Neural Networks and Biological Modeling

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Question set 9

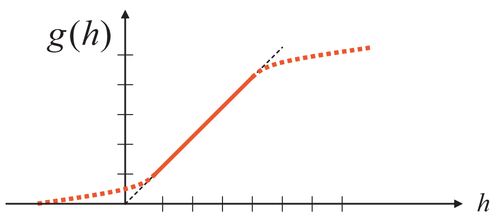
Exercise 1: Nullclines

Consider the following system:

$$\frac{d}{dt}h_1(t) = -h_1(t) + h^{ext} + (w_{ee} - \alpha)g\left(h_1(t)\right) - \alpha g\left(h_2(t)\right)$$

$$\frac{d}{dt}h_2(t) = -h_2(t) + h^{ext} + (w_{ee} - \alpha)g\left(h_2(t)\right) - \alpha g\left(h_1(t)\right)$$

where $\alpha = 1$, $w_{ee} = 1.5$ and $h^{ext} = 0.8$. The function $g(h)$ is a continuous monotonically increasing nonlinear function (schematically shown in Figure 1):



$$\begin{aligned} g(h) &= h \quad \text{for } 0.2 < h < 0.8 \\ g(0) &= 0.1 \\ g(0.9) &= 0.85 \\ g(1) &= 0.9 \end{aligned}$$

1.1 Draw the two nullclines ($\frac{dh_1}{dt} = 0$ and $\frac{dh_2}{dt} = 0$) in the phase plane (h_2 vs h_1 plot). To help you doing this you should start by filling in numerical values in these tables:

h_1	$g(h_2)$	h_2
1		
0.8		
0.2		
0		

h_2	$g(h_1)$	h_1
1		
0.8		
0.2		
0		

1.2 Add arrows on the nullclines.

1.3 Plot qualitatively three trajectories, one starting at (0,0), the second one at (0, 0.1) and the third one at (0.1, 0).

Exercise 2: Stability of the homogenous solution

Assume $h^{ext} = b$

2.1 Consider only the fixed middle fixed point that remain symmetric in the state variables ($h_1 = h_2 = h^*$. Find an expression for $h^*(b)$ for the set of parameters in Ex. 1, under the assumption that the fixed point is in the region where $g(h) = h$. Analyze the stability of this fixed point.

2.2 For arbitrary parameters w_{ee} and h and arbirtrary sufficiently smooth function $g(h)$; give a formula for the symmetric fixed point.

2.3 Assume again that $w_{ee} = 3/2$ and $\alpha = 3/4$. Calculations will be simplified by introducing a parameter $\beta = \frac{3}{4}g'(h^*)$. Calculate the two eigenvalues for arbitrary beta.

2.4 Consider the case $g'(h) = 1$ and $g'(h) = 0$. Show that the fixed point is stable for $g'(h^*) = 0$ and unstable for $g'(h^*) = 1$. At which value of beta does the fixed point change stability?

2.5 Describe in words how the symmetric fixed point gains stability as we decrease b from 0.8. To which of the monkey's experiment does this correspond?