Measuring Systems

Solutions to problem set n° 12

Comparing measured data

Exercise 1 (Capacitive coupling)

We want to assess the quality of a low-pass filter by comparing if the RMS value of the noise after $U_{b,B}$ is smaller than the RMS value of the noise before filtering $U_{b,A}$. For that we use a **unilateral *f-test*** with the following null hypothesis H_0 :

H_0 : "The noise level after filtering is greater than or equal to the noise before filtering" $(U_{b,B} \geq U_{b,A})$

We then find $f_\alpha(\nu_B, \nu_A)$ and f_{obs} :

$$f_{1-\alpha}(\nu_B, \nu_A) = f_{99.9\,\%}(19{,}9) = \frac{1}{f_\alpha(\nu_A,\nu_B)} = \frac{1}{f_{1\,‰}(9{,}19)} = \frac{1}{5.39} = 0.186 \qquad f_{obs} = \frac{{U_{b,B}}^2}{{U_{b,A}}^2} = 0.09$$

$$f_{obs} \notin \left[f_{1-\alpha}(\nu_B, \nu_A); \infty\right[\qquad \Longrightarrow \qquad U_{b,A} > U_{b,B}$$

We therefore reject hypothesis H_0 and we can say that filtering significantly reduces the noise level with a risk of error α.

Exercise 2 (Asymmetric amplifier)

a) Knowing that the theoretical average of the output voltage corresponds to U_s, we find an experimental value $\overline{U}_s$, which depends on temperature T, and a standard deviation σ on the voltage U_s which is :

$$\overline{U}_s = U_s + \varepsilon_{\Delta T} \cdot \Delta T = A \cdot U + \varepsilon_{\Delta T} \cdot (T - T_{ref}) = 1\,V + \Delta T \cdot 0.3\,mV/°C \qquad \sigma = \Phi_b \cdot \sqrt{B_b} = 31.6\,mV$$

b) We must choose the temperature difference ΔT so that the systematic error (*offset*) on the output voltage of the amplifier is not significant. We then use a **bilateral *t-test*** ($N \leq 30$ with known standard deviation). As a matter of fact, we are trying to compare the averaged experimental value $\overline{U}_s$ to the theoretical average U_s. We use the following null hypothesis H_0 :

H_0 : « There is no systematic error on the output voltage » $(\overline{U} = U_s)$

We find $t_{\frac{\alpha}{2}}$ and t_{obs} $(\nu = 19)$:

$$t_{\frac{\alpha}{2}} = t_{0.01} = 2.539 \qquad \text{and} \qquad t_{obs} = \frac{\overline{U} - U_s}{\frac{\sigma}{\sqrt{N}}} = \frac{\varepsilon_{\Delta T} \cdot \Delta T}{\frac{\Phi_b \cdot \sqrt{B_b}}{\sqrt{N}}}$$

We find a temperature range ΔT for which the offset is not significant :

$$|t_{obs}| \leq t_{\frac{\alpha}{2}} \qquad \Longrightarrow \qquad |\Delta T| \leq \frac{\Phi_b \cdot \sqrt{B_b}}{\varepsilon_{\Delta T} \cdot \sqrt{N}} \cdot t_{\frac{\alpha}{2}} = 59.8\,°C$$

$$T \in [T_{min}; T_{max}] = [-34.8; 84.8]\,°C \qquad \text{(with : } T_{min/max} = T_{ref} \pm |\Delta T|\text{)}$$