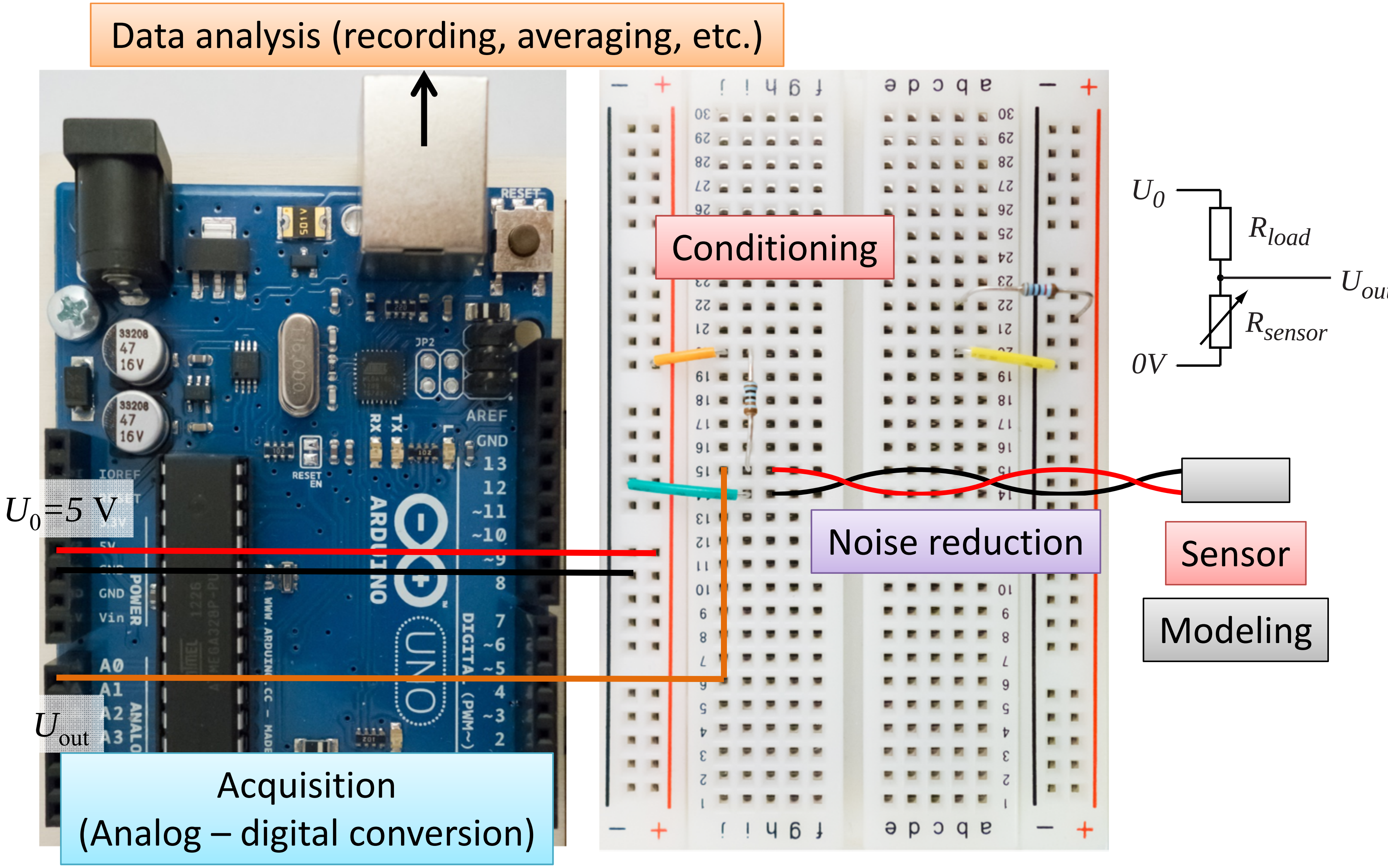
Measurement systems

Lecturer: Andras Kis

Chapter 2: Modeling

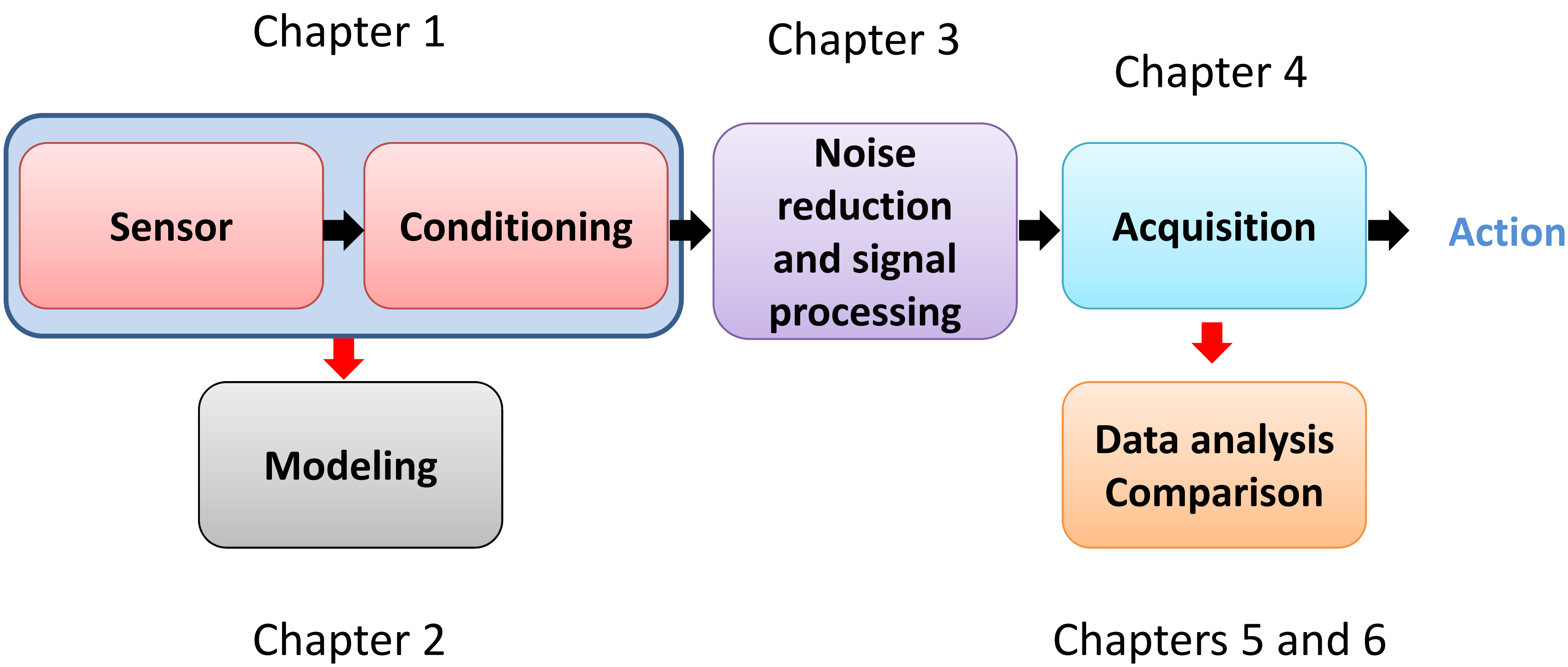
Measurement chain



Arduino UNO board

Conditioning circuit

Measurement chain



Chapter 2: Modeling

- Introduction
- General model
- Static transfer
- Dynamic transfer

What are the important parameters of a measurement system?

SPECIFICATIONS

Excitation: 10 to 50 N: 5 Vdc
≥100 N: 10 Vdc

Output: 10 to 50 N: 1.5 mV/V (nom)
≥100 N: 2 mV/V (nom)

Accuracy: (Linearity and Hysteresis combined)
±0.15% FSO ≤500 N
±0.20% FSO ≥1000 N

Repeatability: 0.20% FSO

5-Point Calibration: (in tension)
0%, 50%, 100%, 50%, 0%

Zero Balance: ±2% FSO

Deflection: 0.025 to 0.075 mm

Operating Temp Range:
-54 to 121°C (-65 to 250°F)

Compensated Temp Range:
16 to 71°C (60 to 160°F)

Thermal Effects:
Span: ±0.009% FSO/°C
Zero: 0.009% FSO/°C

Safe Overload: 150% of Capacity

Ultimate Overload: 300% of Capacity

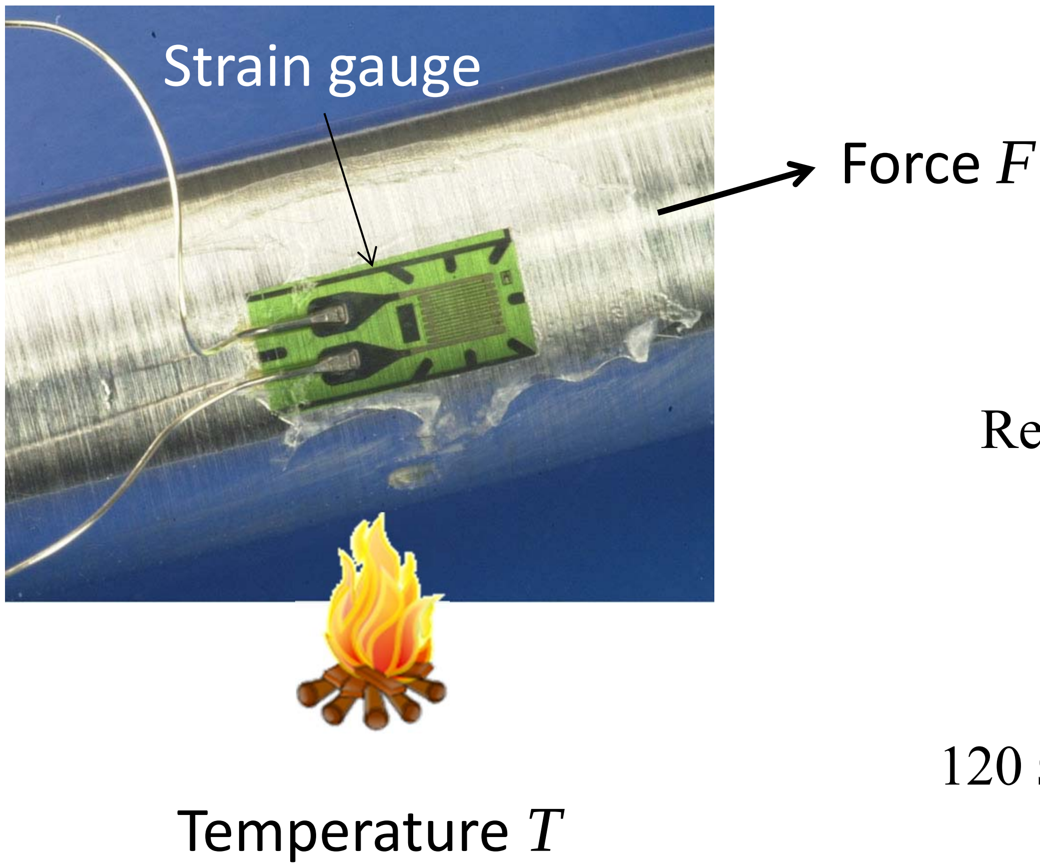
Bridge Resistance: 350 Ω minimum

Construction: Stainless Steel

Electrical Connection: 1.5 m (5 ft)
4 Conductor, Shielded Cable;
≤50N: SS Overbraided with temperature compensation board

Force sensor

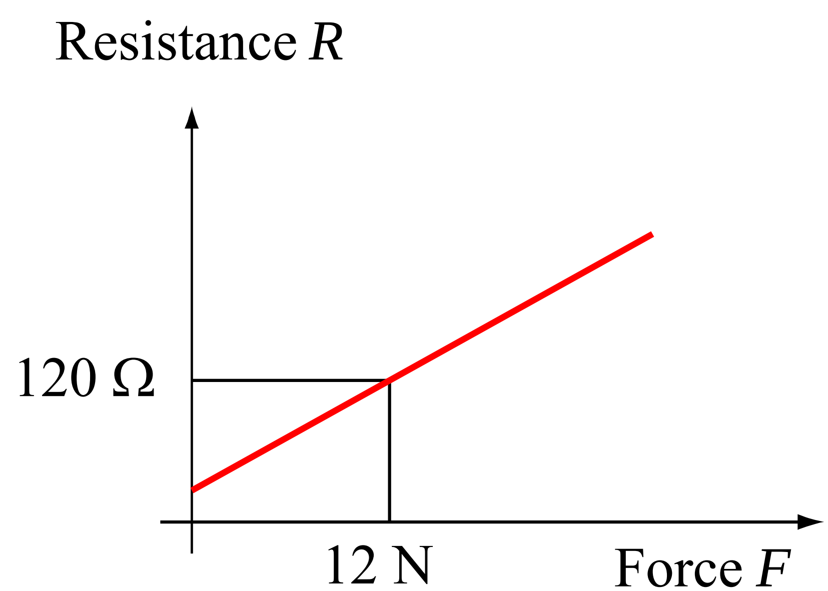
Force sensor: strain gauge on a steel bar



$$\left.\frac{\Delta R}{R}\right|_F = K\varepsilon = \boxed{\frac{a}{Y}}F$$

sensitivity

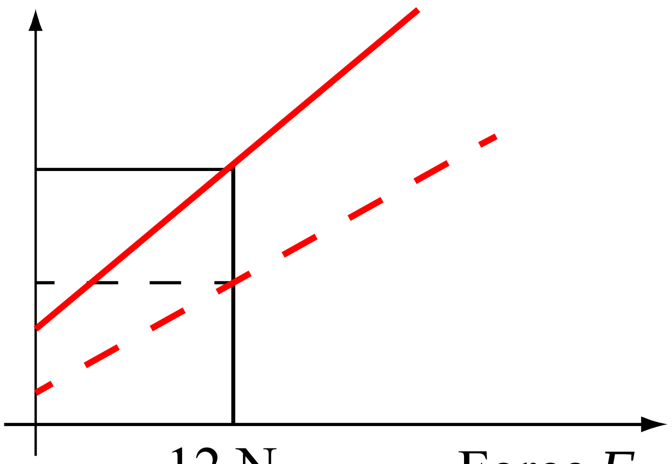
a – proportionality factor
Y – Young's modulus
a/Y – Sensitivity



Force sensor

- Influence of the force (measured quantity) on the sensor output:

$$\left.\frac{\Delta R}{R}\right|_F = K\varepsilon = \frac{a}{Y}F$$



- Influence of the temperature

$$\left.\frac{\Delta R}{R}\right|_{T,offset} = \alpha_T \cdot \Delta T$$

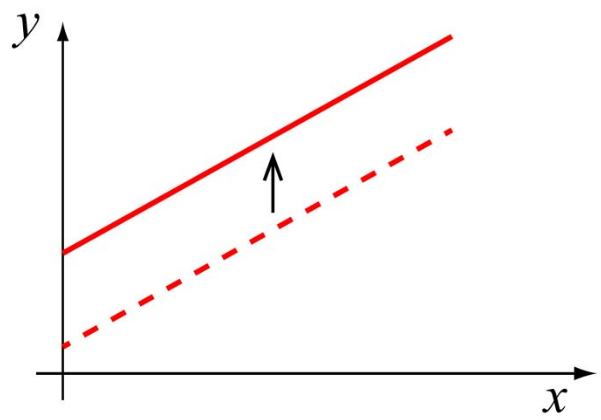
Resistance changes with the temperature:
added offset on the left-hand side

$$\left.\frac{\Delta Y}{Y}\right|_T = -\alpha_Y \Delta T,\ \alpha_Y = 0.26\times 10^{-3}\ /^\circ C \quad (steel)$$

Material's stiffness (Young's modulus of the steel bar) changes with the temperature:
modified slope

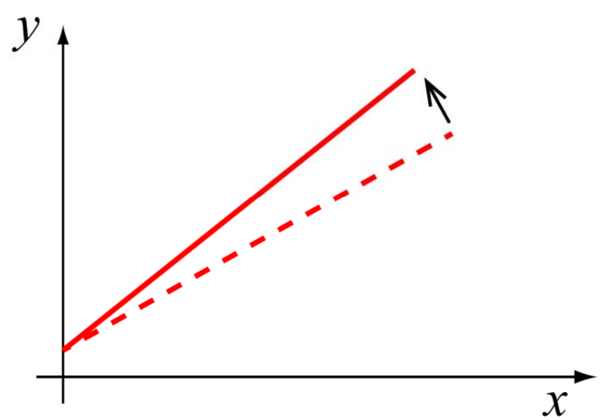
x_i – interfering input

– **adds an offset**

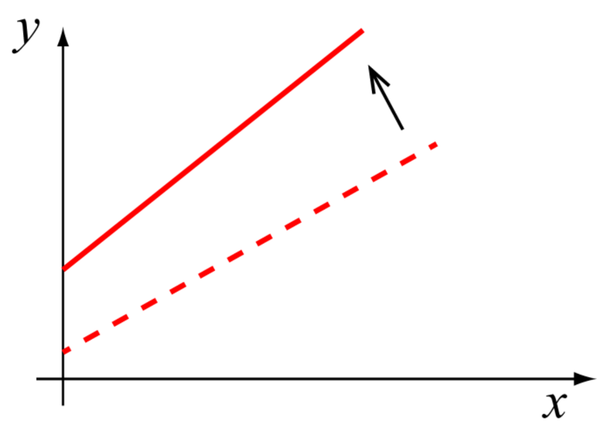


x_m – modifying input

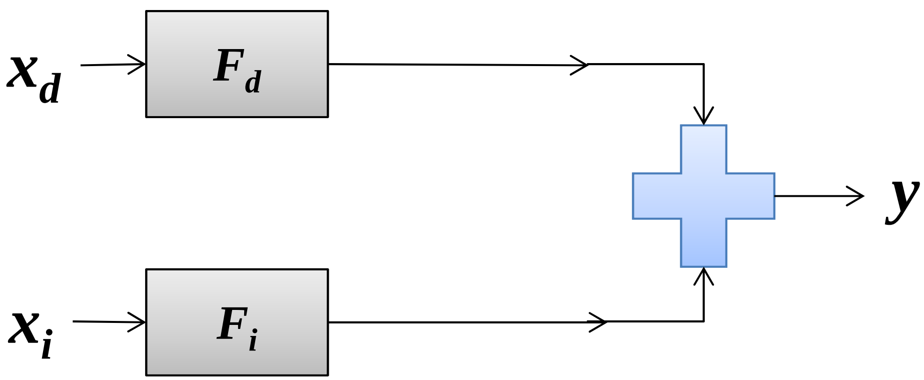
– **changes the slope (sensitivity)**



y – measured quantity



General model

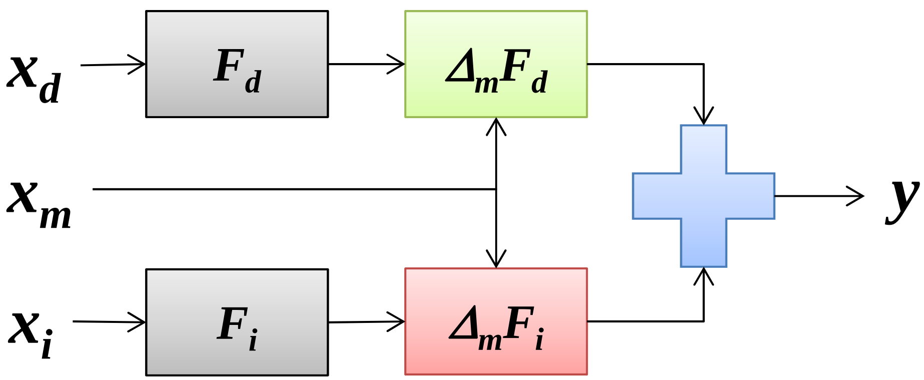


F_d, F_i – transfer functions

x_d – desired quantity

x_i – interfering input

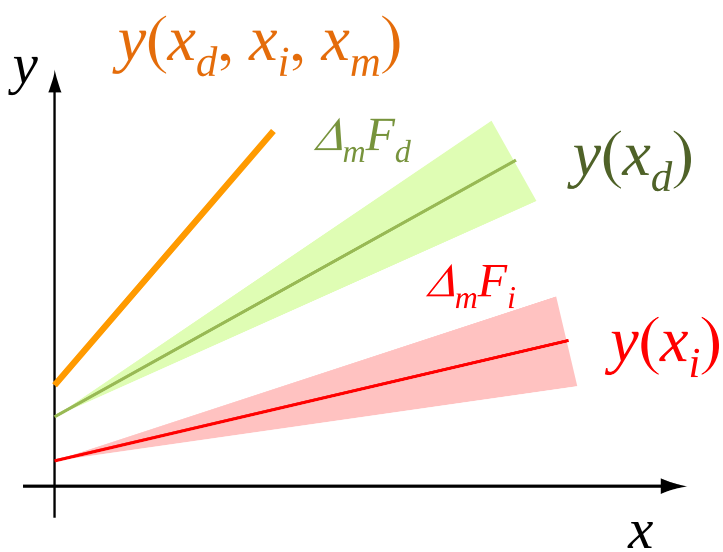
General model



F_d, F_i – transfer functions
$\Delta_m F_d$, $\Delta_m F_i$ – parameters of F_d, F_i

x_d – desired quantity
x_m – modifying input
x_i – interfering input

influencing inputs

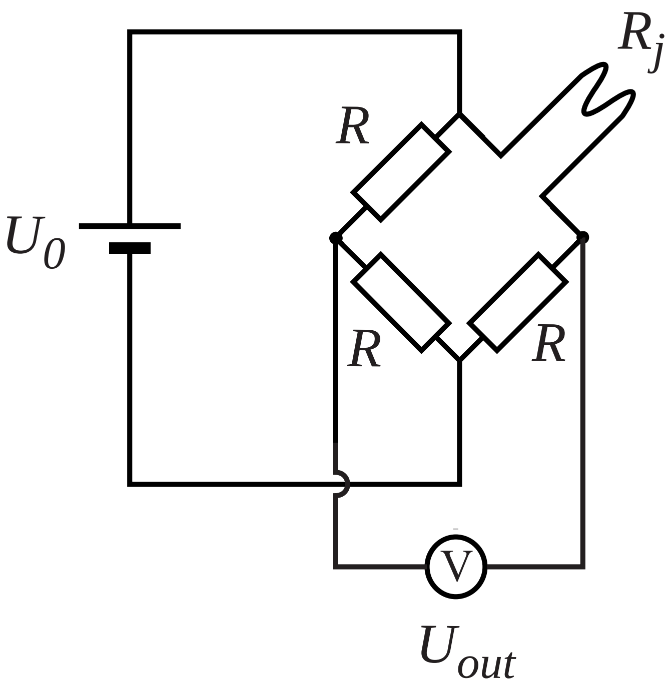


Examples

- Temperature and a strain gauge
- Temperature and a photoresistor
- Temperature and humidity and a pressure sensor (changes in the membrane properties)

Example

- Knowing the modifying and interfering inputs can help us compensate their influence
- Example: strain gauge



$R_j = R$ in the absence of deformation

$$\left.\frac{\Delta R_j}{R_j}\right|_F = K\varepsilon$$

sensitivity

$$U_{out} = \frac{\Delta R}{4R}U_0 = \boxed{\frac{1}{4}KU_0} \cdot \varepsilon = S \cdot \varepsilon$$

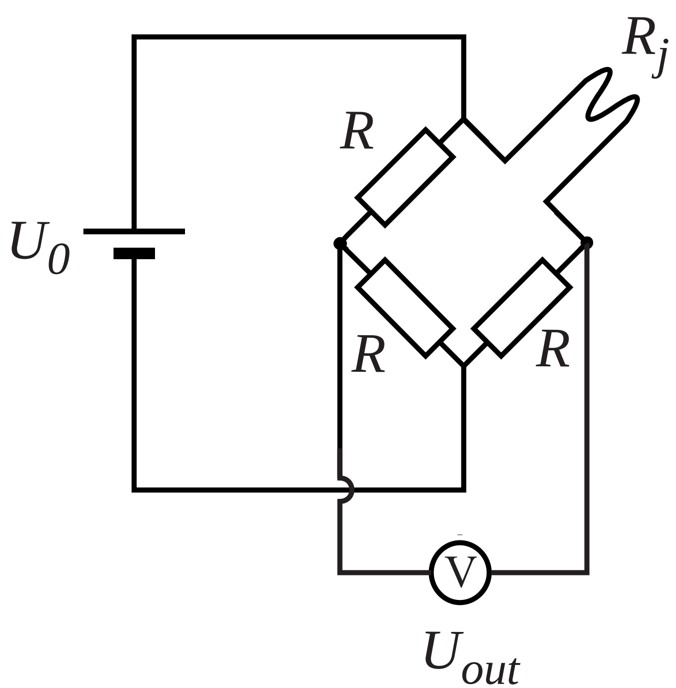
Influence of T

$$\frac{\Delta R_j}{R_j} = \alpha_R \Delta T \longrightarrow U_{out,T} = \alpha_R \Delta T \frac{U_0}{4} + \frac{1}{4}KU_0 \cdot \varepsilon$$

Interfering input

Example

- Knowing the modifying and interfering inputs can help us compensate their influence
- Example: strain gauge



Influence of U_0

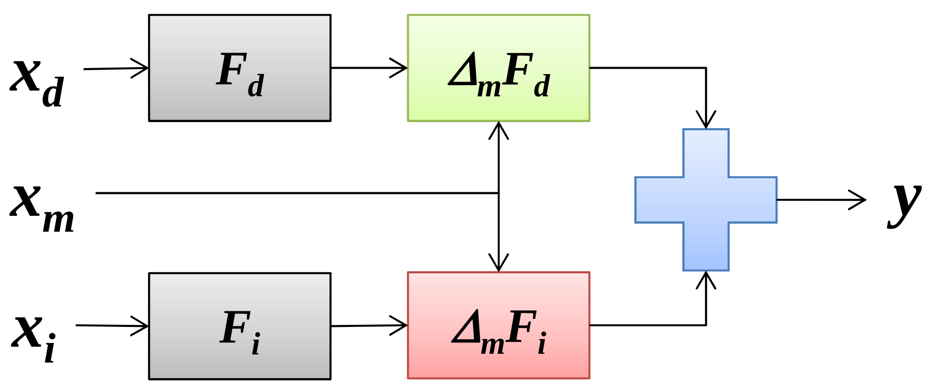
$$U_{out} = \frac{\Delta R}{4R}\left(U_0 + \Delta U_0\right)$$

$$\Delta S = \frac{1}{4} K \Delta U_0$$

Modifying input

Static transfer characteristics

- Acquisition of static transfer characteristics:
  - All the inputs are kept constant, except for one which is varied stepwise (static calibration)
  - Measure the output after all the transients have disappeared
  - This is called *static calibration*



- Components of the static transfer characteristics:
  - Range
  - Sensitivity
  - Offset
  - Drift
  - Linearity
  - Hysteresis
  - Repeatability
  - Resolution
  - Threshold
  - Stability

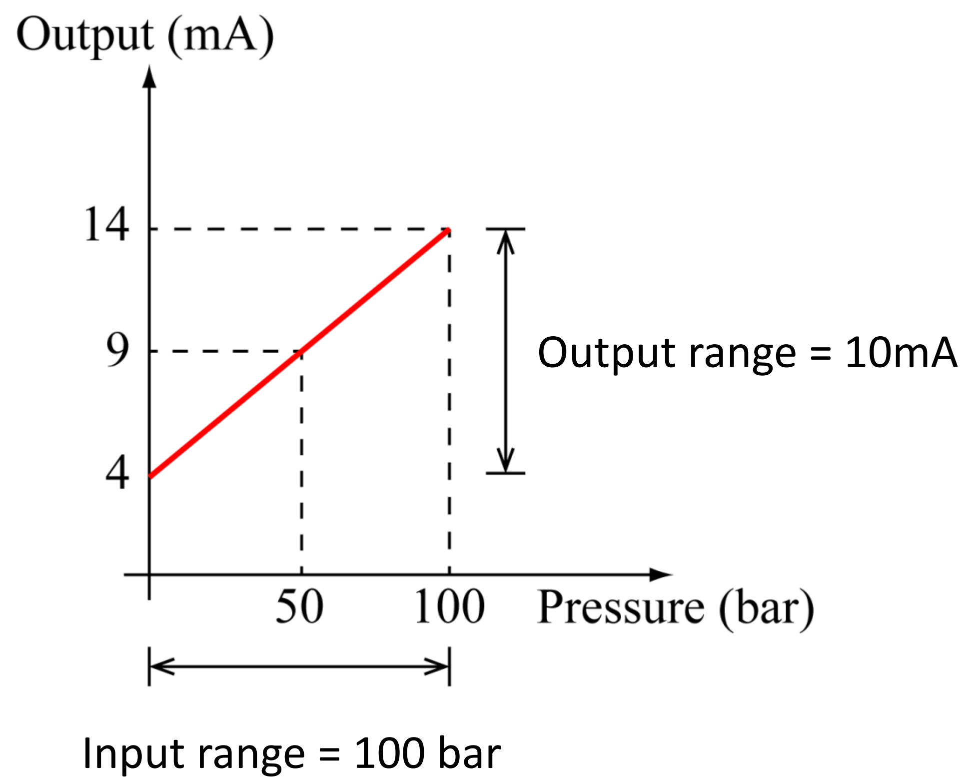
Measurement range

- Input range (input span): the interval within which the input values can vary

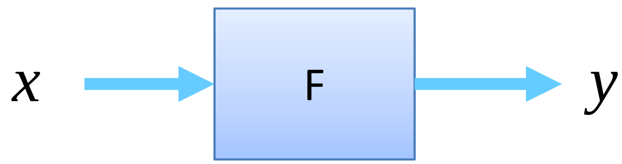
$$Range = x_{\max} - x_{\min}$$

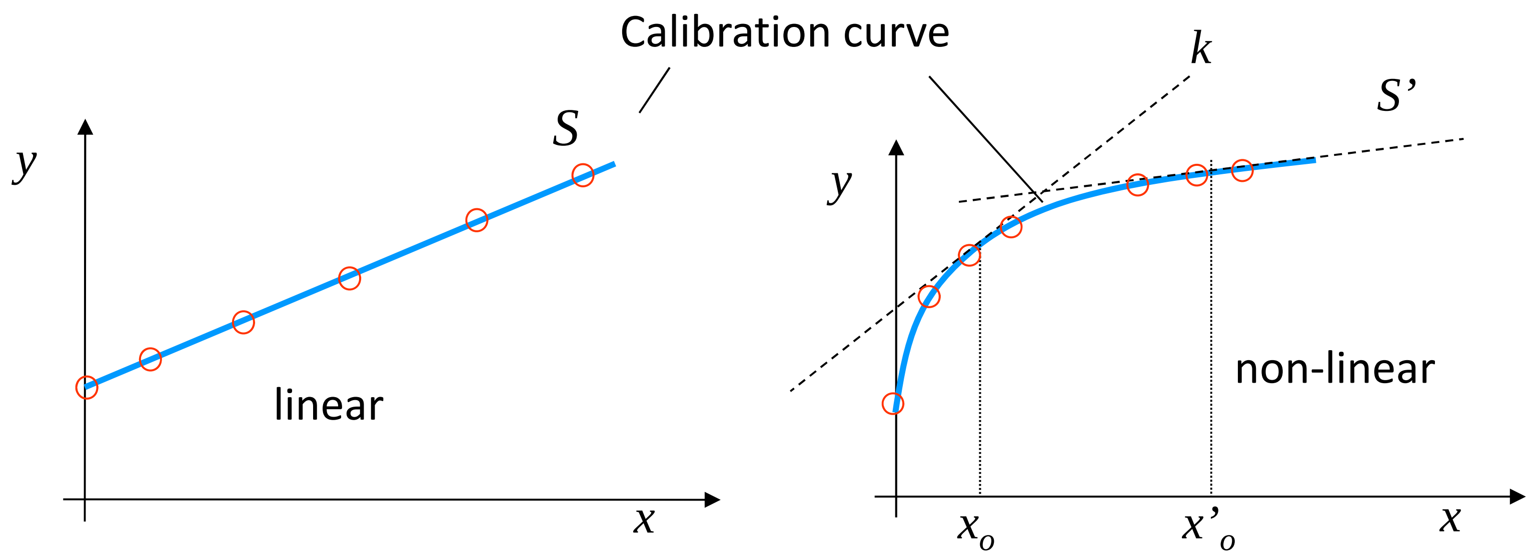
- Output range (output span)

- Measurement range – sometimes also called full scale (FS)



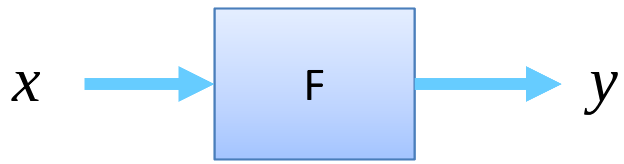
Sensitivity





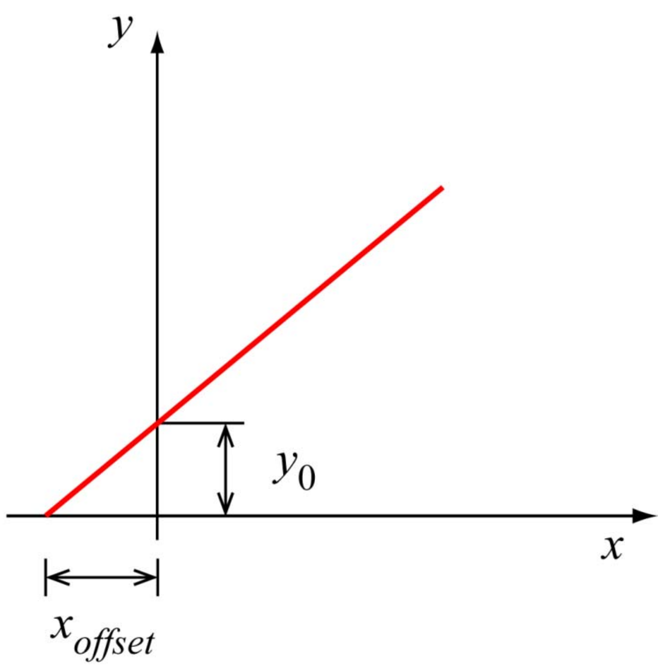
Sensitivity $$S = \left.\frac{dy}{dx}\right|_{x=x_0}$$

Offset



- Value of the output (y_0) for input $x = 0$

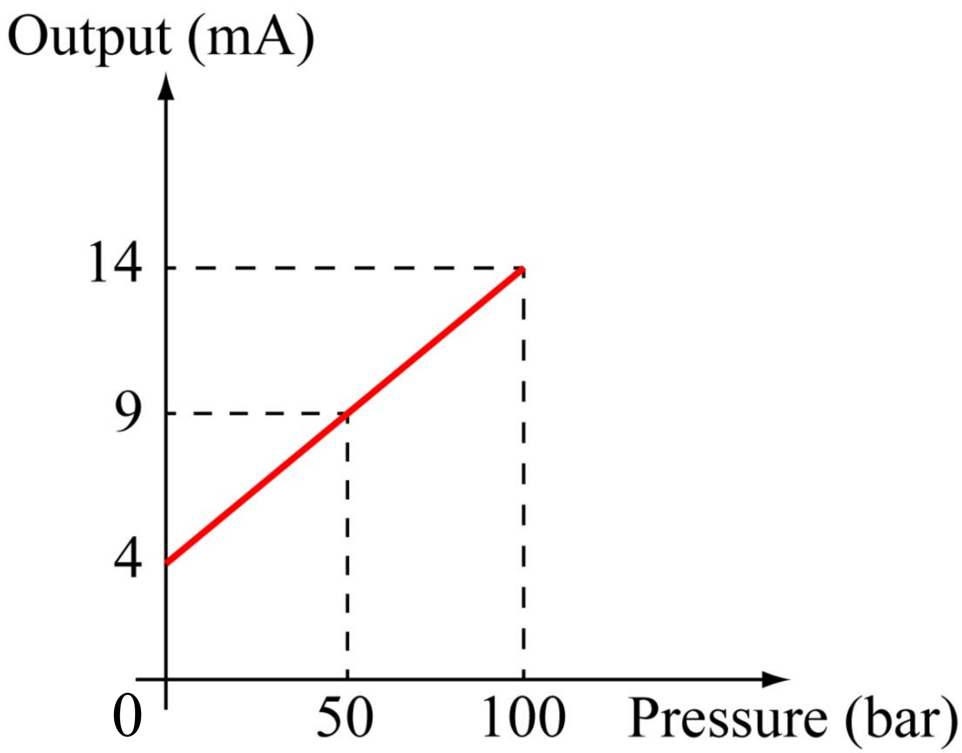
$$y = S \cdot x + y_0$$



- Error in terms of the input value:

$$x_{offset} = \frac{y_0}{S}$$

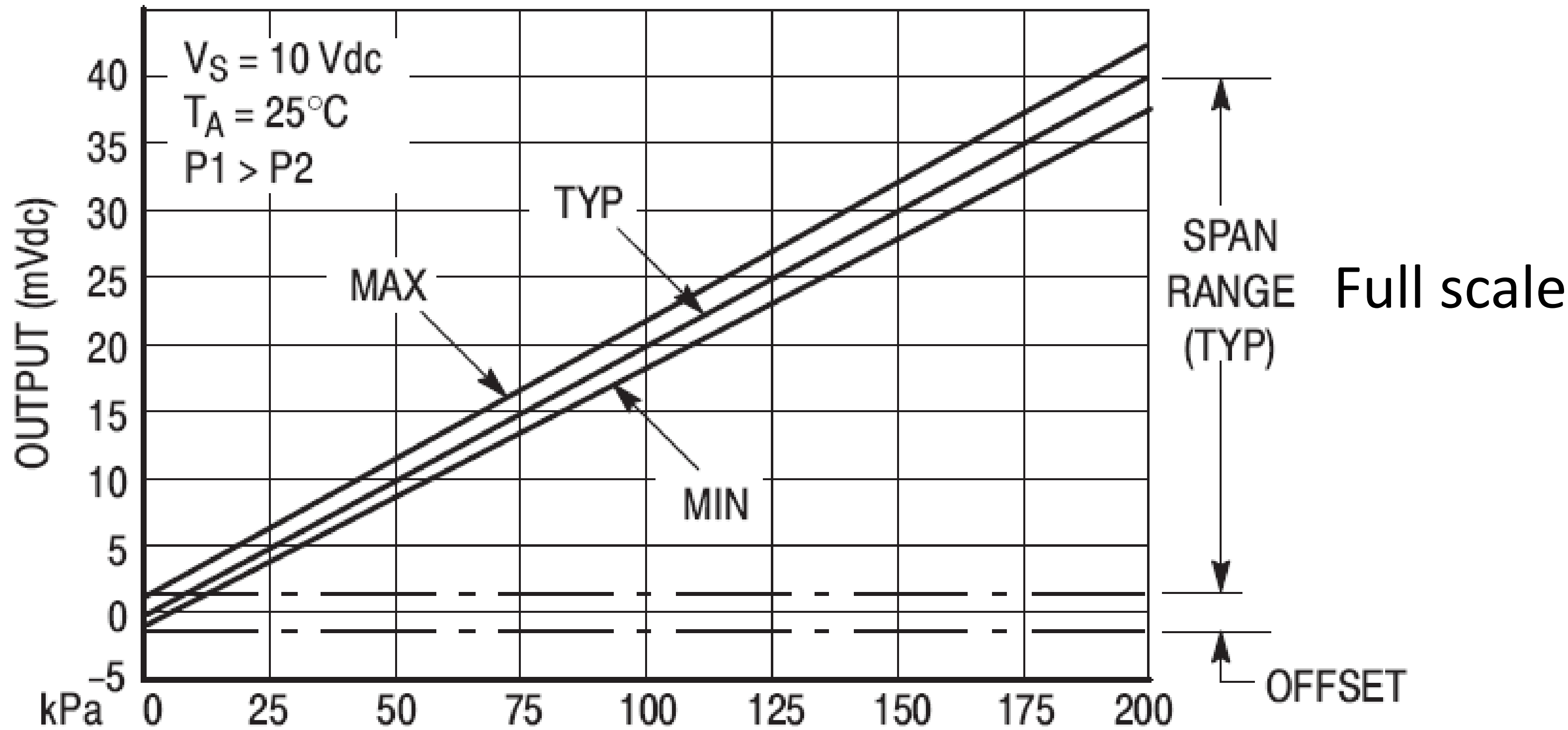
Example: pressure sensor



Sensitivity: S = (14mA-4mA)/100 bar = 0.1 mA/bar

Offset: y_0 = 4mA (40 bar)

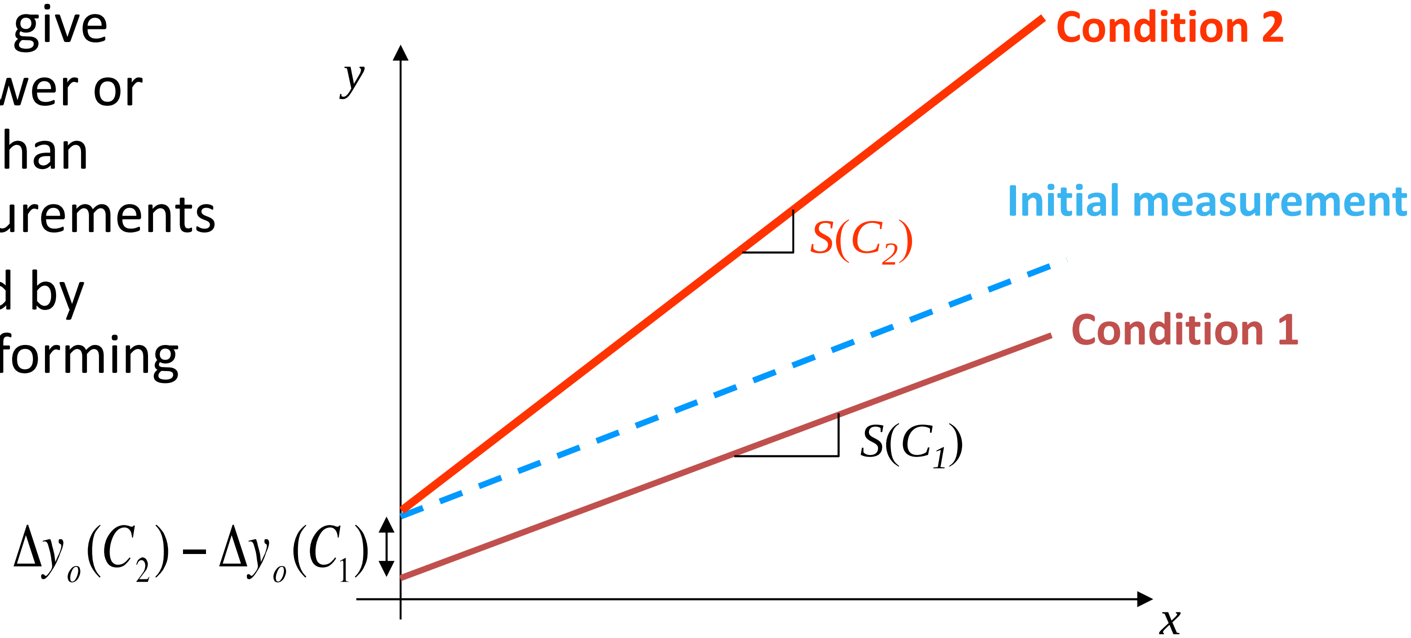
Example: specification sheet for a pressure sensor



Pressure range	FS	0-200	kPa
Offset	Voff	±2	mV
Sensitivity	ΔV/ΔP	0.2	mV/kPa

Drift

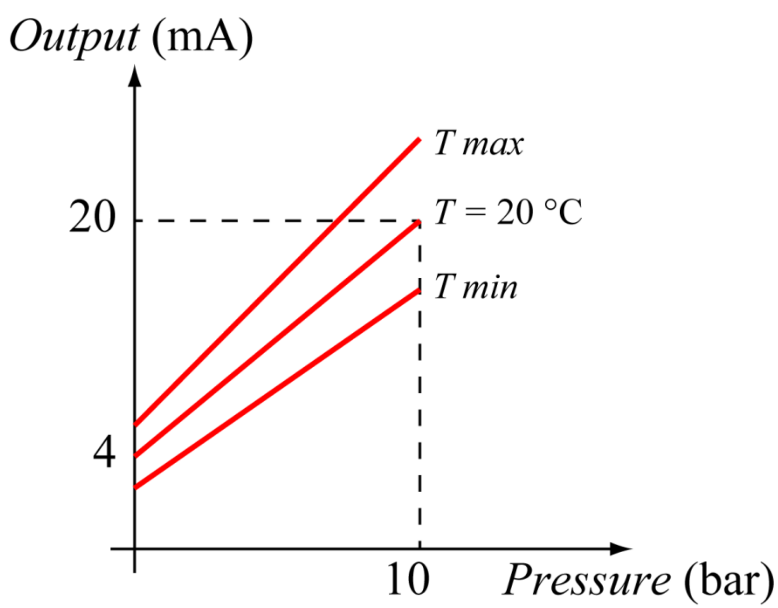
- Slow (in terms of time) variations of the sensitivity or offset
  - Example: instrument warming up
  - Repeated measurements give successively lower or higher results than previous measurements
  - Can be checked by repeatedly performing zero readings

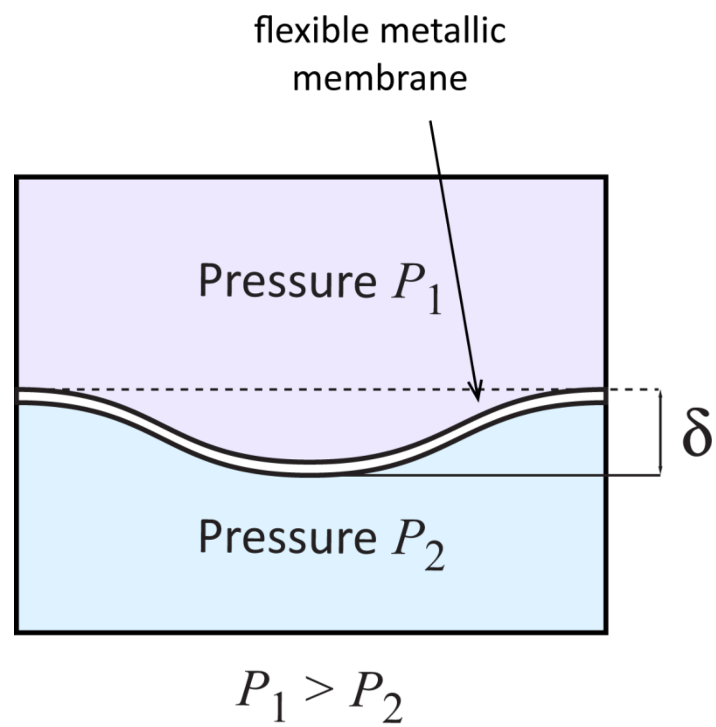


C_2: the perturbation changes the sensitivity (modifying)
C_1: the perturbation changes the offset (interfering)

Example of drift

- Influence of the temperature on a pressure sensor

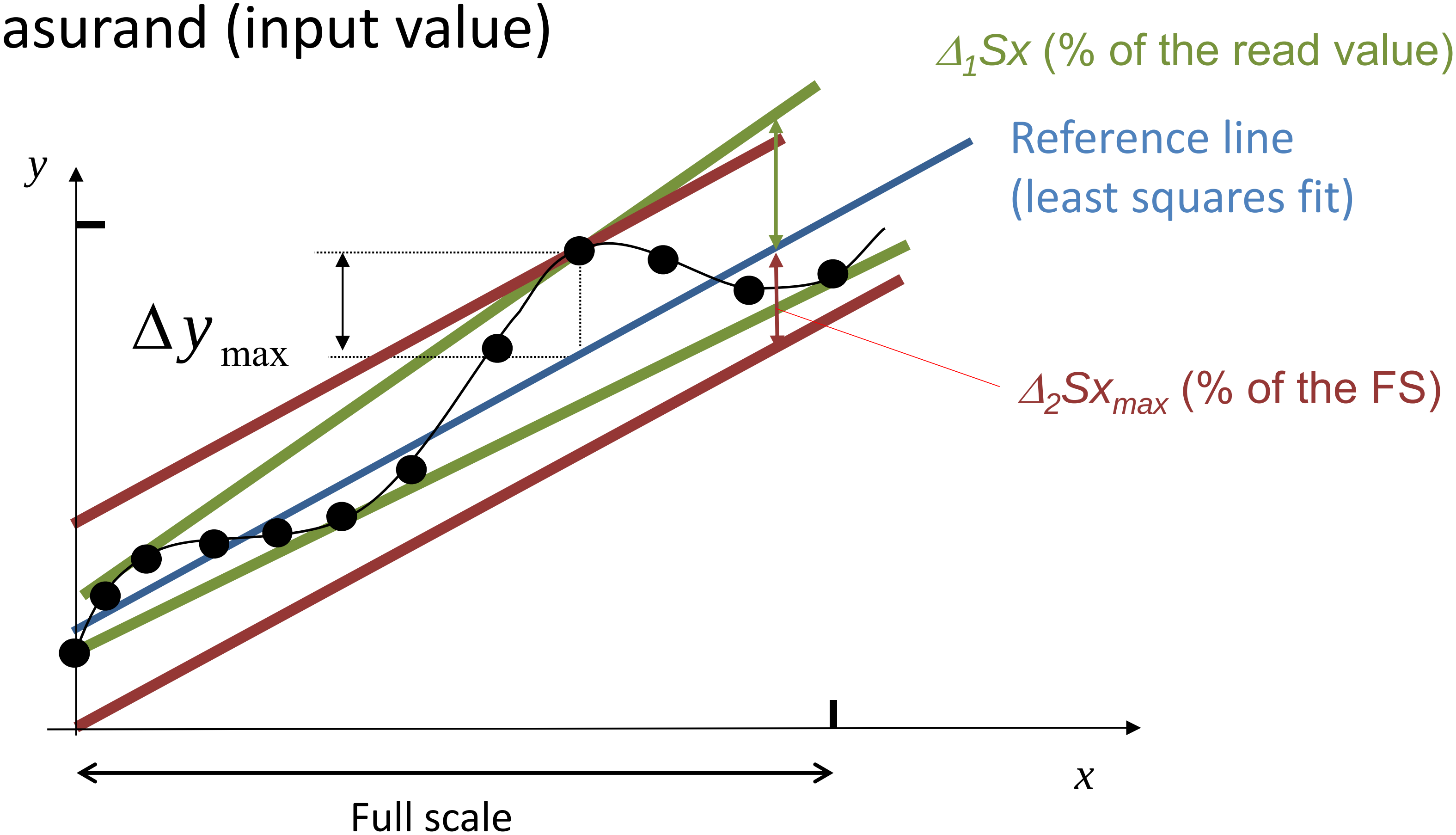




Offset drift: self-heating of the strain gauge (resistor)
Sensitivity drift: heating of the membrane resulting in the change of the Young's modulus

Linearity

- Describes in what measure the sensitivity independent of the measurand (input value)



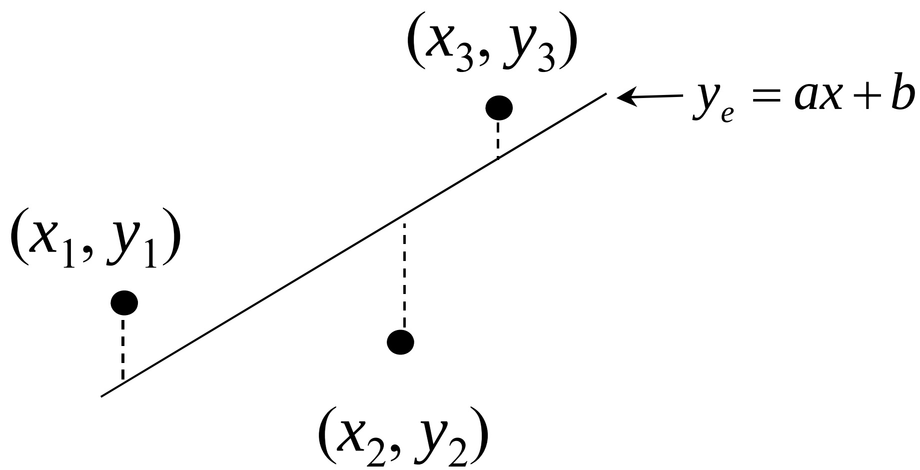
$$y = y_o + S \cdot x \pm (\underbrace{\Delta_1 Sx \text{ or } \Delta_2 Sx_{\max}}_{\text{whichever is bigger}})$$

Reference line – least squares fit

- Assume

$$y_e = ax + b$$

- Minimize total distance



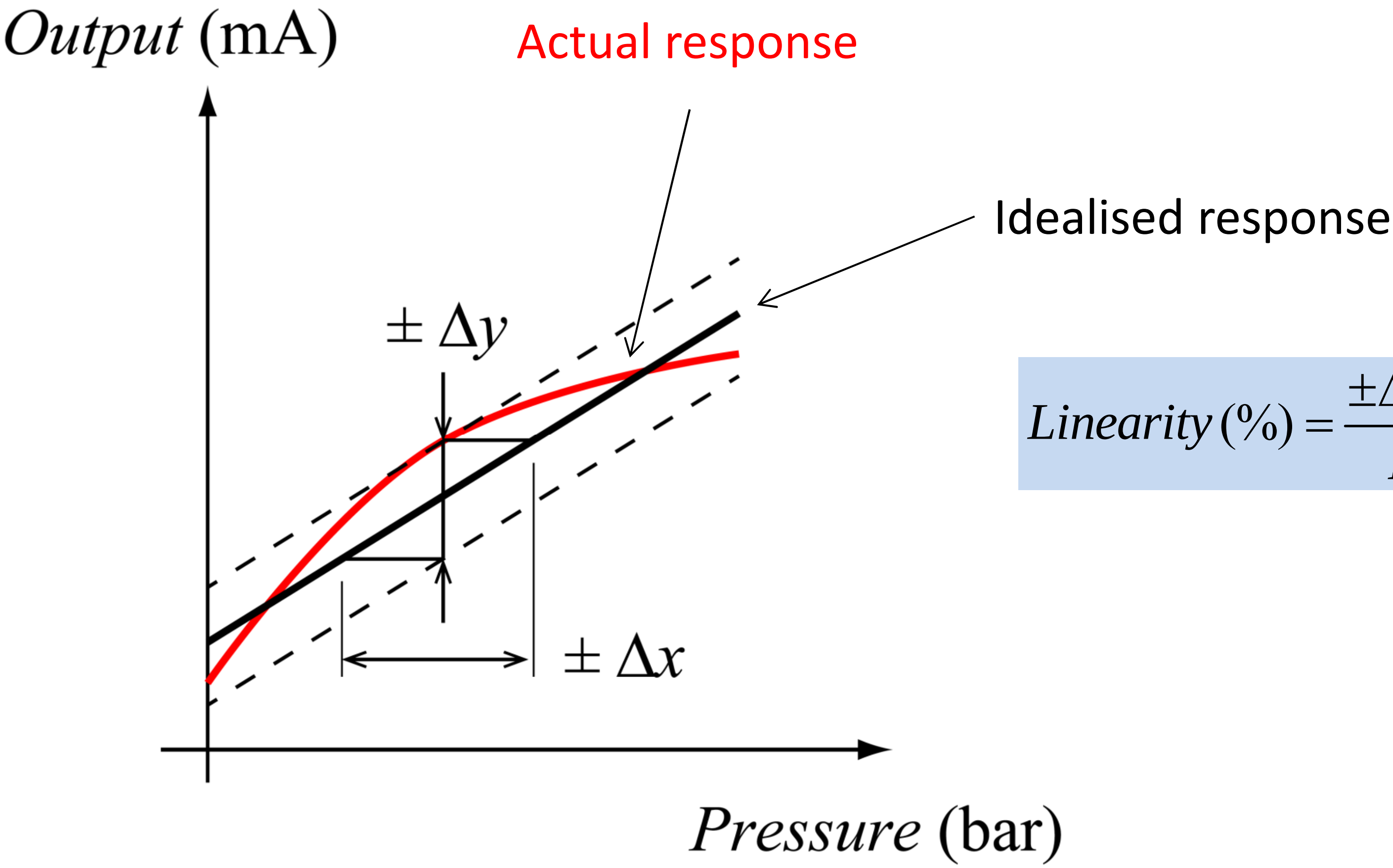
$$D = \sum_{i=1}^{N} (y_i - ax_i - b)^2$$

$$\frac{\partial D}{\partial a} = 0, \frac{\partial D}{\partial b} = 0$$

$$a = \frac{\sum_{i=1}^{N} x_i y_i - \frac{1}{N} \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} x_i^2 - \frac{1}{N} \left(\sum_{i=1}^{N} x_i \right)^2}$$

$$b = \overline{y} - a\overline{x} = \frac{\frac{1}{N} \sum_{i=1}^{N} x_i^2 \sum_{i=1}^{N} y_i - \frac{1}{N} \sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2 - \frac{1}{N} \left(\sum_{i=1}^{N} x_i \right)^2}$$

Example of linearity

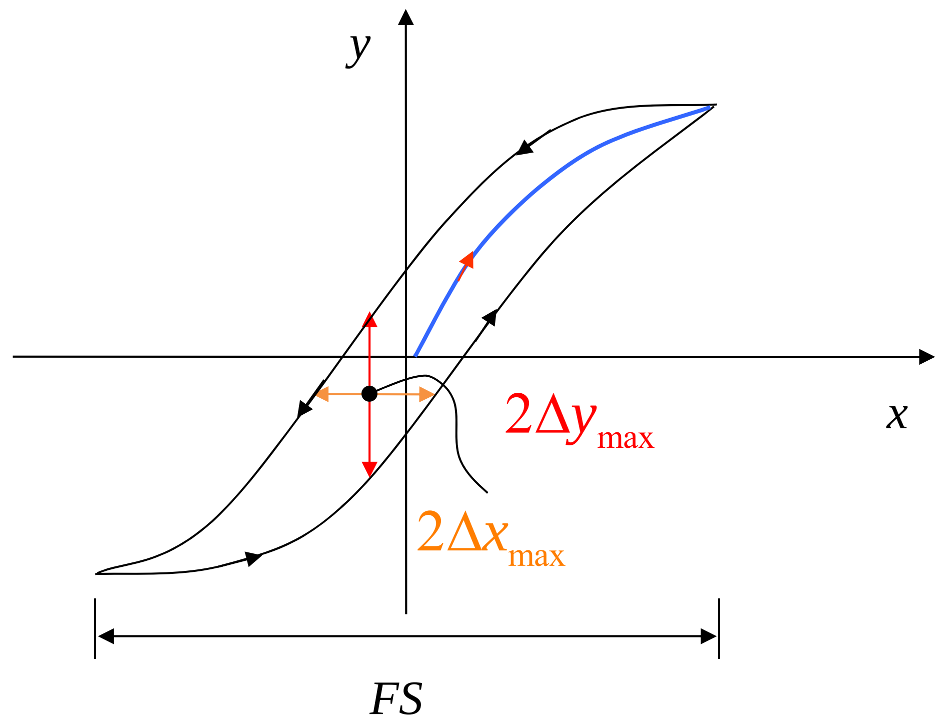


$$Linearity\,(\%) = \frac{\pm \Delta x_{\max}}{FS}$$

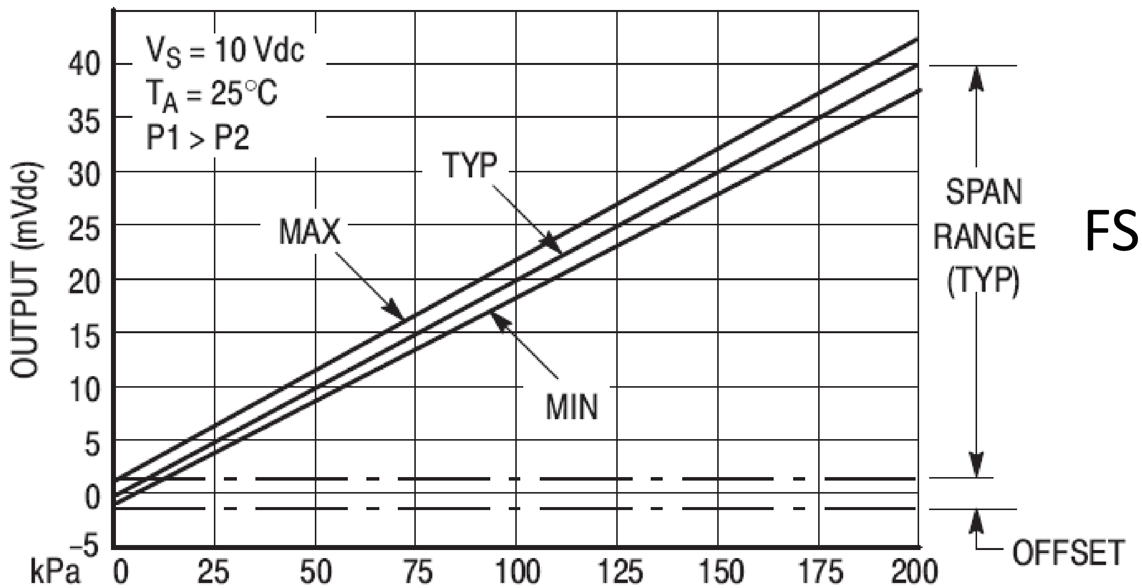
Hysteresis

- Response depends on history
  - Magnetic polarisation
  - Piezoelectric polarisation
  - Friction

$$Hysteresis\,(\%) = \frac{\pm\Delta x_{\max}}{FS}$$

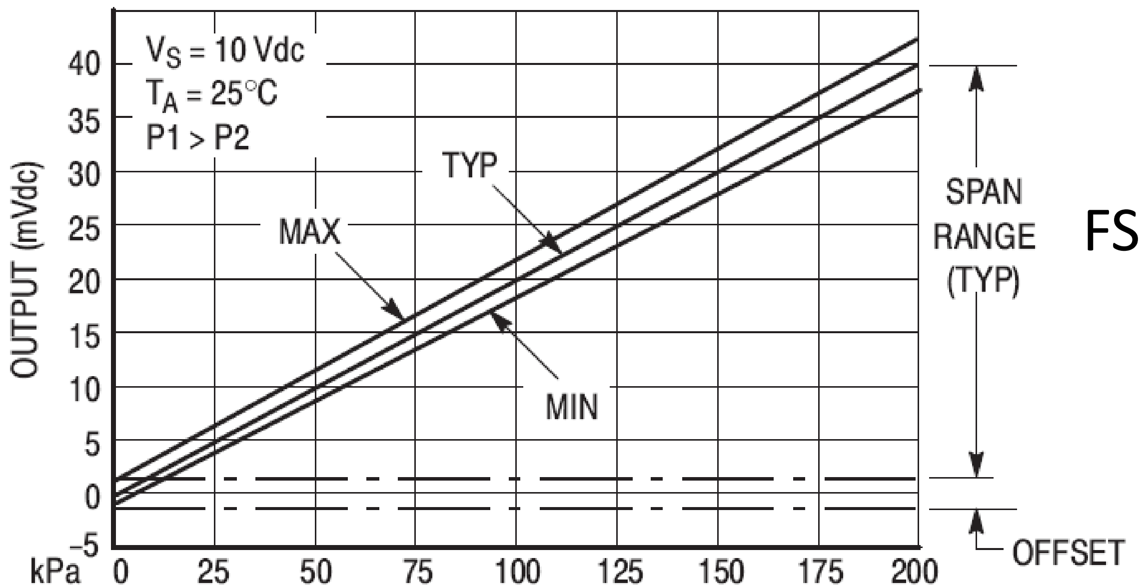


Example: pressure sensor datasheet



Pressure range	FS	0-200	kPa
Offset	V_{off}	±1	mV
Sensitivity	ΔV/ΔP	0.2	mV/kPa
Linearity		±0.5%	FS
Hysteresis		±0.5%	FS

Example: pressure sensor datasheet



Pressure range	FS	0-200	kPa
Offset	Voff	±1	mV
Sensitivity	ΔV/ΔP	0.2	mV/kPa
Temperature effect on FS (0 to 50°C, Tref=25°C)	T_FS	±2%	FS
Temperature effect on Offset (0 to 50°C, Tref=25°C)	T_OFF	±1	mV

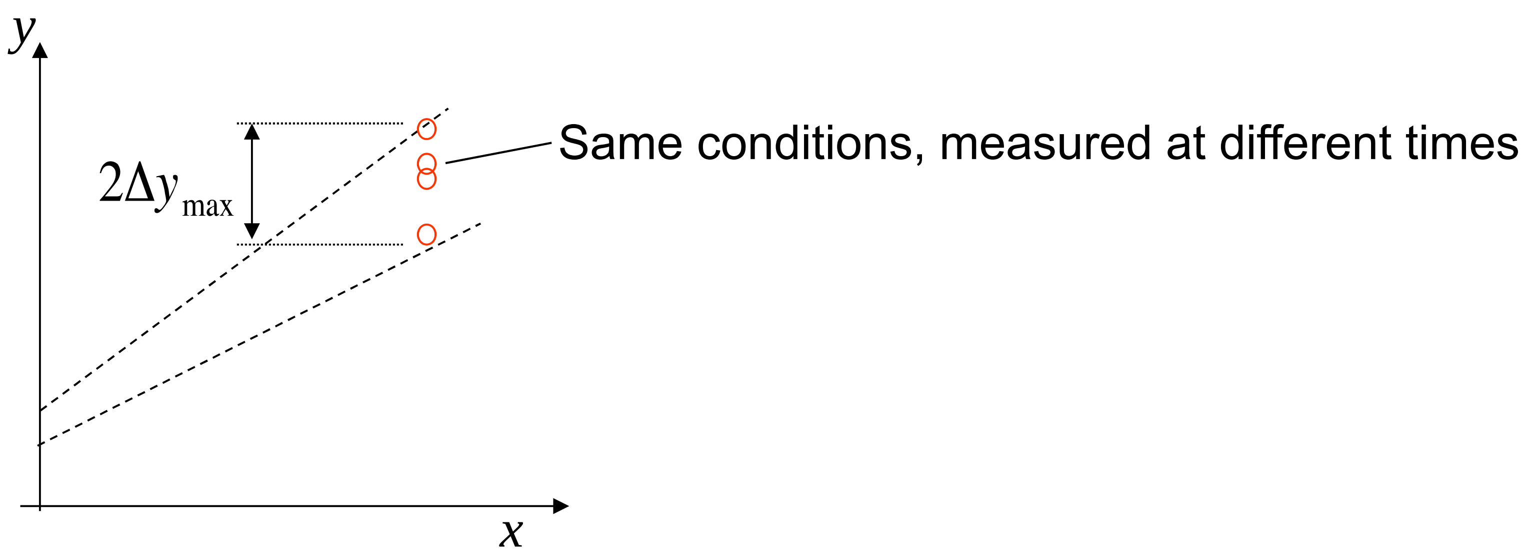
$$Err_{T,\_FS} = \pm 2 \cdot 200/100 = 4\text{kPa}$$

$$Err_{T,\_OFF} = \pm 1/0.2 = 5\text{kPa}$$

Repeatability

- Distribution of successive measurements of y under the same conditions

$$Repeatability\,(\%) = \frac{\pm\Delta x_{\max}}{FS}$$



Stability

- The ability to maintain a response y for a constant x and during a given time period

Pressure range	FS	0-200	kPa
Offset	Voff	±1	mV
Sensitivity	ΔV/ΔP	0.2	mV/kPa
Offset Stability		±0.5%	FS

$$Err_{Stab} = \pm\ 0.5 \cdot 200/100 = \pm 1 kPa$$

Maximal and probable error

- Maximal error

$$Error_{\max} = \pm \sum_i \left| \Delta x_i \right|$$

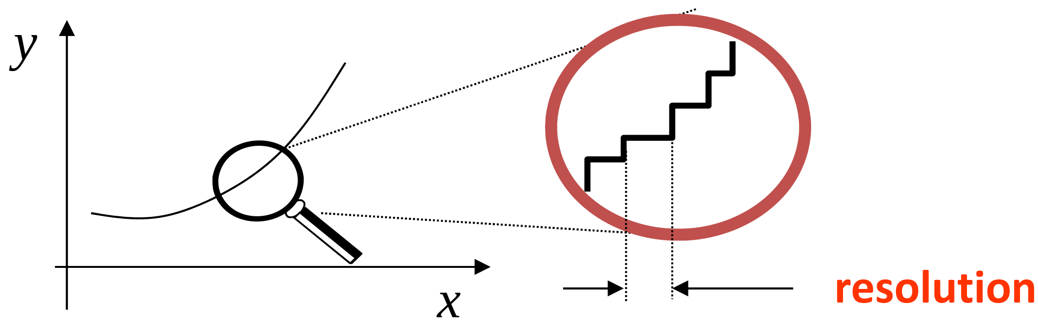
- Probable error

$$Error_{probable} = \pm \sqrt{\sum_i \Delta x_i^{\ 2}} = \pm \sigma$$

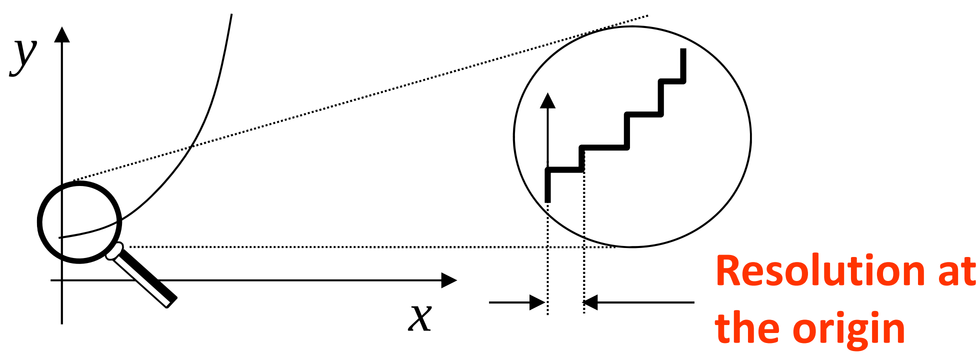
65% probability the actual error is within [-σ,+σ]
98% probability the actual error is within [-2σ,+2σ]
99% probability the actual error is within [-3σ,+3σ]

Resolution and threshold

- Resolution – the smallest detectable change of the input value

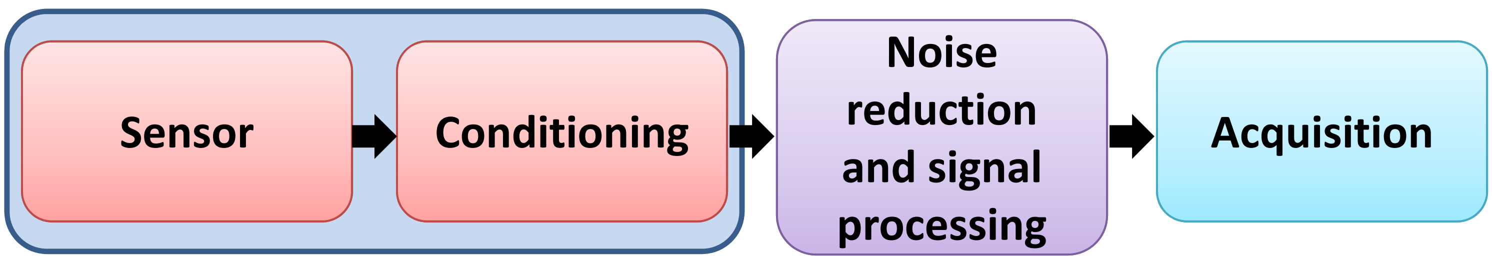


- Threshold – resolution at the origin (input = 0)



Transfer characteristics of conditioning circuits

- A measurement system is not just the sensor but the entire measurement chain
- In order to determine the global transfer characteristics, one must take into account also the transfer characteristics of conditioning circuits
- Achieving the highest sensitivity of the circuit is a common design goal

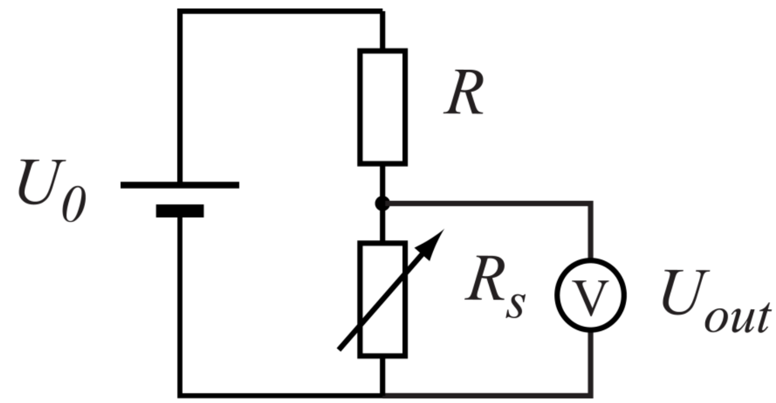


Example: voltage divider

$$U_{out} = U_0 \frac{R_s}{R_s + R}$$

$$U_{out} + \Delta U_{out} = U_0 \frac{R_s + \Delta R_s}{R_s + \Delta R_s + R} =$$

$$= U_0 \frac{R_s + \Delta R_s}{R_s + R} \frac{1}{1 + \frac{\Delta R_s}{R_s + R}}$$



For $\Delta R_s << R + R_s$

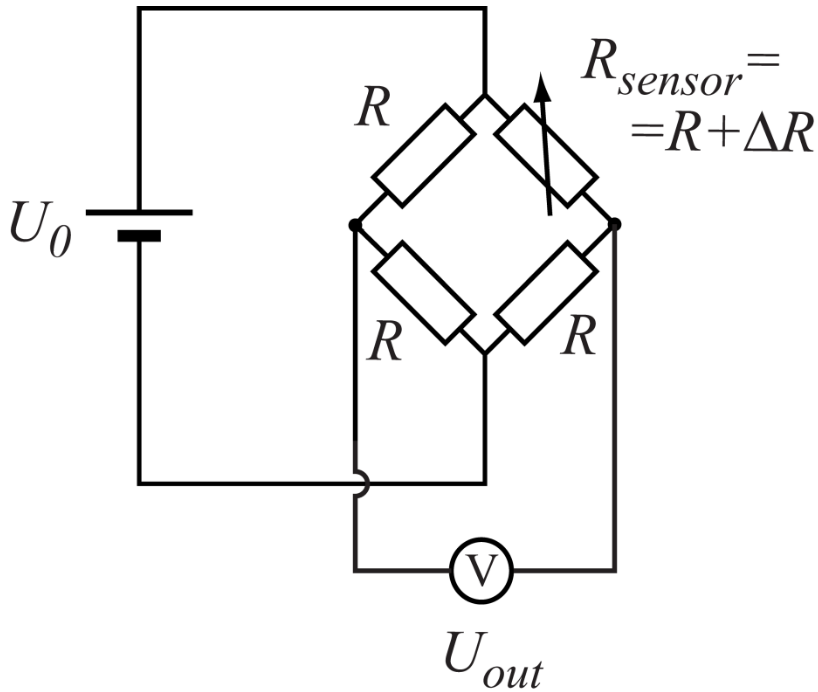
$$U_{out} + \Delta U_{out} = U_0 \frac{R_s + \Delta R_s}{R_s + R}\left(1 - \frac{\Delta R_s}{R_s + R}\right)$$

Linear with ΔR_s

$$\Delta U_{out} = U_0 \frac{R_s + \Delta R_s}{R_s + R}\left(1 - \frac{\Delta R_s}{R_s + R}\right) - U_0 \frac{R_s}{R_s + R} = \frac{U_0 R}{(R_s + R)^2} \Delta R_s$$

Max sensitivity for $R = R_s$. In that case $\Delta U_{out} = U_0 \frac{\Delta R_s}{4R}$

Example: Wheatstone bridge



$$U_{out} = \frac{1}{2}U_0 - \frac{R}{2R+\Delta R}U_0$$

$$= \frac{\Delta R}{4(R+\frac{\Delta R}{2})}U_0$$

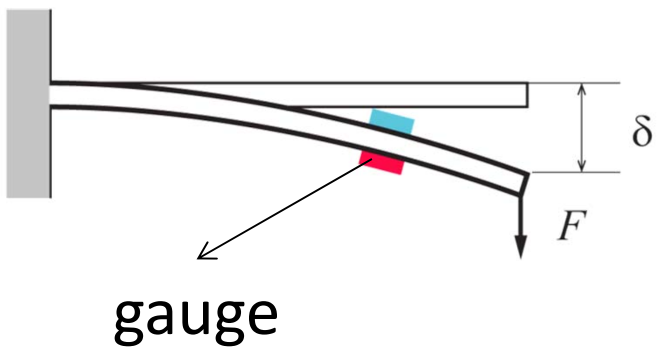
$$\approx \frac{\Delta R}{4R}U_0$$

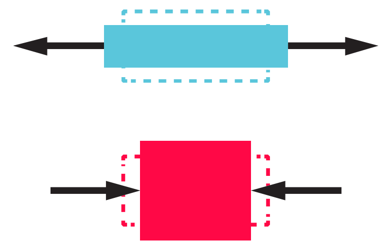
- No power supply noise for a balanced bridge

- The effect of temperature can be compensated by choosing resistors with the same temperature coefficient as the sensor

Force sensor (repeated from Ch 1)

- Based on a strain sensor attached to a test object





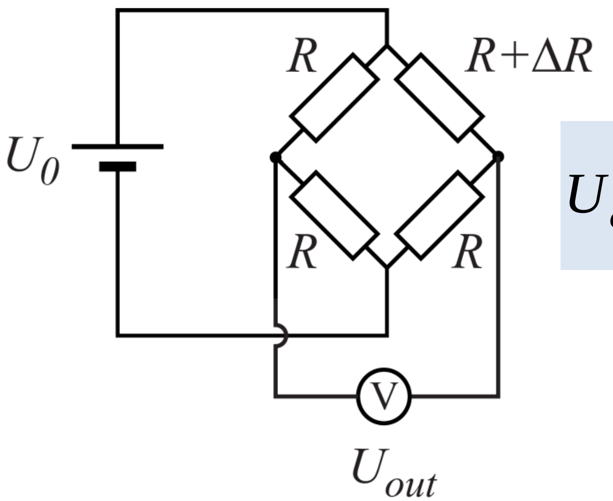
extension $\Delta l / l > 0$

compression $\Delta l / l < 0$

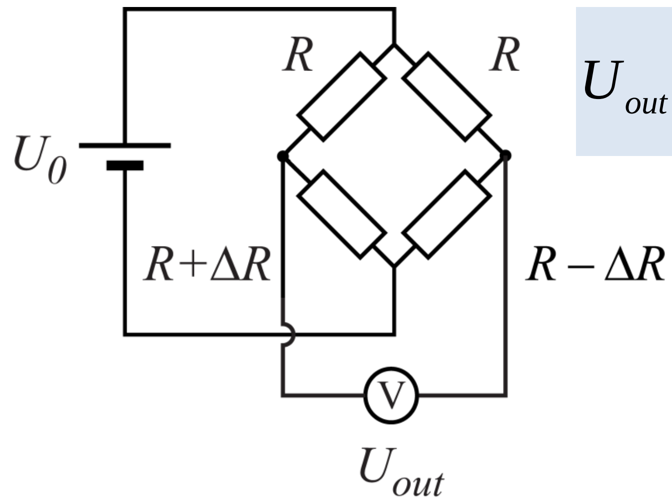
Force F → Deformation of the test object → Resistance change of the gauge → Voltage drop in the circuit

$$\frac{\Delta l}{l} = \frac{F}{A \cdot Y} \qquad \frac{\Delta R}{R} = K \frac{\Delta l}{l} \qquad \frac{\Delta U}{U} = \frac{\Delta R}{R} I$$

Wheatstone bridge – sensitivity optimisation



$$U_{out} \approx \frac{\Delta R}{4R} U_0$$



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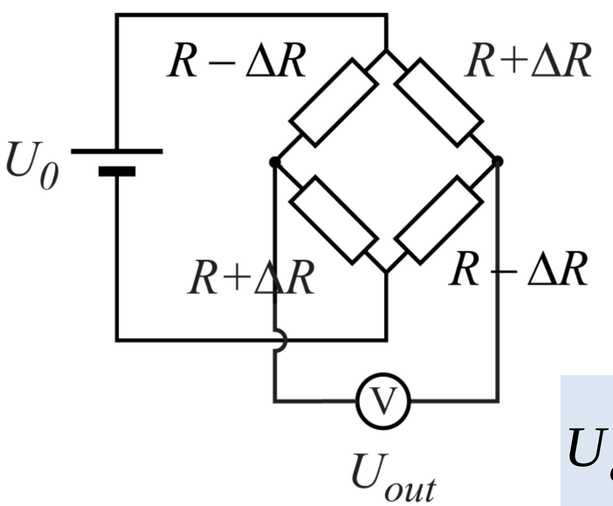

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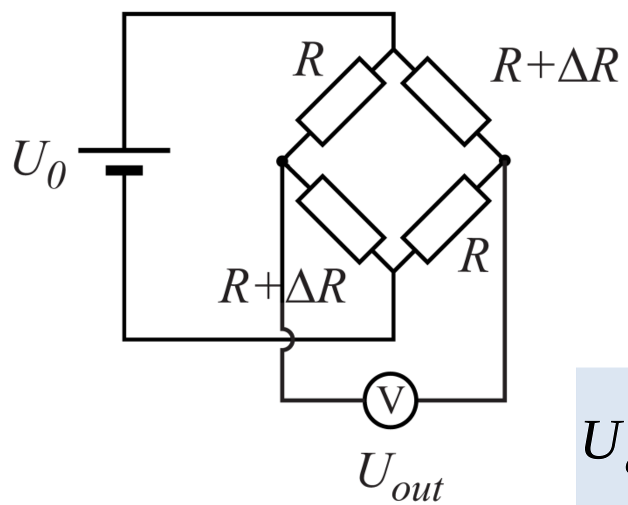
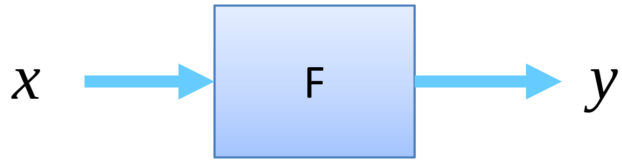


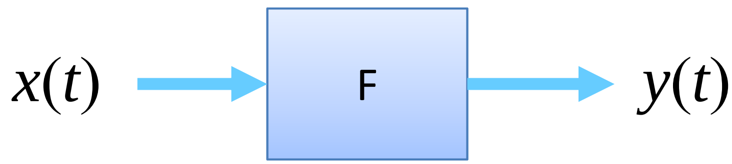
$$U_{out} \approx \frac{\Delta R}{2R} U_0$$

Dynamic transfer characteristics



- Identification of the system order
- Calculating sensitivity
- Time-dependent response
- Frequency-dependent response

Dynamic transfer characteristics



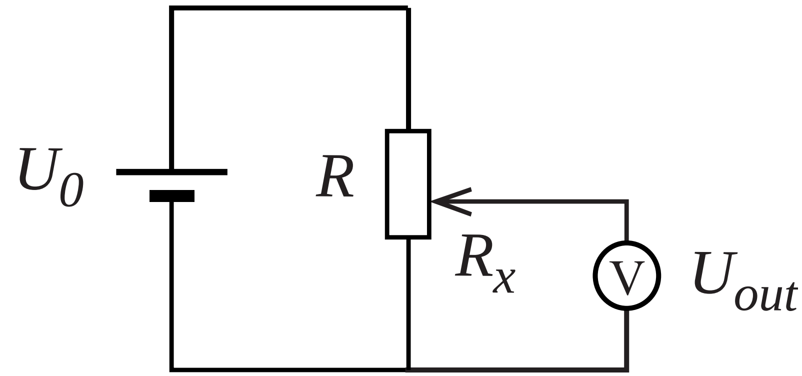
- A time-dependent input will result in a time-dependent output
- This can be described by an ordinary differential equation

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \ldots + a_1 \frac{dy}{dt} + a_o y = x(t)$$

n: order

- Order 0: $y(t) = S \cdot x(t)$ S: sensitivity
- 1st order: $\tau \frac{dy}{dt} + y = S \cdot x$ τ: time constant
- 2nd order: $\frac{1}{\omega_0^2} \frac{d^2 y}{dt^2} + \frac{2\xi}{\omega_0} \frac{dy}{dt} + y = S \cdot x$ ω_0 : undamped frequency; ξ : damping factor

Zero-order system: potentiometer



$$U_{out} = \frac{R_x}{R} U_0 = \frac{\rho \cdot \ell_x / A}{\rho \cdot \ell / A} U_0$$

$$= \frac{\ell_x}{\ell} U_0 = S \cdot \ell_x$$

The model neglects:

- induced EMF in the output loop (dx/dt)
- inductance of the potentiometer
- mechanical properties: mass of the sliding part, friction

ℓ_x : length of section x

ℓ : total length

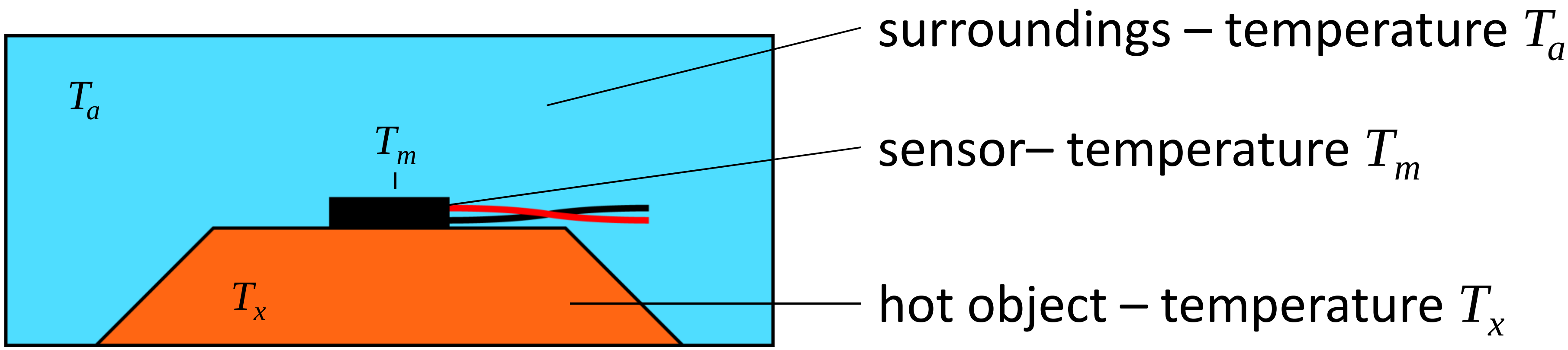
A : wire cross-section

1st order system

$$\tau \frac{dy}{dt} + y = S \cdot x$$

τ: time constant

- Example: temperature measurements



Heating power for object -> sensor

$$P_x = G_{xm}(T_x - T_m)$$

Heating power for surroundings -> sensor

$$P_a = G_{am}(T_a - T_m)$$

G_{xm}: heat conductivity for the sensor/object interface

G_{am}: heat conductivity for the sensor/ambient interface

1^{st} order system

$$\frac{dQ_m}{dt} = C\frac{dT_m}{dt}$$

C : heat capacity of the sensor
Q_m : amount of heat received by the sensor

$$P_a + P_x = \frac{dQ_m}{dt}$$

$$G_{am}(T_a - T_m) + G_{xm}(T_x - T_m) = C\frac{dT_m}{dt}$$

$$\tau\frac{dy}{dt} + y = S\cdot x$$

$$\frac{C}{G_{am}+G_{xm}}\frac{dT_m}{dt} + T_m = \frac{G_{xm}T_x}{G_{am}+G_{xm}} + \boxed{\frac{G_{am}T_a}{G_{am}+G_{xm}}}$$

← Interfering input

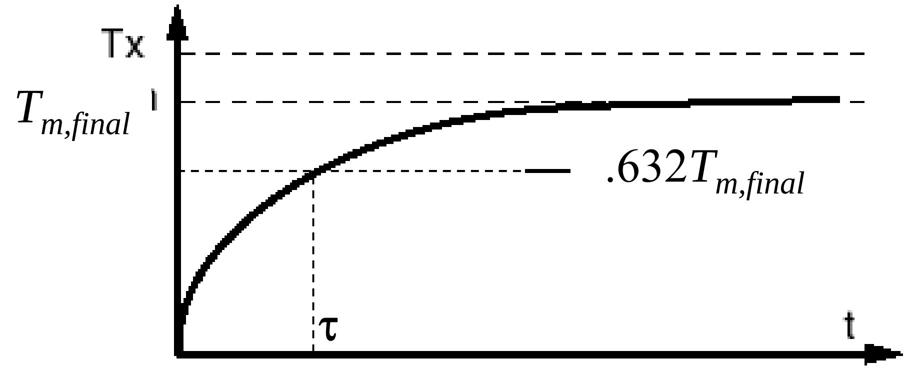
$$\tau = \frac{C}{G_{xm}+G_{am}} \qquad S = \frac{G_{xm}}{G_{am}+G_{xm}}$$

Solution of the equation:

$$T_m = T_{m,\,final}\left(1 - e^{-t/\tau}\right)$$

1^{st} order system

$$T_m = T_{m,\,final}\left(1 - e^{-t/\tau}\right)$$



$T_{m,final}$ from condition $\dfrac{dT_m}{dt} = 0$

$$T_{m,\,final} = \frac{G_{am}T_a + G_{xm}T_x}{G_{am} + G_{xm}} \qquad \tau = \frac{C}{G_{xm} + G_{am}}$$

For $T_{m,final}$ to be close to T_x:

- G_{xm} : as high as possible (good thermal contact sensor/object)
- G_{am} : as low as possible (bad contact sensor/ambient)

For fast measurements (low τ):

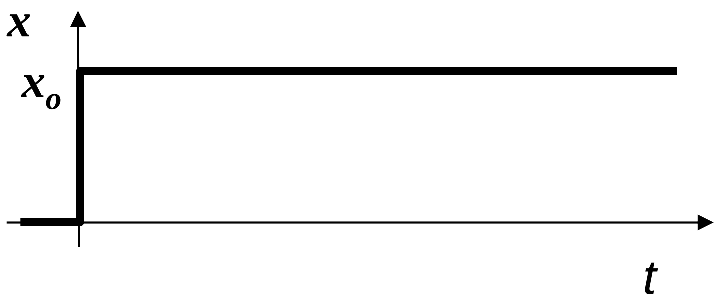
- C : as low as possible (small thermometer)

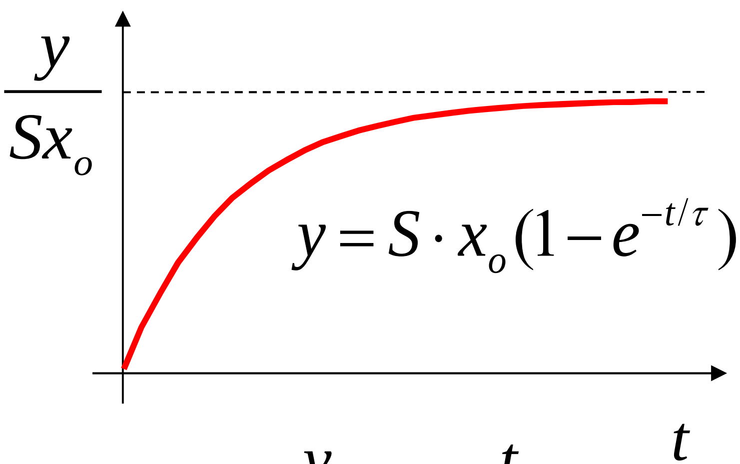
How can we determine S and τ?

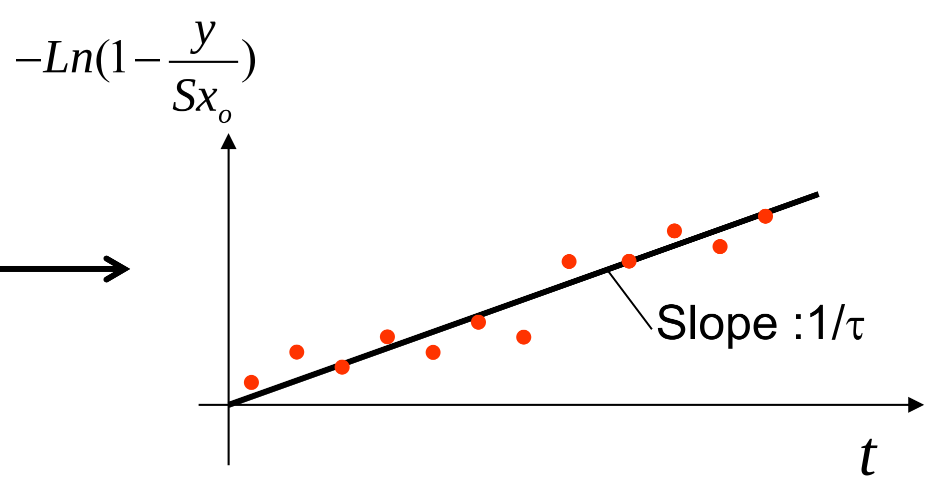
$$\tau \frac{dy}{dt} + y = S \cdot x$$

S : static calibration (keep x constant, wait until y becomes constant)

τ : apply a stepwise change of input



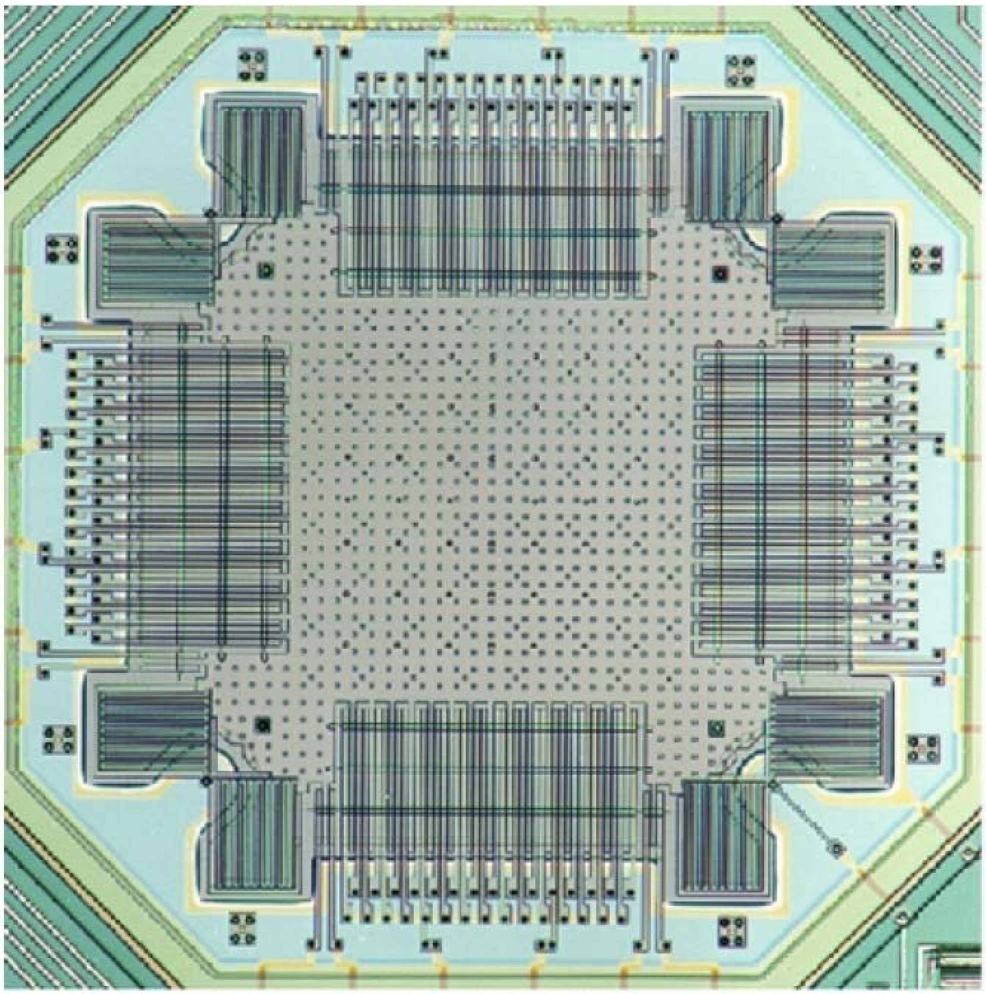


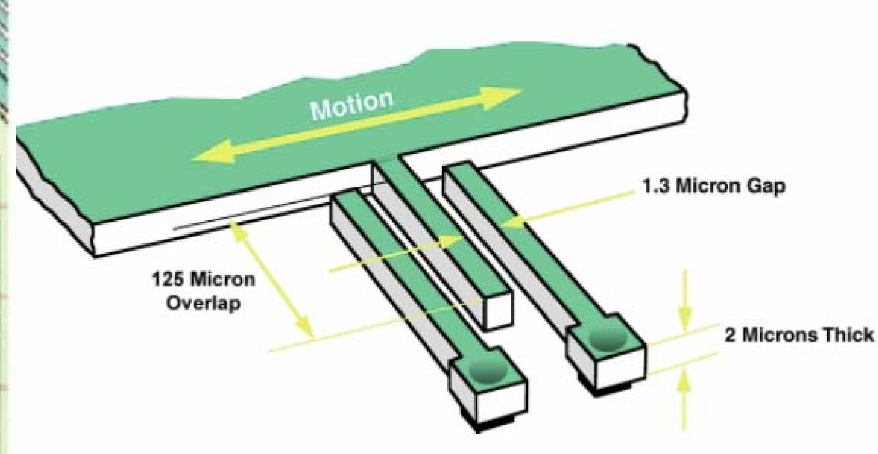


$$Ln(1 - \frac{y}{Sx_o}) = -\frac{t}{\tau}$$

2nd order system: accelerometer

- MEMS (micro-electromechanical system) accelerometer

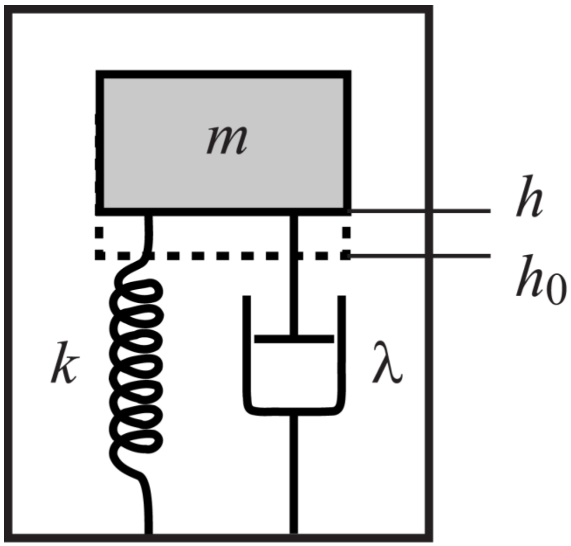




ADXL202 accelerometer
Analog Devices website

- www.analog.com

2^{nd} order system: accelerometer



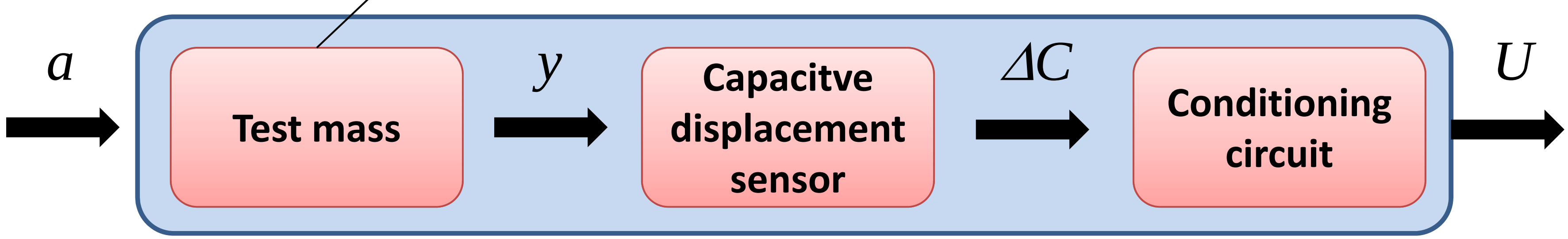
h : position of the test mass

h_0 : position of the accelerometer (package)

$y = h\text{-}h_0$

$$\frac{1}{\omega_0^2}\frac{d^2y}{dt^2} + \frac{2\xi}{\omega_0}\frac{dy}{dt} + y = S \cdot a$$

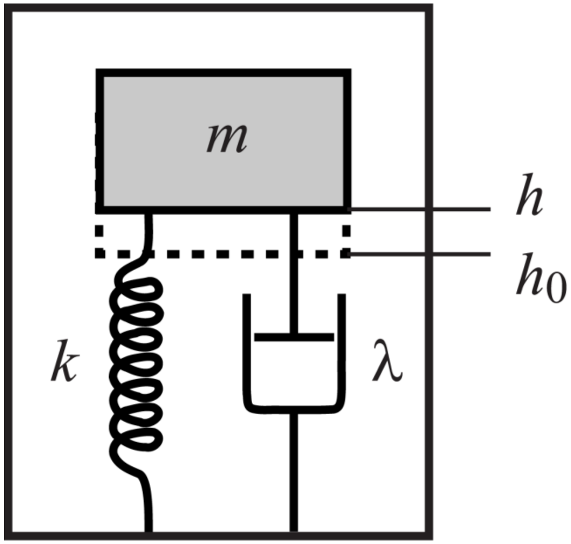
General equation for a 2^{nd} order system



0 order

0 order

2nd order system



$$\frac{1}{\omega_0^2}\frac{d^2y}{dt^2}+\frac{2\xi}{\omega_0}\frac{dy}{dt}+y=S\cdot a$$

$$a=-\frac{d^2h_0}{dt^2}$$ Acceleration of the accelerometer package

$$\sum \vec{F}=m\vec{a}_M \qquad y=h\text{-}h_0$$

$$S=\frac{m}{k}=\frac{1}{\omega_0^{\;2}}$$

$$\lambda\frac{dy}{dt}+ky=-m\frac{d^2h}{dt^2}$$

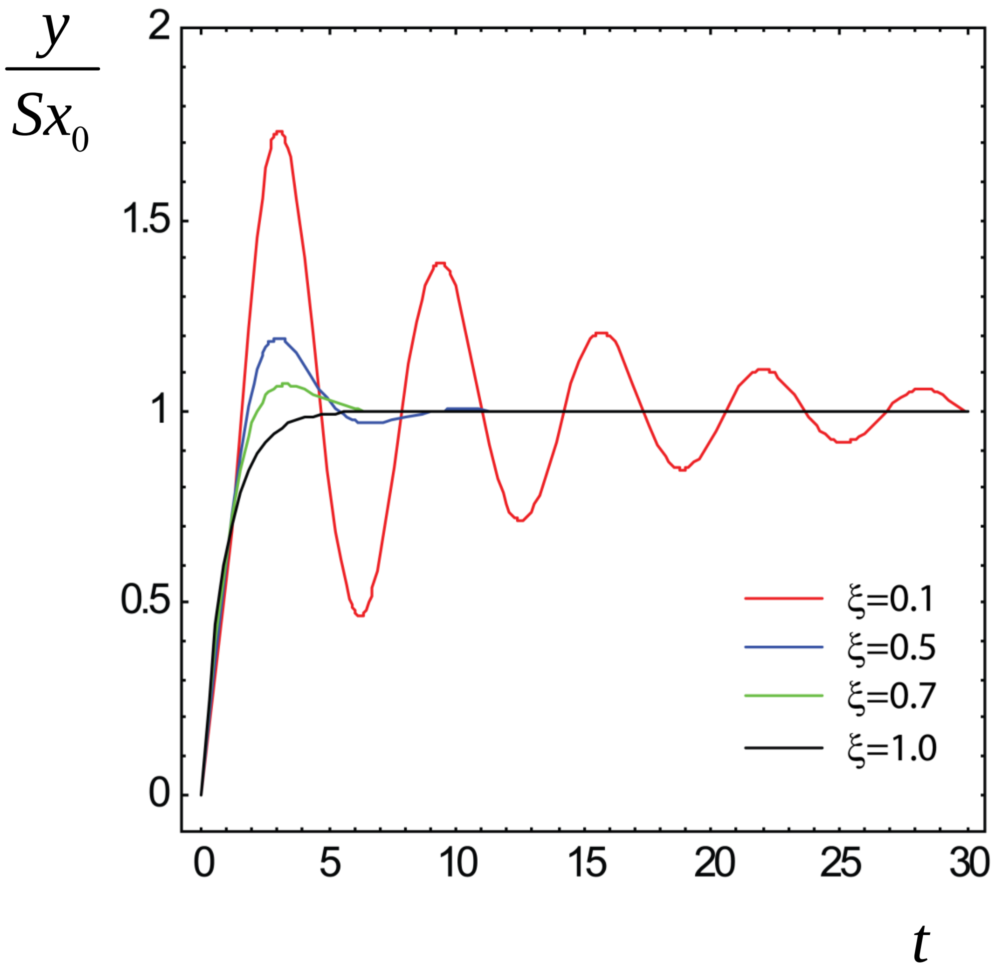
$$\omega_0=\sqrt{\frac{k}{m}}$$

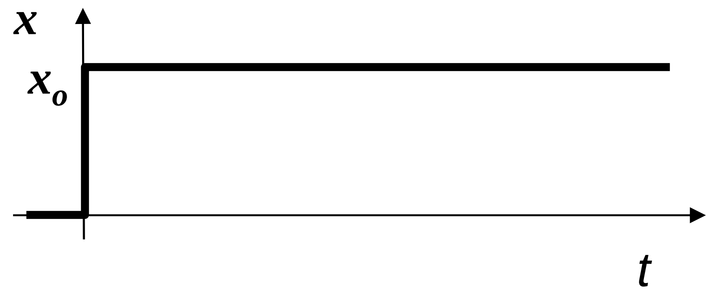
$$m\frac{d^2y}{dt}+\lambda\frac{dy}{dt}+ky=-m\frac{d^2h_o}{dt^2}$$

$$\xi=\frac{\lambda}{2\sqrt{km}}$$

2^{nd} order system

- Response to a step change of input $$\frac{1}{\omega_0^2}\frac{d^2y}{dt^2}+\frac{2\xi}{\omega_0}\frac{dy}{dt}+y=S\cdot x$$





For $\xi < 1$

$$\frac{y}{Sx_o}=1-\frac{e^{-\xi\omega_o t}}{\sqrt{1-\xi^2}}\sin\left(\sqrt{1-\xi^2}\,\omega_o t+\phi\right)$$

$$\phi = Arc\sin\left(\sqrt{1-\xi^2}\right)$$

2nd order system

- Response to a step change of input

overdamped

$\xi > 1$

$$\frac{y}{Sx_o} = 1 + \frac{\xi - \sqrt{\xi^2 - 1}}{2\sqrt{\xi^2 - 1}} e^{-(\xi + \sqrt{\xi^2 - 1})\omega_o t} - \frac{\xi + \sqrt{\xi^2 - 1}}{2\sqrt{\xi^2 - 1}} e^{-(\xi - \sqrt{\xi^2 - 1})\omega_o t}$$

critical damping

$\xi = 1$

$$\frac{y}{Sx_o} = 1 - \omega_o t \cdot e^{-\omega_o t}$$

underdamped

$\xi < 1$

$$\frac{y}{Sx_o} = 1 - \frac{e^{-\xi \omega_o t}}{\sqrt{1 - \xi^2}} \sin\left(\sqrt{1 - \xi^2}\,\omega_o t + \varphi\right)$$

with $\varphi = Arc\sin\left(\sqrt{1 - \xi^2}\right)$

2^{nd} order system

- Response to pulsed change of input (pulse width << period of oscillations)

overdamped
$\xi > 1$

$$\frac{y}{Sx_o\omega_o} = \frac{e^{-\xi\omega_o t}}{\sqrt{\xi^2 - 1}} \sinh(\sqrt{\xi^2 - 1})\omega_o t)$$

critical damping
$\xi = 1$

$$\frac{y}{Sx_o\omega_o} = \omega_o t e^{-\omega_o t}$$

underdamped
$\xi < 1$

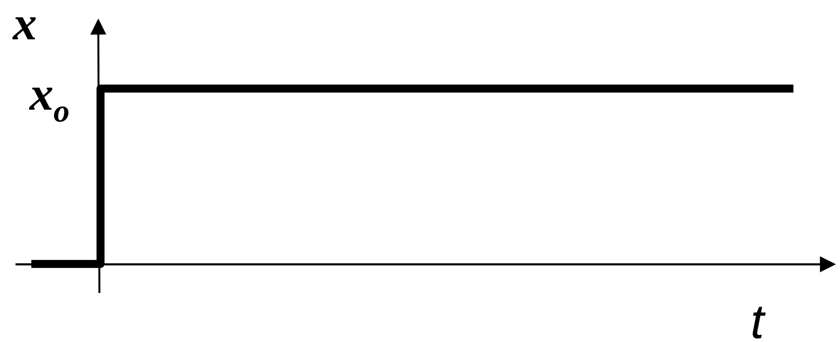
$$\frac{y}{Sx_o\omega_o} = \frac{e^{-\xi\omega_o t}}{\sqrt{1 - \xi^2}} \sin\left(\sqrt{1 - \xi^2}\,\omega_o t\right)$$

How can we determine these parameters?

$\boldsymbol{S}$: static calibration (keep input constant, wait until output becomes constant)

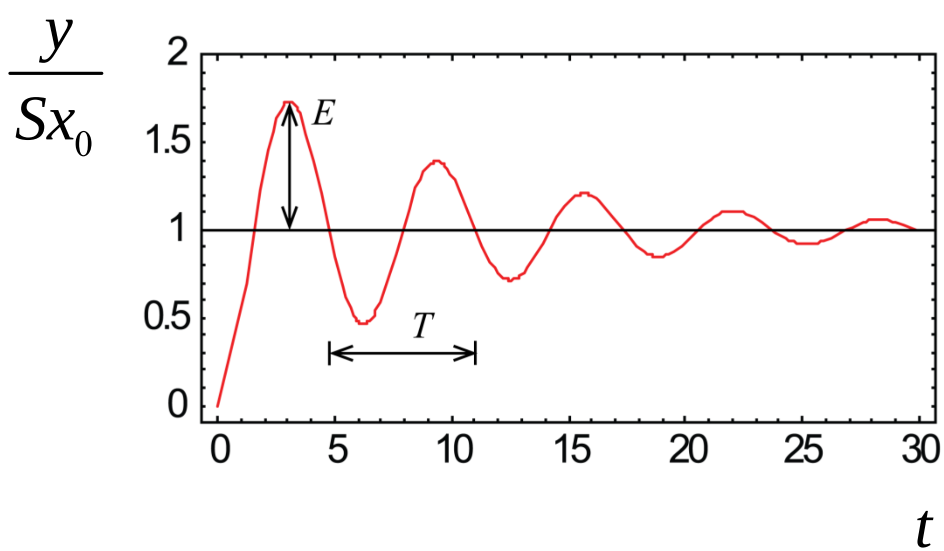
ω_0 **and** ξ :apply a step-wise change of the input

$$\frac{1}{\omega_0^2}\frac{d^2y}{dt^2}+\frac{2\xi}{\omega_0}\frac{dy}{dt}+y=S\cdot x$$



Other parameters:

Measure E (excursion) and T (period)



2^{nd} order system – stepped input

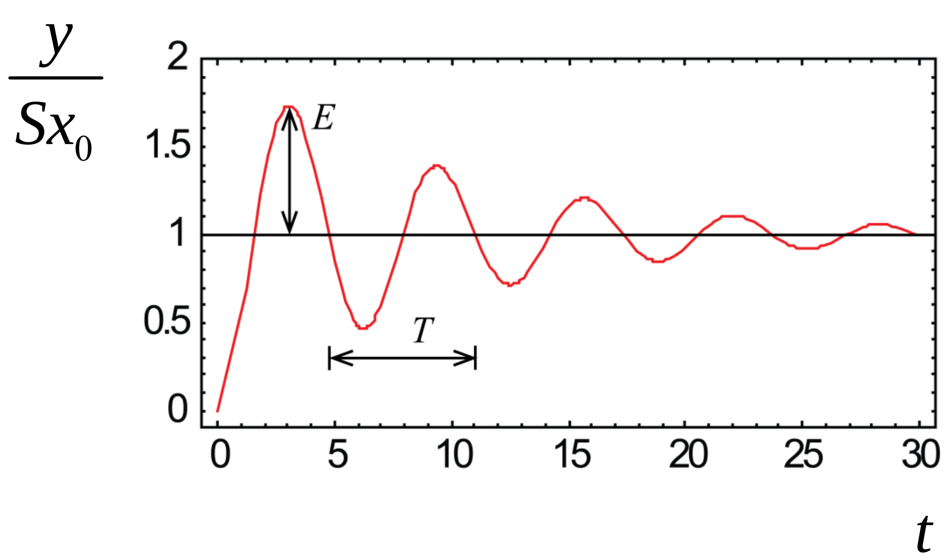
$$\frac{y}{Sx_o} = 1 - \frac{e^{-\xi\omega_o t}}{\sqrt{1-\xi^2}} \sin\left(\underbrace{\sqrt{1-\xi^2}\,\omega_o}_{\frac{2\pi}{T}} t + \varphi \right) \qquad \omega_0 = \frac{2\pi}{T\sqrt{1-\xi^2}}$$

Calculate ξ:

- find E from the condition $\frac{d}{dt}\left(\frac{y}{Sx_o}\right) = 0$

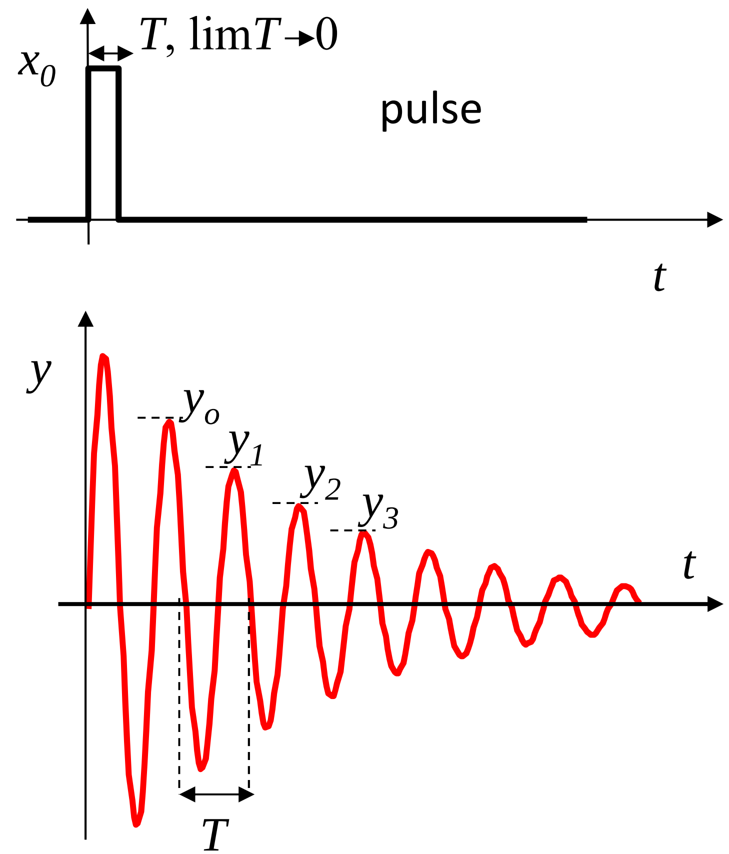
$$\xi = \frac{1}{\sqrt{\left(\frac{\pi}{LnE}\right)^2 + 1}}$$

$$\omega_0 = \frac{2\pi}{T\sqrt{1-\xi^2}}$$



2^{nd} order system – pulsed input

$$\frac{1}{\omega_0^2}\frac{d^2y}{dt^2}+\frac{2\xi}{\omega_0}\frac{dy}{dt}+y=S\cdot x$$



S : static calibration (keep input constant, wait until output becomes constant)

ω_0 **and** ξ :apply a pulsed input

Measure y_0/y_i and T

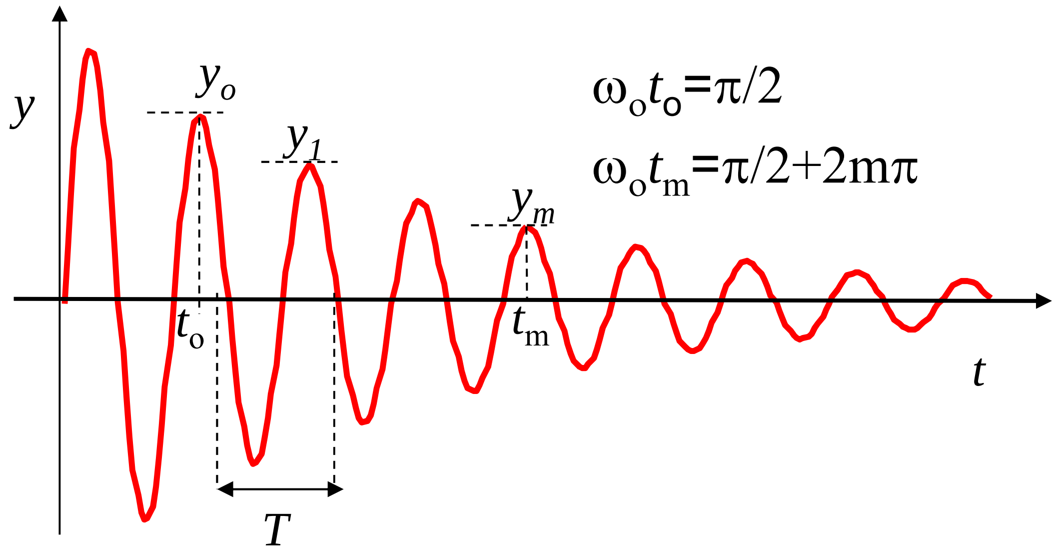
2nd order system – pulsed input

$$\frac{y}{Sx_o\omega_o} = \frac{e^{-\xi\omega_o t}}{\sqrt{1-\xi^2}} \sin\left(\underbrace{\sqrt{1-\xi^2}\,\omega_o}_{\frac{2\pi}{T}} t \right)$$

$$\omega_0 = \frac{2\pi}{T\sqrt{1-\xi^2}}$$ for $\xi < 1$

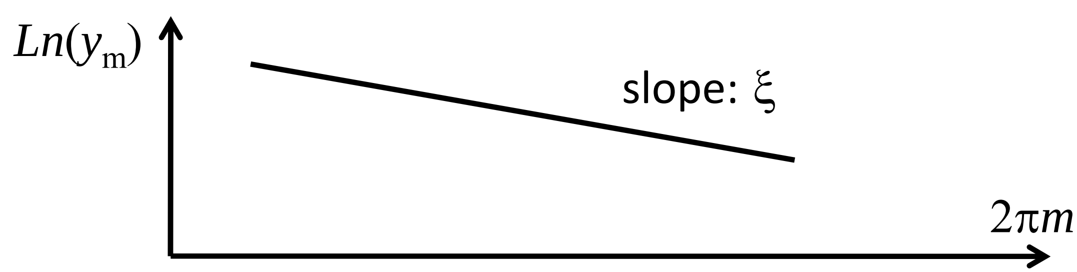
Calculating ξ: deduce y_i from the maxima of y

$$\xi \approx \frac{Ln\dfrac{y_o}{y_m}}{2\pi m}$$



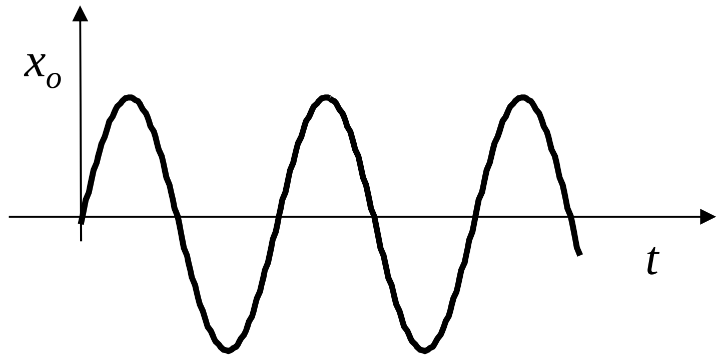
$$Lny_m = Lny_0 - 2\pi m\xi$$

assuming $\sqrt{1-\xi^2} \approx 1$



2^{nd} order system – oscillating input

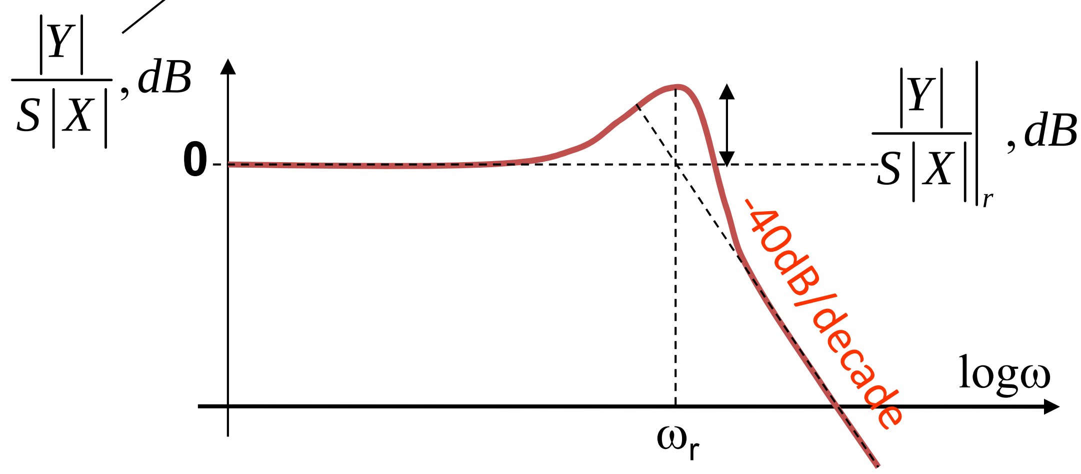
$$\frac{1}{\omega_0^2}\frac{d^2y}{dt^2}+\frac{2\xi}{\omega_0}\frac{dy}{dt}+y=S\cdot x$$



S : static calibration (keep input constant, wait until output becomes constant)

ω_0 **and** ξ :apply an oscillating input with a variable frequency ω

$|Y|, |X|$: output and input amplitudes



Measure the maximum

$$\frac{|Y|}{S|X|}=\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_o^2}\right)^2+4\xi^2\frac{\omega^2}{\omega_o^2}}}$$

2nd order system – oscillating input

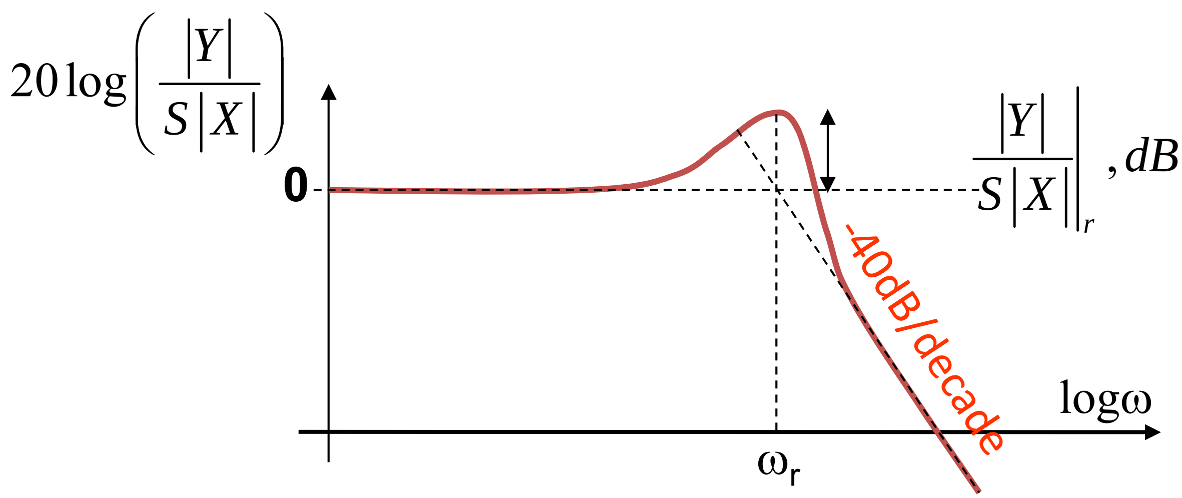
$$\frac{|Y|}{S|X|} = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_o^2}\right)^2 + 4\xi^2\frac{\omega^2}{\omega_o^2}}}$$

$\omega = \omega_r$ for $\frac{|Y|}{S|X|} = max$ or $\left(1-\frac{\omega^2}{\omega_o^2}\right)^2 + 4\xi^2\frac{\omega^2}{\omega_o^2} = min$

$$\omega_0 = \frac{\omega_r}{\sqrt{1-2\xi^2}}$$

For $\omega \gg \omega_r$

$$\frac{|Y|}{S|X|} \approx \frac{1}{\sqrt{\frac{\omega^4}{\omega_0^4} + 4\xi\frac{\omega^2}{\omega_0^2}}} \approx \frac{\omega_0^2}{\omega^2}$$



$$20\cdot\log(100) = 40$$

Key points

- A measurement system is often modelled by a linear differential equation of the order 0, 1 or 2
- Static transfer parameters allow us to predict the measurement error
- The sensitivity and dynamic transfer parameters allow us to identify the order of the system