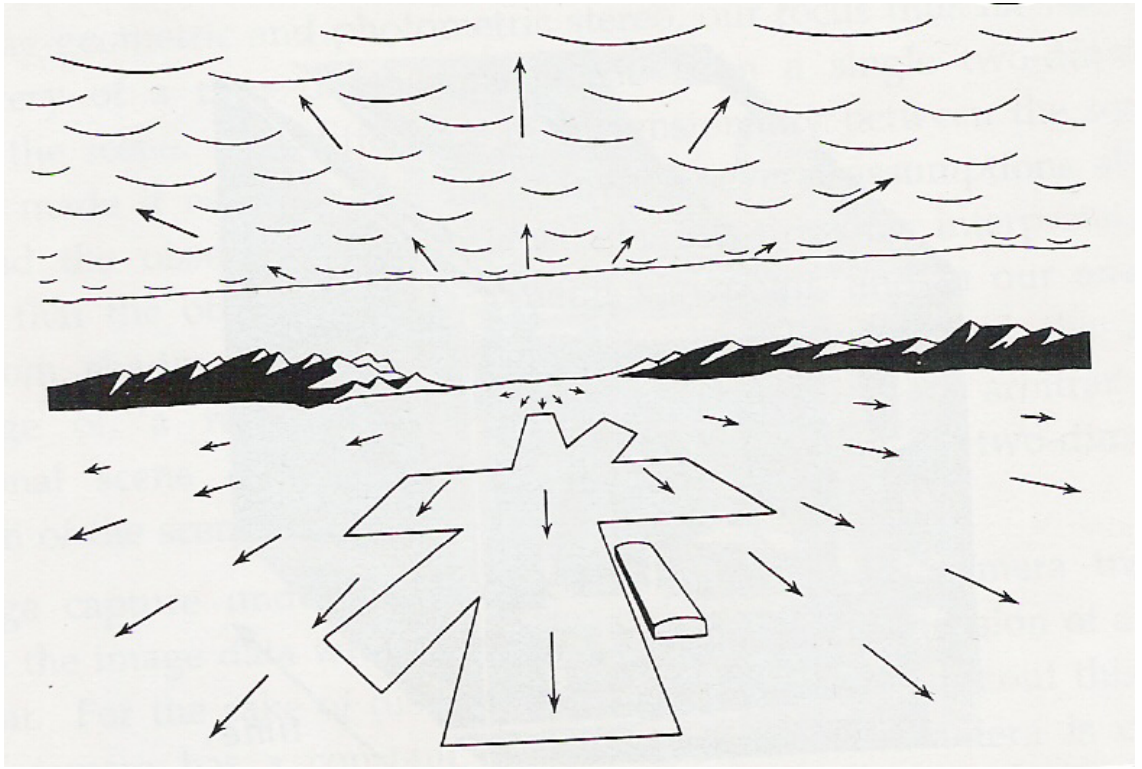
Shape from X

- One image:
  - Texture
  - Shading
- Two images or more:
  - Stereo
  - Contours
  - **Motion**

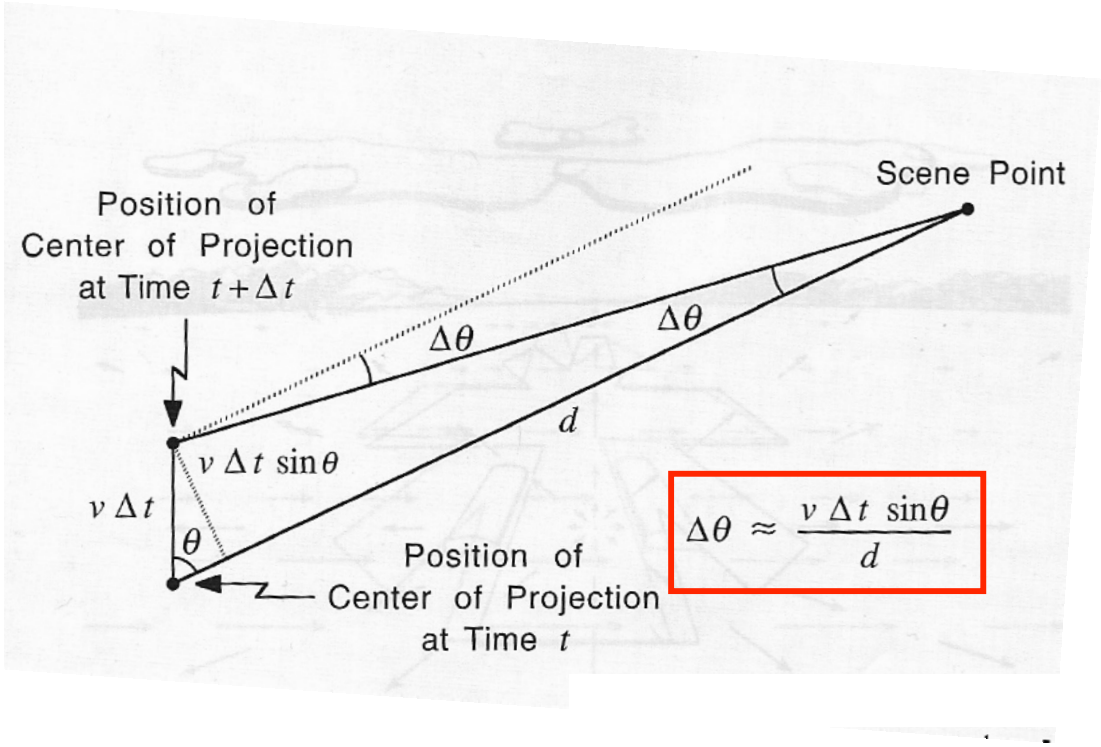
Motion



When objects move at equal speed, those more remote seem to move more slowly.

Euclid, 300 BC

Velocity vs Distance



Apparent velocity is:

- Inversely proportional to the distance of the point to the observer.
- Proportional to the sine of the angle between the line of sight and the direction of translation.

Epipolar Plane Analysis

Image sequence

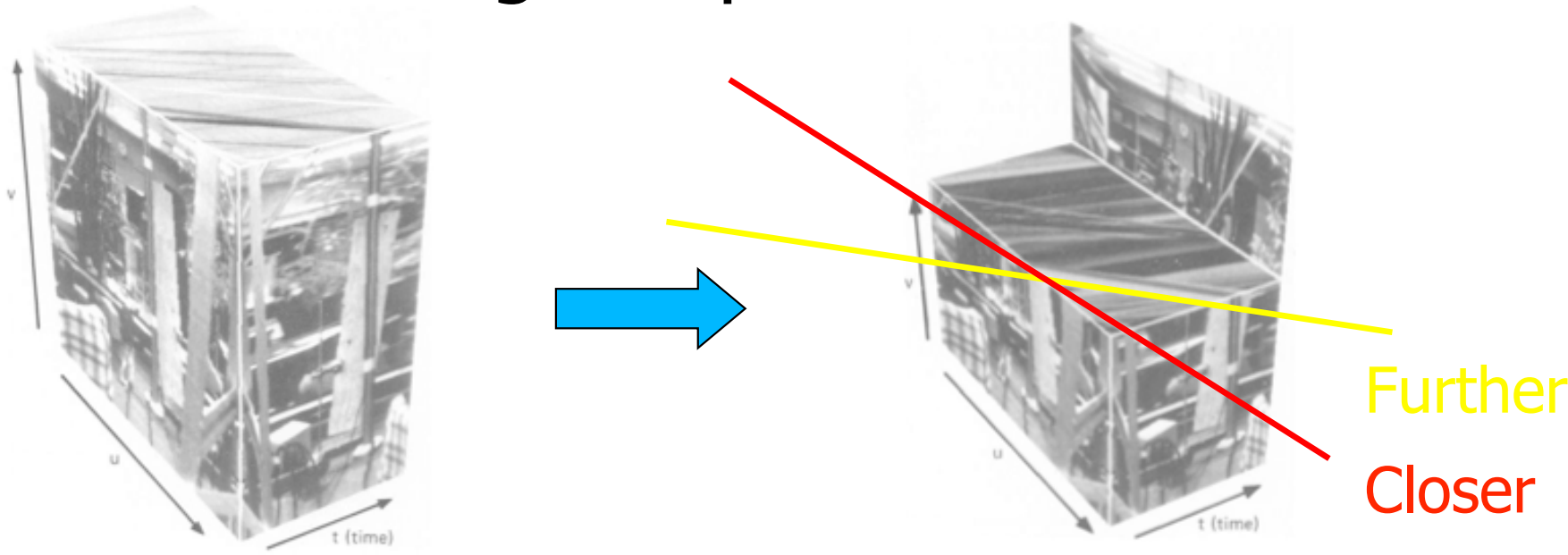
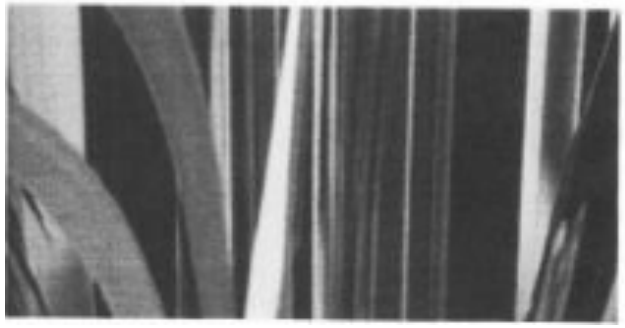


Image cube

Generalized Motion

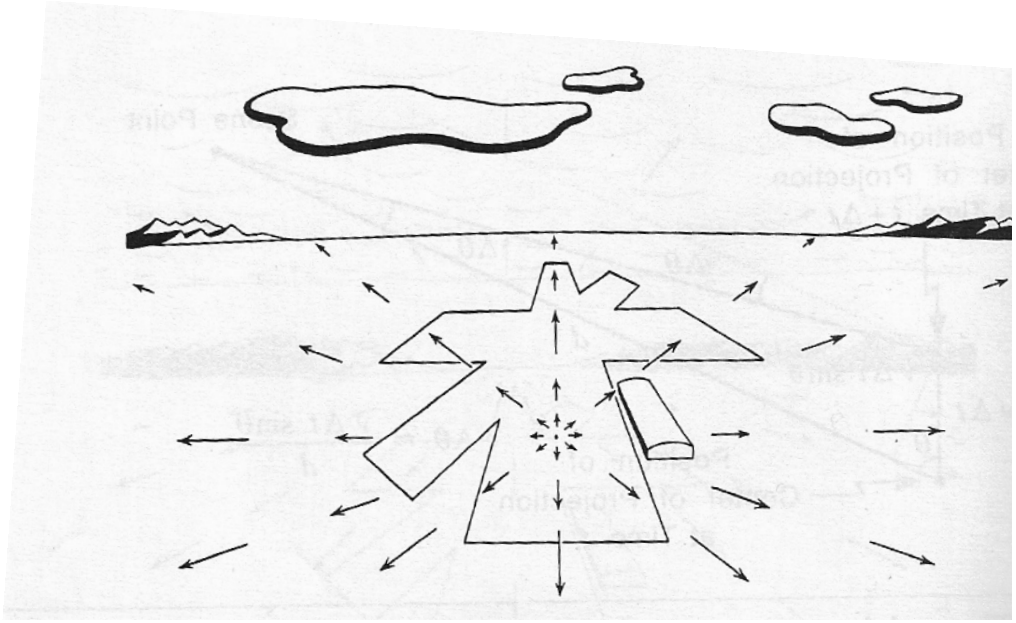
Orthogonal viewing



Non-orthogonal viewing

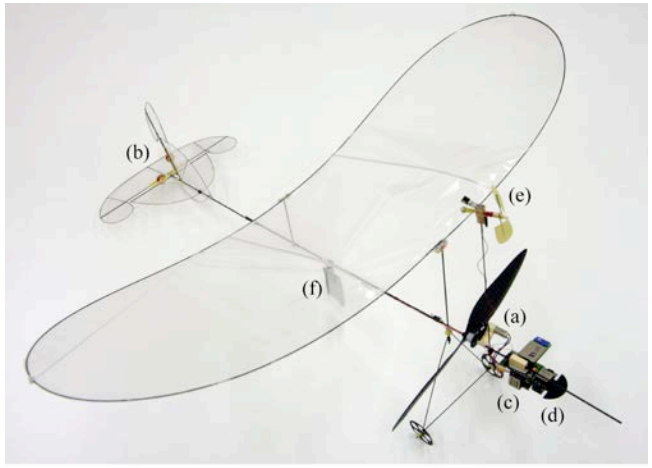
View direction varying

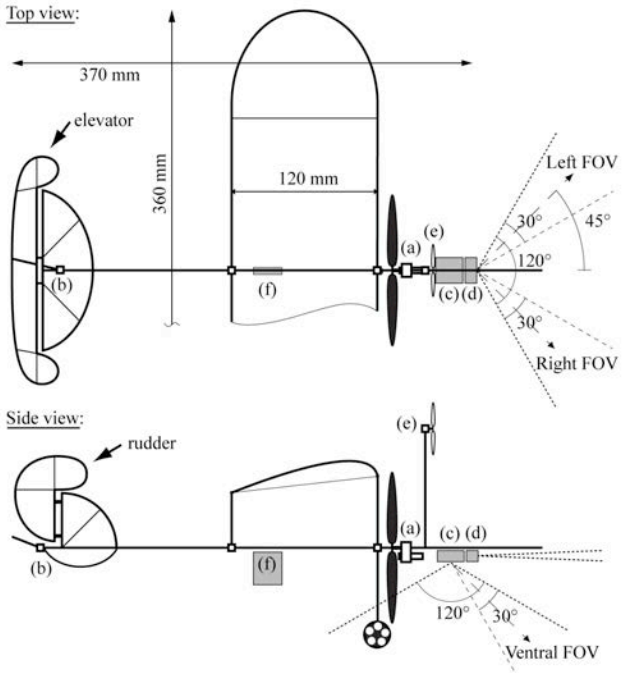
Focus of Expansion



For a translational motion of the camera, all the **motion-field** vectors converge or diverge from a single point: The focus of expansion (FOE) or contraction (FOC).

Microflyer





The plane detects POEs and uses them to avoid collisions.

Motion Field Estimation

Approaches can be classified with respect to the assumptions they make about the scene:

- Images properties remain invariant under relative motion between the camera and the scene.
- Feature points can be tracked across frames.

Assumption 1: Brightness Constancy

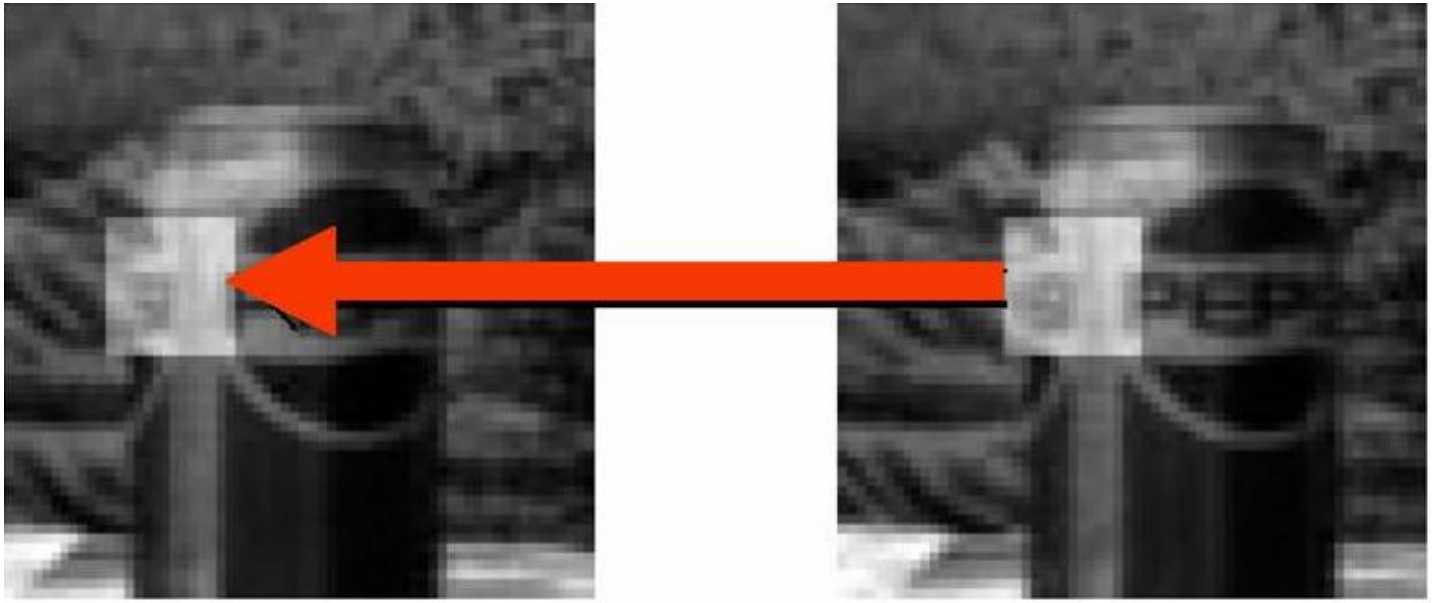
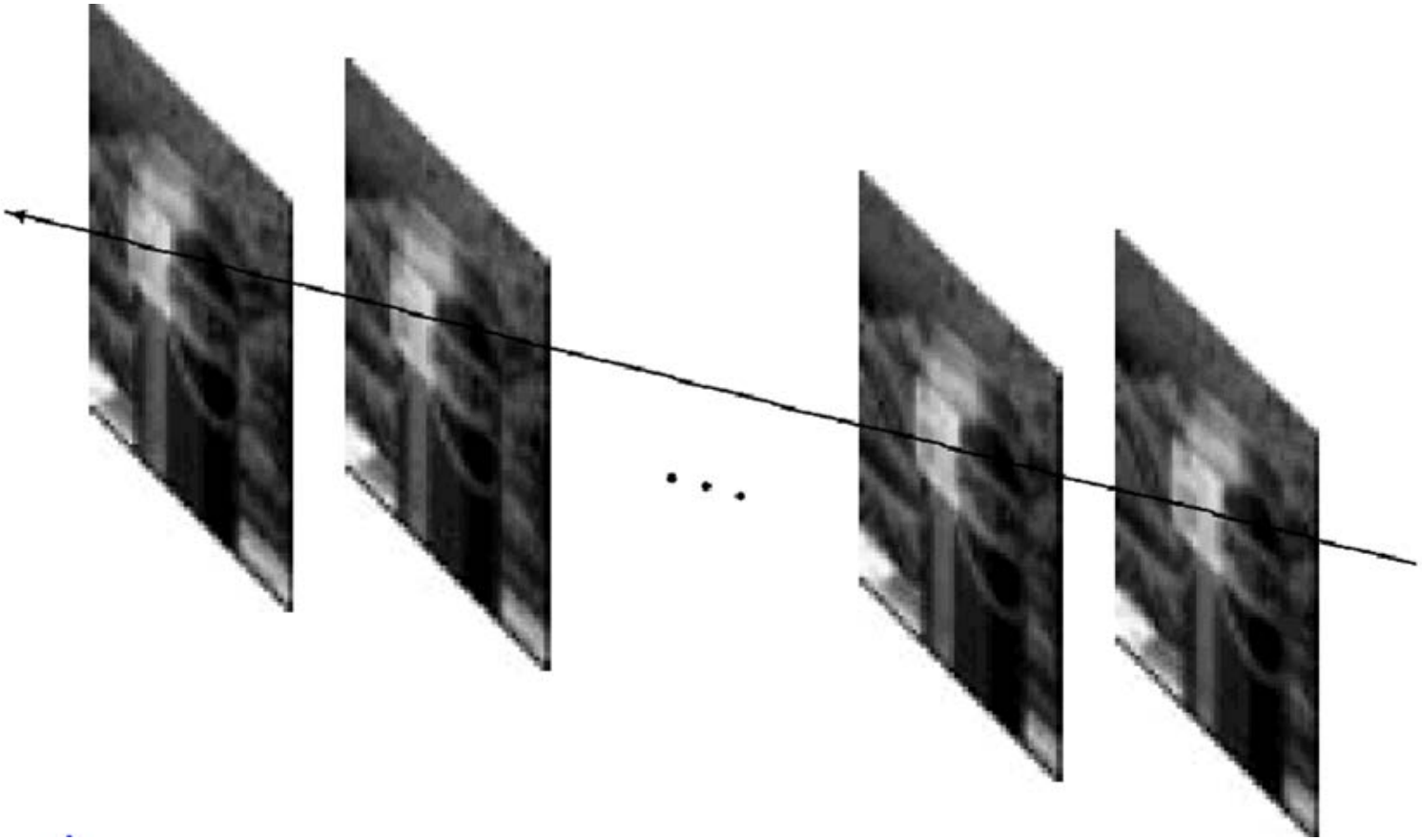


Image measurements (e.g. brightness) in a small region remain the same although its location may change.

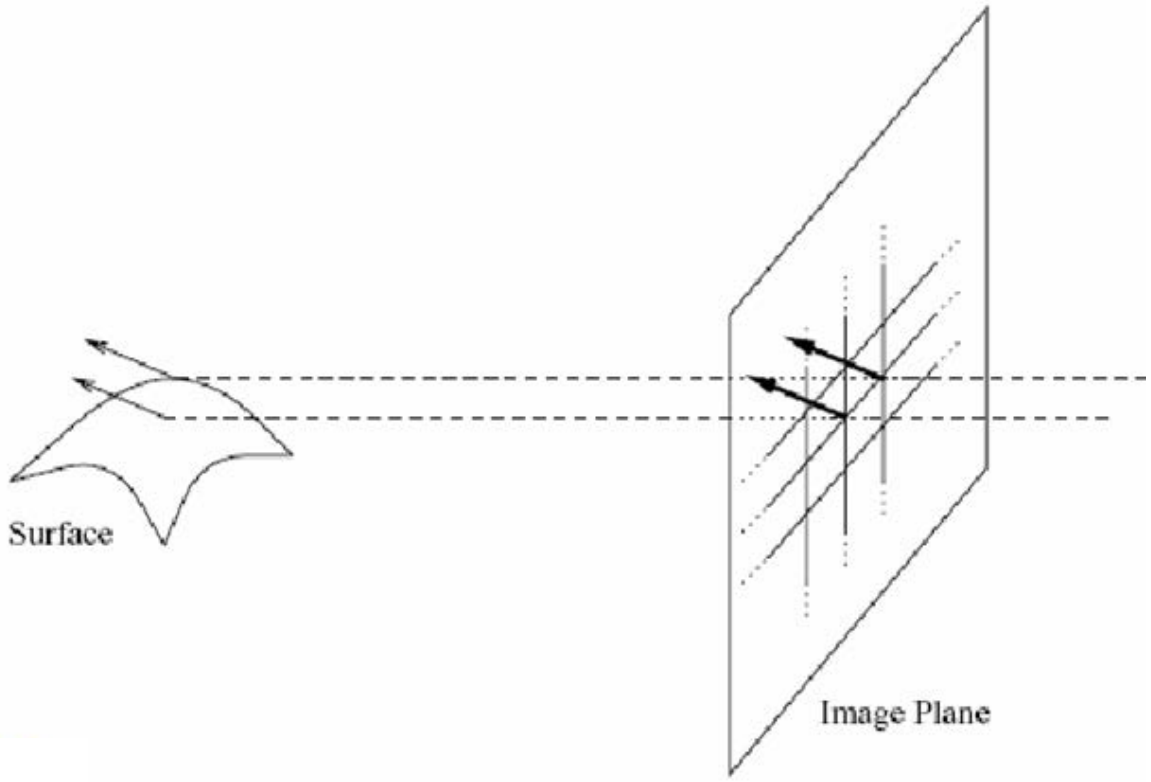
$$I(x + dx, y + dy, t + dt) = I(x, y, t)$$

Assumption 2: Temporal Consistency



The image speed of a surface patch only changes gradually over time.

Assumption 3: Spatial Consistency



- Neighboring points in the scene typically belong to the same surface and hence have similar motions.
- Since they also project to nearby image locations, we expect spatial coherence of the flow.

Spatio Temporal Derivatives

Under the assumptions of

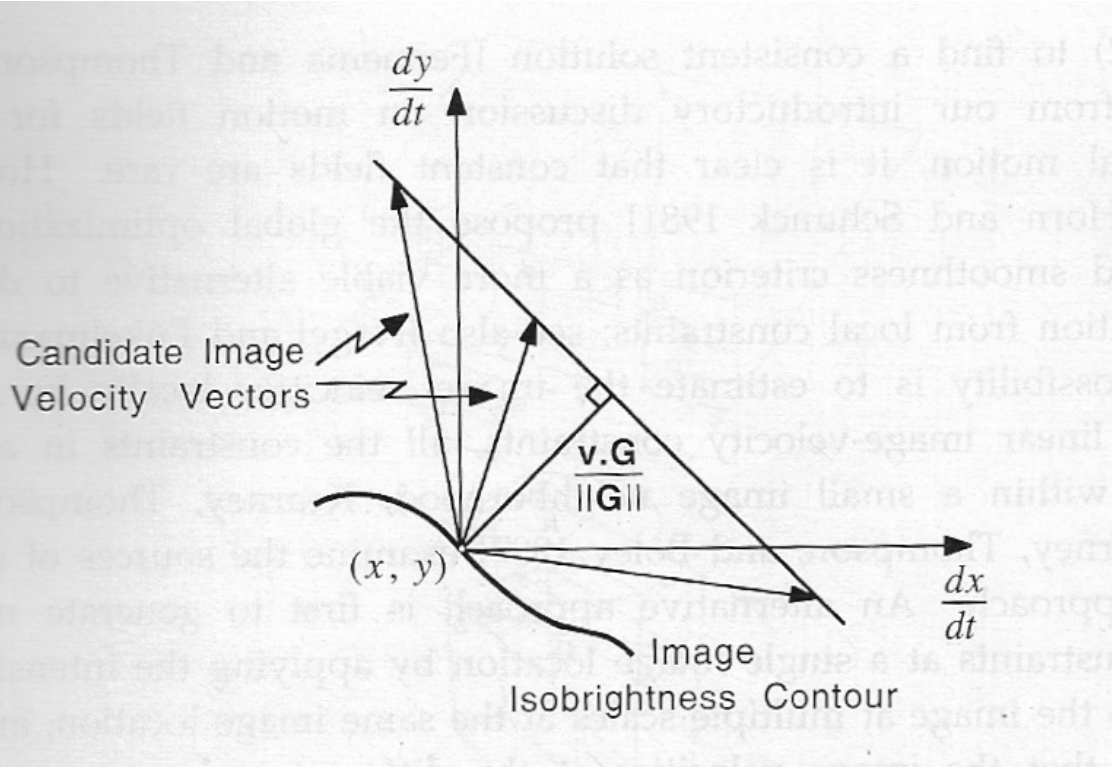
- Brightness constancy,
- Temporal consistency,

Image projection at time t

we write:

$$\begin{aligned} \text{cst} &= I(x(t), y(t), t) \\ \Rightarrow 0 &= \frac{\delta I}{\delta x}\frac{dx}{dt} + \frac{\delta I}{\delta y}\frac{dy}{dt} + \frac{\delta I}{\delta t} \end{aligned}$$

Normal Flow Equation



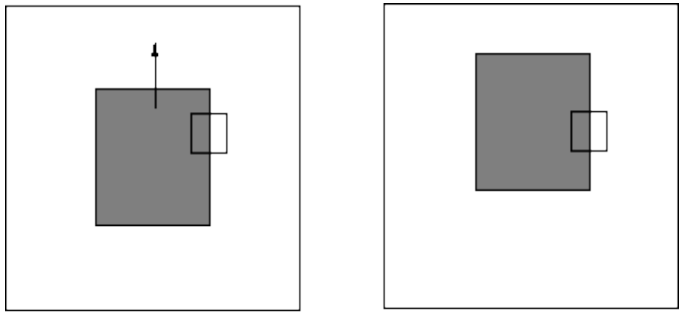
$$v\frac{G}{\|G\|} = -\frac{\frac{\partial I}{\partial t}}{\sqrt{\frac{\partial I}{\partial x}^2 + \frac{\partial I}{\partial y}^2}}$$

$$G = \left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right]$$

$$v = \left[\frac{dx}{dt}, \frac{dy}{dt}\right]$$

Ambiguities

- At each pixel, we have 1 equation and 2 unknowns.
- Only the flow component in the gradient direction can be determined locally.



The motion is parallel to the edge, and it cannot be determined.

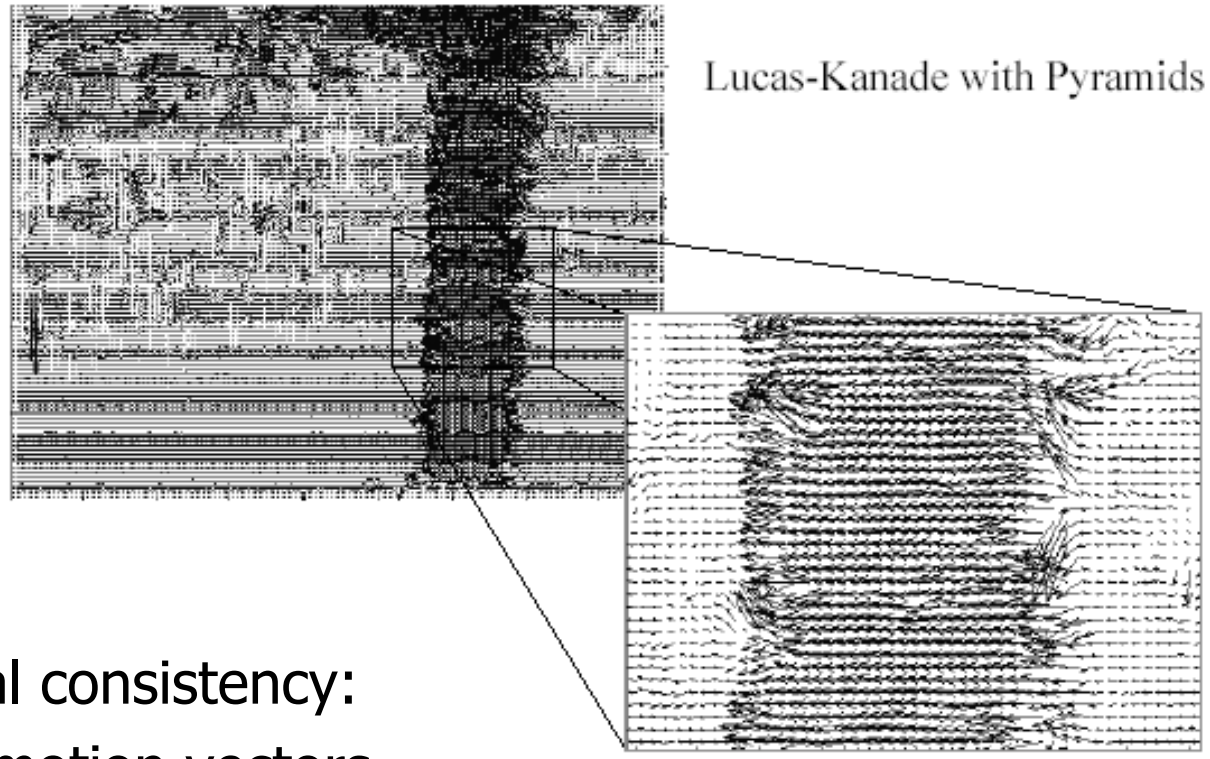
Local Constancy

Assume the flow to be constant is a 5x5 window:

$$\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}$$

--> 25 equations for 2 unknown, which can be solved in the least squares sense.

Enforcing Consistency



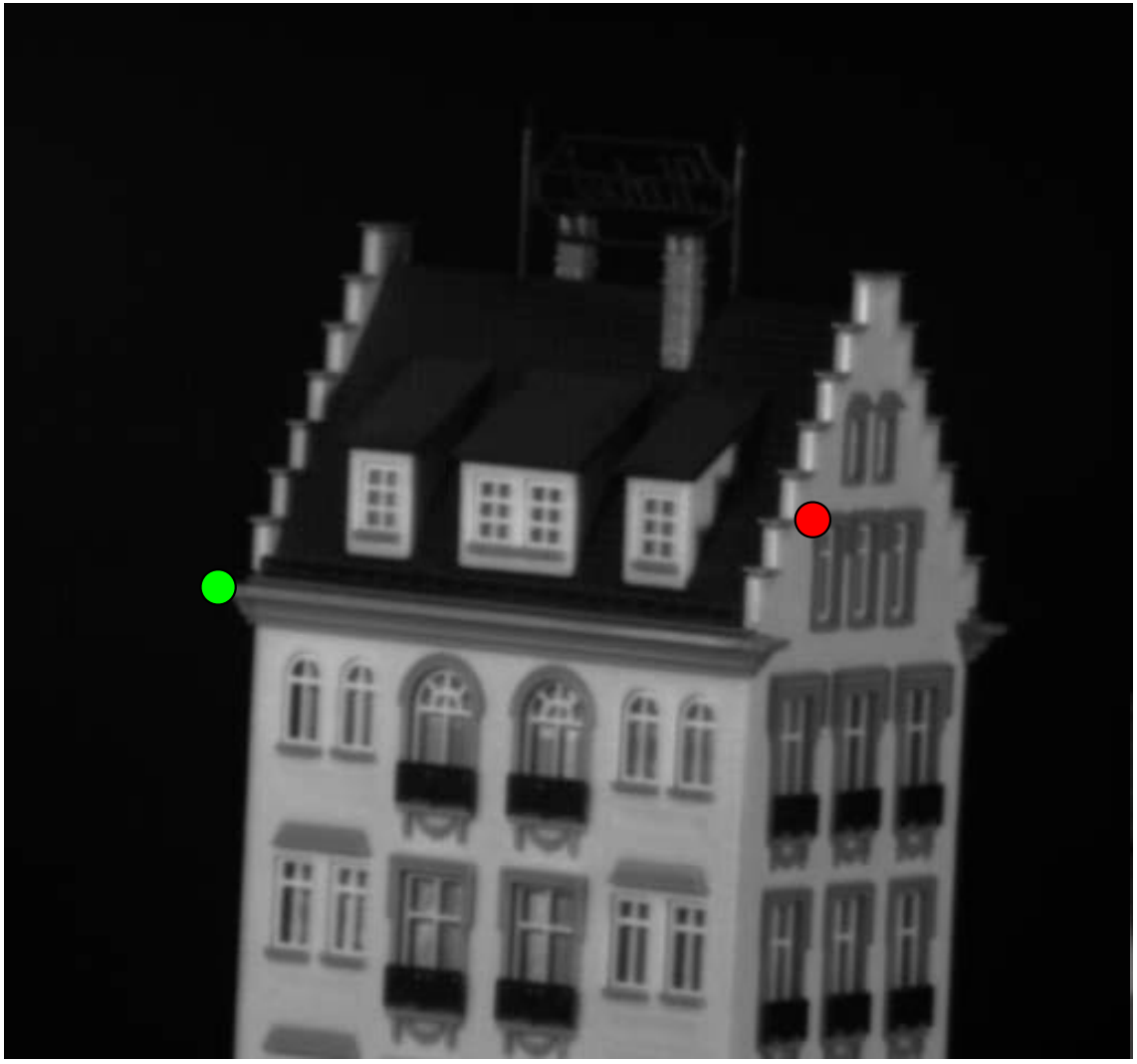
Under the assumption of spatial consistency:

- Hough Transform on the motion vectors.
- Regularization of the motion field.
- Multi scale approach.

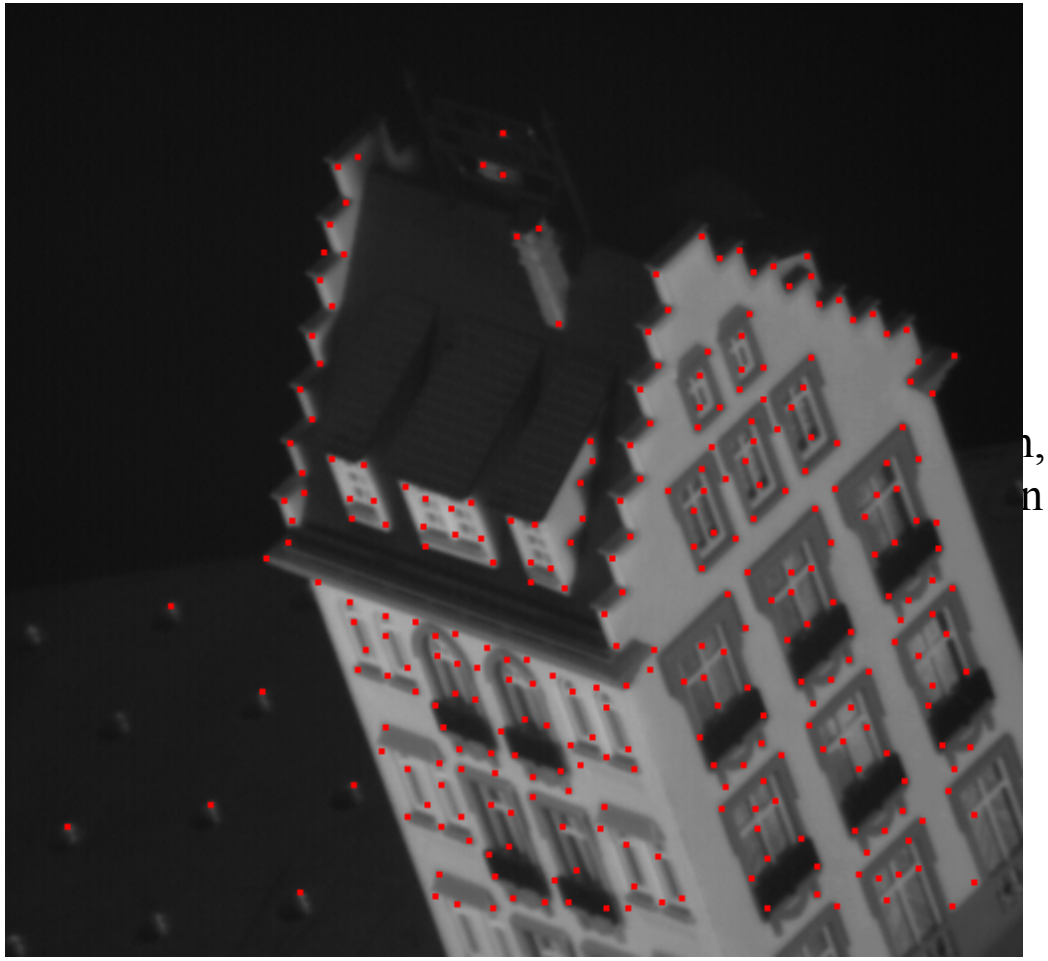
But, the world is neither Lambertian nor smooth.

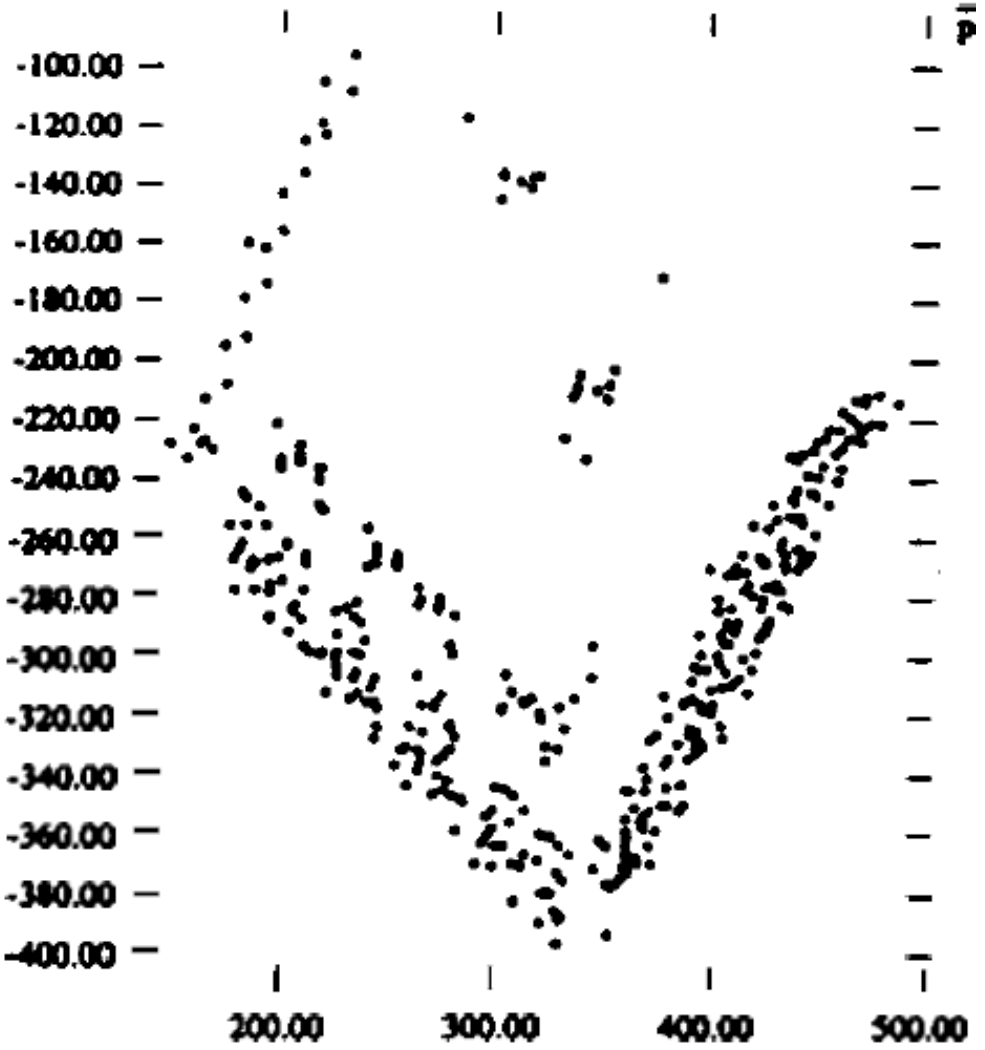
→ These assumptions are rarely valid.

Tracking Points across Images



3D Shape Reconstruction





Multi-View Projection

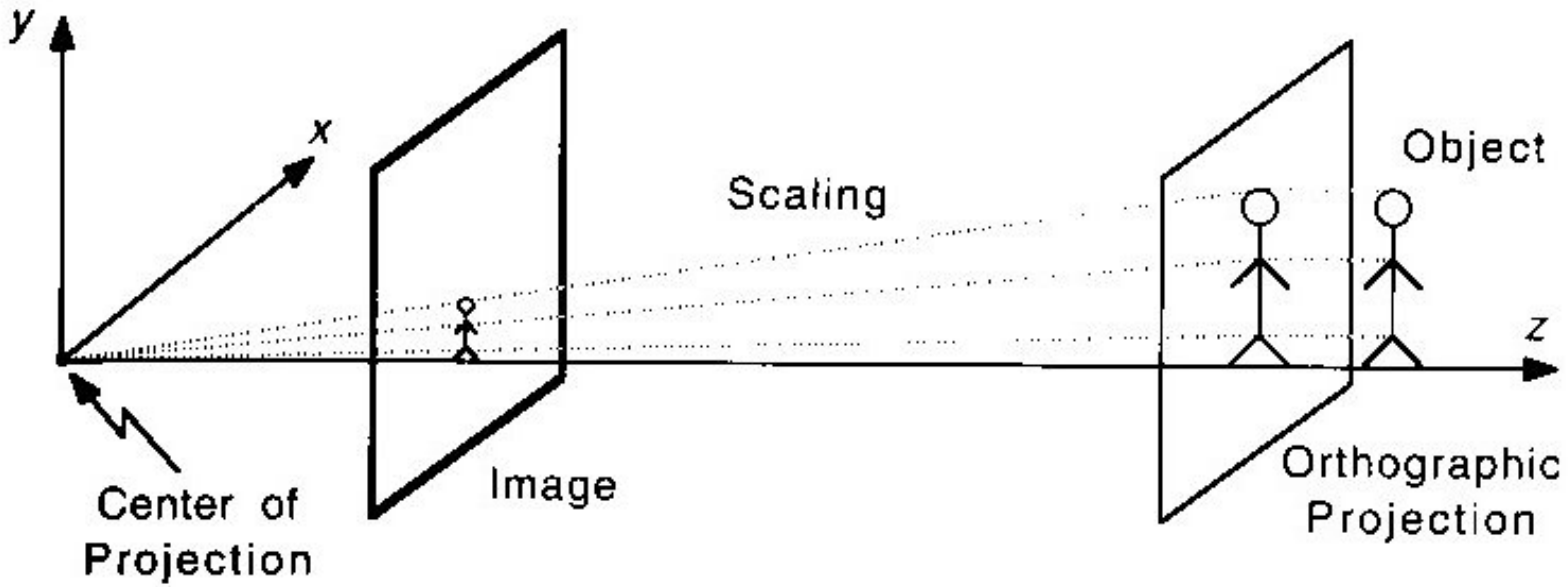
- n image points are projected from 3-D scene points over m views via

$$\mathbf{x}_j^i = \mathbf{P}^i \mathbf{X}_j$$

where i = 1, ... , m and j = 1, ... , n.

- Here each $\mathbf{P}^i$ is a 3 x 4 matrix and each $\mathbf{X}_j$ is a homogeneous 4-vector.

Orthographic Projection



$$u = sx$$

$$v = sy$$

MULTI-VIEW ORTHOGRAPHIC PROJECTION

- The last row of each $\mathbf{P}^i$ is $(0, 0, 0, 1)$ for affine cameras, so we can "ignore" it and write the orthographic projection as:

$$\mathbf{x}_j^i = \mathbf{M}^i \mathbf{X}_j + \mathbf{t}^i$$

where each $\mathbf{X}_j$ is now an inhomogeneous 3-vector.

- Here, each $\mathbf{M}^i$ a 2 x 3 matrix, and each $\mathbf{t}^i$ a 2-vector.

Reconstruction Problem

- Estimate affine cameras $\mathbf{M}^i$, translations $\mathbf{t}^i$, and 3-D points $\mathbf{X}_j$ that minimize the geometric error in image coordinates:

$$\min_{\mathbf{M}^i,\mathbf{t}^i,\mathbf{X}_j} \sum_{i,j} \left(\mathbf{x}_j^i - (\mathbf{M}^i\mathbf{X}_j + \mathbf{t}^i)\right)^2$$

Simplifying the Problem

- Normalization: We can eliminate the translation vectors $\mathbf{t}^i$ by choosing the centroid of the image points in each image as the coordinate system origin

$$\mathbf{x}_j^i \leftarrow \mathbf{x}_j^i - \frac{1}{n}\sum_j \mathbf{x}_j^i$$

- Working in "centered coordinates", the minimization problem becomes:

$$\min_{\mathbf{M}^i, \mathbf{X}_j} \sum_{i,j} \left(\mathbf{x}_j^i - \mathbf{M}^i \mathbf{X}_j\right)^2$$

- This works because the centroid of the 3-D points is preserved under affine transformations

Matrix Formulation

- Let the measurement matrix be:

$$\mathbf{W} = \begin{pmatrix} \mathbf{x}_1^1 & \mathbf{x}_2^1 & \dots & \mathbf{x}_n^1 \\ \mathbf{x}_1^2 & \mathbf{x}_2^2 & \dots & \mathbf{x}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_1^m & \mathbf{x}_2^m & \dots & \mathbf{x}_n^m \end{pmatrix}$$

- Since $\mathbf{x}_j^i = \mathbf{M}^i \mathbf{X}_j$, this means solving

$$\mathbf{W} = \begin{bmatrix} \mathbf{M}^1 \\ \vdots \\ \mathbf{M}^m \end{bmatrix} [\mathbf{X}_1, \dots, \mathbf{X}_n]$$

2m x 3

3 x n

in the least squares sense.

Solving with SVD

- There will be no exact solution with noisy points, so we want the nearest $\mathbf{W}'$ to $\mathbf{W}$ that is an exact solution
  - $\mathbf{W}'$ is rank 3 since it's the product of a $2m$ x 3 motion matrix $\mathbf{M}'$ and a 3 x n structure matrix $\mathbf{X}'$

- Use singular value decomposition to get rank 3 matrix $\mathbf{W}'$ closest to $\mathbf{W}$
  - Let SVD of $\mathbf{W} = \mathbf{U}\mathbf{D}\mathbf{V}^T$
  - Then $\mathbf{W}' = \mathbf{U}_{2mx3}\mathbf{D}_{3x3}\mathbf{V}_{nx3}{}^T$, where
    - $\mathbf{U}_{2mx3}$ is the first 3 columns of $\mathbf{U}$, $\mathbf{D}_{3x3}$ is an upper-left 3 x 3 submatrix of $\mathbf{D}$,
    - $\mathbf{V}_{nx3}{}^T$ is first three columns of $\mathbf{V}$.

Structure and Motion

- Set stacked camera matrix as

$$\mathbf{M}' = \mathbf{U}_{2mx3}\ \mathrm{sqrt}(\mathbf{D}_{3x3})$$

- Set stacked 3-D structure matrix as

$$\mathbf{X}' = \mathrm{sqrt}(\mathbf{D}_{3x3})\mathbf{V}_{nx3}^{T}$$

so that $\mathbf{W}' = \mathbf{M}'\mathbf{X}'$

Metric Upgrade

- There is an affine ambiguity since an arbitrary 3 x 3 rank 3 matrix **A** can be inserted as:

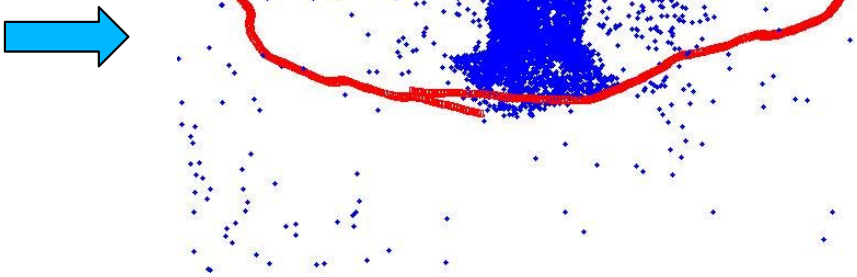
$$\mathbf{W'} = (\mathbf{M'A})(\mathbf{A}^{-1}\mathbf{X'})$$

- Get rid of ambiguity by finding **A** that performs "metric rectification"

- Affine camera provides orthonormality constraints on **A**:
  - Rows of **M=M'A** are unit vectors: $\mathbf{m}_i \cdot \mathbf{m}_i = 1$.
  - Rows of **M=M'A** are orthogonal: $\mathbf{m}_i \cdot \mathbf{m}_j = 0$.

- Everything relies on linear algebra but is limited to orthographic cameras.

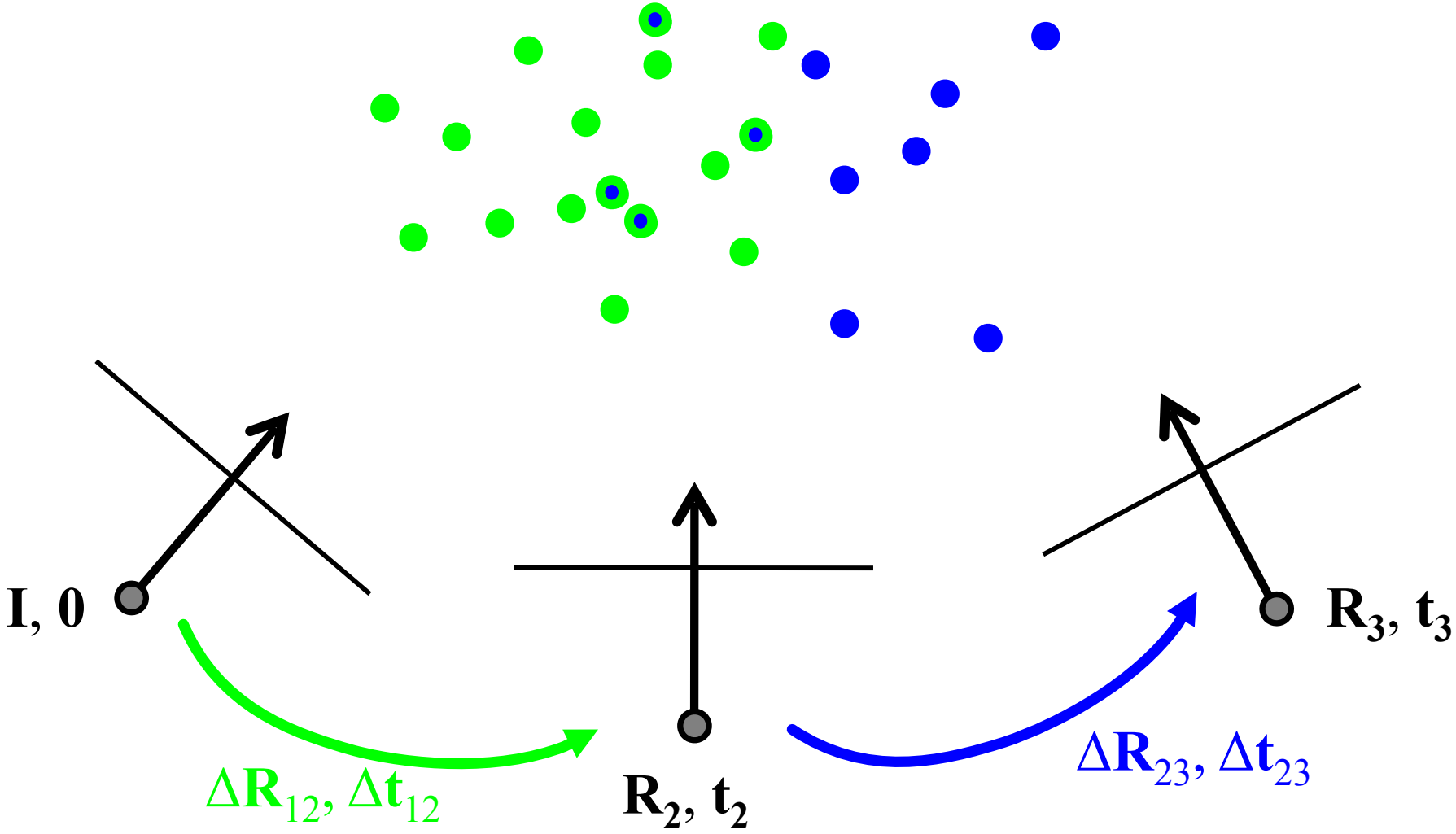
Simultaneous Localization And Mapping



- Compute point tracks.
- Infer both camera motion and 3D structure.

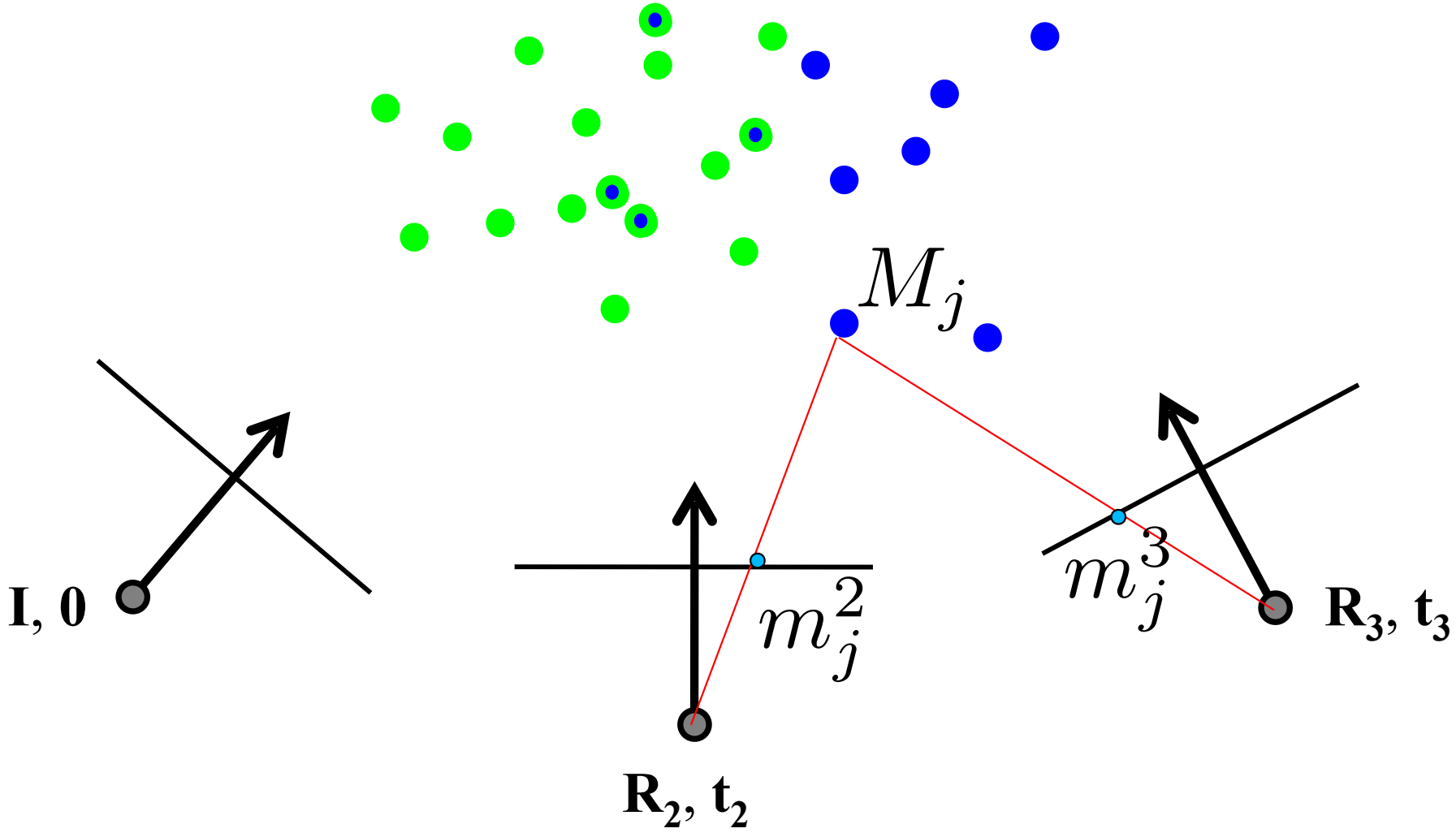
Steedly et al., ICCV'03

Sequential Structure from Motion



-> Trajectory and 3D points defined up to a Euclidean motion and scale

Bundle Adjustment



$$\text{argmin}_{R_i,t_i,M_j} \sum_i \sum_j \|\text{proj}(R_i, t_i, M_j) - m_j^i\|^2$$

Global Non-Linear Optimization

$$\text{argmin}_{R_i, t_i, M_j} \sum_i \sum_j \|\text{proj}(R_i, t_i, M_j) - m_j^i\|^2$$

- Often performed using the Levenberg-Marquardt algorithm.
- Many parameters to estimate, but sparse Jacobian matrix.
- Initial estimates computed using the eight point algorithm:

  - Given 8 point correspondences between a pair of images, ΔR and ΔT can be estimated in closed form by solving an SVD.

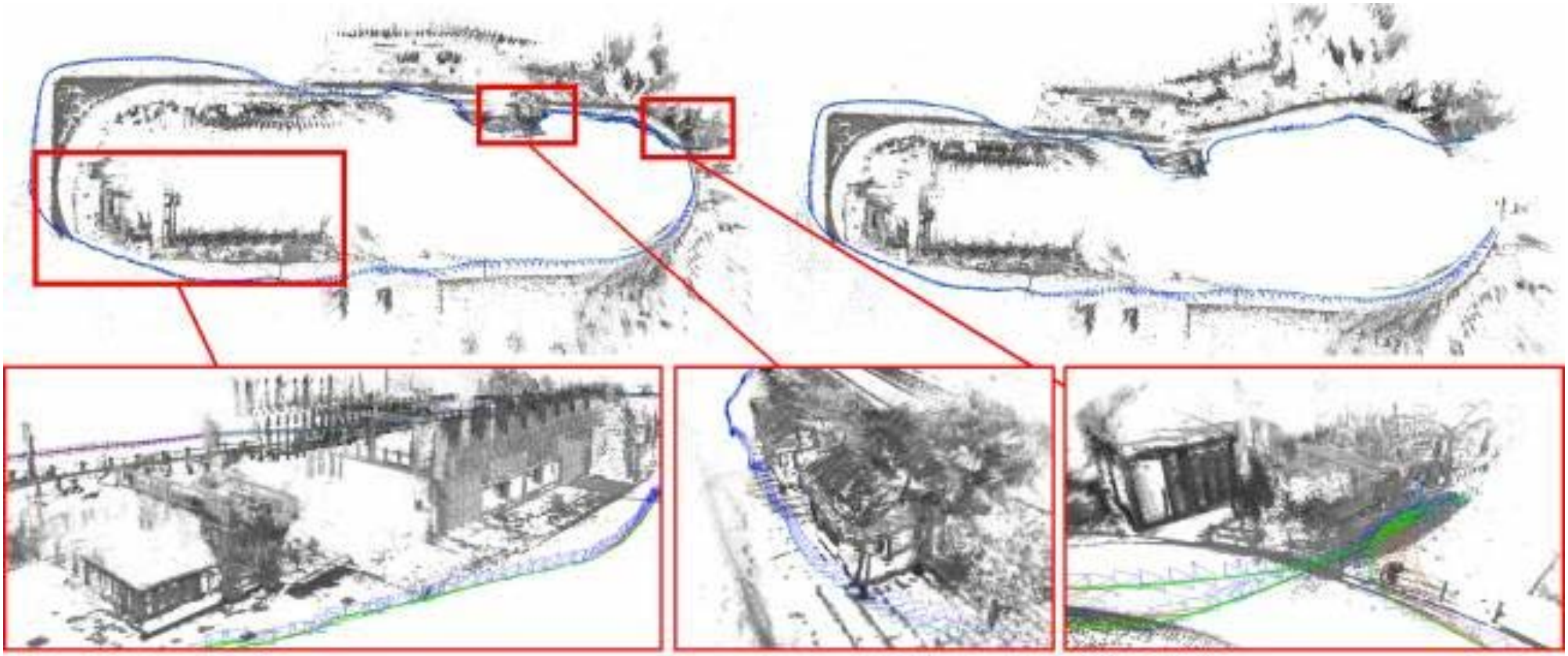
Augmented Reality

Parallel Tracking and Mapping
for Small AR Workspaces

Extra video results made for
ISMAR 2007 conference

Georg Klein and David Murray
Active Vision Laboratory
University of Oxford

Simultaneous Localization And Mapping



A robot can reconstruct its environment and position itself at the same time.

Engel et al. , ECCV'14

Fusing Depth Maps

- Both the depth camera and the person are moving.
- Use a deformable model to combine the data over time.
- Real-time implementation.

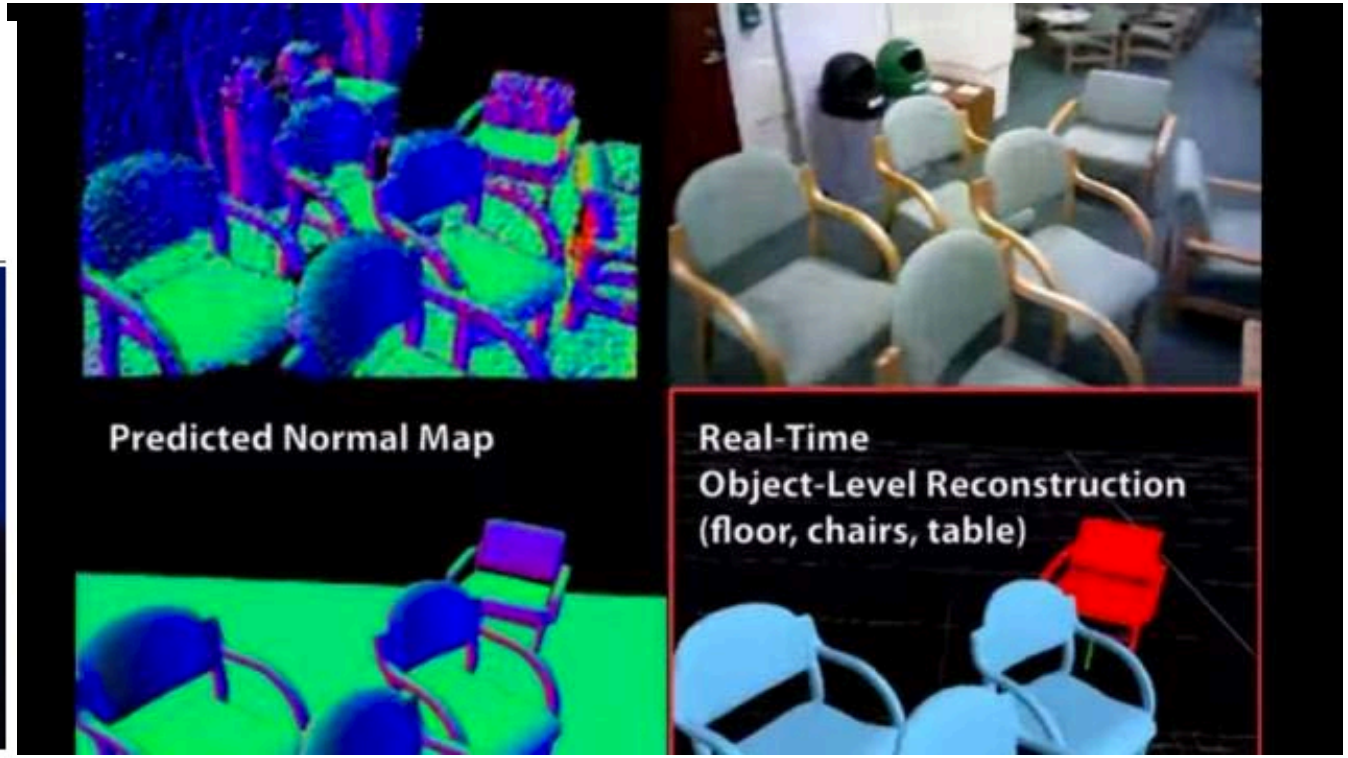
Into the Commercial World

Facebook buys British virtual reality start-up Surreal Vision

Surreal Vision aims to make a computerised version of the world so real that users are unable to distinguish between the two

614 156 0 27 797 Email

Oculus Rift is expected to be launched next year Photo: AFP



The Telegraph. May 2015

Into the Commercial World

Microsoft Hololens

… and they are both being worked on in Zurich!

Strengths And Limitations

Strengths:

- Combine information from many images.

Limitations:

- Requires multiple views.
- Requires either texture or a depth camera.