Measuring Systems

Problem set n° 7

Exercise 1 (Acquisition and Nyquist sampling theorem)

a) The signal can be the superposition of three AC signals with the following frequencies:

$F_1 = 1000 \text{ Hz}$
$F_2 = 3000 \text{ Hz}$
$F_3 = 6000 \text{ Hz}$

Thus $F_{max} = 6000 \text{ Hz}$ and according to the sampling theorem:

$F_s > 2F_{max} = 12000 \text{ Hz}$. The Nyquist rate is: $F_N = 2F_{max} = 12000 \text{ Hz}$

b) The discrete-time signal of the signal sampled at $5000\ samples/sec$ is

$$u(n) = 3\cos\left(\frac{2000\pi}{5000}\right)n + 5\sin\left(\frac{6000\pi}{5000}\right)n + 10\cos\left(\frac{12000\pi}{5000}\right)n$$
$$= 3\cos 2\pi\left(\frac{1}{5}\right)n + 5\sin 2\pi\left(\frac{3}{5}\right)n + 10\cos 2\pi\left(\frac{6}{5}\right)n$$
$$= 3\cos 2\pi\left(\frac{1}{5}\right)n + 5\sin 2\pi\left(1-\frac{2}{5}\right)n + 10\cos 2\pi\left(1+\frac{1}{5}\right)n$$
$$= 3\cos 2\pi\left(\frac{1}{5}\right)n + 5\sin 2\pi\left(-\frac{2}{5}\right)n + 10\cos 2\pi\left(\frac{1}{5}\right)n$$

Finally we obtain:

$$u(n) = 13\cos 2\pi\left(\frac{1}{5}\right)n - 5\sin 2\pi\left(\frac{2}{5}\right)n$$

Since $F_s = 5000\ samples/sec$, the cut-off frequency of the reconstruction filter is $\frac{F_s}{2} = 2500$ Hz. This is the maximum frequency that can be represented uniquely by the sampled signal.

c) Since only the frequency components at 1000 Hz and 2000 Hz are present in the sampled signal, the analog signal that can be recovered is

U(t) = 13cos 2π (1000)t – 5sin 2π (2000)t

which is obviously different from the original signal u(t). This distortion of the original analog signal was caused by the aliasing effect, due to the low sampling rate used.

d) One can find the maximum of the signal from $\frac{dU(t)}{dt} = 0$, however solving this equation is not straightforward. One strategy can be to plot the signal using Excel, MATLAB or Mathematica and then extract the maximum values (see Figure 1).

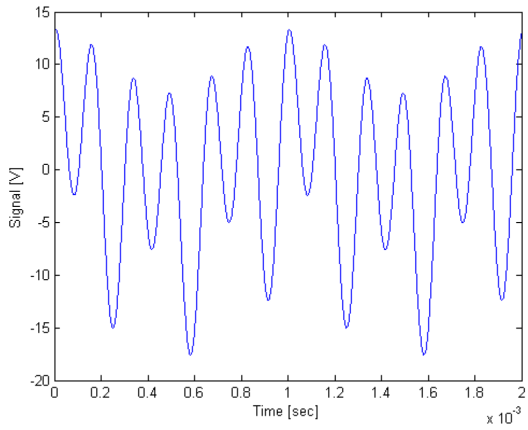

Figure 1.

➔ In the worst case, the maximum absolute value of U(t) cannot be more than the sum of the amplitudes of each sinusoidal component. Thus, the maximum peak to peak range is 2×(10+3+8) = 36 V. This means that in the worst case the full scale range should be 36V. If we use an A/D converter with 8 bits the corresponding resolution is:

$$\mathrm{R} = \frac{\mathrm{FS}}{2^{\mathrm{N}}} = \frac{36\mathrm{V}}{256} = 125\mathrm{mV}$$

Exercise 2 (extrinsic noise and asymmetric assembly)

The superposition principle is used for calculating the influence of powerline network $\underline{U}_{network}$ on the voltage at the input of the amplifier $\underline{U}_{i,n}\big|_{sec}$ (cf. Figure 2).

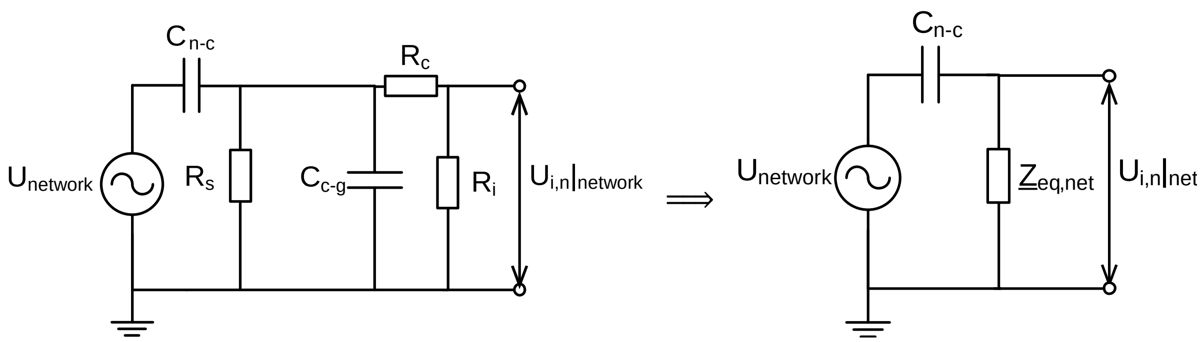

Figure 2: Simplified scheme for the input of the amplifier

It was assumed that:

$$R_i \gg R_w \qquad \text{and} \qquad \frac{1}{R_s} \gg \frac{1}{R_i}$$

We find the voltage $\underline{U}_{i,b}\big|_{network}$:

$$\underline{U}_{i,n}\big|_{net} = \frac{\underline{Z}_{eq,net}}{\underline{Z}_{eq,net} + \frac{1}{j \cdot \omega \cdot C_{n-c}}} \cdot \underline{U}_{net} \qquad \text{with: } \underline{Z}_{eq,net} = \frac{1}{\frac{1}{R_s} + j \cdot \omega \cdot C_{c-g}} = \frac{R_s}{1 + j \cdot \omega \cdot R_s \cdot C_{c-g}}$$

$$\underline{U}_{i,n}\big|_{net} = \frac{j \cdot \omega \cdot R_s \cdot C_{n-c}}{1 + j \cdot \omega \cdot R_s \cdot (C_{s-w} + C_{c-g})} \cdot \underline{U}_{net} = \frac{j \cdot 2 \cdot \pi \cdot f \cdot R_s \cdot C_{n-c}}{1 + j \cdot \frac{f}{fc}} \cdot \underline{U}_{net}$$

with: $f_c = \frac{1}{2 \cdot \pi \cdot R_s \cdot (C_{n-c} + C_{c-g})} = 1.45\ MHz$

Note that the cutoff frequency f_c is well above the frequency of the disturbance $f_{network}$. The parasitic voltage $\underline{U_{i,n}}\big|_{net}$ then is expressed as:

$$\underline{U_{i,n}}\big|_{net} = j \cdot 2 \cdot \pi \cdot f_{net} \cdot R_s \cdot C_{s-w} \cdot \underline{U_{net}}$$

Notice that we can simply calculate the voltage $\underline{U_{i,n}}\big|_{net}$ considering the current $\underline{I_c}$ of the capacitance C_{n-c} through the resistor R_s if the coupling C_{c-g} is considered negligible :

$$\underline{U_{i,n}}\big|_{sec} = R_s \cdot \underline{I_c} = R_s \cdot C_{n-c} \cdot \frac{d\underline{U_{network}}}{dt} = R_s \cdot C_{n-c} \cdot (j \cdot 2 \cdot \pi \cdot f_{net}) \cdot \underline{U_{net}}$$

The superposition principle is applied to find the contribution of the voltage $\underline{U_s}$ to the input voltage of the amplifier ($\underline{U_i}\big|_s$) and to analyze if it is influenced by the parasitic coupling:

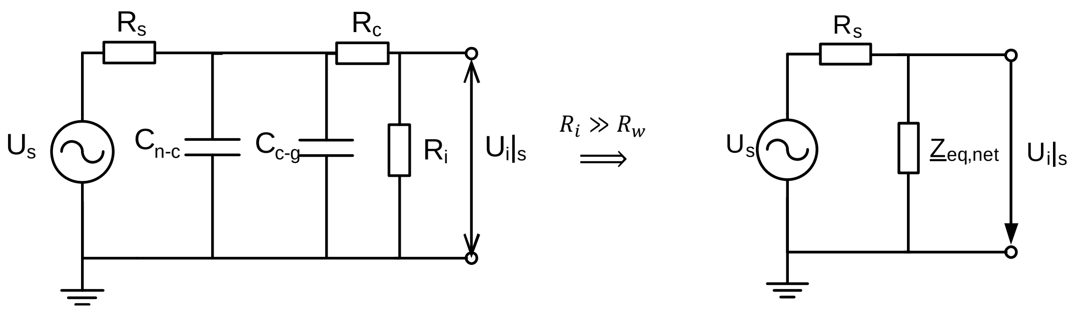

Figure 3: Simplified scheme for the input of the amplifier

$$\underline{U_i}\big|_s = \frac{\underline{Z_{eq,s}}}{\underline{Z_{eq,s}} + R_s} \cdot \underline{U_s} \text{ with : } \underline{Z_{eq,s}} = \frac{1}{\frac{1}{R_i} + j \cdot \omega \cdot (C_{c-g} + C_{n-c})} = \frac{R_i}{1 + j \cdot \omega \cdot R_i \cdot (C_{c-g} + C_{n-c})}$$

$$\underline{U_i}\big|_s = \frac{R_i}{R_i + R_s} \cdot \frac{1}{1 + j \cdot \omega \cdot \frac{R_i \cdot R_s}{R_i + R_s} \cdot (C_{c-g} + C_{n-c})} \cdot \underline{U_s} \xrightarrow{R_i \gg R_s} \underline{U_i}\big|_s = \frac{1}{1 + j \cdot \omega \cdot R_s \cdot (C_{c-g} + C_{n-c})} \cdot \underline{U_s} = \frac{1}{1 + j \cdot \frac{f}{f_c}} \cdot \underline{U_s}$$

We know that the cutoff frequency f_c is well above the source frequency f_s . We see that the input voltage of the amplifier $\underline{U_i}\big|_s$ is:

$$\underline{U_i}\big|_s = \underline{U_s}$$

Therefore, the parasitic capacity due to the voltage caused by the source is negligible!

- The output of the amplifier corresponds to the voltage measured by the voltmeter (see Figure 4). The parasitic contribution to the measured voltage $\underline{U_{m,n}}$ is then:

$$\underline{U_{m,n}} = A \cdot \underline{U_{i/n}} = j \cdot A \cdot 2 \cdot \pi \cdot f_{network} \cdot R_s \cdot C_{n-c} \cdot \underline{U_{network}}$$

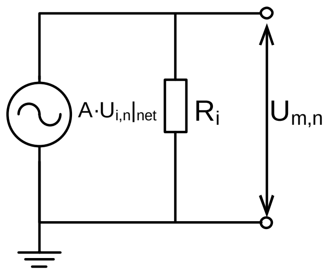

Figure 4: Output of the amplifier

- The voltage $|\underline{U}_{m,n}|$ and $SNR|_{dB}$ obtained are:

$$|\underline{U}_{m,n}| = 72\ mV$$

$$SNR|_{dB} = 20 \cdot \log\left(\frac{A \cdot U_{s,eff}}{|\underline{U}_{m,n}|}\right) = 42.8\ dB$$

  Note that the noise at the input of the amplifier is amplified along with the signal.

- To minimize noise voltage due to capacitive coupling, we propose to use a shield and a symmetric amplifier (see Figure 5). The circuit corresponds to Figure 6:

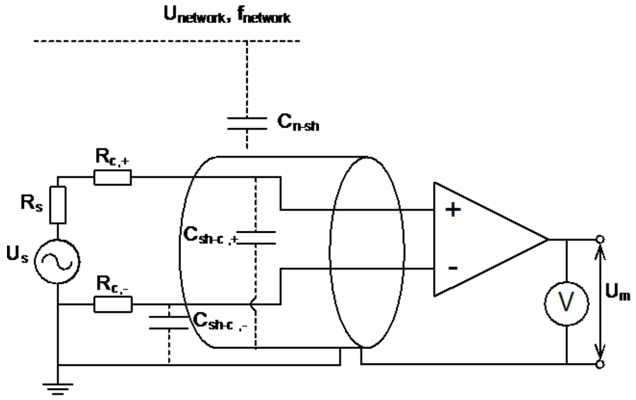

Figure 5: shielded and symmetric amplifier

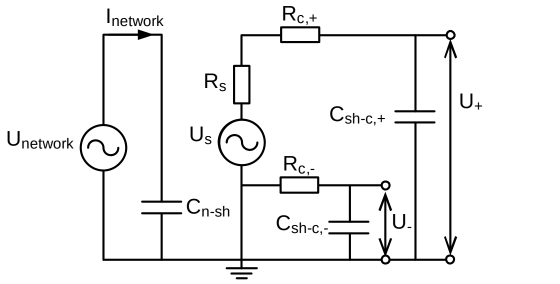

Figure 6: Circuit with shield and symmetric amplifier

  Note that in the circuit the current $I_{network}$ from the AC coupling is going to ground without influencing the voltage measurement. The differential measurement, maximizing the symmetry between the wires, minimizes electrostatic coupling between the wires and the shield.