Biological Modeling of Neural Networks

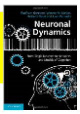
Week 5

NETWORKS of NEURONS and ASSOCIATIVE MEMORY

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Reading for week 5:
NEURONAL DYNAMICS
- Ch. 17.1 - 17.2.4

Cambridge Univ. Press



5.1 Introduction
- networks of neuron
- systems for computing
- associative memory

5.2 Classification by similarity

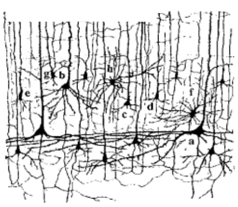
5.3 Detour: Magnetic Materials

5.4 Hopfield Model

5.5 Learning of Associations

5.6 Storage Capacity

5.1. memory

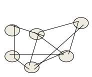


-president
-first day at EPFL
-apple

Our memory has multiple aspects
- recent and far-back
- events, places, facts, concepts

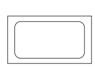
5.1. Systems for computing and information processing

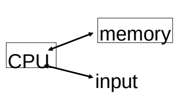
Brain



Distributed architecture
(10^{10} proc. Elements/neurons)
No separation of
processing and memory

Computer



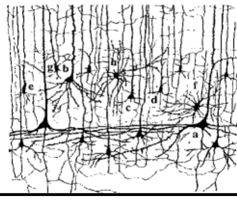


Von Neumann architecture
1 CPU
(10^{10} transistors)

5.1. Systems for computing and information processing

1mm

10 000 neurons
3 km of wire

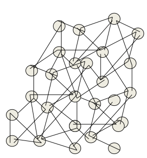


5.1. Systems for computing and information processing

Brain

1mm

10 000 neurons
3 km of wire



Distributed architecture

10^{10} neurons

10^{4} connections/neurons

No separation of processing and memory

5.1. Associations, Associative memory

*Read this text **NOW**!*

5.1. Associations, Associative memory

pattern completion/word recognition

brai* → atom / brave / brain / brass → brain

Noisy word | List of words | Output the closest one

Your brain fills in missing information: 'associative memory'

Week 5: Networks of Neurons-Introduction

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- √ 5.1 Introduction
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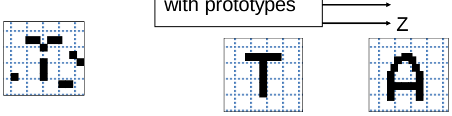
- 5.3 Detour: Magnetic Materials
- 5.4 Hopfield Model
- 5.5 Learning of Associations
- 5.6 Storage Capacity

5.2 Classification by similarity: **pattern recognition**

image → Classification: comparison with prototypes → A, B, T, Z



Noisy image

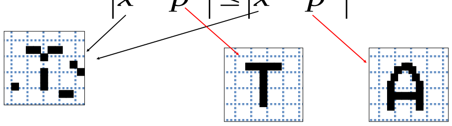
Prototypes

5.2 Classification by similarity: **pattern recognition**

Classification by closest prototype

Blackboard:

$$\left|x - p^{T}\right| \leq \left|x - p^{A}\right|$$



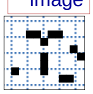
Noisy image

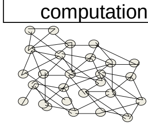
Prototypes

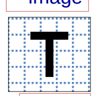
5.2 **pattern recognition and Pattern completion**

Aim: Understand Associative Memory

Noisy image → Associative memory/ collective computation → Full image







Partial word

Full word

Brain-style computation

Quiz 5.1: Connectivity

A typical neuron in the brain makes connections
- To 6-20 neighbors
- To 100-200 neurons nearby
- To more than 1000 neurons nearby
- To more than 1000 neurons nearby or far away.

In a typical cristal in nature, each atom interacts
- with 6-20 neighbors
- with 100-200 atoms nearby
- with more than 1000 atoms nearby
- with more than 1000 atoms nearby or far away.

Week 5: Networks of Neurons-Introduction

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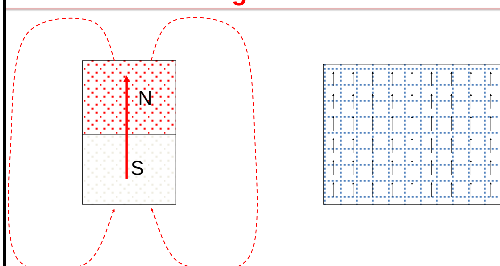
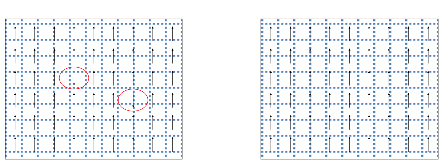
5.3 Detour: magnetism

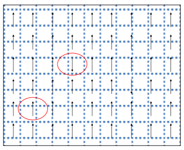


5.3 Detour: magnetism



Noisy magnet → pure magnet

5.3 Detour: magnetism



Elementary magnet

$S_i = +1$

$S_i = -1$

Blackboard: example

dynamics

$$S_i(t+1) = \text{sgn}[\sum_j S_j(t)]$$

Sum over all interactions with i

5.3 Detour: magnetism

Anti-ferromagnet

Elementary magnet

$S_i = +1$ $w_{ij} = +1$

$S_i = -1$ $w_{ij} = -1$

dynamics

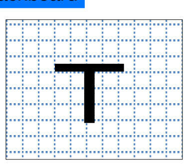
$$S_i(t+1) = \text{sgn}[\sum_j w_{ij} S_j(t)]$$

blackboard

Sum over all interactions with i

5.3 Magnetism and memory patterns

blackboard



Hopfield model:
Several patterns→ next section

Elementary pixel

■ $S_i = +1$ ■–■ $w_{ij} = +1$

□ $S_i = -1$ □–□ $w_{ij} = +1$

□–■ $w_{ij} = -1$

dynamics

$$S_i(t+1) = \text{sgn}[\sum_j w_{ij} S_j(t)]$$

Sum over all interactions with i

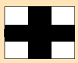
Exercise 1: Associative memory (1 pattern)

Exercises: later

Elementary pixel

■ $S_i = +1$ ■—■ $w_{ij} = +1$

□ $S_i = -1$ □—□ $w_{ij} = +1$



dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

Sum over all interactions with i

9 neurons
- define appropriate weights
- what happens if one neuron wrong?
- what happens if *n* neurons wrong?

Week 5: Networks of Neurons-Introduction

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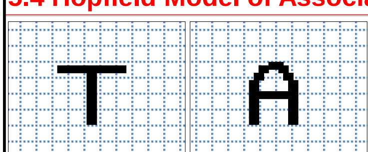
5.4 Hopfield Model

5.5 Learning of Associations

5.6 Storage Capacity

5.4 Hopfield Model of Associative Memory

Blackboard: $p_i p_j$



Prototype $\vec{p}^1$ Prototype $\vec{p}^2$

Hopfield model

interactions

$$w_{ij} = \sum_\mu p_i^\mu p_j^\mu$$

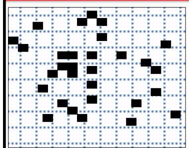
Sum over all prototypes

dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

Sum over all interactions with i

5.4 Hopfield Model of Associative Memory



Prototype
$\vec{p}^1$

interactions

$$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Sum over all prototypes

This rule is very good for **random** patterns

It does not work well for correlated patters

DEMO

dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

all interactions with i

Random patterns, fully connected:
Hopfield model

5.4 Hopfield Model of Associative Memory

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

$$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Blackboard

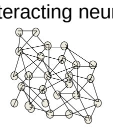
overlap $m^{\mu}(t) = \frac{1}{N} \sum_j p_j^{\mu} S_j(t)$

$$m^{\mu}(t+1) = \frac{1}{N} \sum_j p_j^{\mu} S_j(t+1)$$

5.4 Hopfield Model of Associative Memory

Prototype
$\vec{p}^1$

Interacting neurons



Finds the closest prototype i.e. maximal overlap m^{μ} (similarity)

Hopfield model

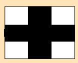
Computation
- without CPU,
- without explicit memory unit

Exercise 1: Associative memory (1 pattern)

Elementary pixel

■ $S_i = +1$ ■—■ $w_{ij} = +1$

□ $S_i = -1$ □—□ $w_{ij} = +1$



dynamics

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

Sum over all interactions with i

9 neurons
- define appropriate weights
- what happens if one neuron wrong?
- what happens if n neurons wrong?

Exercise 2.1 (now) and start with 2.2

Prototype $\vec{p}^1$

Next lecture at
11h15

$$w_{ij} = \frac{1}{N}\sum_\mu p_i^\mu p_j^\mu$$

$$S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$$

Sum over all interactions with i

Assume 4 patterns. At time t=0, overlap with Pattern 3, no overlap with other patterns.
discuss temporal evolution (of overlaps)
(assume that patterns are orthogonal)

Week 5-5: Learning of Associations

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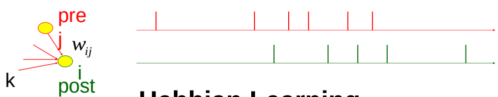
5.5 Learning of Associations

Where do the connections come from?



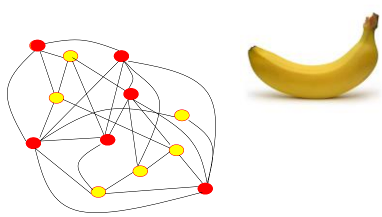
Hebbian Learning

When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then j's efficiency as one of the cells firing i is increased

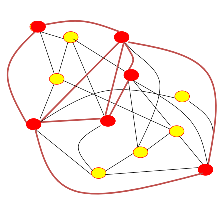
Hebb, 1949

- local rule
- simultaneously active (correlations)

5.5 Hebbian Learning of Associations



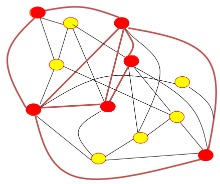
5.5 Hebbian Learning of Associations



item memorized

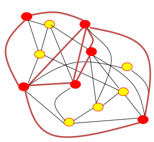
5.5 Hebbian Learning: Associative Recall

Recall:
Partial info



item recalled

5.5 Learned concepts



Activity of neurons in human brain

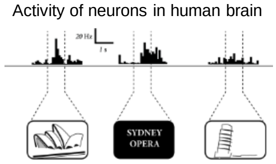
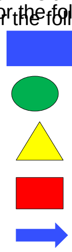


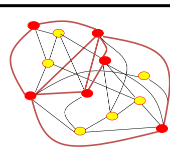
Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014), Adapted from Quiroga et al. (2005), Nature 435:1102-1107

5.5 Associative Recall

Tell me the object **shape**
for the following list of 5 items:

Tell me the **color**
for the following list of 5 items:



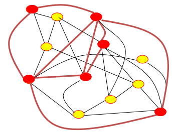


be as fast as possible:

time

5.5 Associative Recall

Tell me the **color**
for the following list of 5 items:



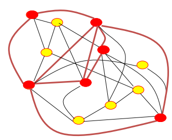
be as fast as possible:

time

***Stroop effect**:*
Slow response: hard to work
Against natural associations

5.5 Associative Recall

Hierarchical organization of
Associative memory



animals

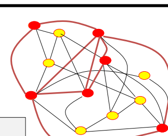
birds fish

Name as fast as possible
an example of a bird
swan (or goose or raven or …)

Write down first letter: *s* for *swan* or *r* for *raven* …

5.5 Associative Recall

Nommez au plus vite possible
un exemple d'un /d'une



Week 5-5: Learning of Associations

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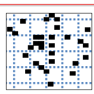
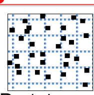
5.6. learning of several prototypes



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

interactions

(1) $w_{ij} = \frac{1}{N} \sum_{\mu} p_i^{\mu} p_j^{\mu}$

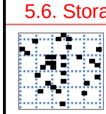
Sum over all prototypes

Question: How many prototypes can be stored?

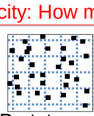
dynamics $S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$

all interactions with i

5.6. Storage capacity: How many prototypes can be stored?



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

blackboard

Random patterns

Interactions (1) $w_{ij} = \frac{1}{N} \sum_{\mu} p_i^{\mu} p_j^{\mu}$

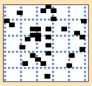
Dynamics (2) $S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$

Minimal condition: pattern is fixed point of dynamics
-Assume we start directly in one pattern
-Pattern stays

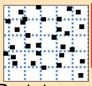
Attention: Retrieval requires more (pattern completion)

Exercise 4 now: Associative memory

Q; How many prototypes can be stored?



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

End of lecture, exercise+ Computer exercise : 12:00

Interactions (1) $w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$

Dynamics (2) $S_i(t+1) = \mathrm{sgn}[\sum_j w_{ij} S_j(t)]$

Random patterns → random walk

a) show relation to erf function: importance of p/N

b) network of 1000 neurons – allow at most 1 wrong pixel?

c) network of N neurons – at most 1 promille wrong pixels?

The end