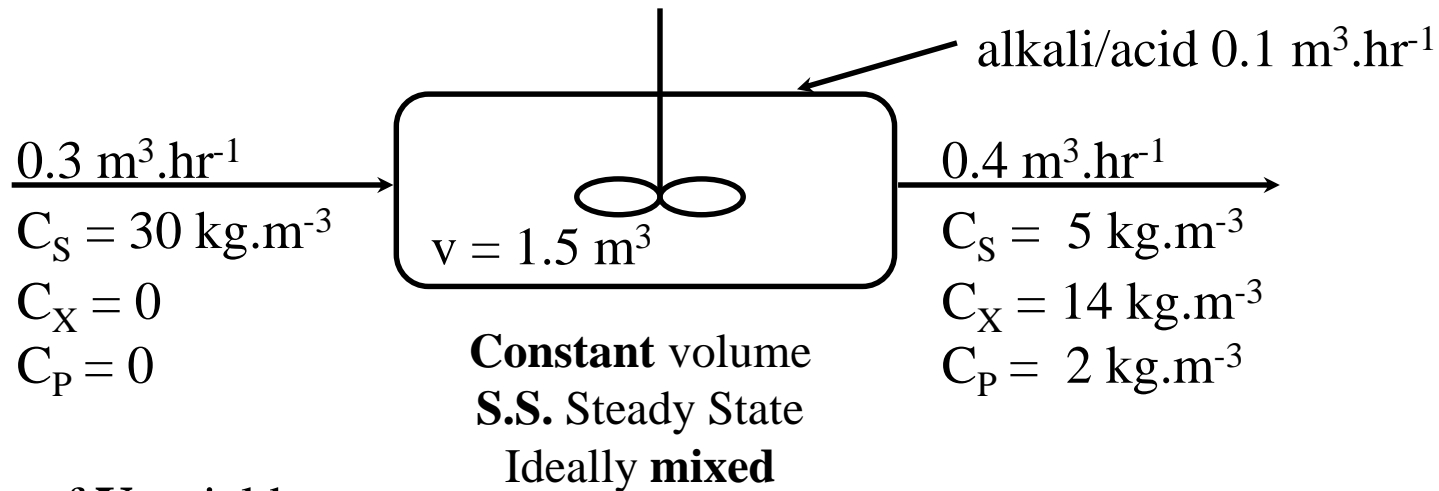
$$r_P = 0.53 \text{ kg [kgP} \cdot \text{m}^{-3} \cdot \text{h}^{-1}]$$

$$\text{X: } q_X = \mu = r_X / C_X = 3.73 / 14 = 0.26 \text{ [kgX} \cdot \text{m}^{-3} \cdot \text{h}^{-1} \cdot (\text{kgX} \cdot \text{m}^{-3})^{-1}] = 0.26 \text{ [h}^{-1}]$$

$$\text{S: } q_S = r_S / C_X = -4.66 / 14 = -0.33 \text{ [kgS} \cdot \text{kgX}^{-1} \cdot \text{h}^{-1}]$$

$$\text{P: } q_P = r_P / C_X = 0.53 / 14 = 0.037 \text{ [kgP} \cdot \text{kgX}^{-1} \cdot \text{h}^{-1}]$$

# Quantification of $r_i$ , $q_i$ , $Y_{ij}$ in CHEMOSTAT reactor



## (3) Calculation of $Y_{ij}$ yields

$$\begin{array}{ll}
 q_X = 0.26 \text{ [h}^{-1}\text{]} & r_X = 3.73 \text{ [kgX} \cdot \text{m}^{-3} \cdot \text{h}^{-1}\text{]} \\
 q_S = -0.33 \text{ [kgS} \cdot \text{kgX}^{-1} \cdot \text{h}^{-1}\text{]} & r_S = -4.66 \text{ [kgS} \cdot \text{m}^{-3} \cdot \text{h}^{-1}\text{]} \\
 q_P = 0.037 \text{ [kgPS} \cdot \text{kgX}^{-1} \cdot \text{h}^{-1}\text{]} & r_P = 0.53 \text{ kg [kgP} \cdot \text{m}^{-3} \cdot \text{h}^{-1}\text{]}
 \end{array}$$

$$Y_{ij} = \left| \frac{R_j}{R_i} \right| = \left| \frac{r_j}{-r_i} \right| = \left| \frac{q_j}{q_i} \right|$$

$$Y_{SX} = |\mu/q_S| = |0.26/-0.33| = 0.8 = |r_X/r_S| = |3.73/-4.66| = 0.8 \text{ [kgX} \cdot \text{kgS}^{-1}\text{]}$$

$$Y_{SP} = |r_P/r_S| = |0.53/-4.66| = 0.11 \text{ [kgP} \cdot \text{kgS}^{-1}\text{]}$$

$$Y_{XP} = |r_P/r_X| = |0.53/3.73| = 0.14 \text{ [kgP} \cdot \text{kgX}^{-1}\text{]}$$