

Exploratory data analysis

- Also called *descriptive statistics*, this term is used to describe the process of ‘looking at the data’ prior to formal analysis
- In this phase of analysis, data are examined for *quality* and ‘cleaned’ as well as *displayed* to provide an overall impression of results
- We will look at two types of summaries:
 - Graphical summaries
 - Numerical summaries
- Necessary to use *statistical software*

Why R?

- Powerful, flexible, and extensible statistical computing language and environment
- Wide range of built-in statistical functions and add-on packages available, including a growing number specifically for microarray data analysis
- High quality, customizable graphics capabilities
- Available for Unix/Linux, Windows, Mac
- All this and ... R is free!

Variables (I)

- Statisticians call characteristics which can differ across individuals *variables*
- Types of variables
 - *Categorical* (also called *qualitative*)
 - Examples: eye color, favorite television program
 - *Numerical* (also called *quantitative*)
 - Examples: height, number of children, fluorescence intensity

Variables (II)

- Categorical variables may be
 - *Nominal* – the categories have names, but no ordering (e.g. eye color)
 - *Ordinal* – categories have an ordering (e.g. ‘Always’, ‘Sometimes’, ‘Never’)
- Numerical variables may be
 - *Discrete* – possible values can differ only by fixed amounts (most commonly counting values)
 - *Continuous* – can take on any value within a range (e.g. any positive value)

Univariate Data

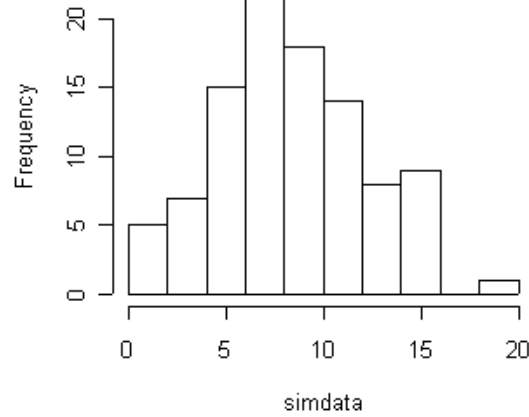
- Measurements on *a single (continuous)* variable X
- Summarizing X
 - Graphically:
 - Distribution: histogram, QQ plot, dotplot, boxplot
 - Quality: cluster analysis, PCA, spatial plots
 - Numerically:
 - Distribution: quantiles
 - Center: mean, median
 - Spread: SD, IQR, MAD

Bivariate / Multivariate Data

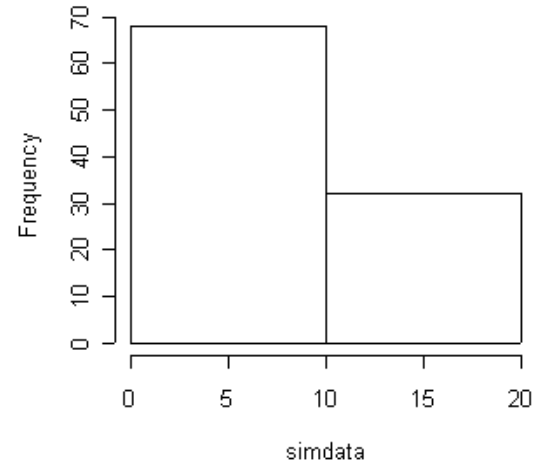
- *Bivariate (or multivariate) data* – data with measurements on *two (or more)* variables
- Here, we will look at two *continuous* variables
- Want to explore the *relationship* between the two variables
 - Graphically: scatterplot
 - Numerically: correlation coefficient

Histogram: same data

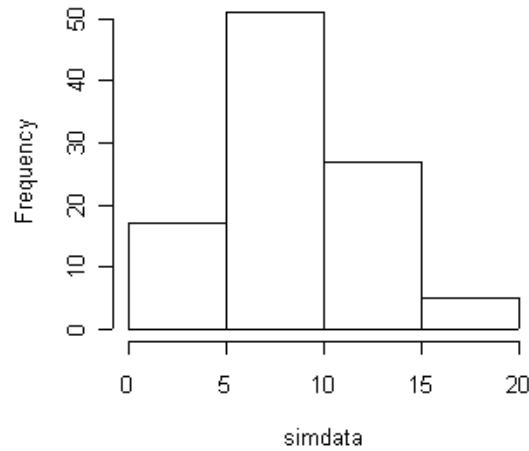
Histogram of simdata



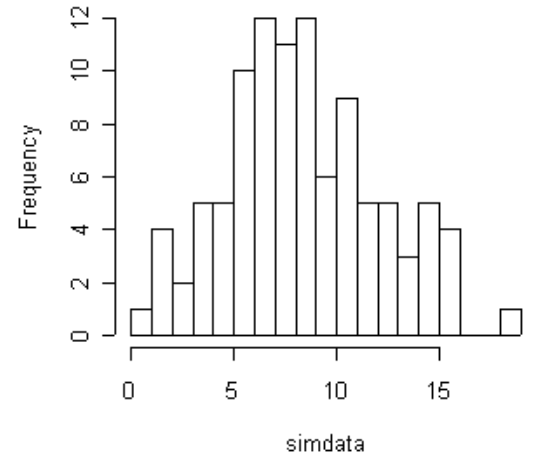
Histogram of simdata



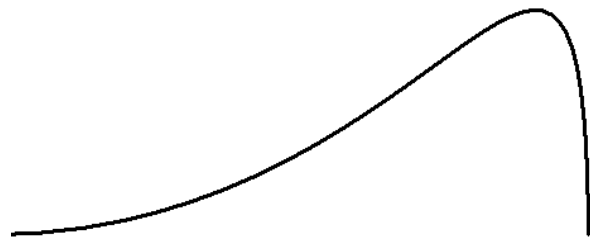
Histogram of simdata



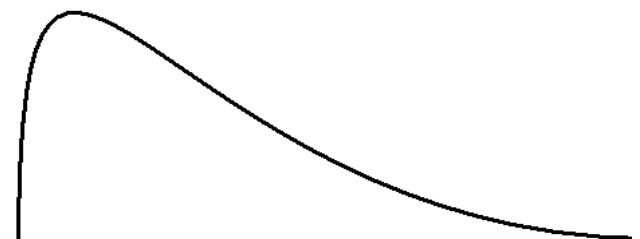
Histogram of simdata



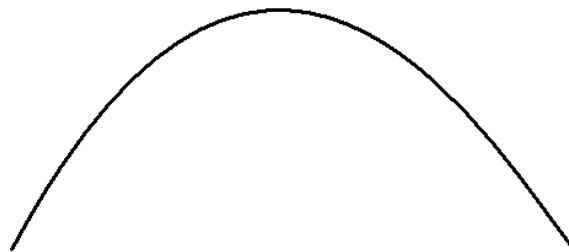
Some general histogram forms



left-skewed

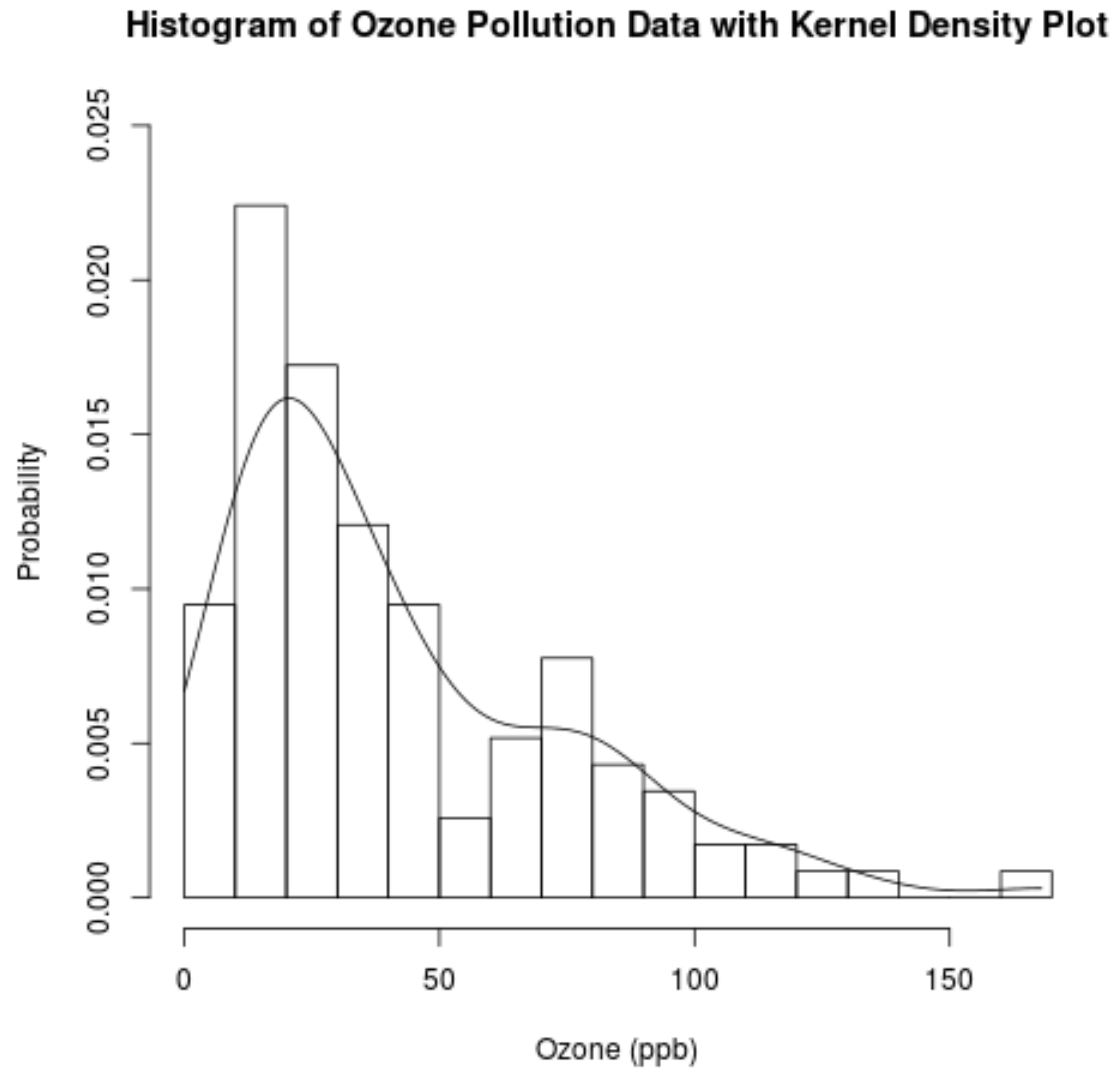


right-skewed

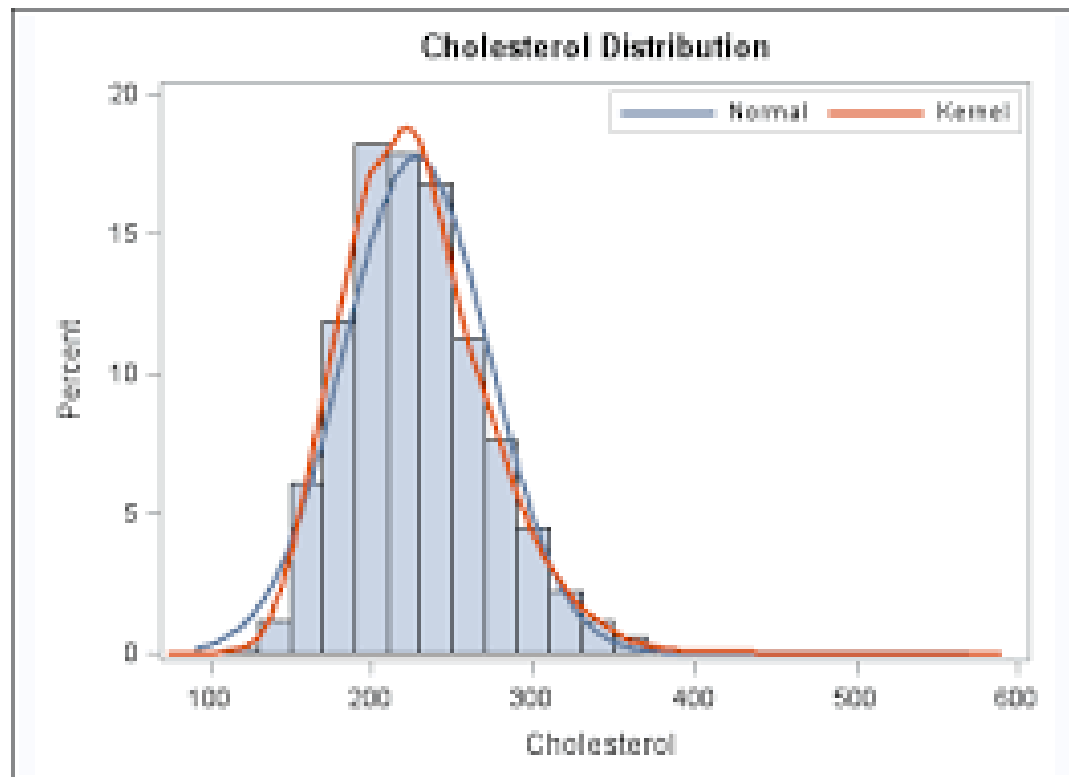


symmetric

Histogram: bars and smoothed



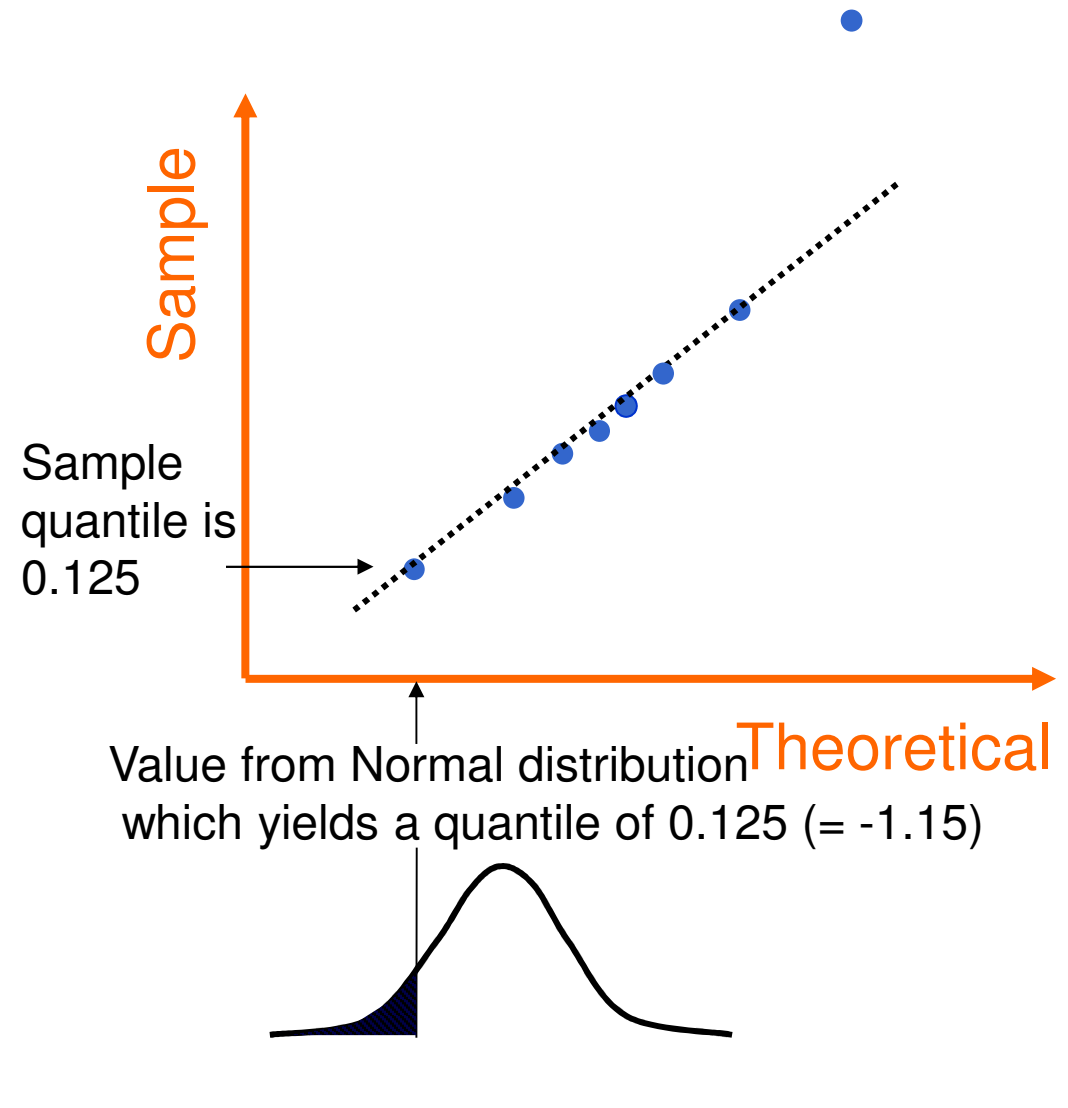
Histogram: comparing distributions



- Histogram, smoothed histogram (kernel), normal density
- NOT the best way to compare distributions (use QQ plot)

QQ-Plot

- Quantile-quantile plot
- Used to assess whether a sample follows a particular (e.g. normal) distribution (or to compare two samples)
- A method for looking for outliers when data are mostly normal

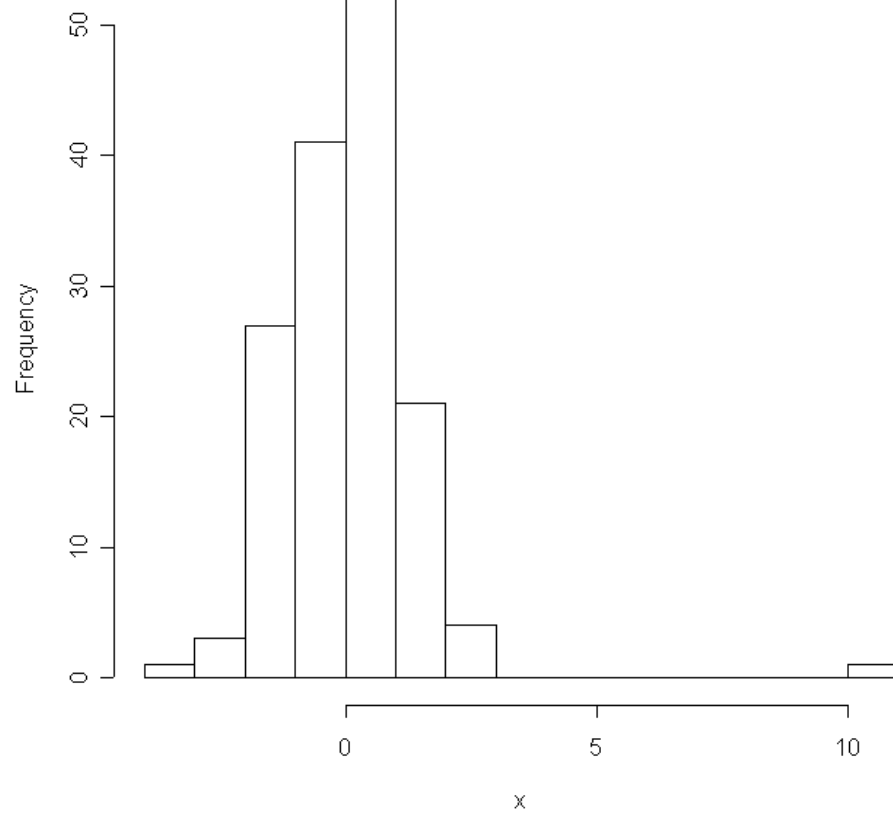


Typical deviations from straight line patterns

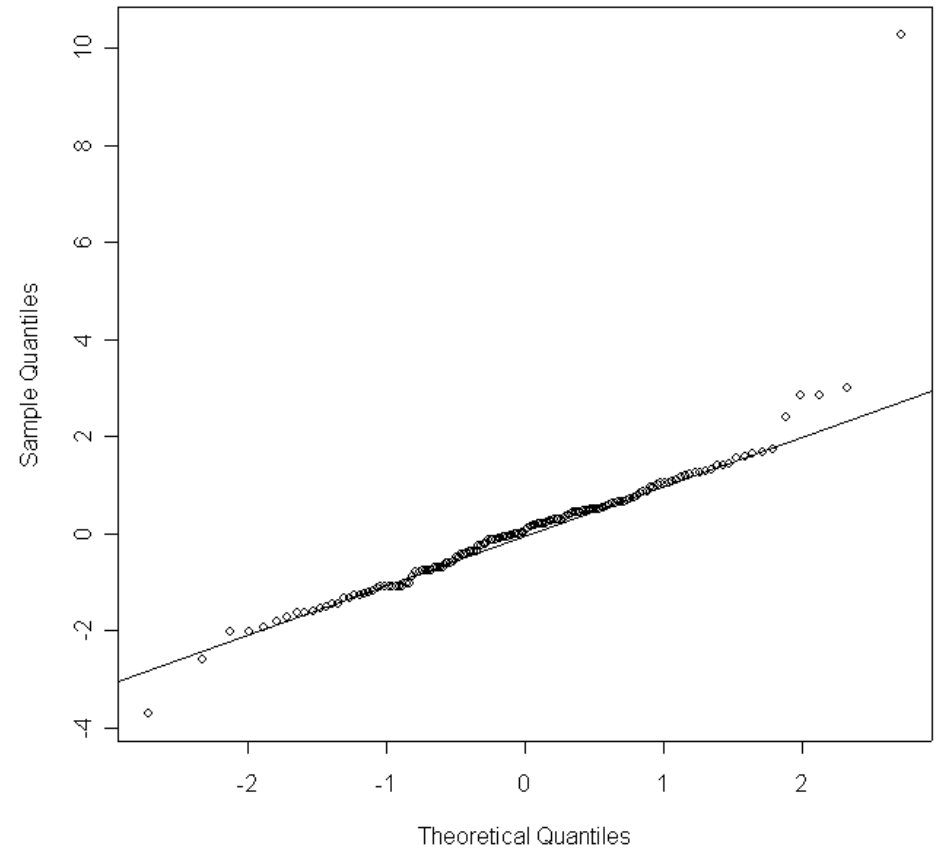
- Outliers
- Curvature at both ends (long or short tails)
- Convex/concave curvature (asymmetry)
- Horizontal segments, plateaus, gaps

Outliers

Histogram of x

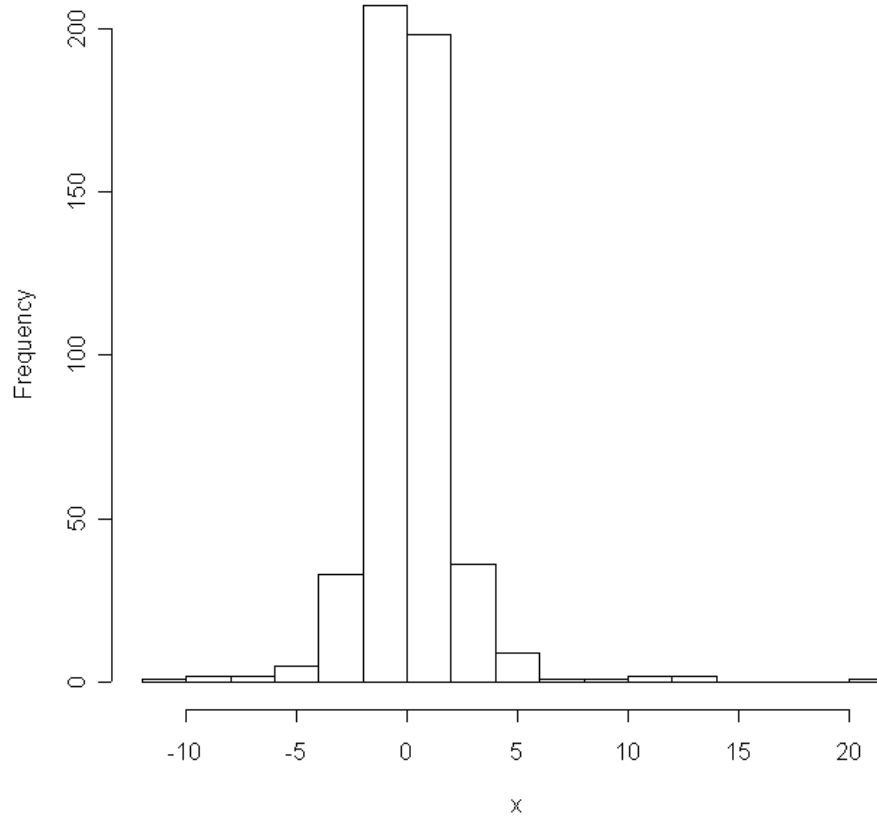


Normal Q-Q Plot

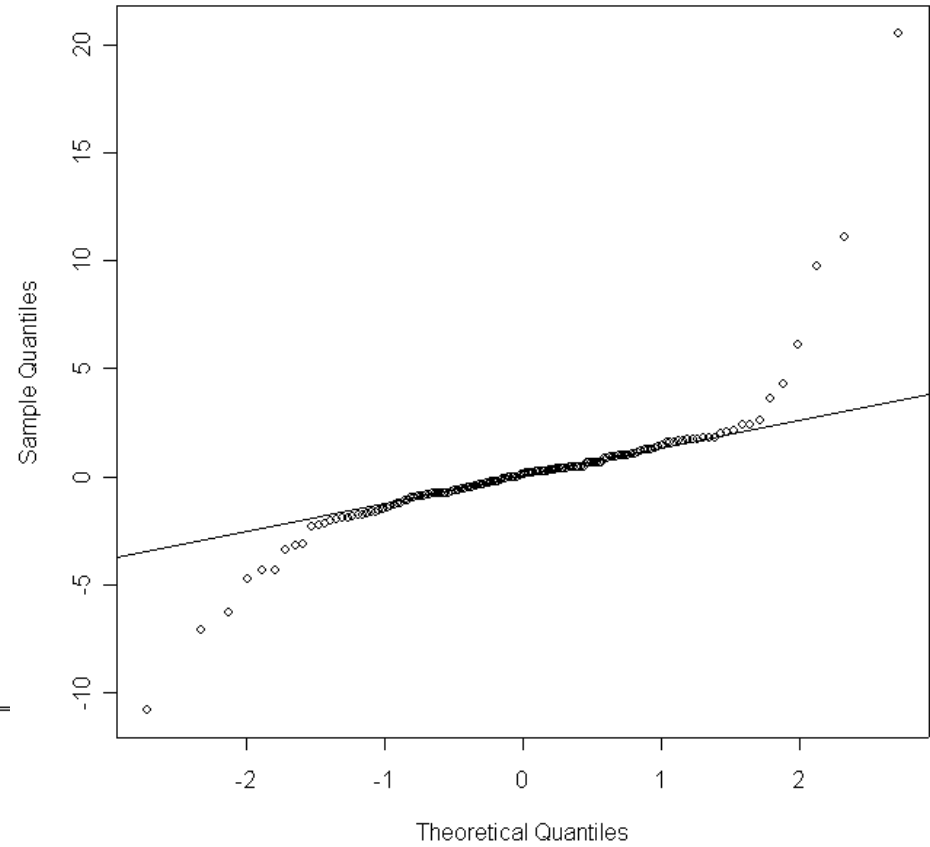


Long Tails

Histogram of x

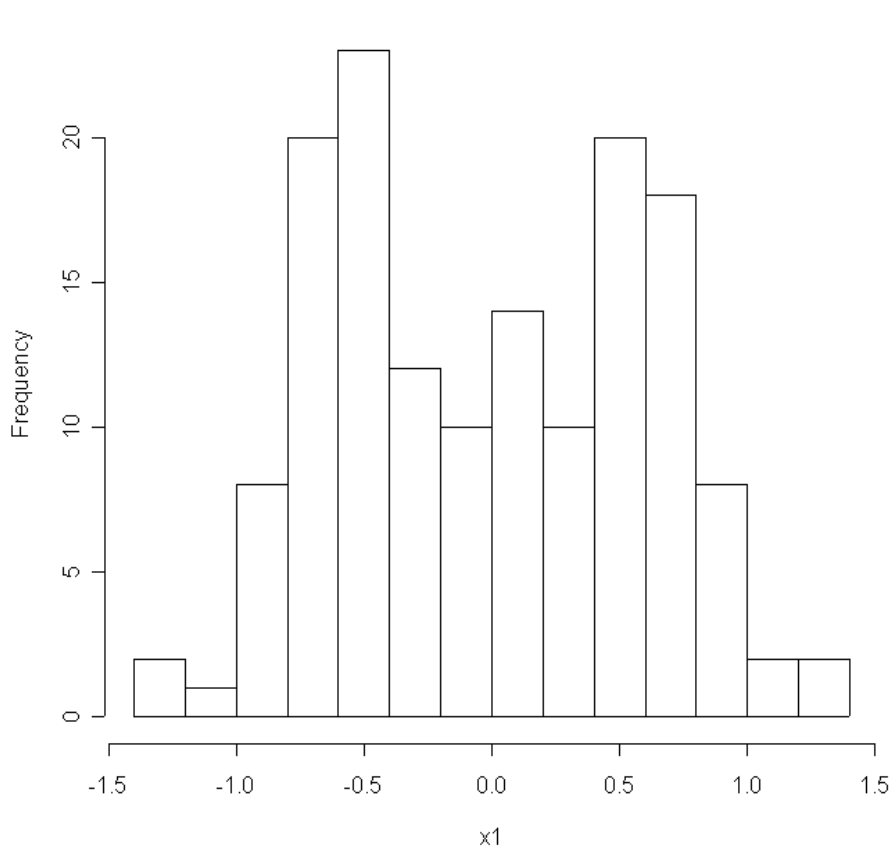


Normal Q-Q Plot

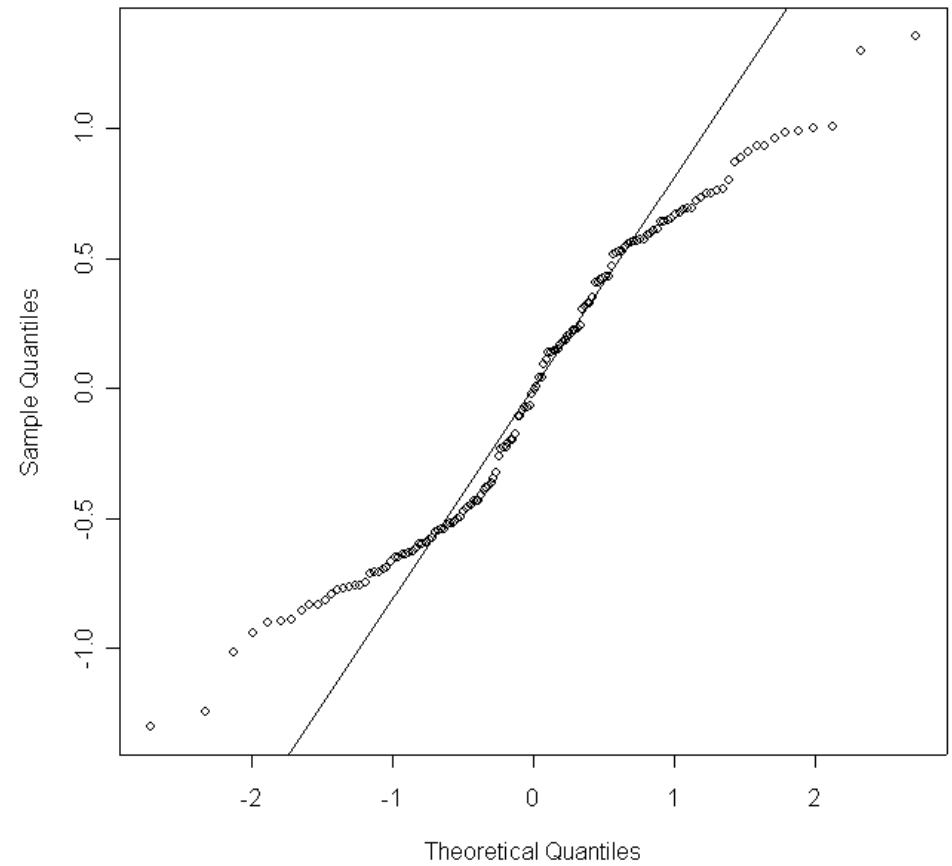


Short Tails

Histogram of x1

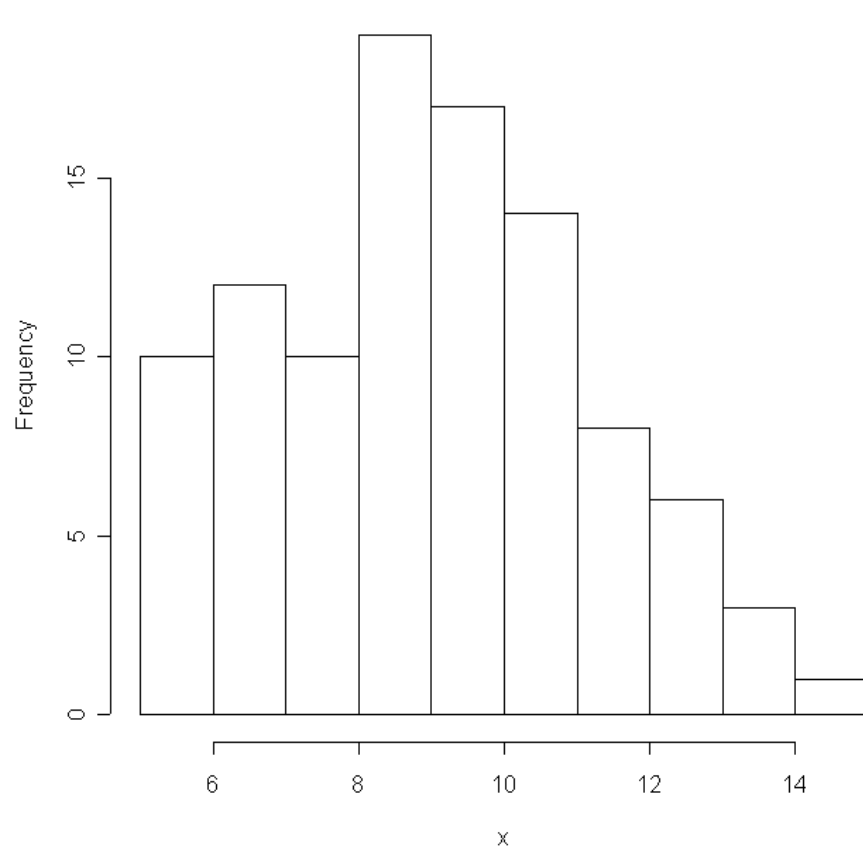


Normal Q-Q Plot

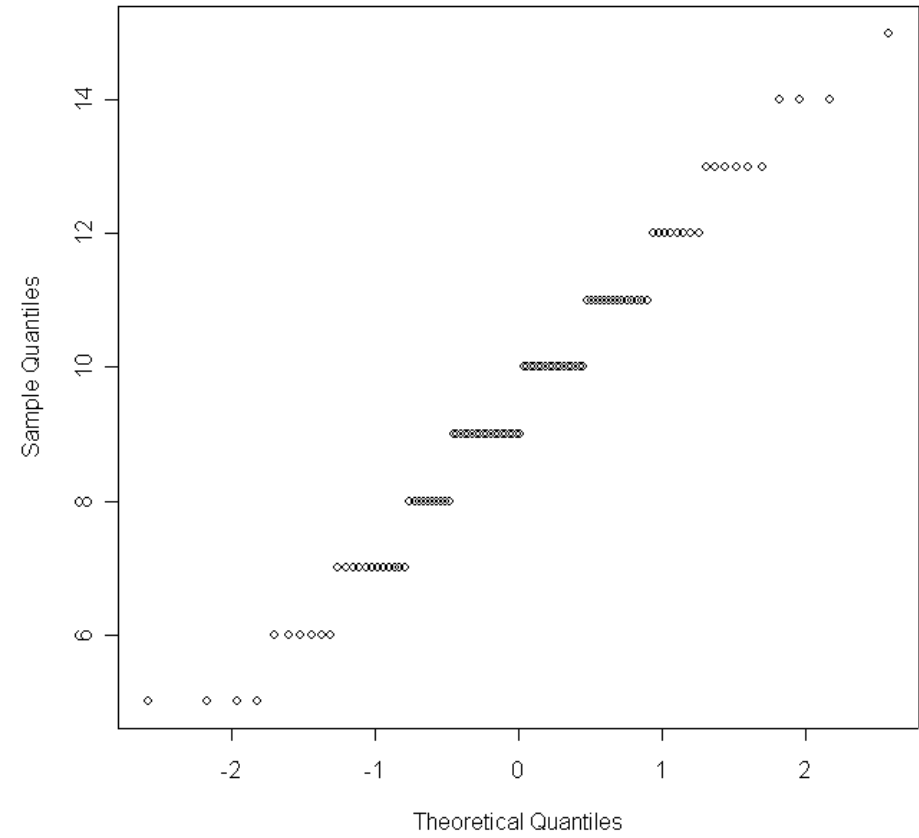


Plateaus/Gaps

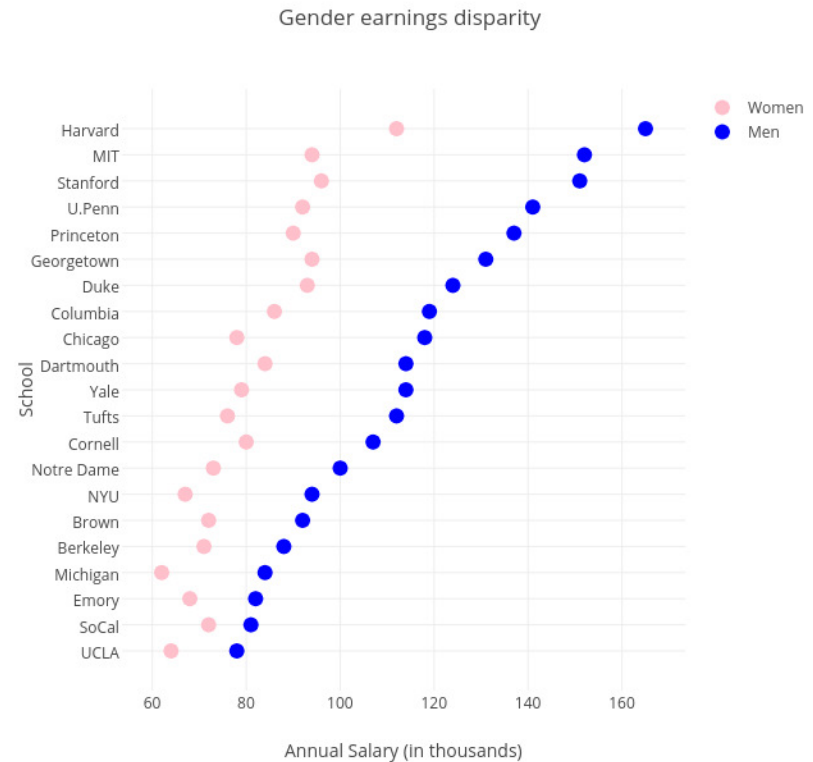
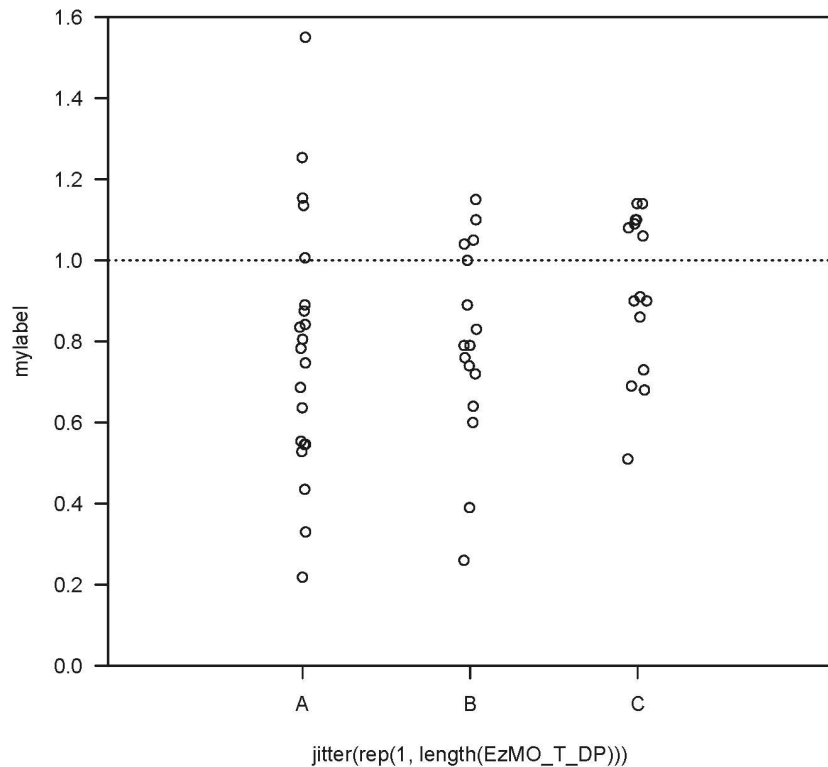
Histogram of x



Normal Q-Q Plot



Dot plot



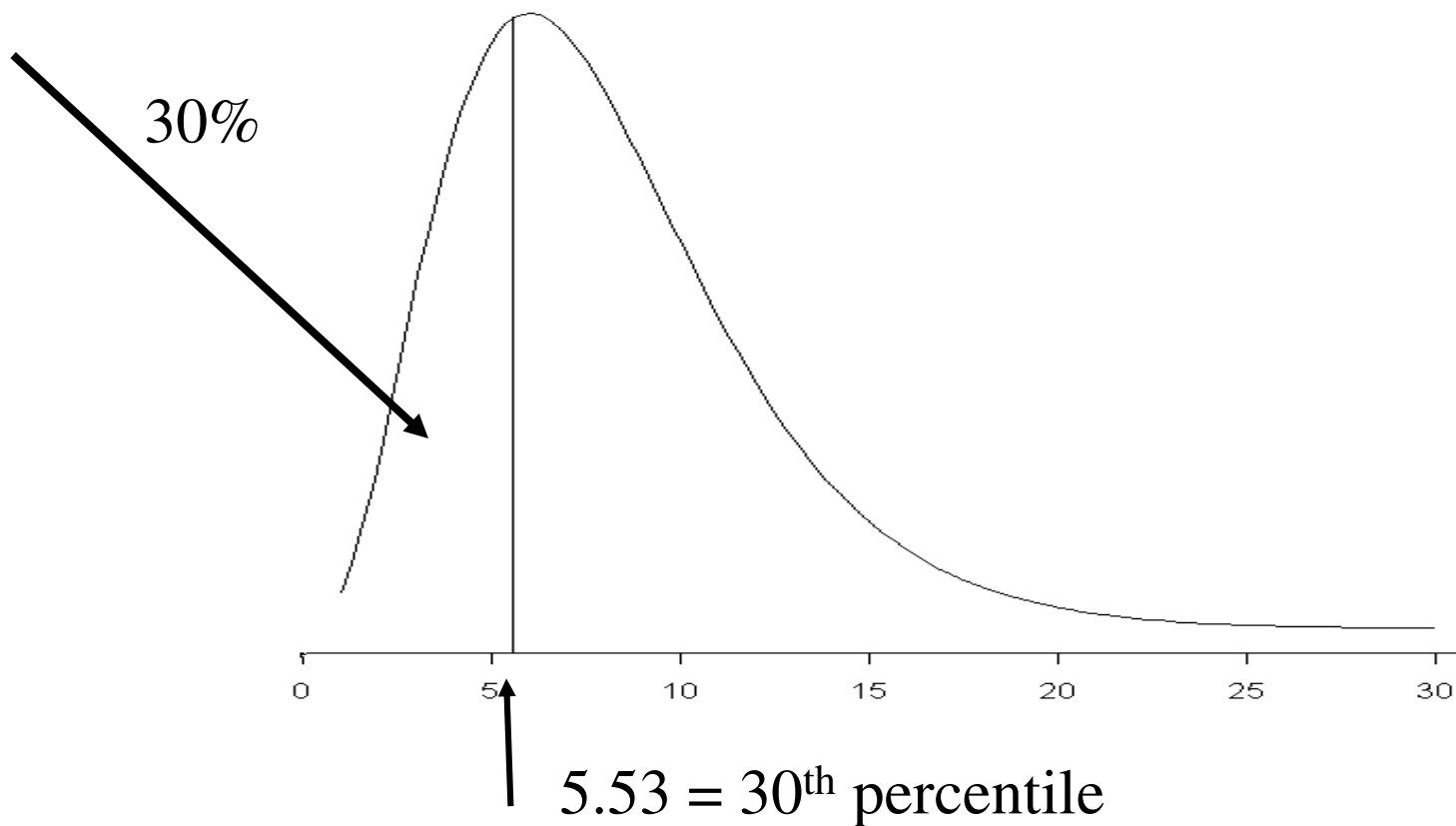
- *Values* plotted separately (as dots) for each group
- Most useful when there *aren't too many* observations

Numerical Summaries

- To provide *objectivity* (put in same objects to same methods, get out same classification)
 - This is in contrast to *experts* deciding
- To provide *stability*
 - Would like classification to be ‘robust’ to a wide variety of additions of objects, or characteristics
- Categorical/Qualitative variables
 - frequency table
- Numerical/Quantitative variables
 - Distribution: quantiles
 - Center: mean, median
 - Spread: SD, IQR, MAD

Quantiles

- The p^{th} *quantile* is the number that has the proportion p of the data values smaller than it
-



Measures of center

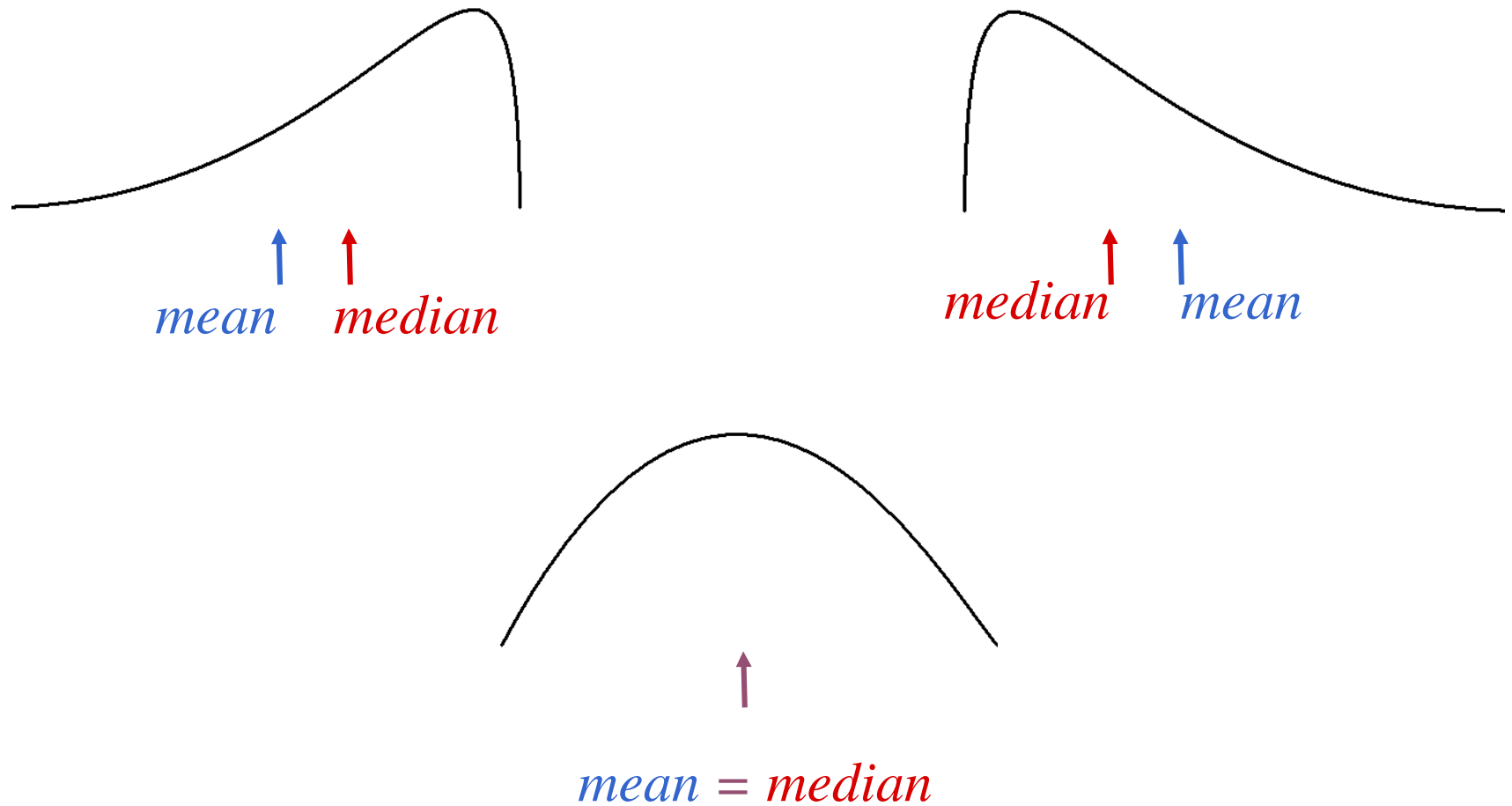
■ Mean

- Total of the values divided by the number of values
- Appropriate for distributions that are fairly *symmetric*
- *Sensitive* to outliers (since all values contribute equally)
- ‘Balance-point’ for a histogram

■ Median

- The *median* value of a variable is the ‘middlemost number: 50% (half) of the values are smaller than it, 50% bigger
- NOT sensitive to outliers (since it ‘ignores’ most values)
- Appropriate summary for *skewed distributions*

Relative location of mean and median

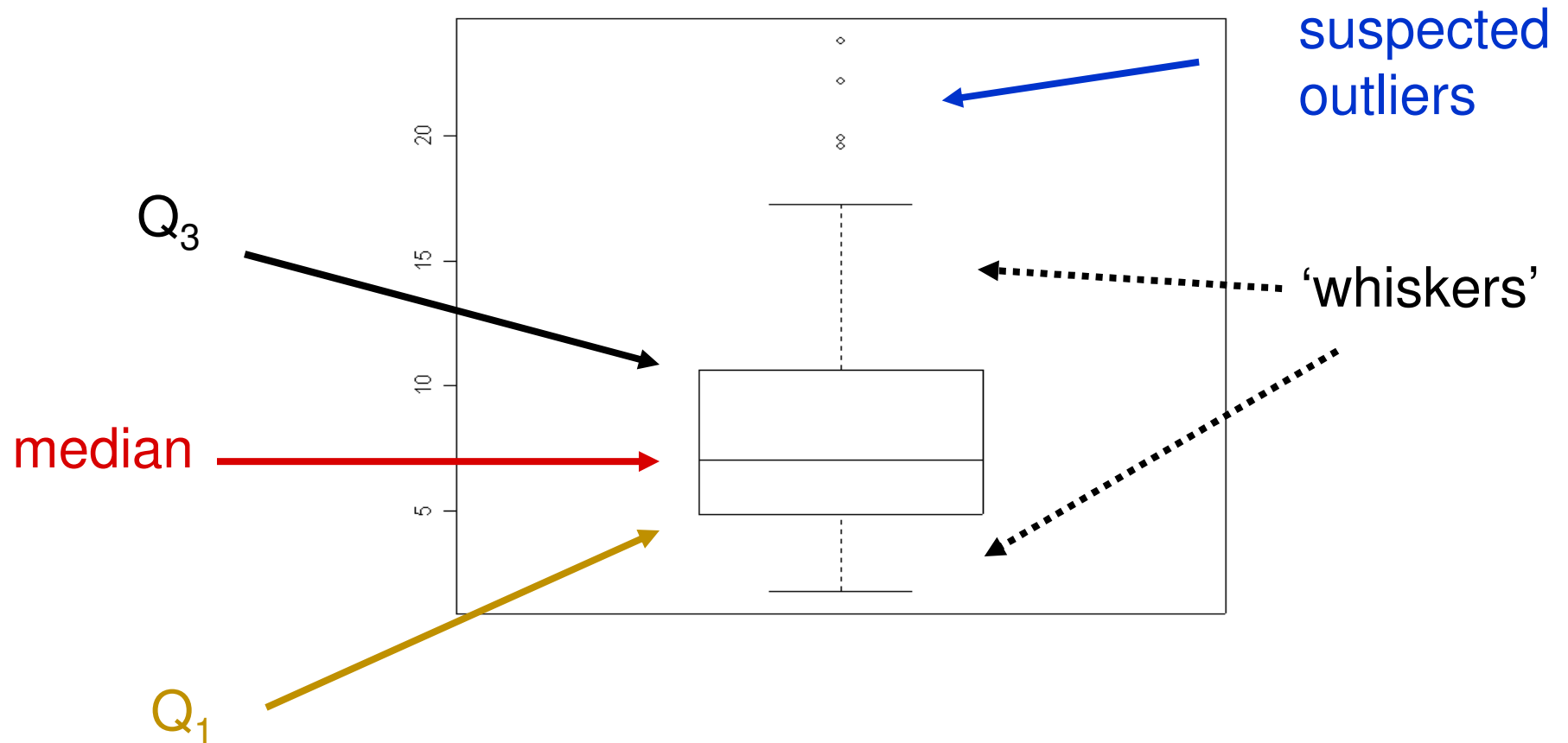


Measures of spread

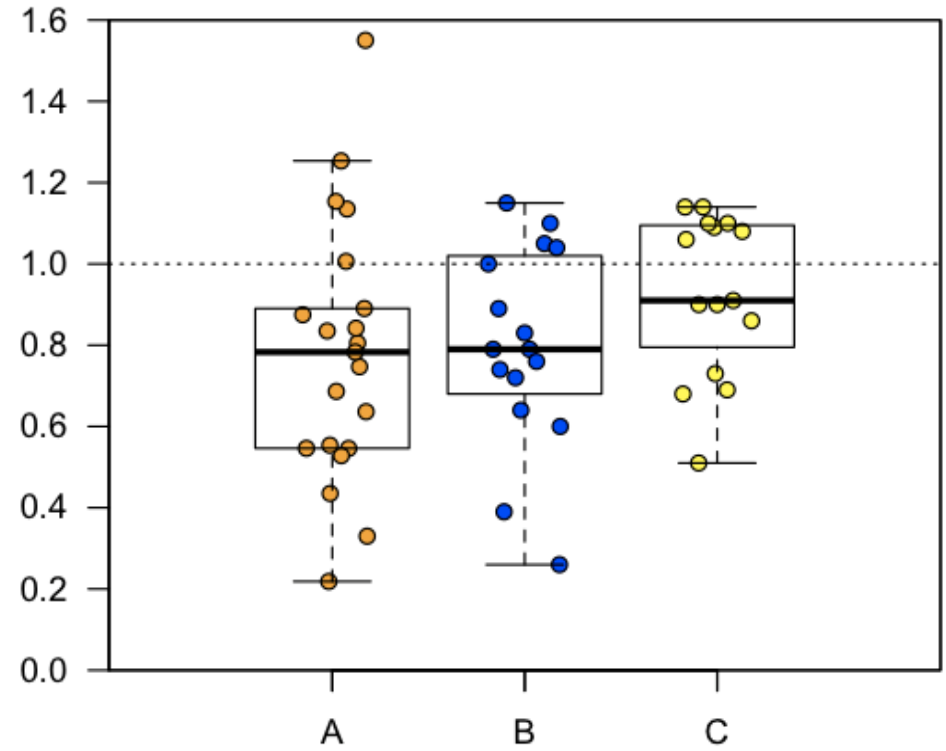
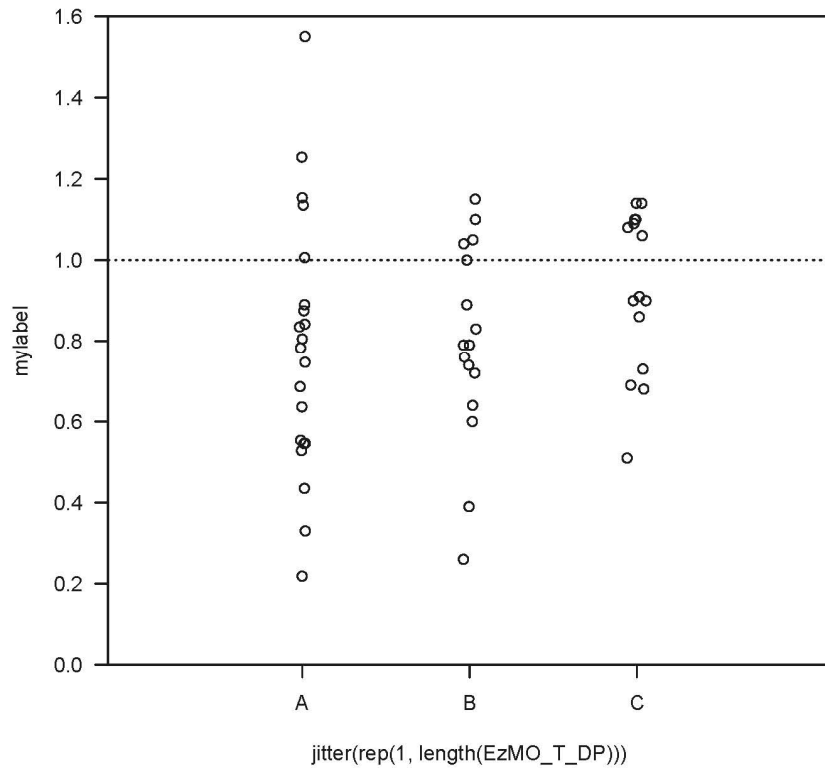
- Standard deviation (SD)
 - Square root of the average* of squared deviations from mean
 - Appropriate when center measured with the *mean*
- Interquartile range (IQR)
 - Distance between 25th (Q_1) and 75th (Q_3) percentiles:
$$\text{IQR} = Q_3 - Q_1$$
 - One measure of spread when center measured with *median*
- Median Absolute Deviation (MAD)
 - *Median* of *absolute* values of *deviations* from median
 - More *robust* measure of spread than SD
 - Another way (besides IQR) to measure spread when center measured with *median*

Five-number summary and boxplot

- Overall summary of the distribution: Min, Q_1 , Median, Q_3 , Max
- A *boxplot* provides a visual summary:



Box plot combined with dot plot



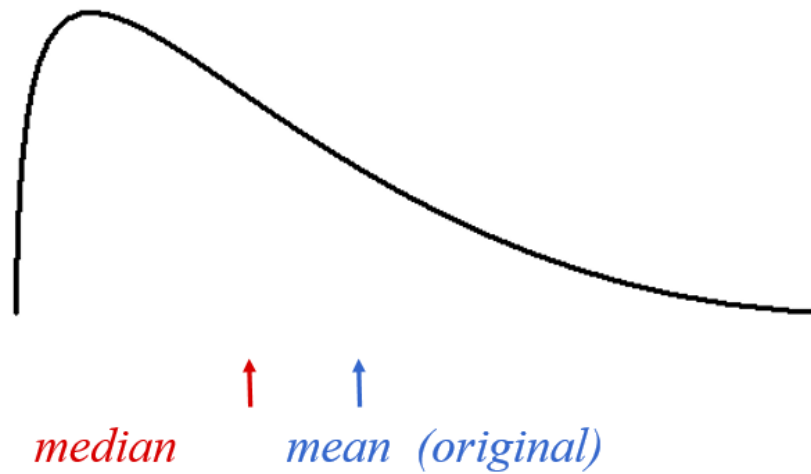
- *'jitter'*, *size* and *color* aid in the comparison of groups

Robustness and resistance

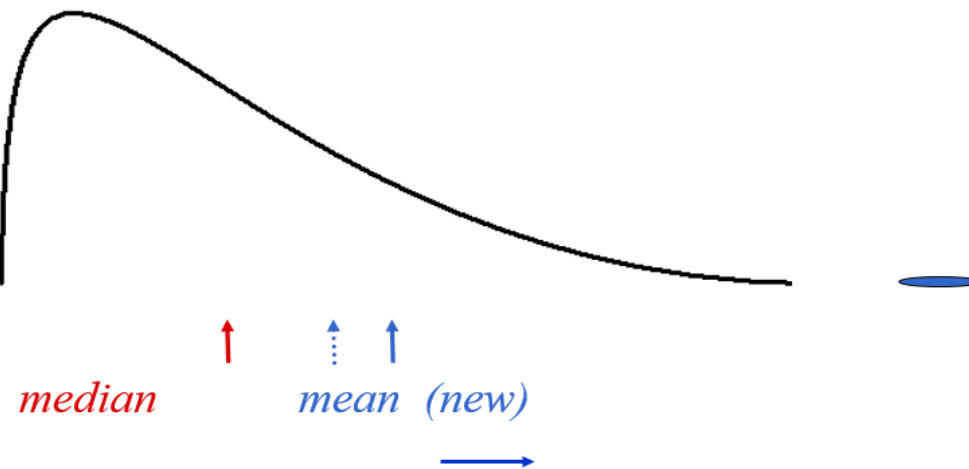
- These concepts refer to *lack of sensitivity* to assumed distributions and effects of a small number of values or outliers
- These qualities are *desirable*: you don't want inferences to be strongly influenced by only a small part of the data set
- The mean is very sensitive to outlying values, the median is very resistant

Robustness of mean, median

Just us:



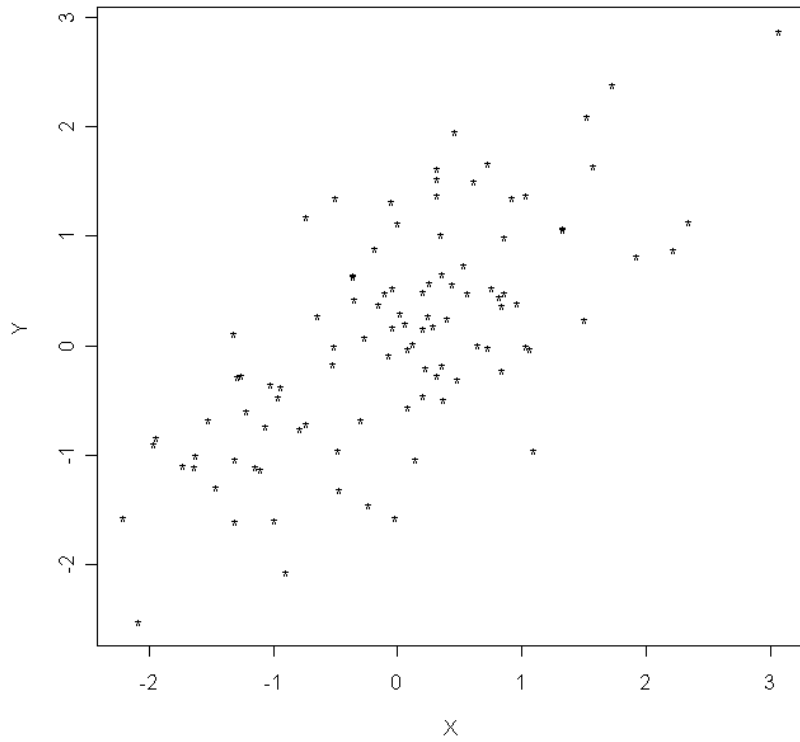
With Mark:



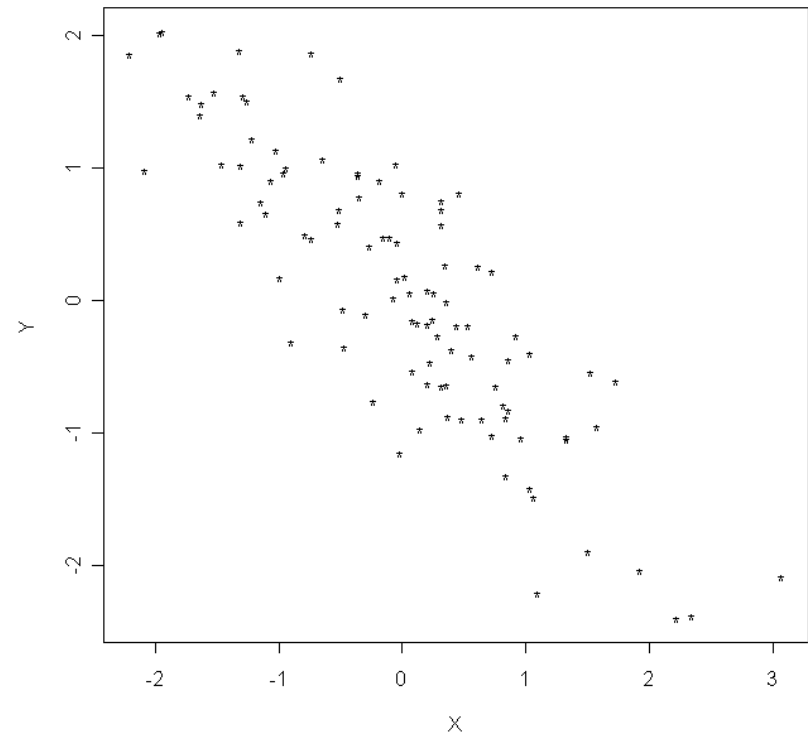
Scatterplot

- We can graphically summarize a bivariate data set with a *scatterplot* (also sometimes called a *scatter diagram*)
- Plots values of one variable on the horizontal axis and values of the other on the vertical axis
- Can be used to see how values of 2 variables tend to move with each other (*i.e.* how the variables are *associated*)

Scatterplots

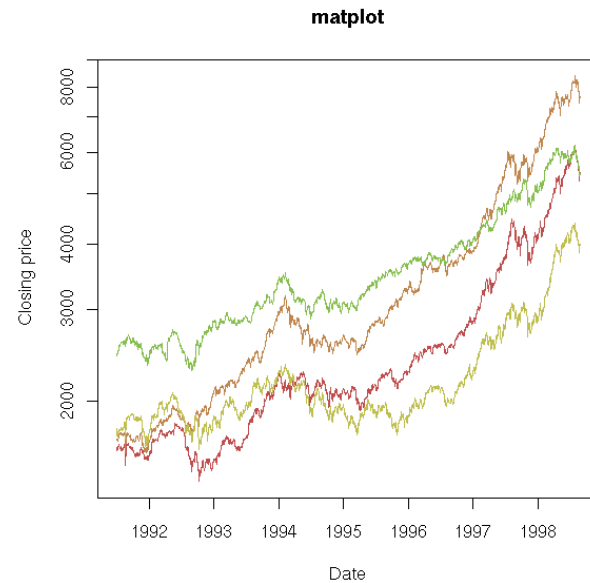
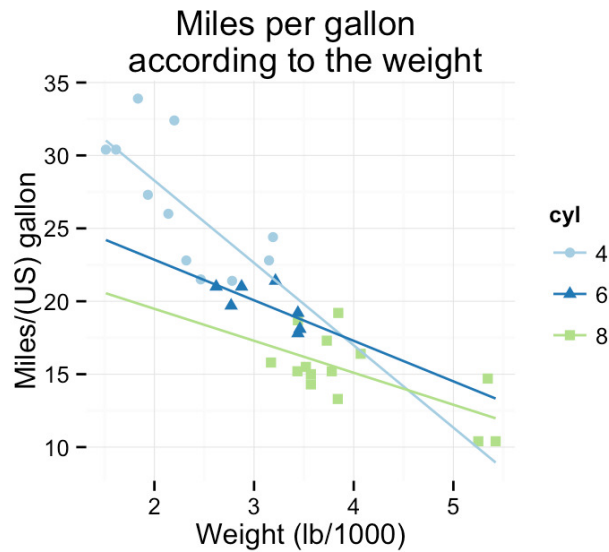
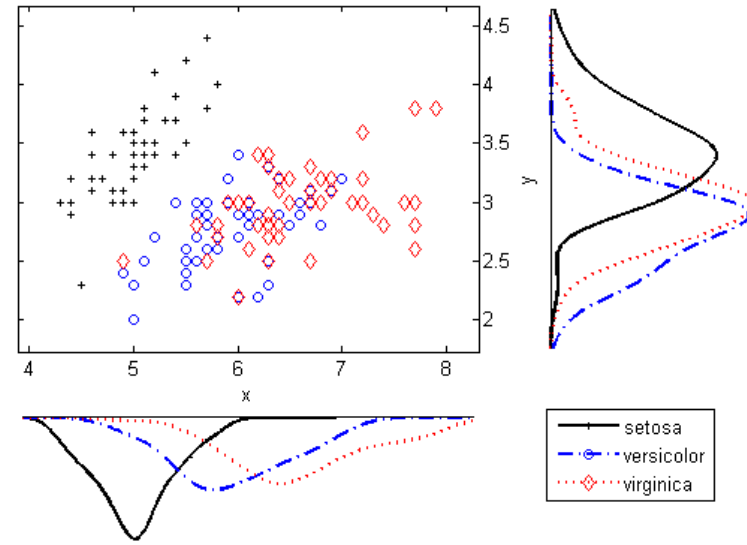
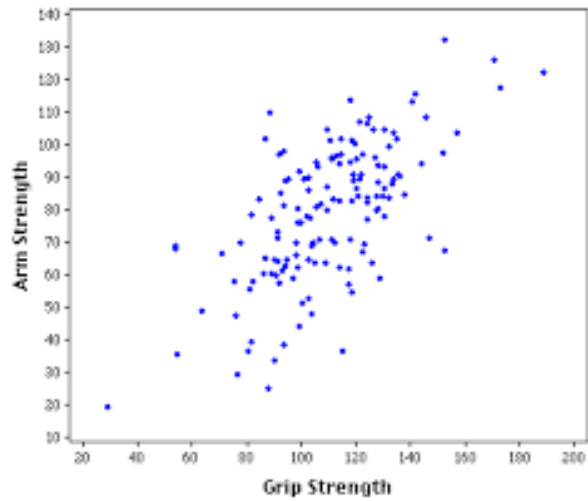


positive association

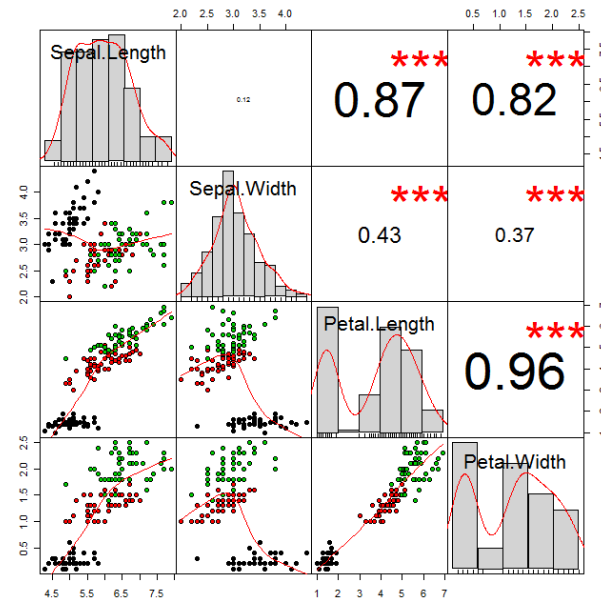
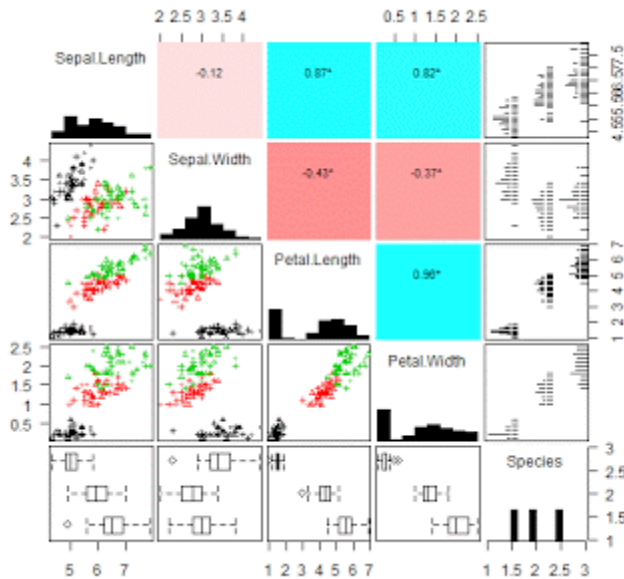
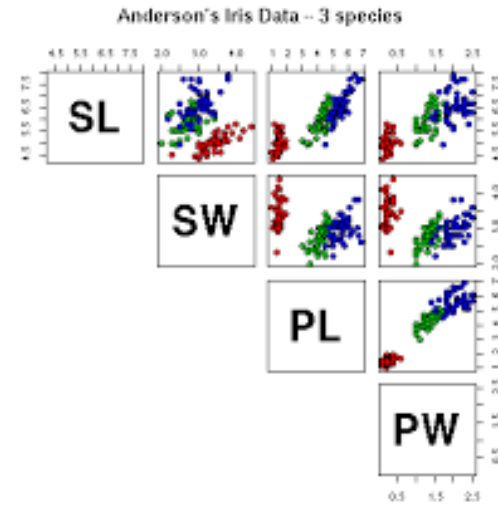
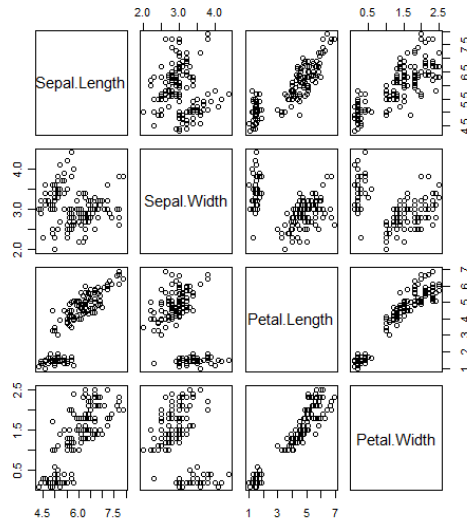


negative association

Scatterplots: customized



All pairwise plots: pairs / splom



Numerical Summary

- Typically, a bivariate data set is summarized numerically with 5 *summary statistics*
- These provide a fair summary for scatterplots with the same general shape as we just saw, like an oval or an ellipse
- We can summarize each variable *separately* : X mean, X SD; Y mean, Y SD
- But these numbers don't tell us how the values of X and Y vary together

Correlation Coefficient

- The (sample) *correlation coefficient* r is defined as the average value of the product

$$(X \text{ in SUs}) * (Y \text{ in SUs})$$

- [SU = standard units = (value-mean)/SD]
- r is a *unitless* quantity
- $-1 \leq r \leq 1$
- r is a measure of *LINEAR ASSOCIATION*

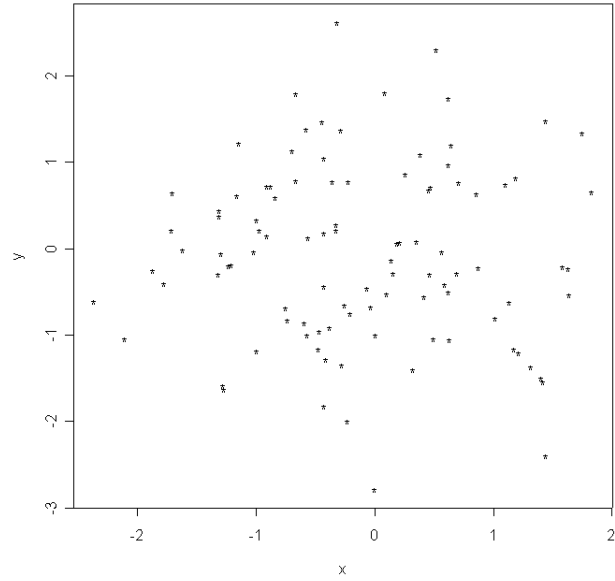
What r is...

- r is a measure of *LINEAR ASSOCIATION*
- The closer r is to -1 or 1 , the more tightly the points on the scatterplot are clustered around a line
- The sign of r (+ or -) is the same as the sign of the slope of the line
- When $r = 0$, the points are not *LINEARLY ASSOCIATED* – this does *NOT* mean there is *NO ASSOCIATION*

...and what r is *NOT*

- r *is* a measure of *LINEAR ASSOCIATION*
- r does *NOT* tell us if Y is a function of X
- r does *NOT* tell us if X causes Y
- r does *NOT* tell us if Y causes X
- r does *NOT* tell us what the scatterplot looks like

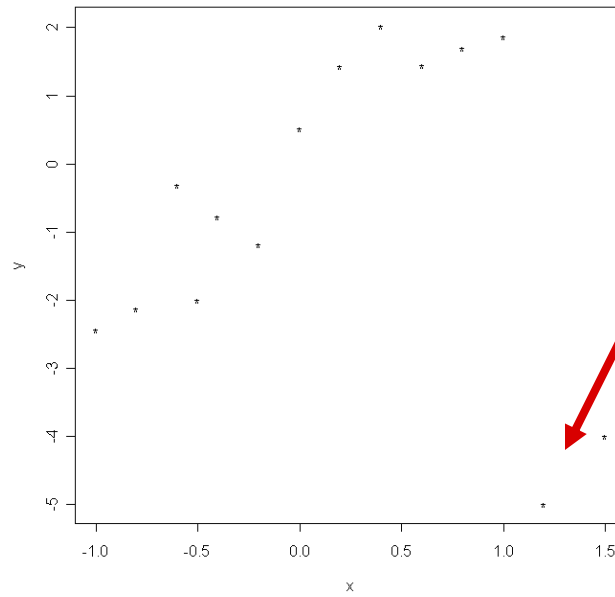
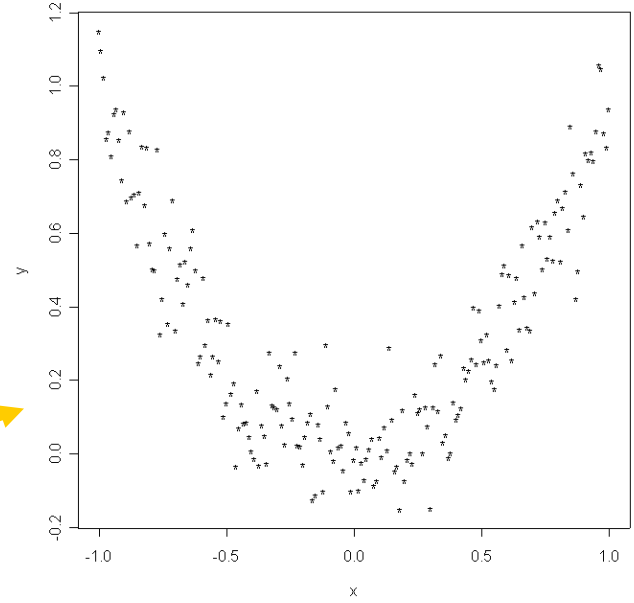
$$r \approx 0$$



random
scatter



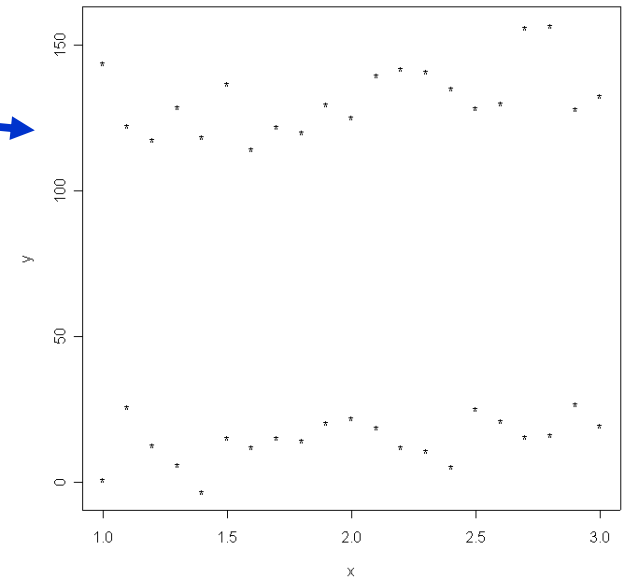
curved
pattern



parallel
lines



outliers



Categorical data

- So far, we have been looking at *continuous* response variables
- Sometimes, the response is *categorical*
 - male/female
 - yes/no
- In this case, we are often interested in questions dealing with *proportions* (rather than means)

Two-way tables

- Table below is from a blind 5 year randomized study of physicians testing whether regular aspirin use reduces mortality from cardiovascular disease
- Every other day, participants took an aspirin or a placebo

| | MI | | |
|---------|-----|--------|--------|
| Group | Yes | No | Total |
| Placebo | 189 | 10,845 | 11,034 |
| Aspirin | 104 | 10,933 | 11,037 |

Table layout

- Tables often better than words to convey quantitative data
- Avoid too many decimal places
- Usually better to use *space* to separate columns (rather than lines):

| Subject | Time 1 | Time 2 |
|---------|---------|---------|
| Joe | 3.67390 | 2.79495 |
| Mary | 4.75435 | 1.23578 |
| Nancy | 3.96456 | 2.84379 |

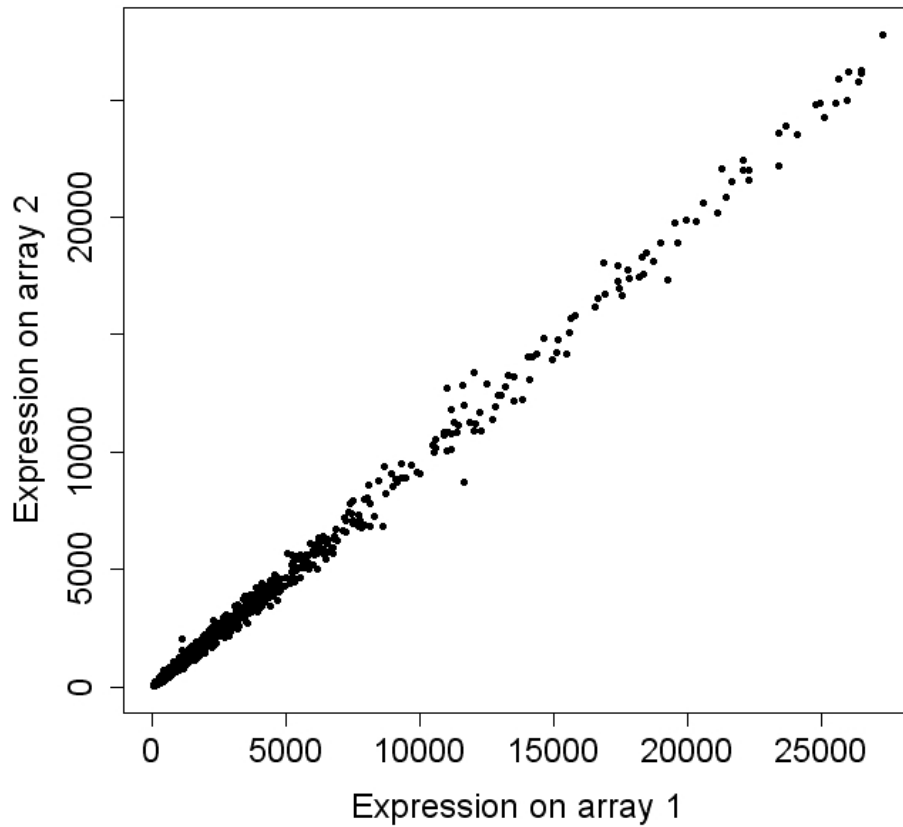
| Subject | Time 1 | Time 2 |
|---------|--------|--------|
| Joe | 3.67 | 2.79 |
| Mary | 4.75 | 1.24 |
| Nancy | 3.96 | 2.84 |

Application: microarray EDA

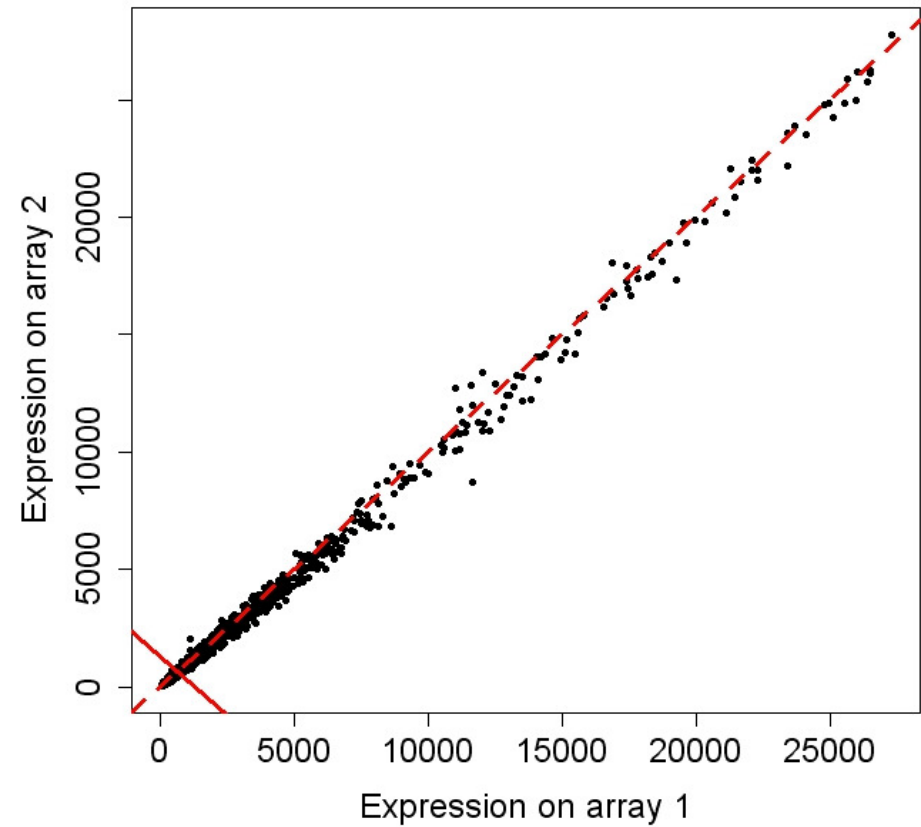
- We are interested in finding true *biologically meaningful differences* between sample types
- Due to other sources of systematic variation, there are also usually *artifactual differences*
- Sources of artifacts include:
 - print tips - differences in subarrays
 - plate effects – differences in rows within subarray
 - batch effects
 - hybridization artifacts
- Exploratory data analysis (EDA) is an important component of microarray data preprocessing

Scatterplots

Expression data from 2 arrays

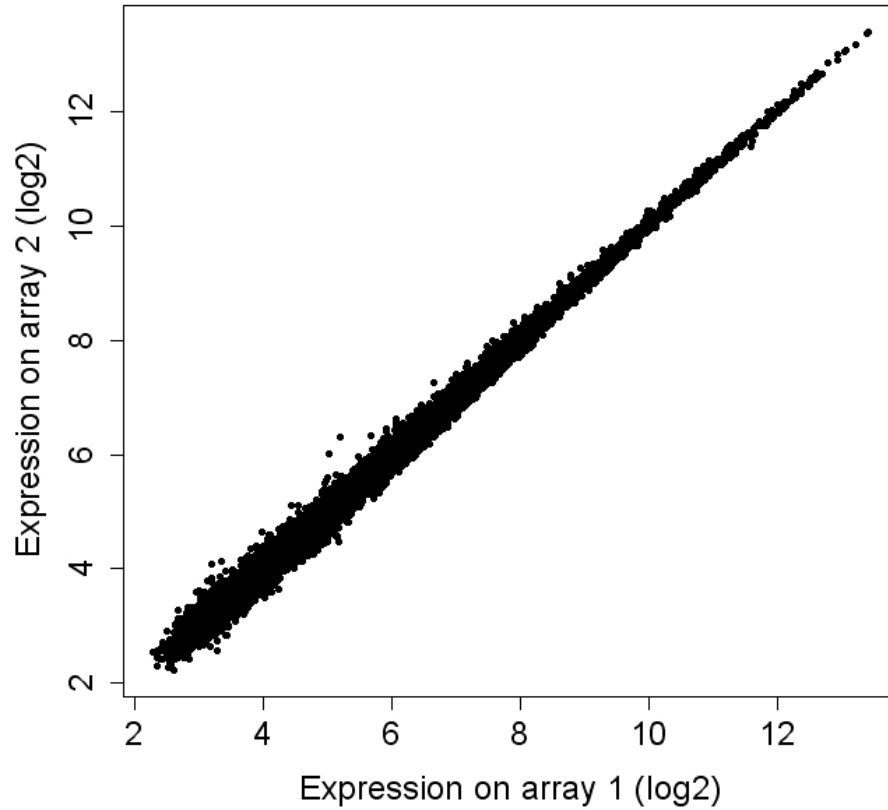


Expression data from 2 arrays

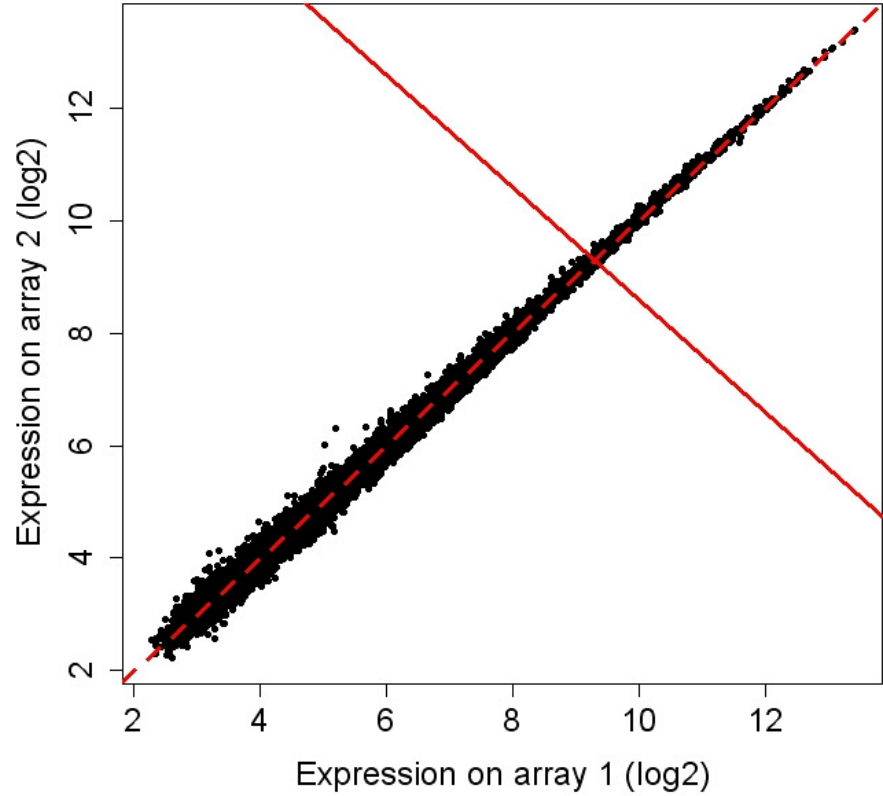


Take logs...

log2 Expression data from 2 arrays

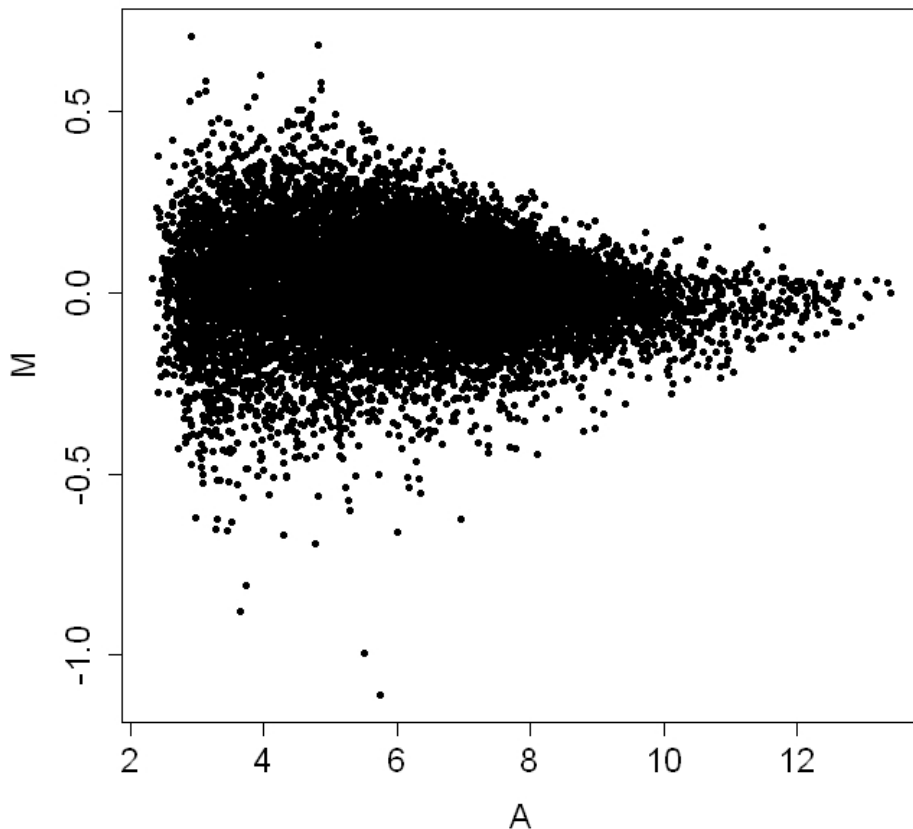


log2 Expression data from 2 arrays

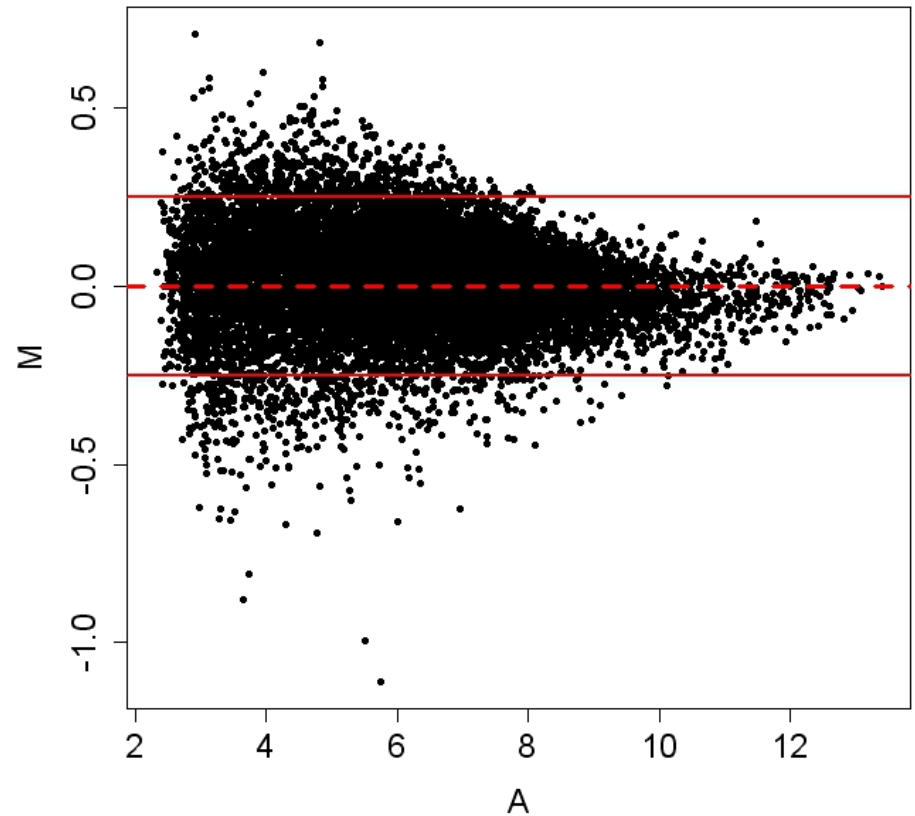


... and rotate (plot Diff vs. Avg.)

MA plot



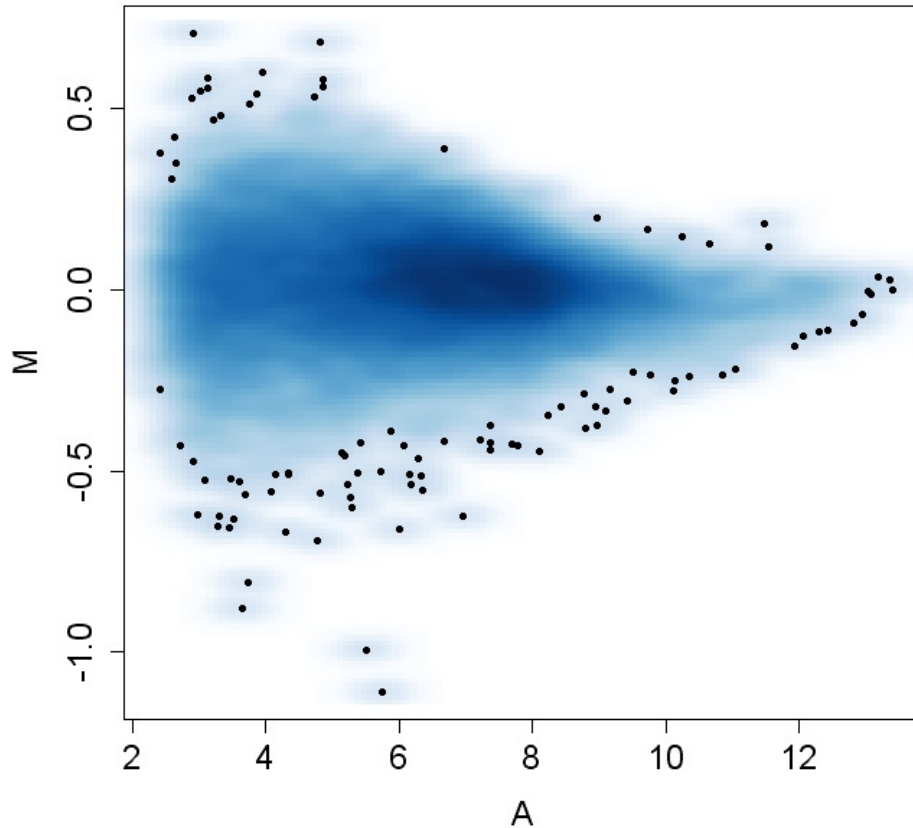
MA plot



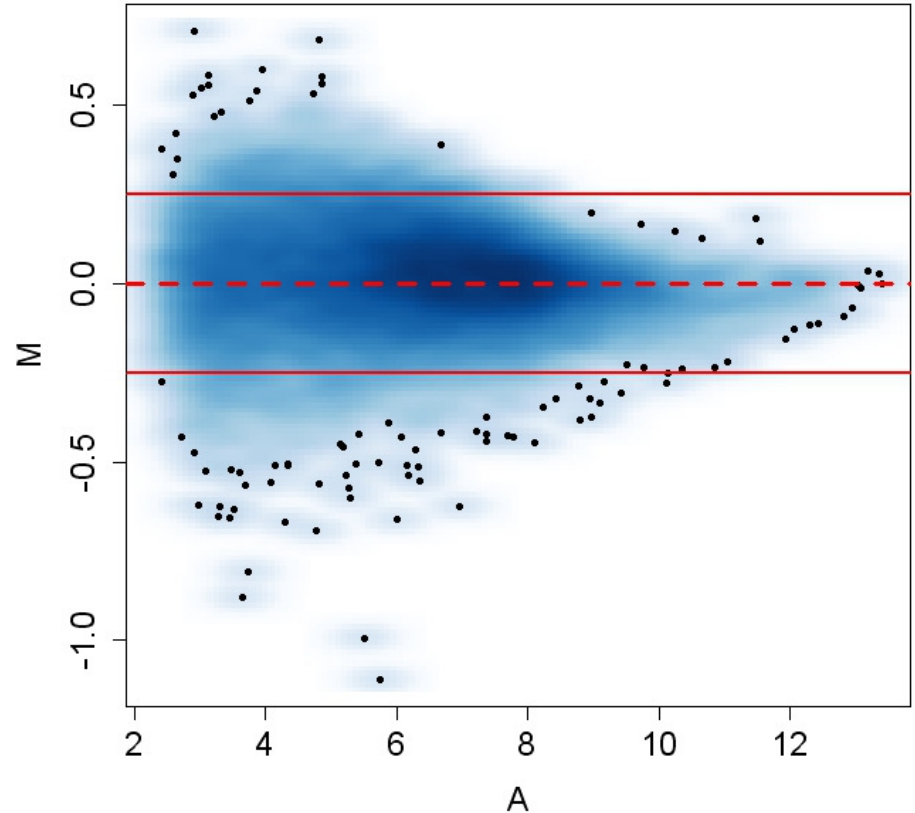
- $M = \text{'minus' (difference)} = \log_2 (\text{expression 2}) - \log_2 (\text{expression 1})$
- $A = \text{'average'} = [\log_2 (\text{expression 1}) + \log_2 (\text{expression 2})]/2$

smoothScatter

MA plot

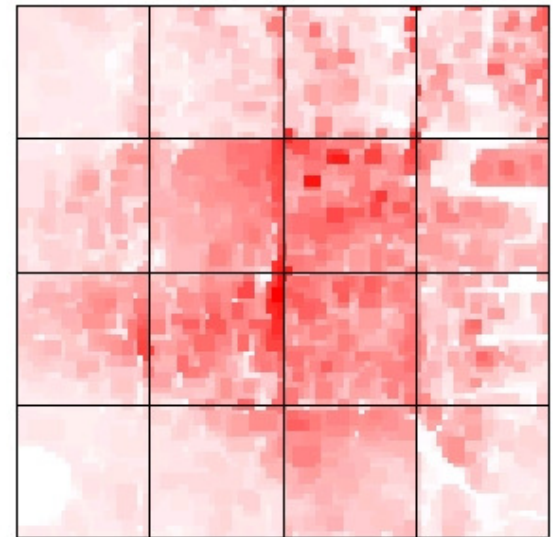
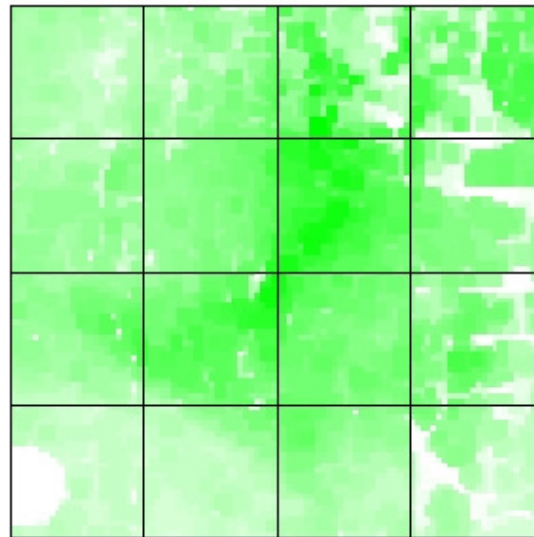
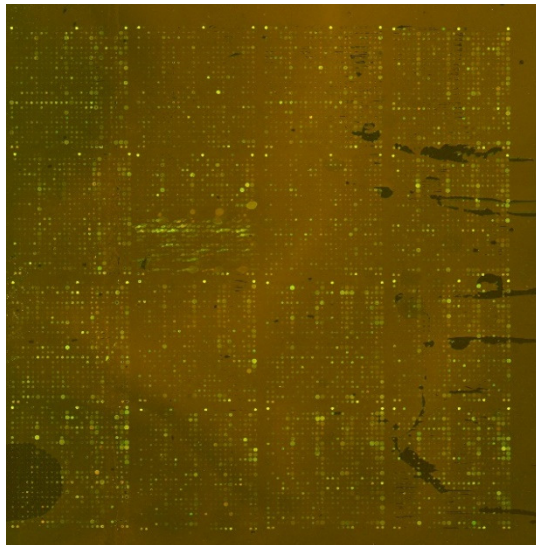
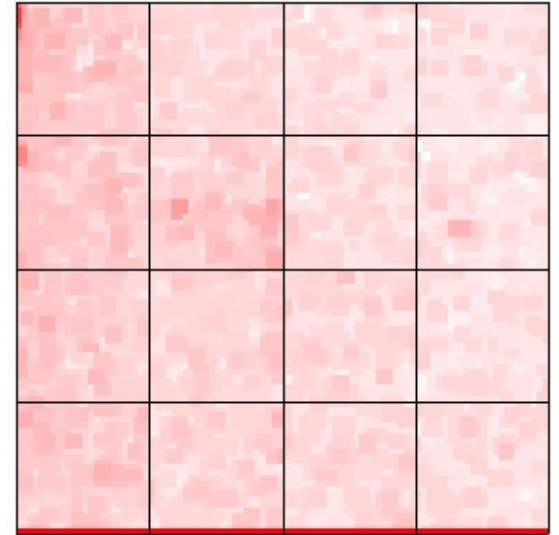
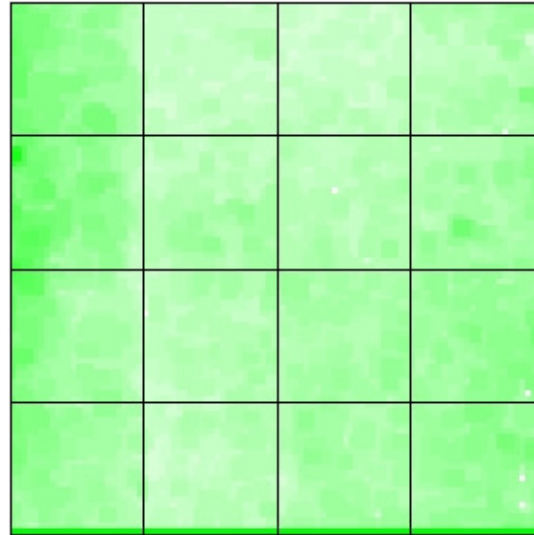
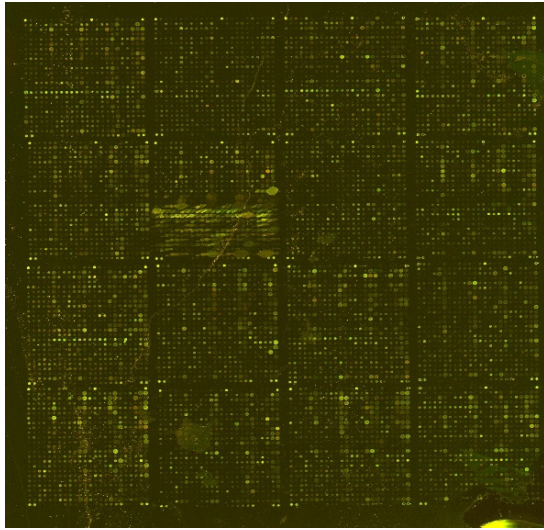


MA plot

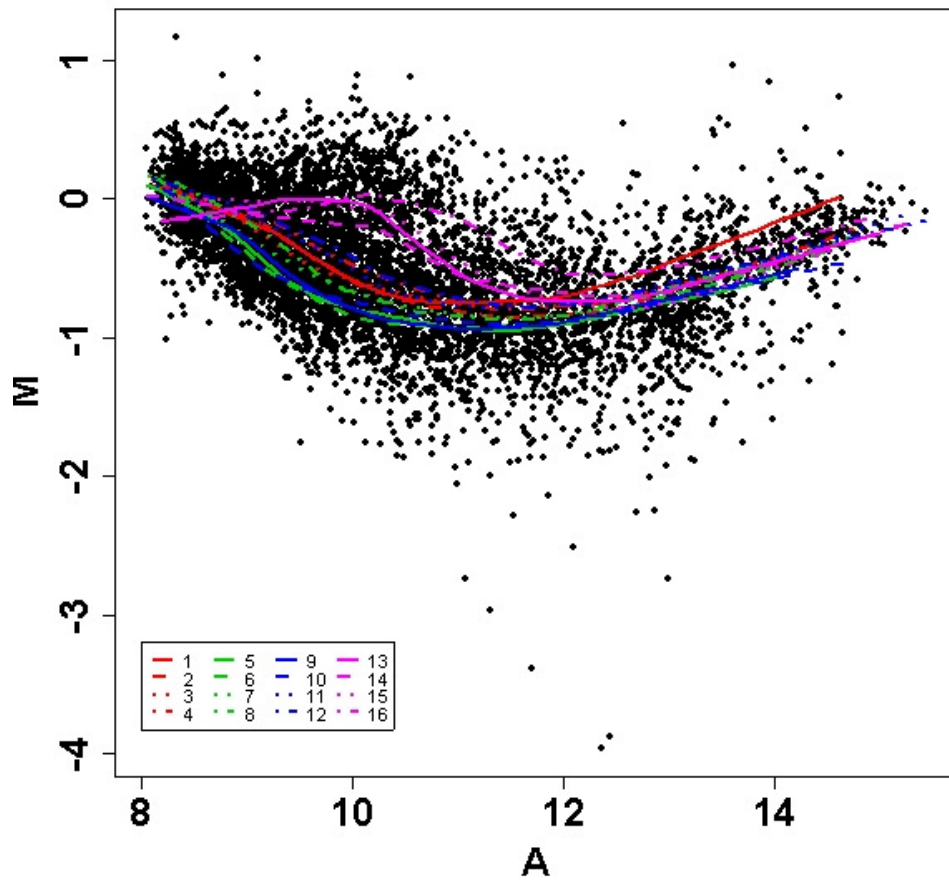


- Rather than plotting all individual points, color plot according to the *density* of points
- Useful when there are many points (here several thousand)

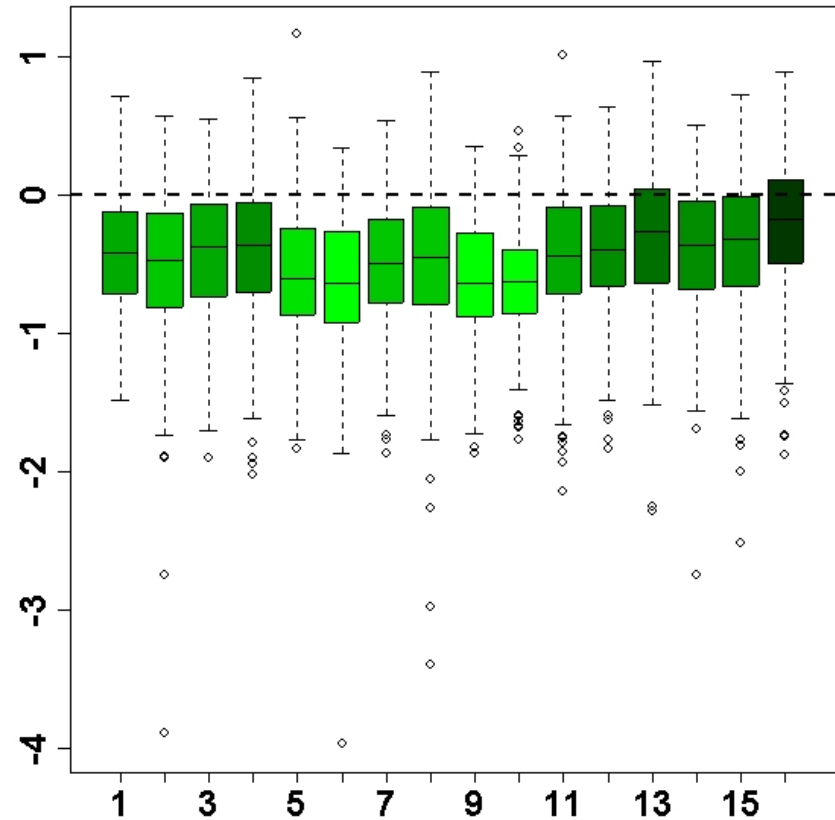
Spatial plots: background from two slides



Pin group (sub-array) effects

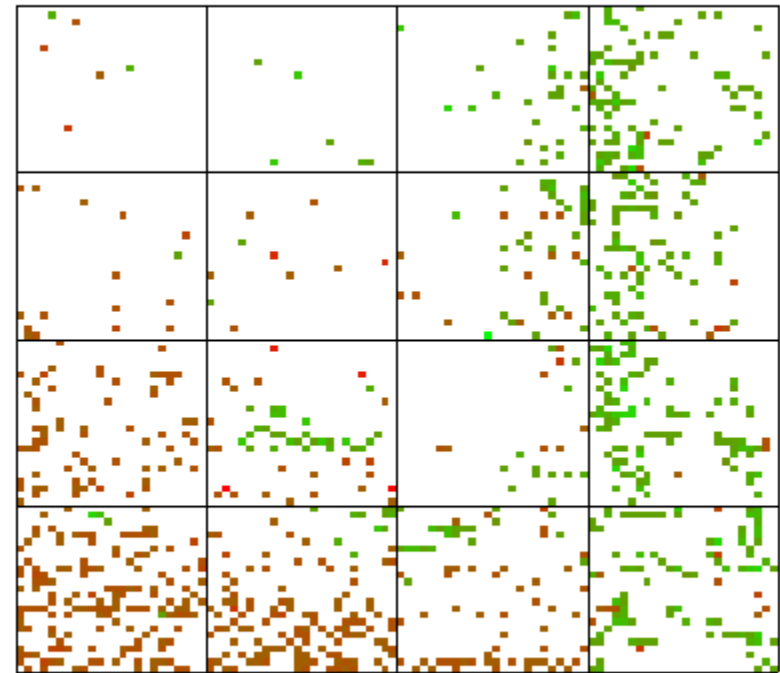
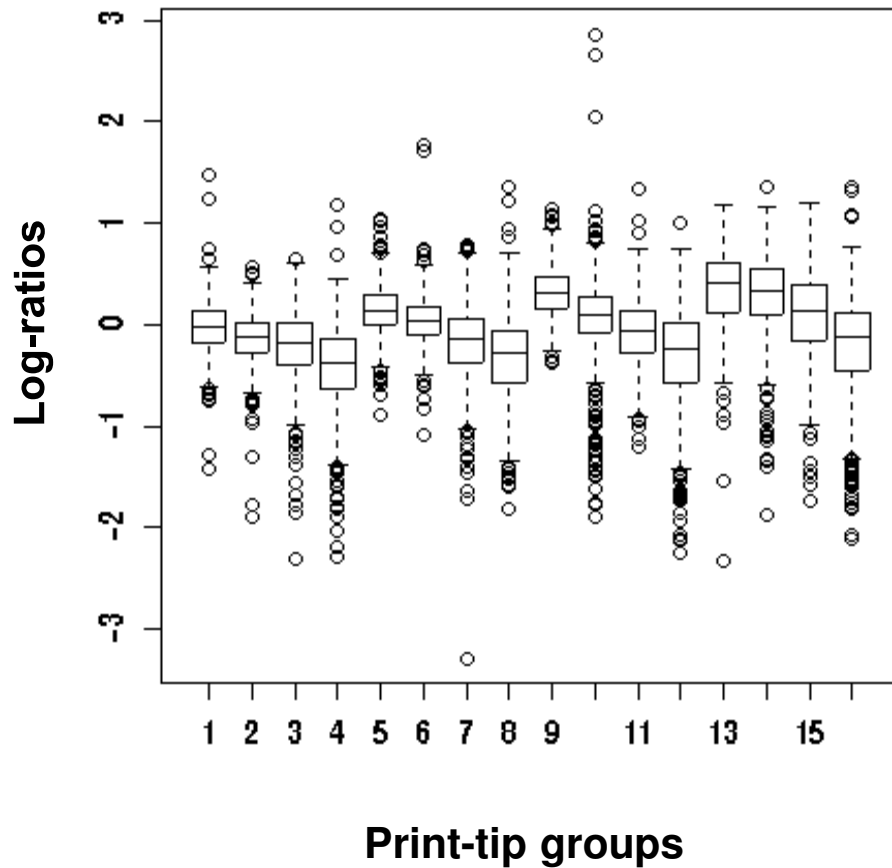


Lowess (local regression) lines through points from pin groups

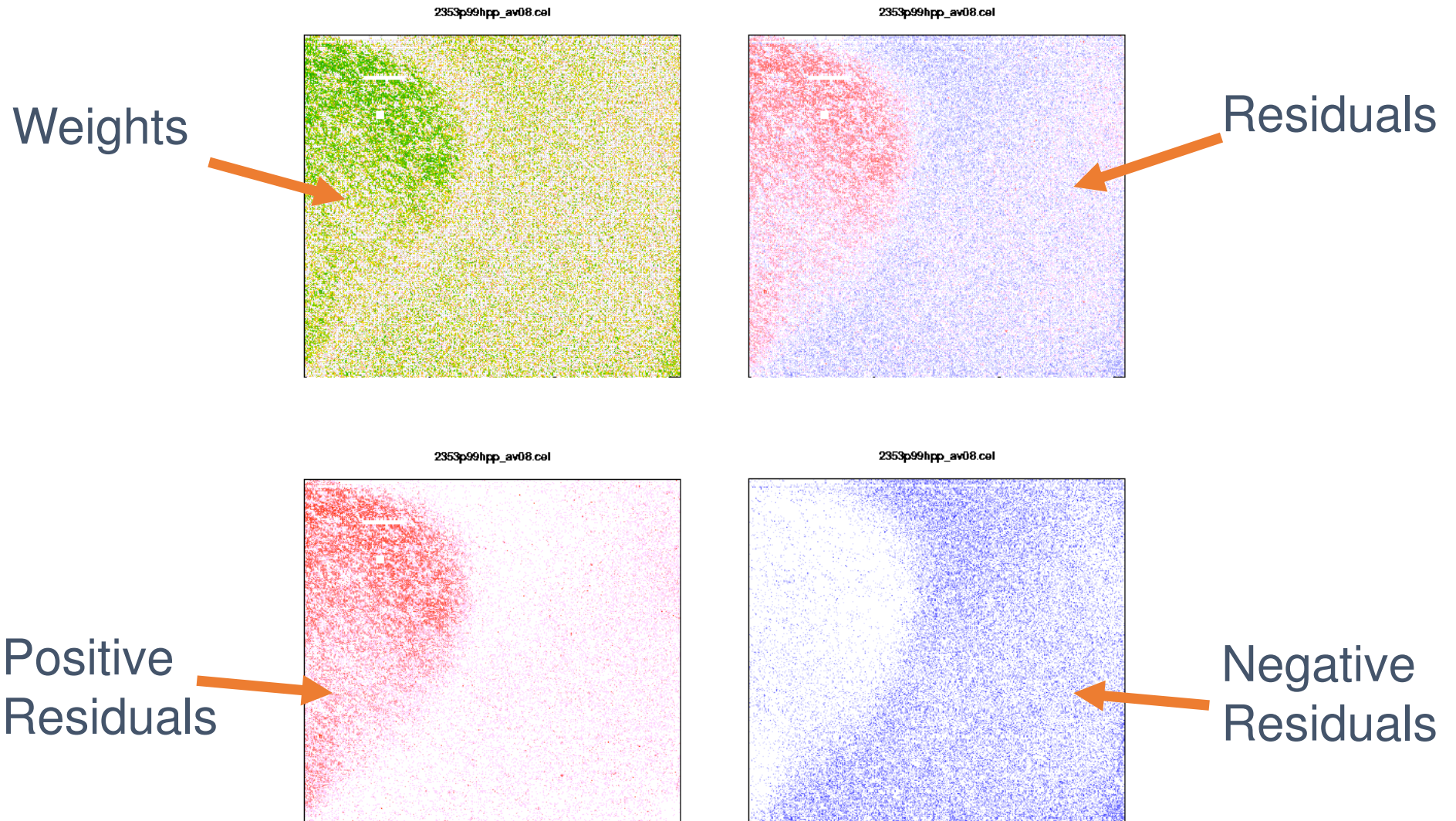


Boxplots of log ratios by pin group

Highlighting pin group effects: Clear example of *spatial bias*



Pseudo-chip images for QC



Presenting results

- *Communicating results* is an important part of science
- There is no magic 'formula' for how to present results!
- You need to think carefully about the message you wish to give and how to present it *clearly* and *convincingly*
- Avoid excessive computer output

Edward Tufte on graphics

- ‘Excellence in statistical graphics consists of complex ideas communicated with clarity, precision and efficiency’; should
 - show the data
 - make the reader think about substance
 - avoid data distortion
 - present many numbers in a small space
 - encourage the eye to make comparisons
 - reveal several levels of detail
 - serve a clear purpose
- See also work by Karl Broman

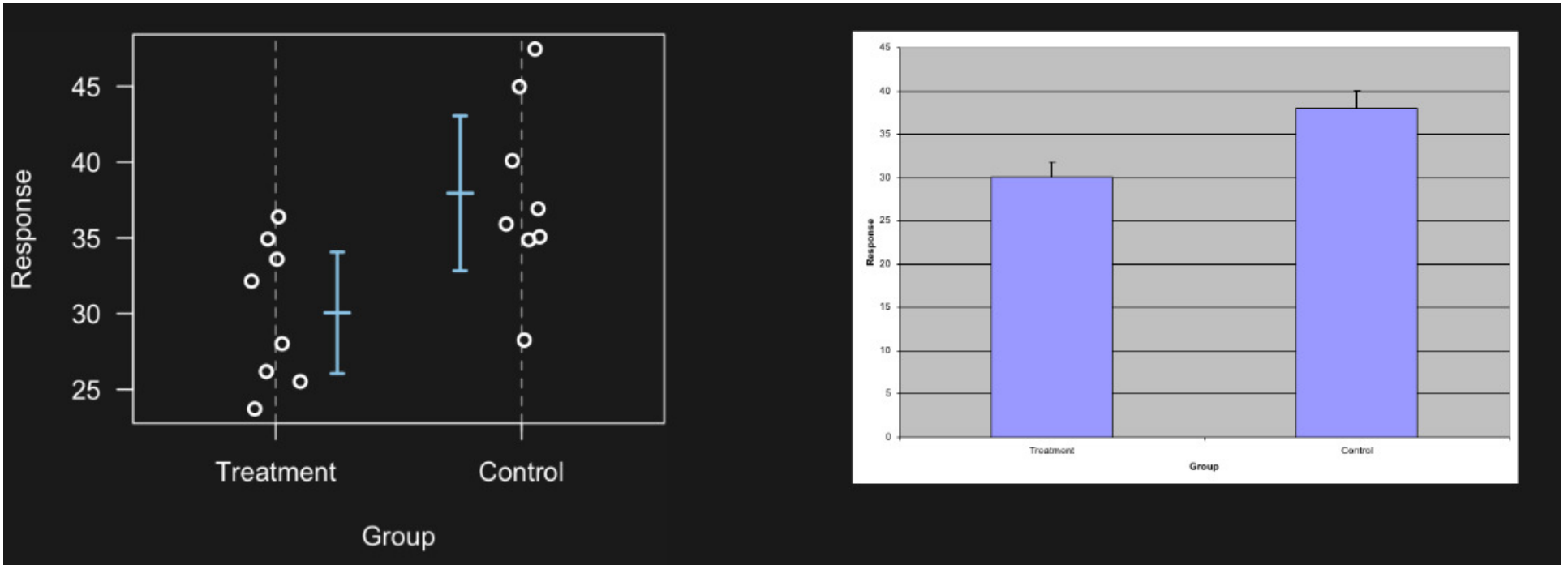
Graphical display tips

- Show the data (!!)
- Don't use pie charts
- Consider logs
- Take differences
- Ease comparisons
 - Things to be compared should be adjacent
 - Align vertically
 - Common axes
 - Labels not legends (where possible)
 - Should sorting really be alphabetical?
 - Consider whether the 0 is needed

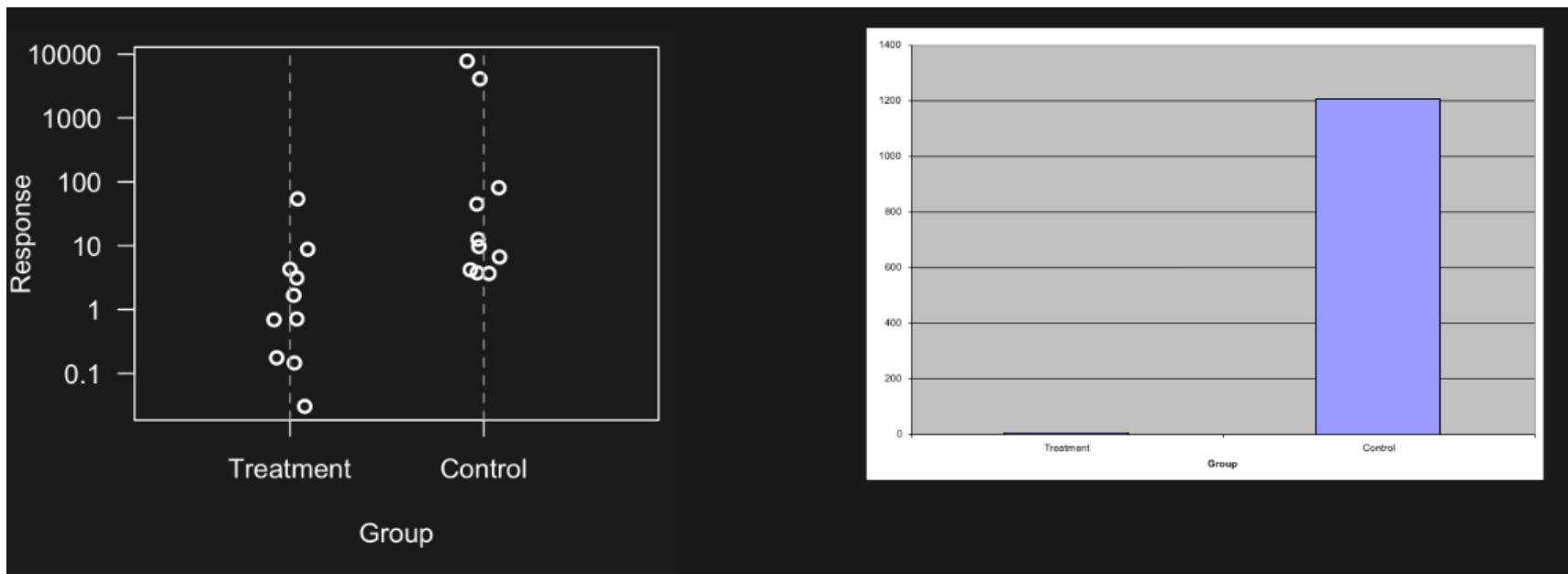
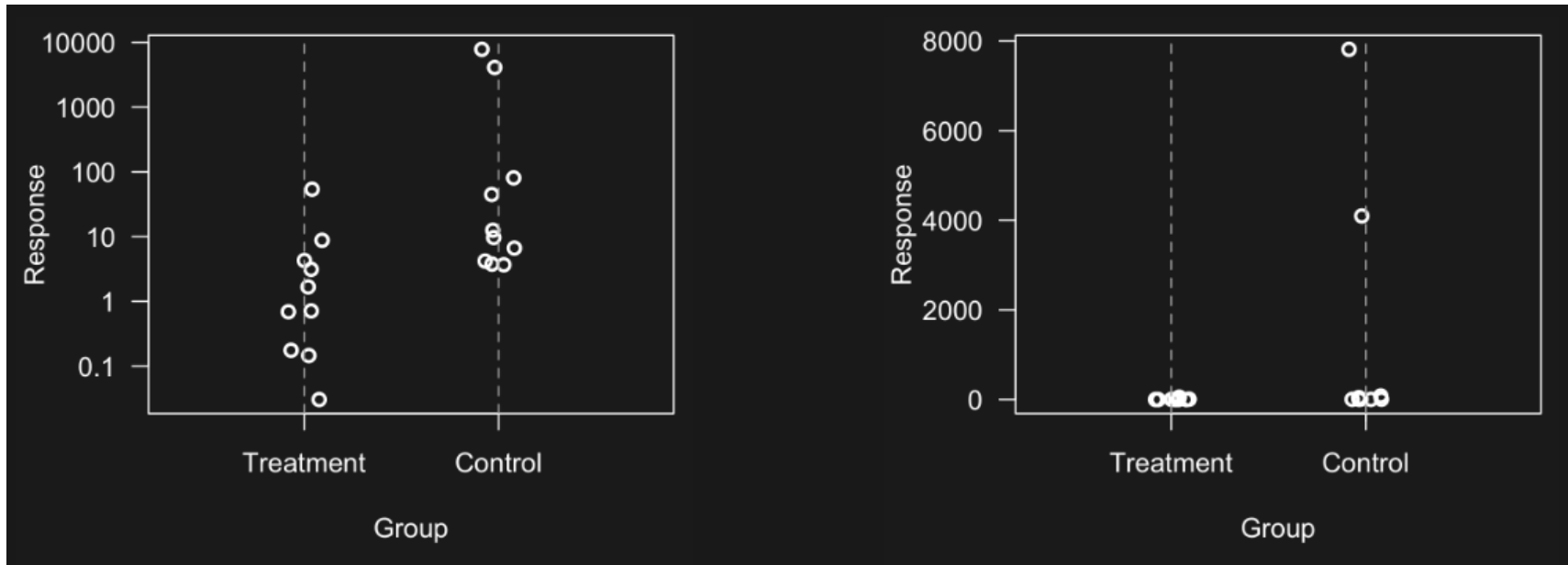
More graphical display tips

- Data density – for example, number of data points per square centimeter
- Avoid ‘chartjunk’ – decoration that provides no data
- Use color to convey information
- Use appropriate dimensionality
- Did I say Don't use pie charts ?? 😊
- And now: a *graphics tour* for discussion ...

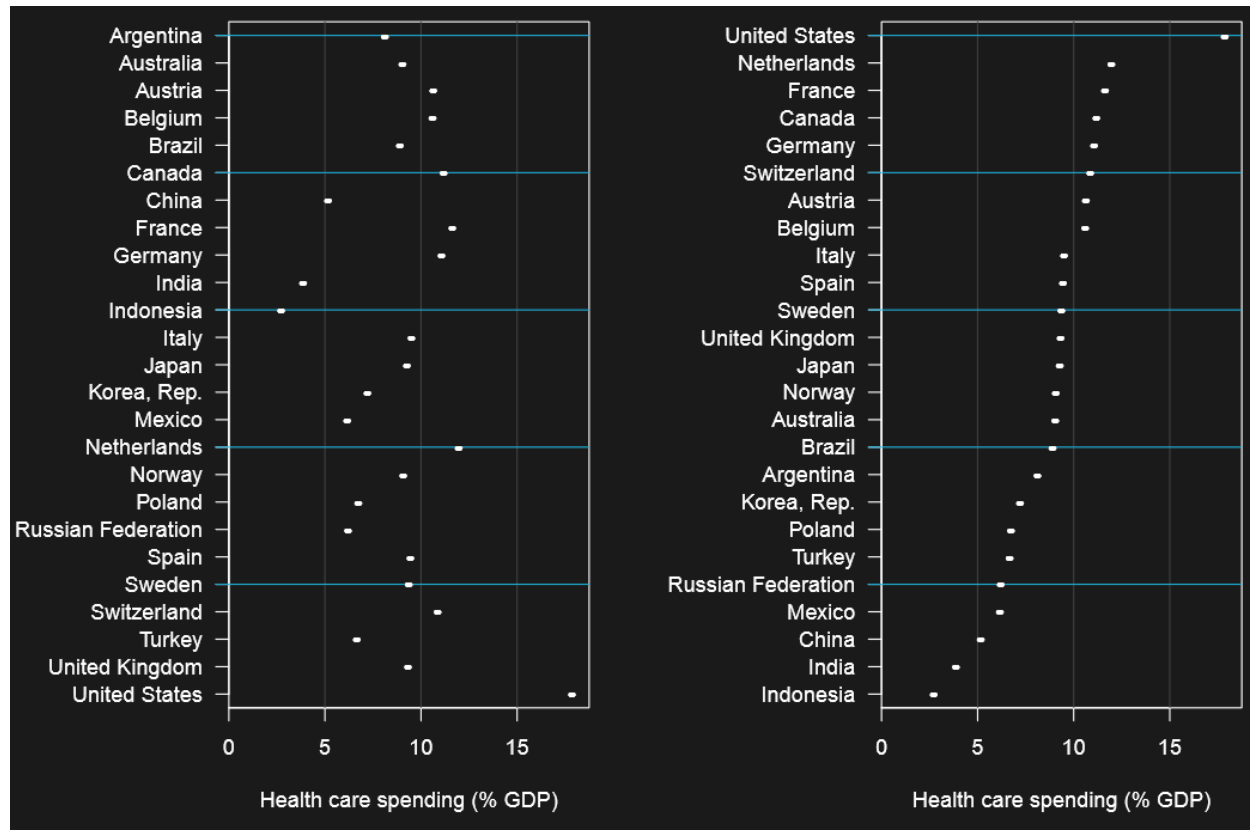
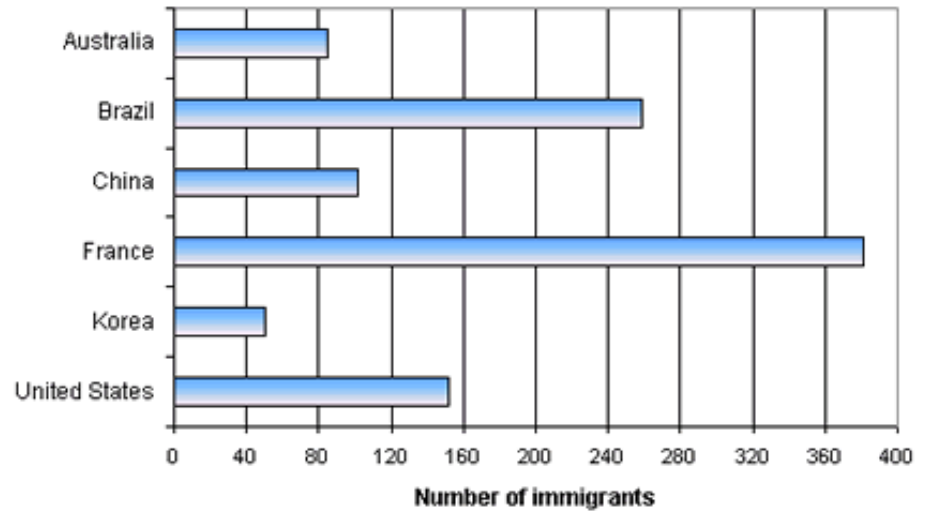
Show the data



Consider logs

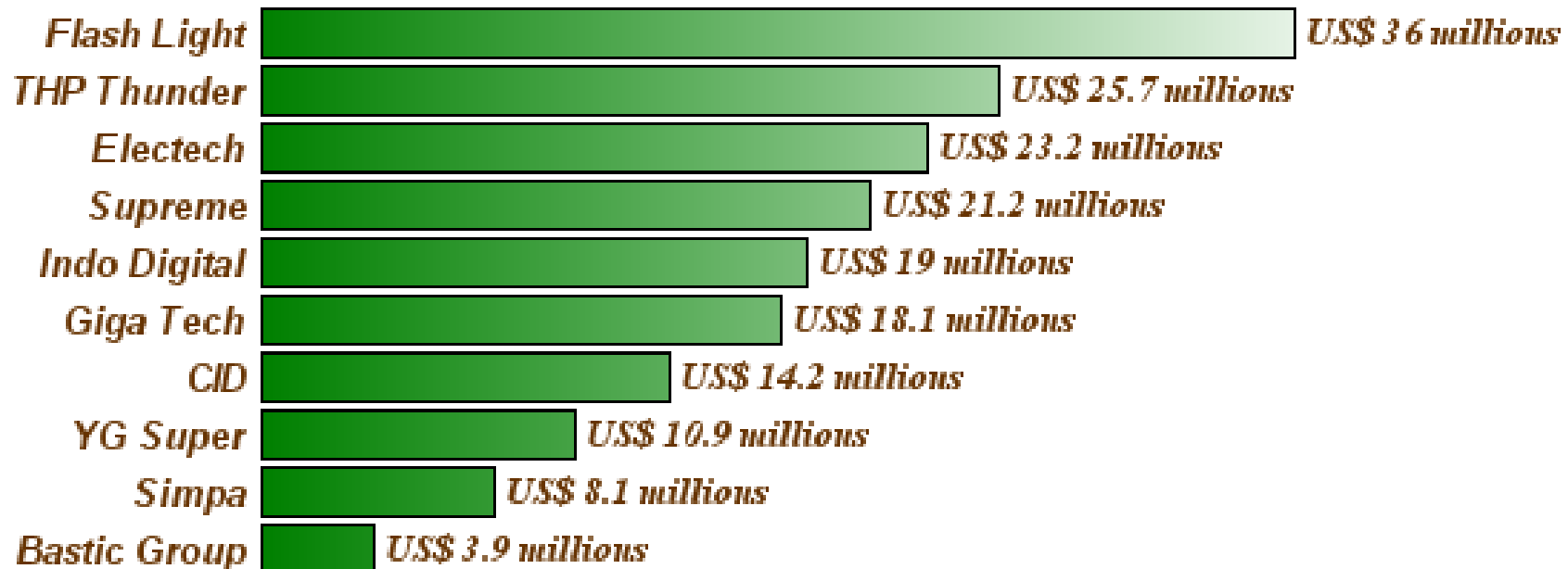


Alphabetical?

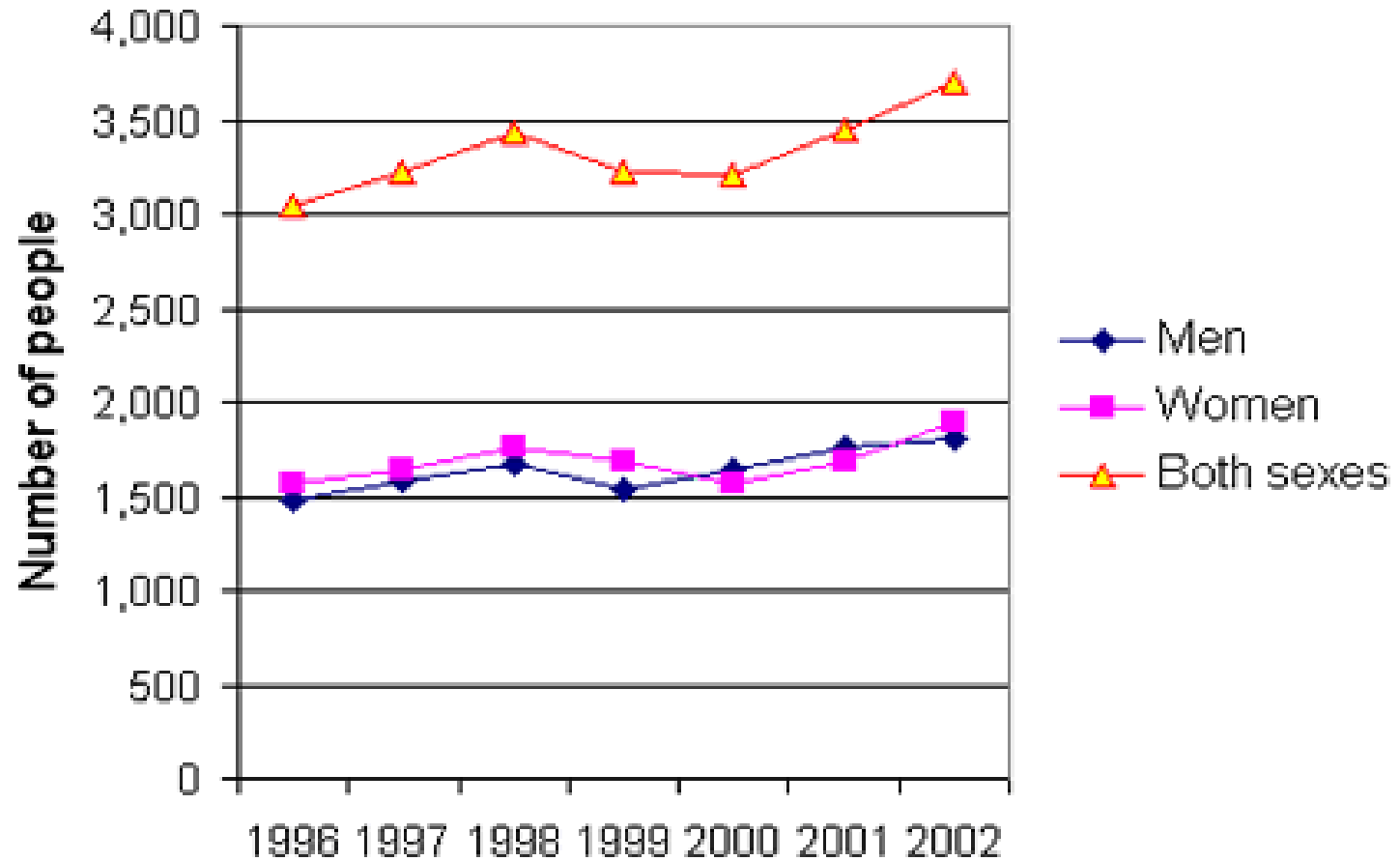


Do we really need color here?

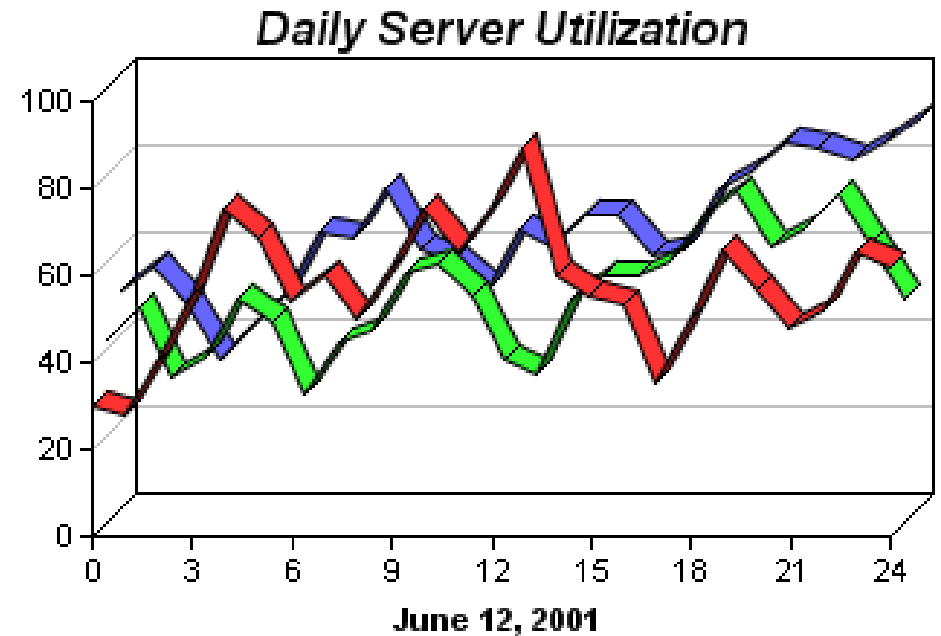
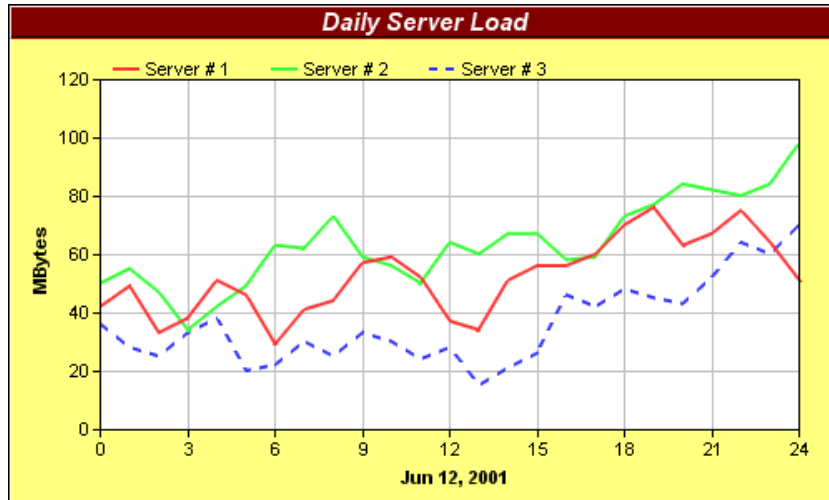
Revenue Estimation - Year 2002



3 lines?

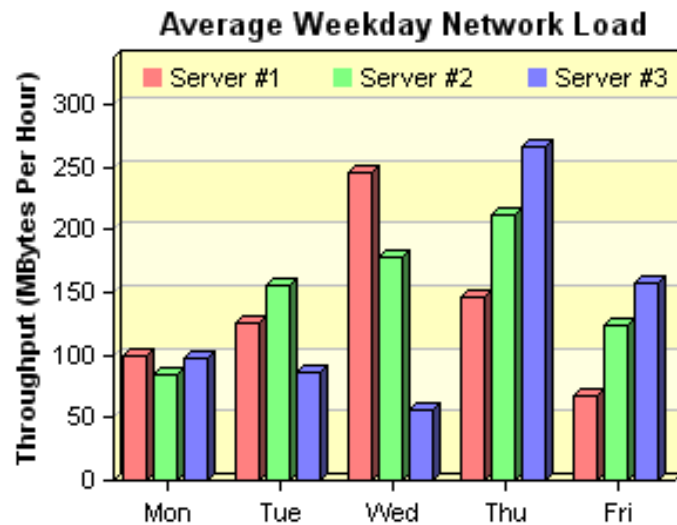
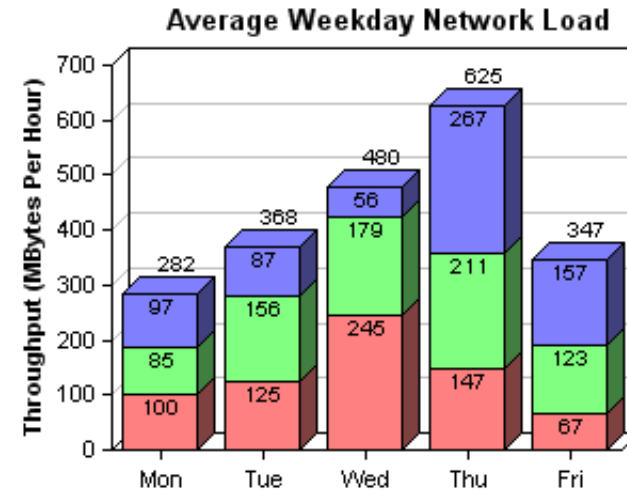
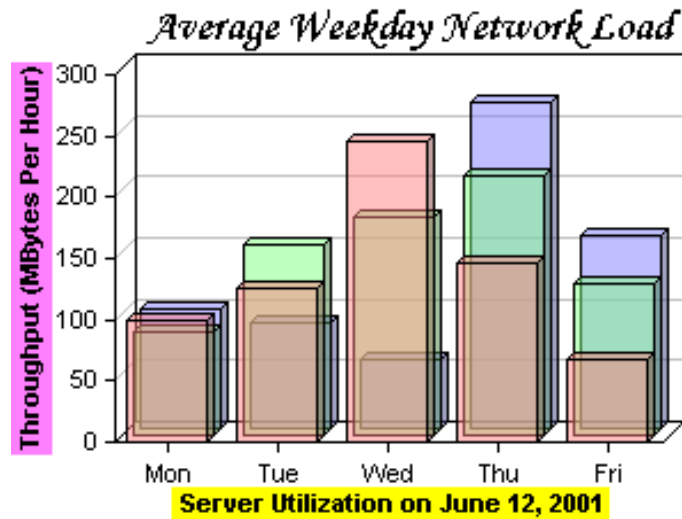


More about lines



- Different types (solid, dotted)?
- Colors?
- 3D??

What the **^*\$%#* are these saying?

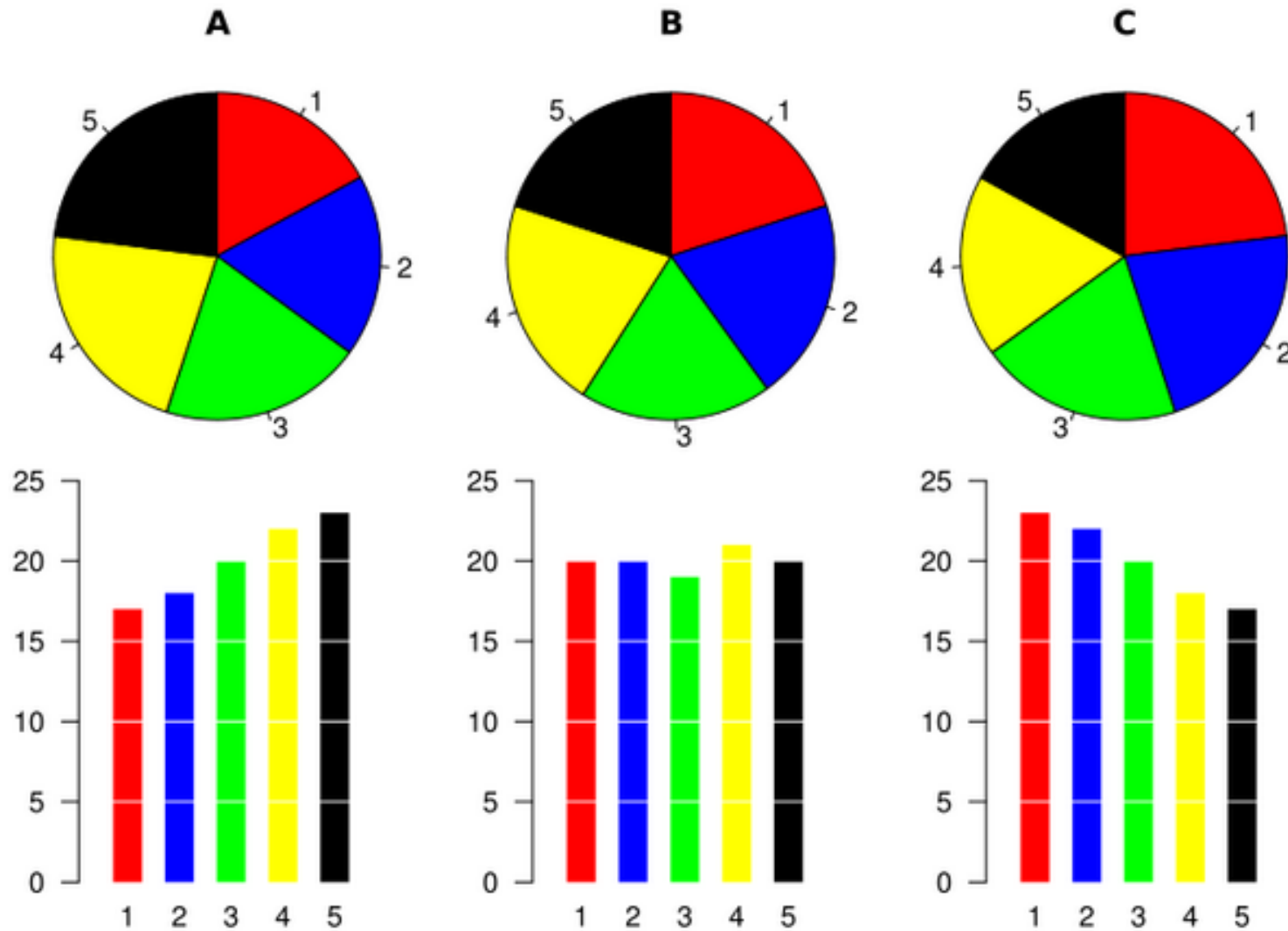


What improvements might be made?

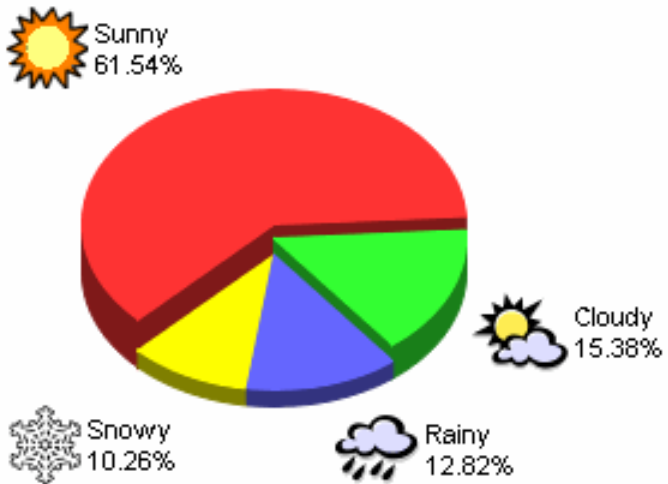
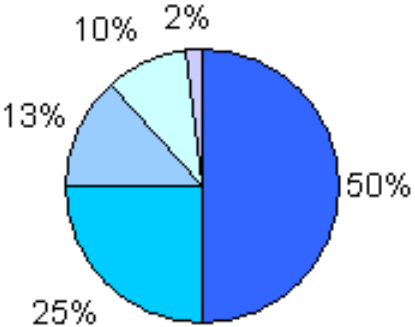
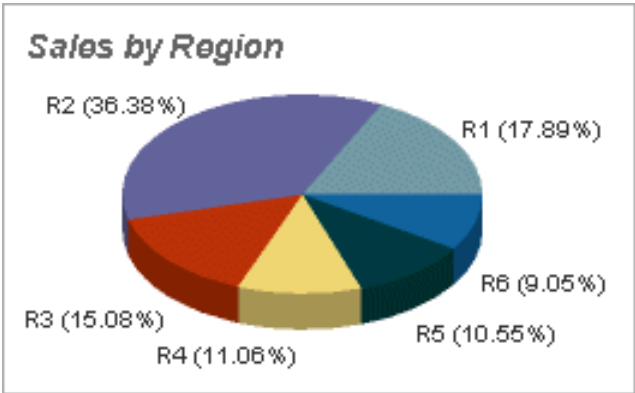
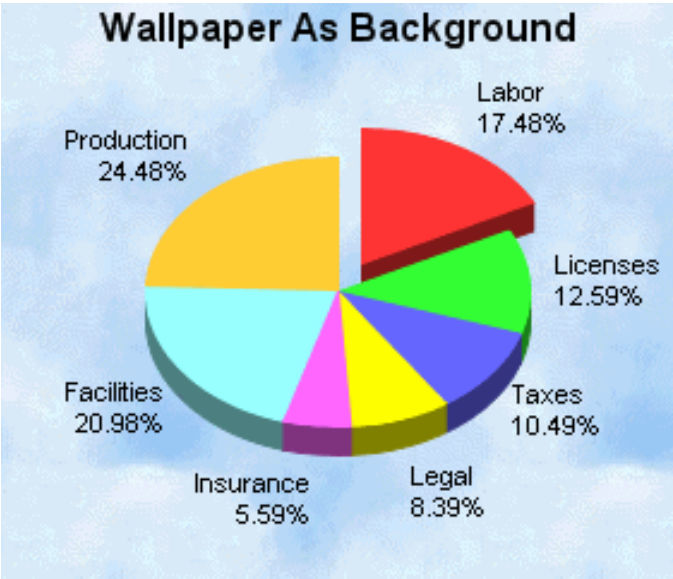
Pie Charts: **JUST SAY NO !!!**

- Pie charts are a **bad way** to display information
- The eye is
 - **good** at judging *linear measures* and
 - **bad** at judging *relative areas, volumes or angles*
- A pie chart is *never necessary* - data that can be shown by pie charts *always* can be shown by a dot plot (or bar chart, or table)
- 3D version even worse!

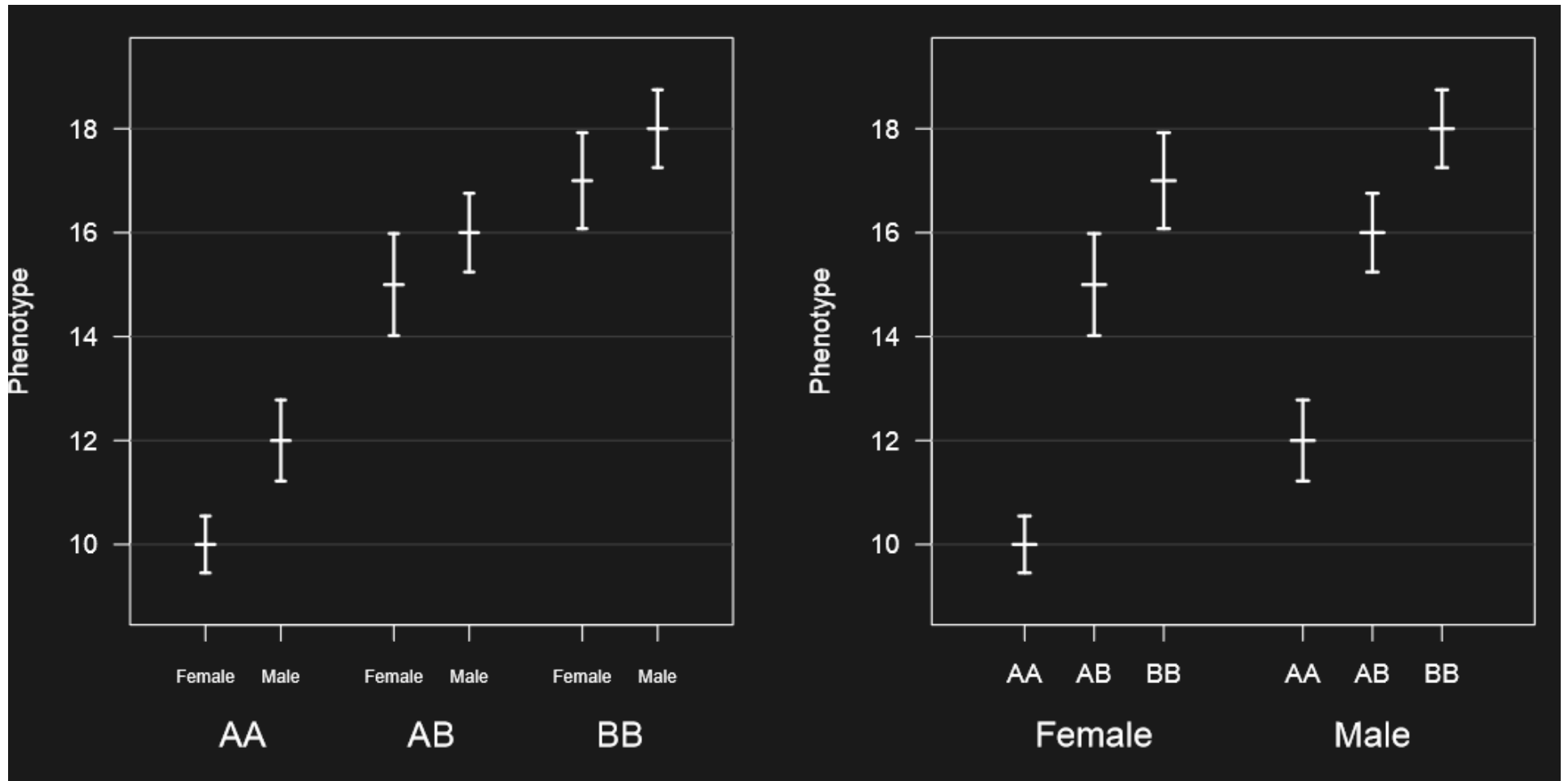
Spot the differences: pie vs. bar



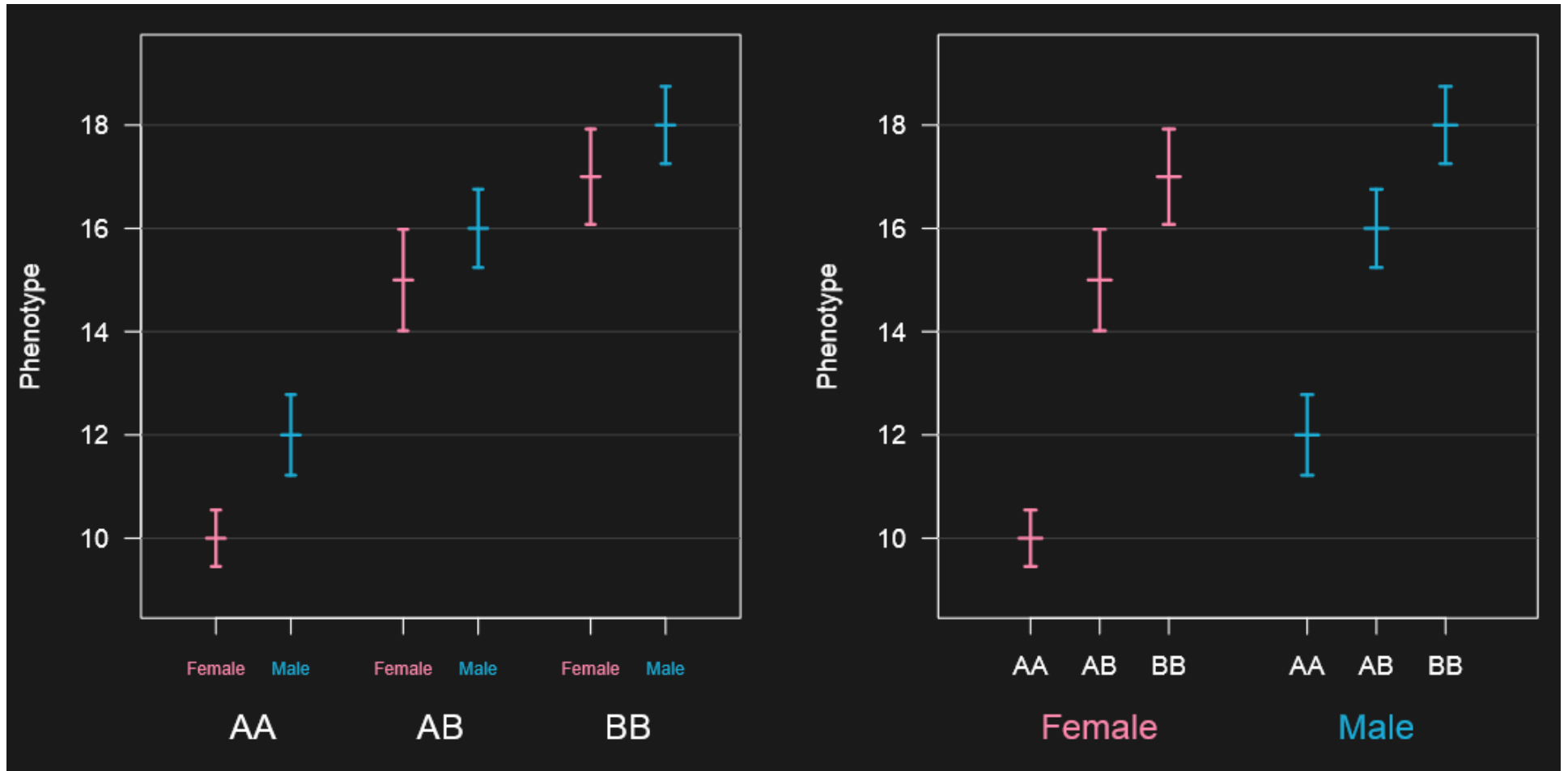
Even worse examples of pie charts



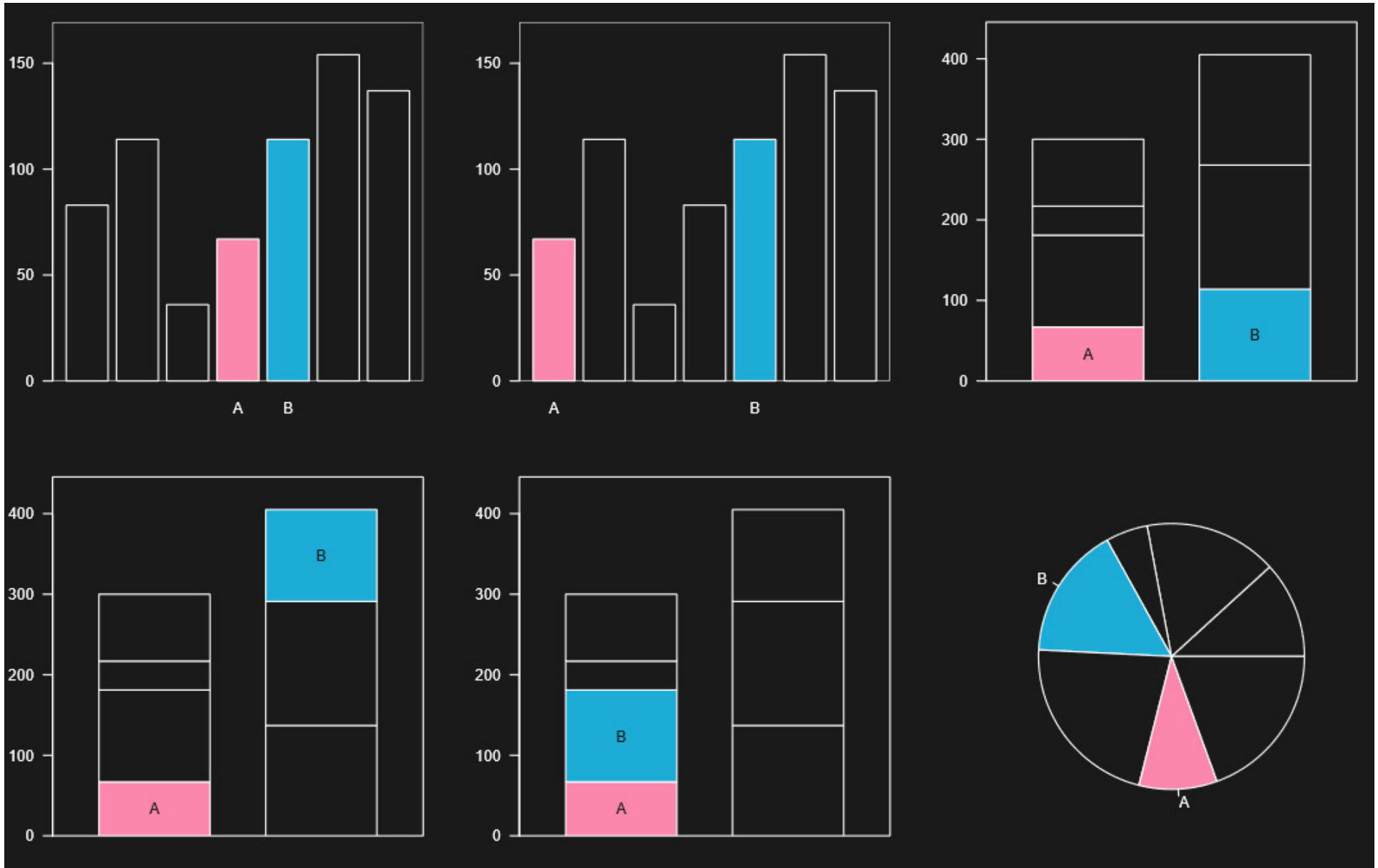
Things to be compared: adjacent



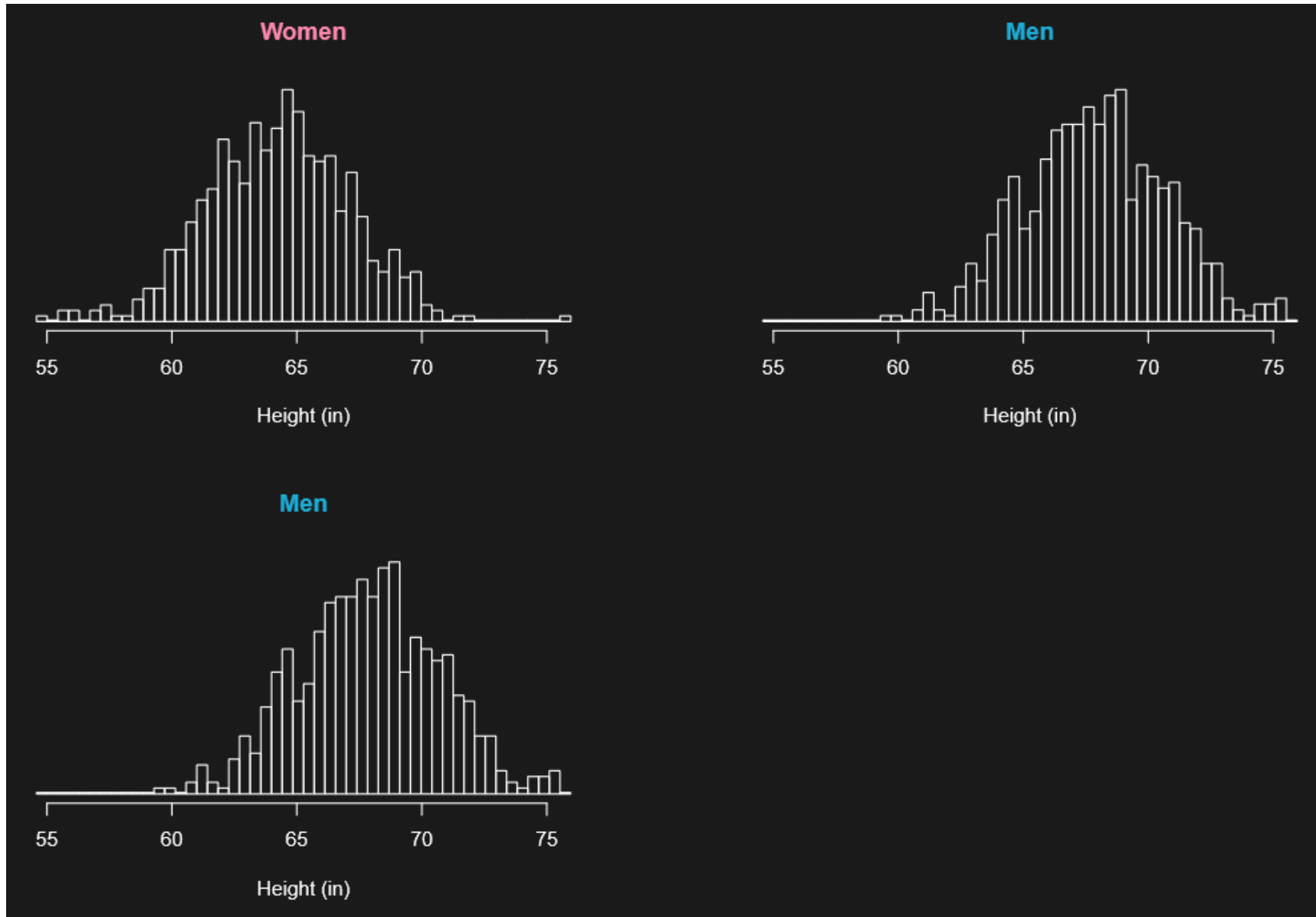
Use color where helpful



Where easiest to compare A and B?



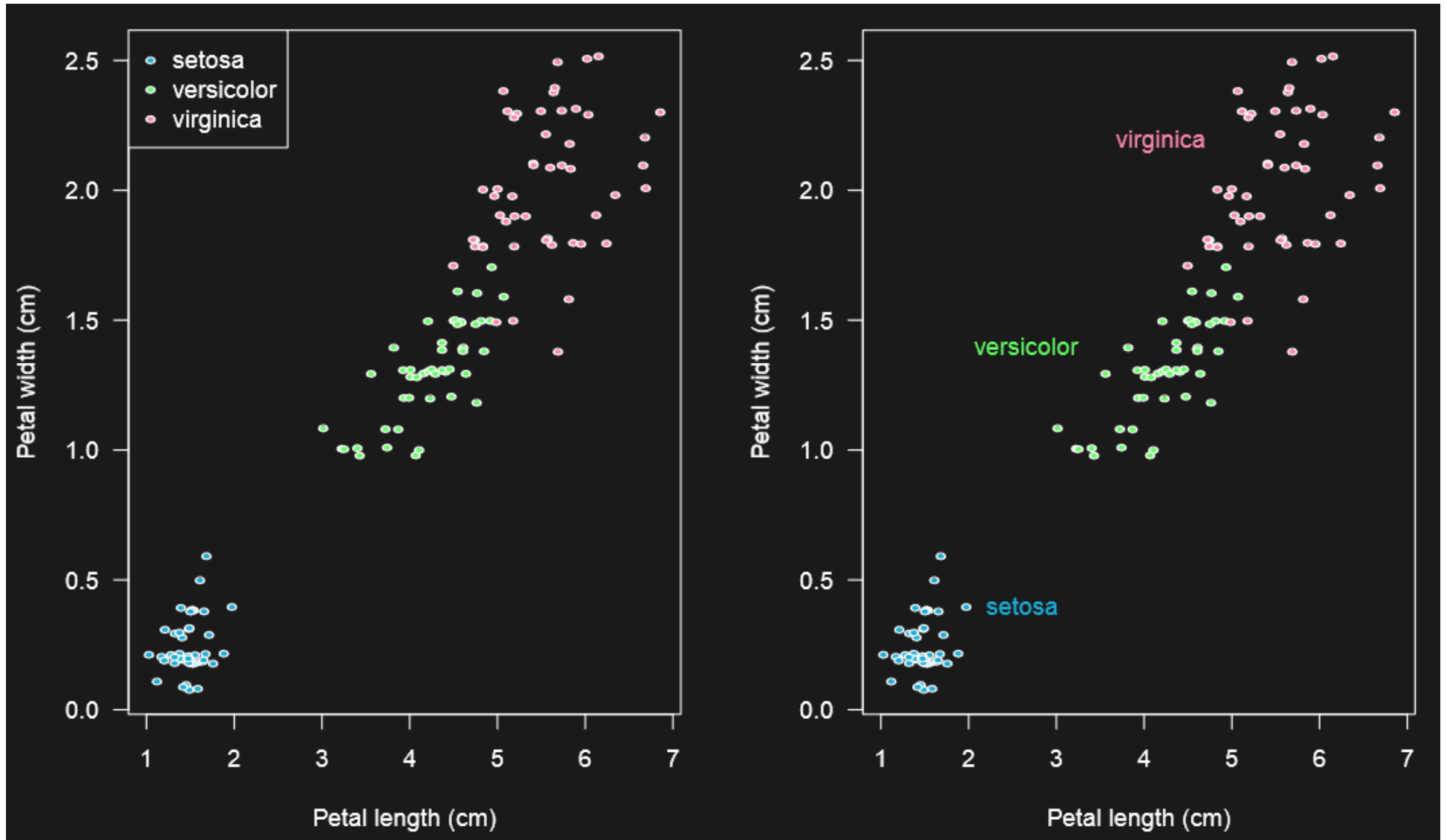
Easier to compare vertical aligned



Use common axes

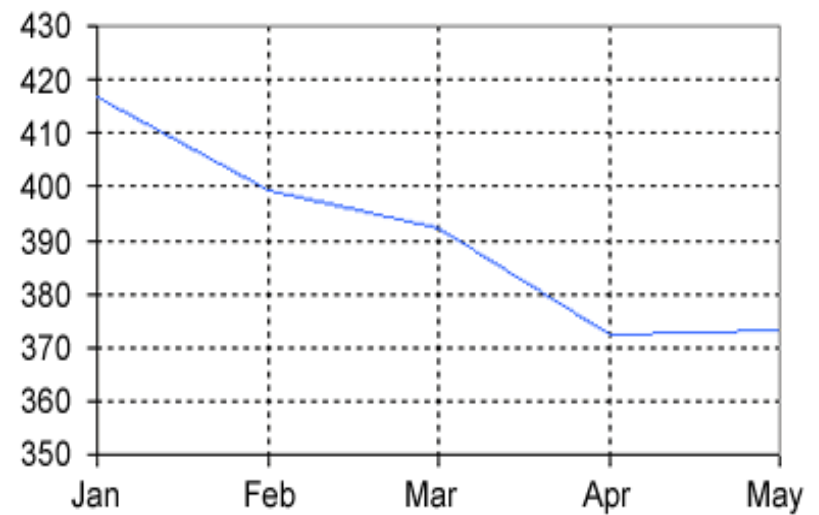
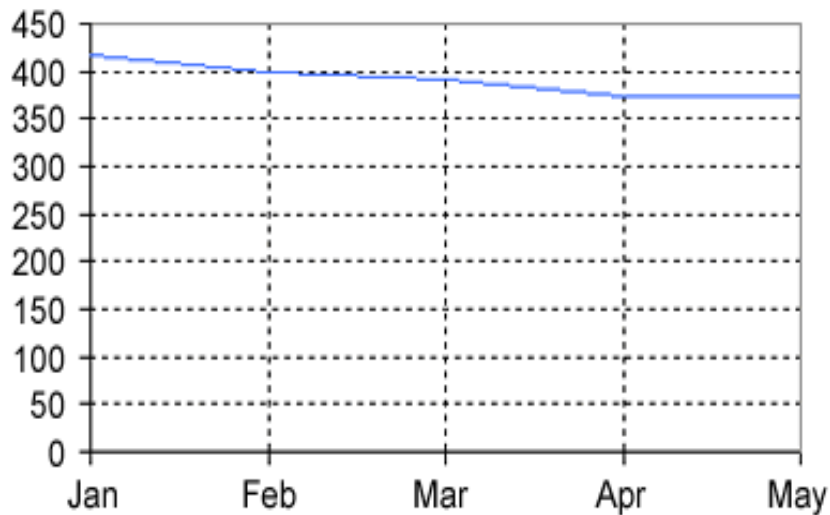
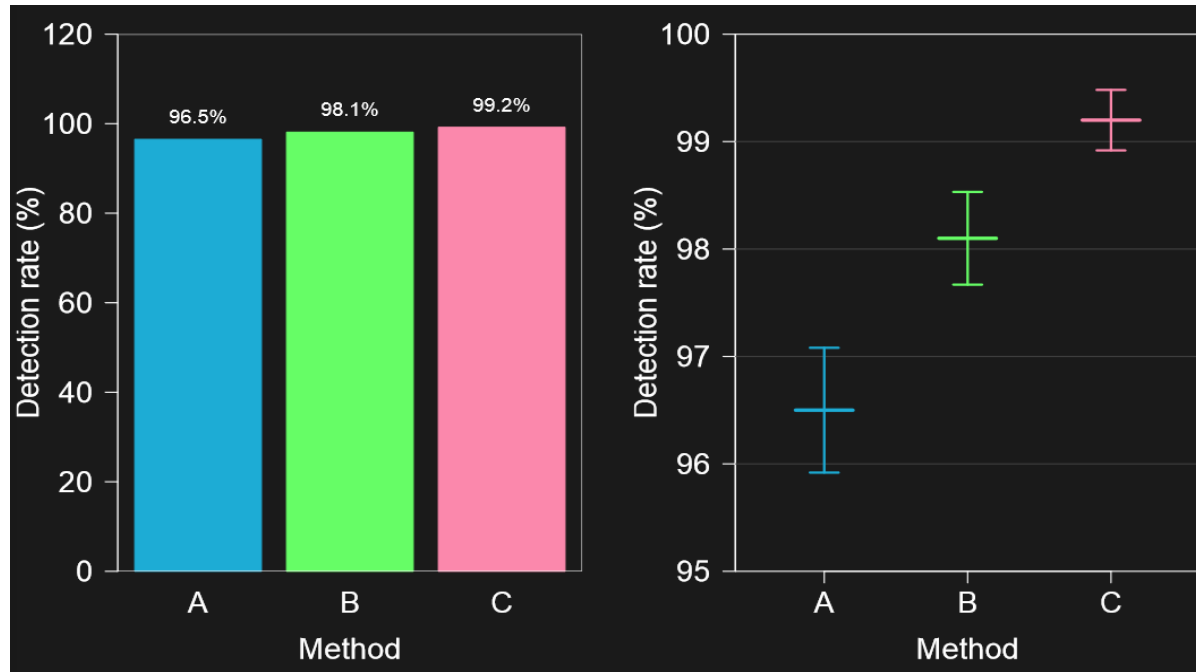


Use labels not legends *

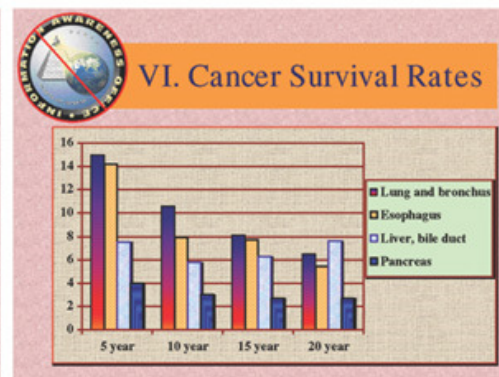
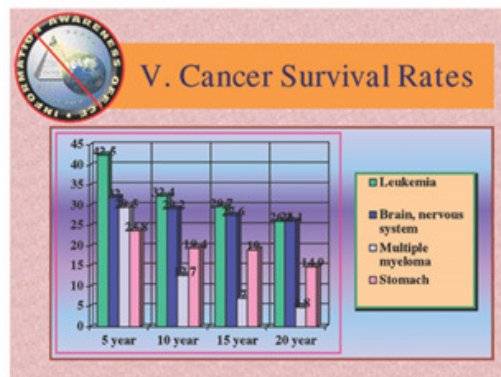
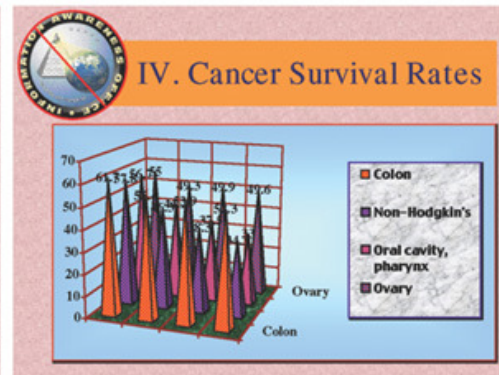
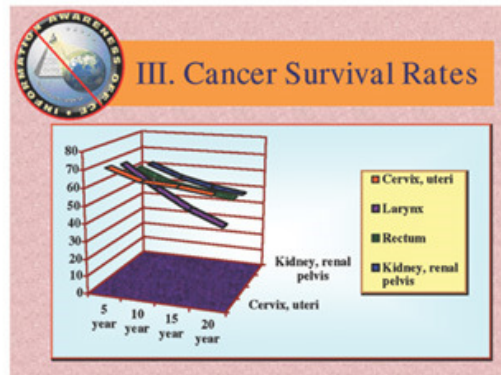
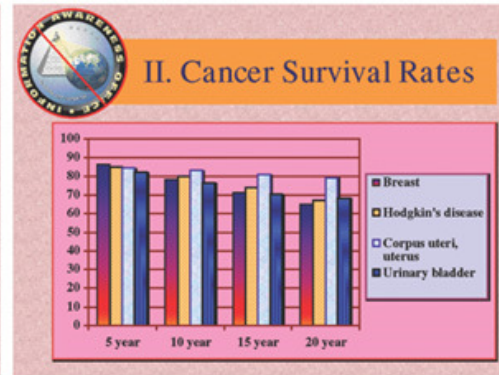
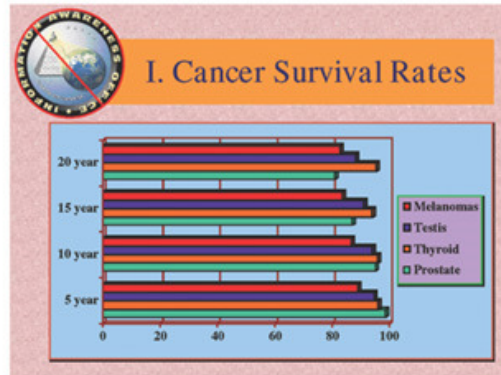


* Where possible

Consider whether you need 0



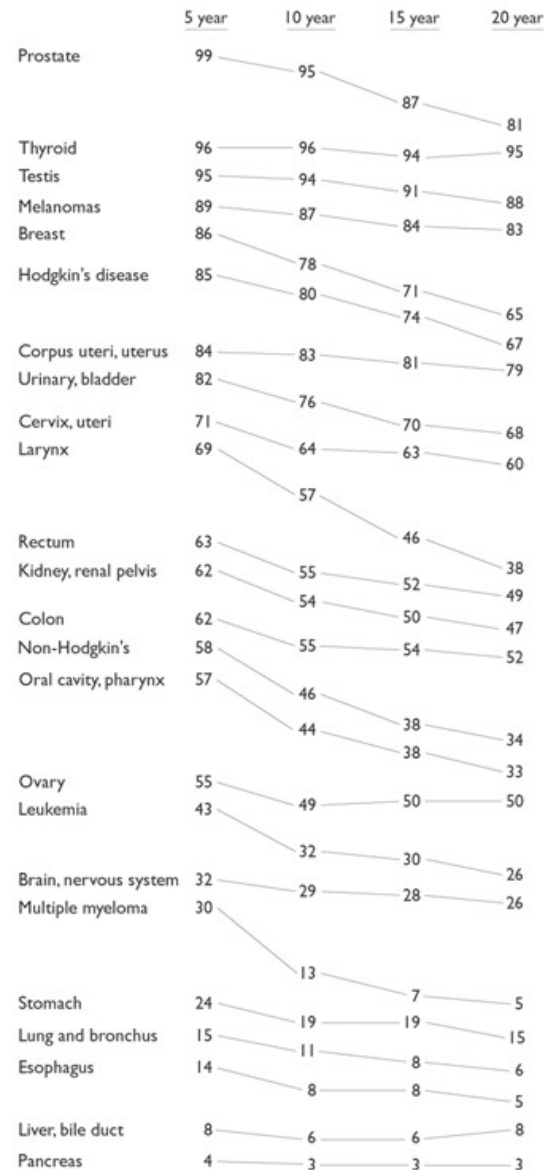
Several types of problems



The same data

Estimates of relative survival rates, by cancer site

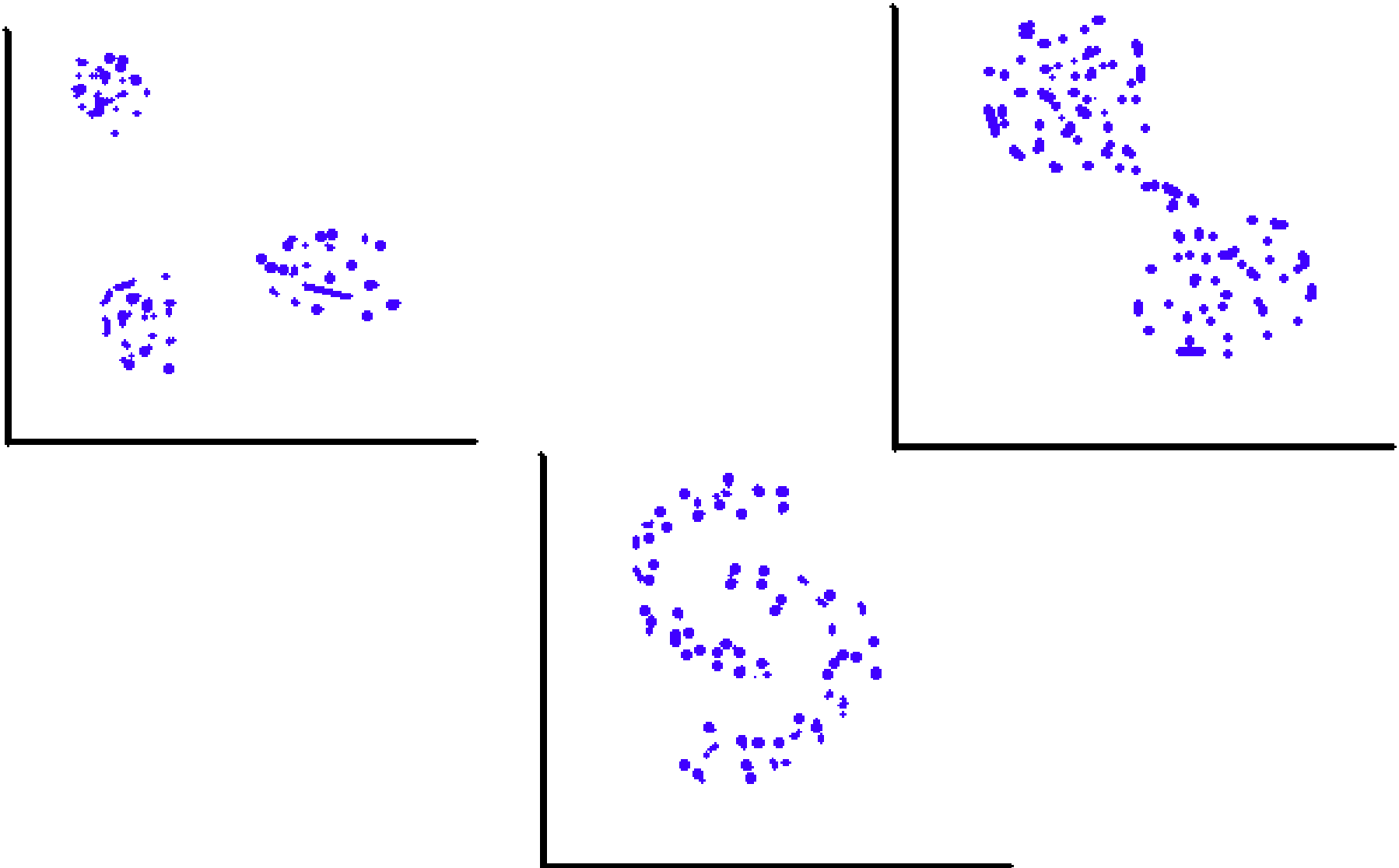
| | % survival rates and standard errors | | | | | | | |
|-----------------------|--------------------------------------|-----|---------|-----|---------|-----|---------|-----|
| | 5 year | | 10 year | | 15 year | | 20 year | |
| Prostate | 98.8 | 0.4 | 95.2 | 0.9 | 87.1 | 1.7 | 81.1 | 3.0 |
| Thyroid | 96.0 | 0.8 | 95.8 | 1.2 | 94.0 | 1.6 | 95.4 | 2.1 |
| Testis | 94.7 | 1.1 | 94.0 | 1.3 | 91.1 | 1.8 | 88.2 | 2.3 |
| Melanomas | 89.0 | 0.8 | 86.7 | 1.1 | 83.5 | 1.5 | 82.8 | 1.9 |
| Breast | 86.4 | 0.4 | 78.3 | 0.6 | 71.3 | 0.7 | 65.0 | 1.0 |
| Hodgkin's disease | 85.1 | 1.7 | 79.8 | 2.0 | 73.8 | 2.4 | 67.1 | 2.8 |
| Corpus uteri, uterus | 84.3 | 1.0 | 83.2 | 1.3 | 80.8 | 1.7 | 79.2 | 2.0 |
| Urinary, bladder | 82.1 | 1.0 | 76.2 | 1.4 | 70.3 | 1.9 | 67.9 | 2.4 |
| Cervix, uteri | 70.5 | 1.6 | 64.1 | 1.8 | 62.8 | 2.1 | 60.0 | 2.4 |
| Larynx | 68.8 | 2.1 | 56.7 | 2.5 | 45.8 | 2.8 | 37.8 | 3.1 |
| Rectum | 62.6 | 1.2 | 55.2 | 1.4 | 51.8 | 1.8 | 49.2 | 2.3 |
| Kidney, renal pelvis | 61.8 | 1.3 | 54.4 | 1.6 | 49.8 | 2.0 | 47.3 | 2.6 |
| Colon | 61.7 | 0.8 | 55.4 | 1.0 | 53.9 | 1.2 | 52.3 | 1.6 |
| Non-Hodgkin's | 57.8 | 1.0 | 46.3 | 1.2 | 38.3 | 1.4 | 34.3 | 1.7 |
| Oral cavity, pharynx | 56.7 | 1.3 | 44.2 | 1.4 | 37.5 | 1.6 | 33.0 | 1.8 |
| Ovary | 55.0 | 1.3 | 49.3 | 1.6 | 49.9 | 1.9 | 49.6 | 2.4 |
| Leukemia | 42.5 | 1.2 | 32.4 | 1.3 | 29.7 | 1.5 | 26.2 | 1.7 |
| Brain, nervous system | 32.0 | 1.4 | 29.2 | 1.5 | 27.6 | 1.6 | 26.1 | 1.9 |
| Multiple myeloma | 29.5 | 1.6 | 12.7 | 1.5 | 7.0 | 1.3 | 4.8 | 1.5 |
| Stomach | 23.8 | 1.3 | 19.4 | 1.4 | 19.0 | 1.7 | 14.9 | 1.9 |
| Lung and bronchus | 15.0 | 0.4 | 10.6 | 0.4 | 8.1 | 0.4 | 6.5 | 0.4 |
| Esophagus | 14.2 | 1.4 | 7.9 | 1.3 | 7.7 | 1.6 | 5.4 | 2.0 |
| Liver, bile duct | 7.5 | 1.1 | 5.8 | 1.2 | 6.3 | 1.5 | 7.6 | 2.0 |
| Pancreas | 4.0 | 0.5 | 3.0 | 1.5 | 2.7 | 0.6 | 2.7 | 0.8 |



More advanced techniques

- Cluster analysis
 - Leads to readily interpretable figures
 - Can be helpful for identifying patterns in time or space
 - Can be used for exploratory purposes
 - Used to find groups of objects when not already known
- Principal components analysis
 - Often used as exploratory tool
 - Dimensionality reduction
- Useful for EDA and quality assessment of high-dimensional datasets
- *Briefly* outline the main ideas here

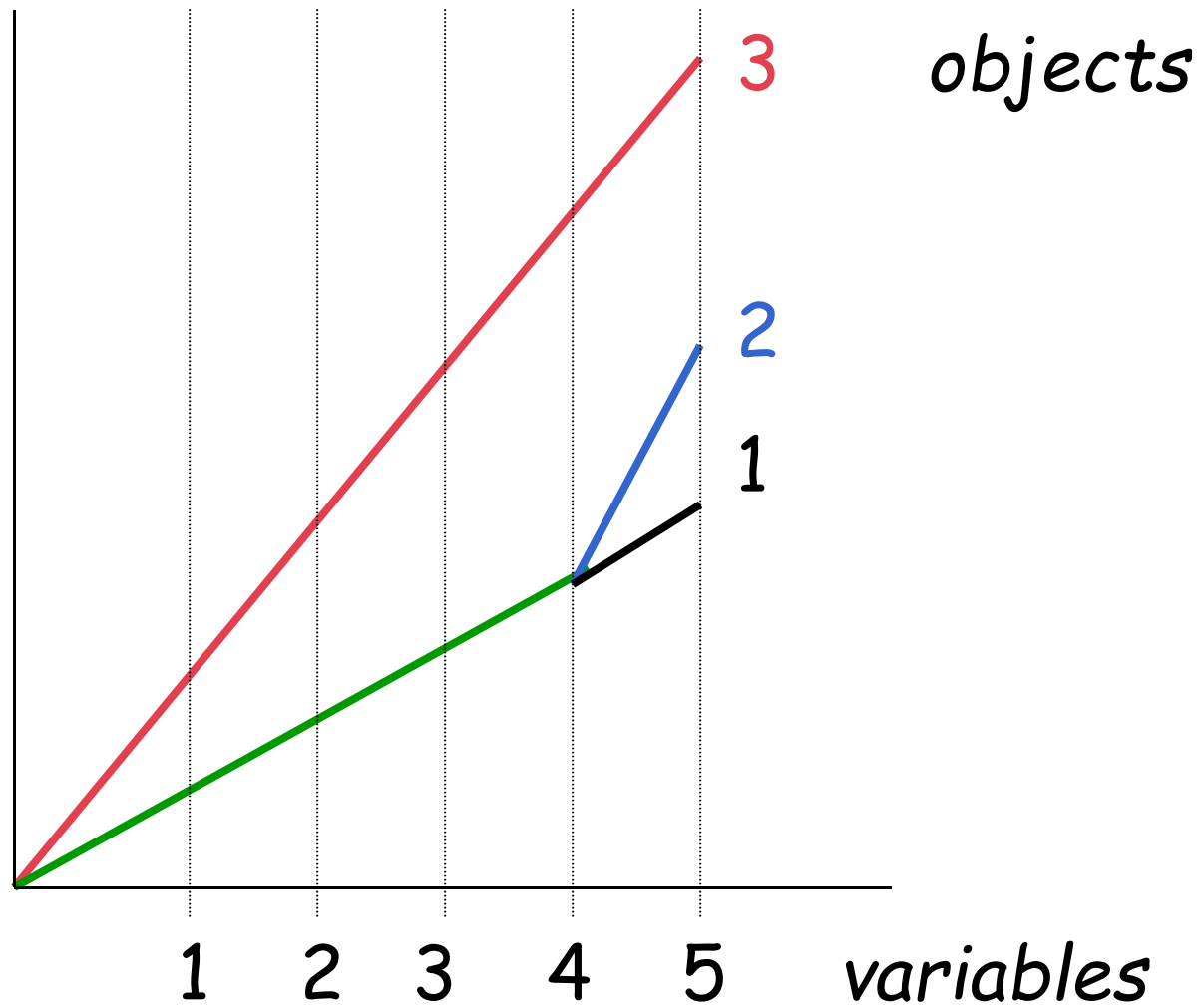
Difficulties in defining 'cluster'



Similarity

- *Similarity* s_{ij} indicates the strength of relationship between two objects i and j
- Usually $0 \leq s_{ij} \leq 1$
- Correlation-based similarity ranges from -1 to 1
- Use of correlation-based similarity is quite common in gene expression studies but is in general contentious...

Problems using correlation



Dissimilarity and Distance

- Associated with similarity measures s_{ij} bounded by 0 and 1 is a *dissimilarity* $d_{ij} = 1 - s_{ij}$
- *Distance* measures have the metric property $(d_{ij} + d_{ik} \geq d_{jk})$
- Many examples: Euclidean ('as the crow flies'), Manhattan ('city block'), *etc.*
- Distance measure has a *large effect* on performance
- Behavior of distance measure related to *scale* of measurement

Partitioning Methods

- Partition the objects into a *prespecified* number of groups K
- Iteratively reallocate objects to clusters until some criterion is met (e.g. minimize within cluster sums of squares)
- Examples: k-means, self-organizing maps (SOM), partitioning around medoids (PAM), model-based clustering

Hierarchical Clustering

- Produce a *dendrogram*
- Avoid prespecification of the number of clusters K
- The tree can be built in two distinct ways:
 - Bottom-up: *agglomerative* clustering
 - Top-down: *divisive* clustering

Agglomerative Methods

- Start with n sample clusters
- At each step, *merge* the two closest clusters using a measure of between-cluster dissimilarity which reflects the shape of the clusters
- Examples of *between-cluster* dissimilarities:
 - *Unweighted Pair Group Method with Arithmetic Mean (UPGMA)*: average of pairwise dissimilarities
 - *Single-link (NN)*: minimum of pairwise dissimilarities
 - *Complete-link (FN)*: maximum of pairwise dissimilarities

Divisive Methods

- Start with only *one* cluster
- At each step, *split* clusters into two parts
- Advantage: Obtain the main structure of the data (*i.e.* focus on upper levels of dendrogram)
- Disadvantage: Computational difficulties when considering all possible divisions into two groups

Partitioning vs. Hierarchical

- *Partitioning*

- Advantage: Provides clusters that satisfy some optimality criterion (approximately)
- Disadvantages: Need initial K, long computation time

- *Hierarchical*

- Advantage: Fast computation (agglomerative)
- Disadvantages: Rigid, cannot correct later for erroneous decisions made earlier

Generic Clustering Tasks

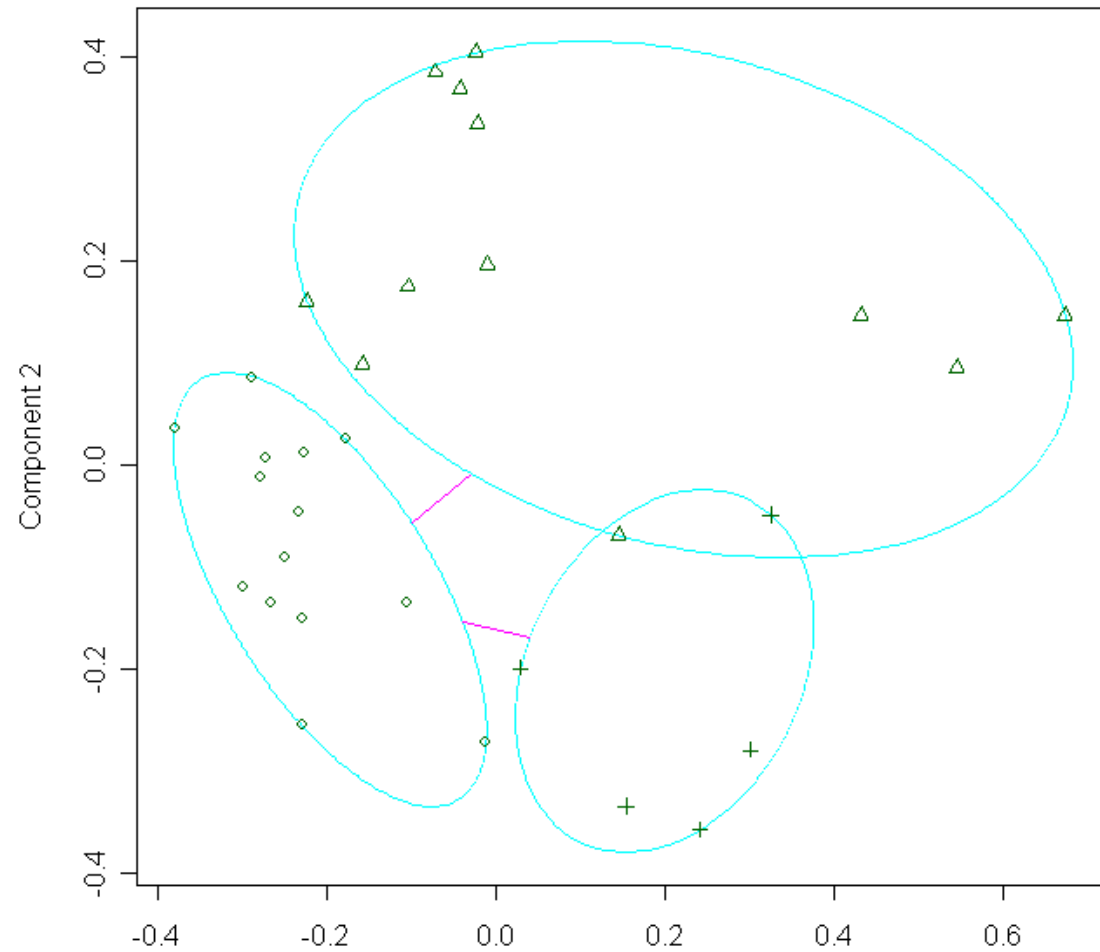
- Estimating number of clusters
- Assigning each object to a cluster
- Assessing strength/confidence of cluster assignments for individual objects
- Assessing cluster homogeneity

Issues in Clustering

- Data pre-processing
- Which genes (variables) are used
- Which samples are used
- Which distance measure is used
- Which algorithm is applied
- How to decide the number of clusters K

Visualizing partition

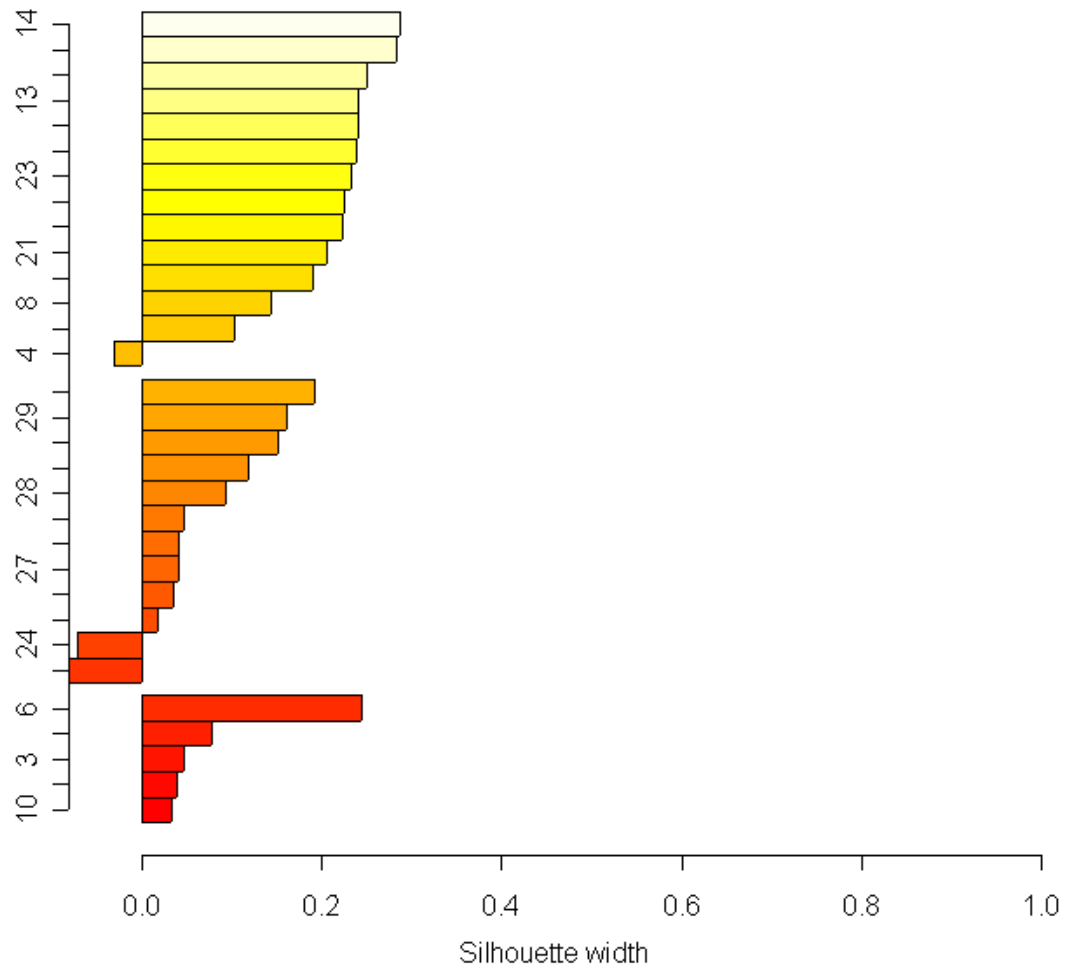
```
clusplot(pam(x = as.dist(1 - cor(mel.data)), k = 3, diss = TRUE))
```



Component 1
These two components explain 37.03 % of the point variability.

Estimating Number of Clusters

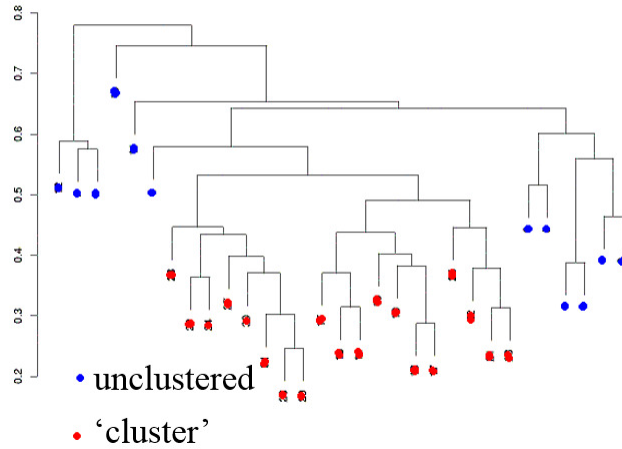
Silhouette plot of `pam(x = as.dist(1 - cor(mel.data)), k = 3, diss = 1)`



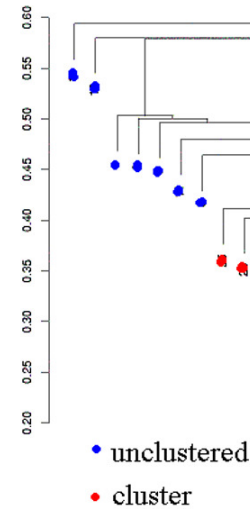
Average silhouette width : 0.13

Hierarchical, agglomerative: different methods

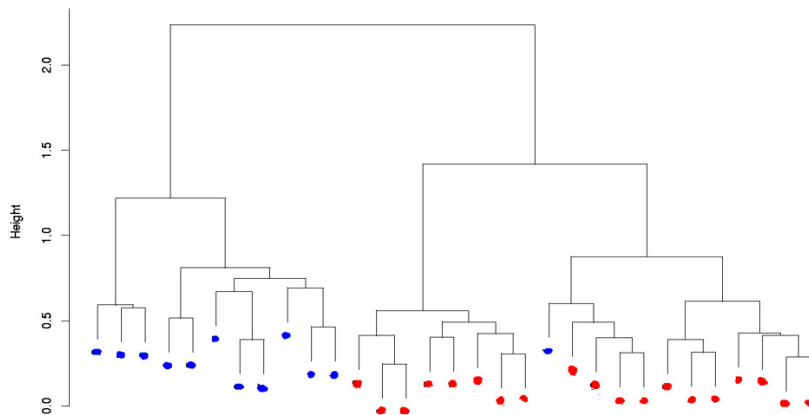
Average linkage, *melanoma only*



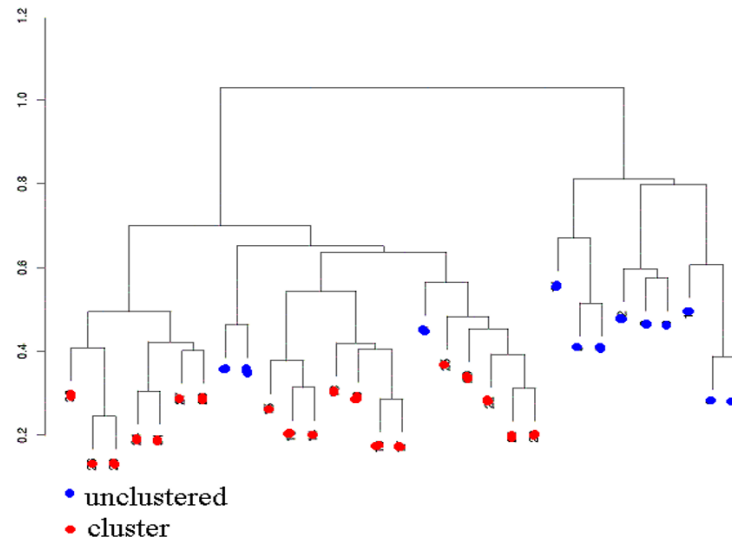
Complete linkage (FN)



Ward's method (information loss)

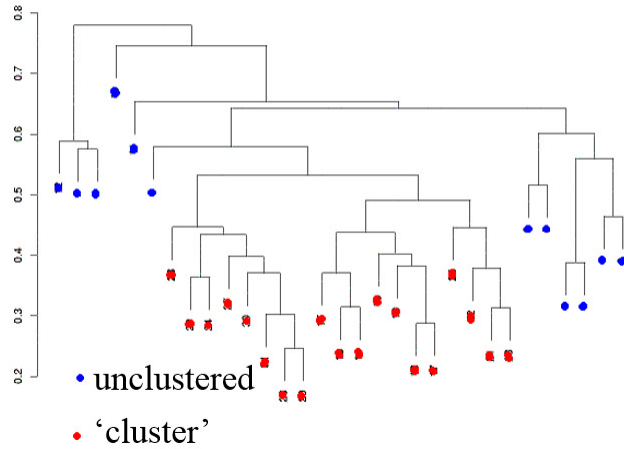


Single linkage (NN)

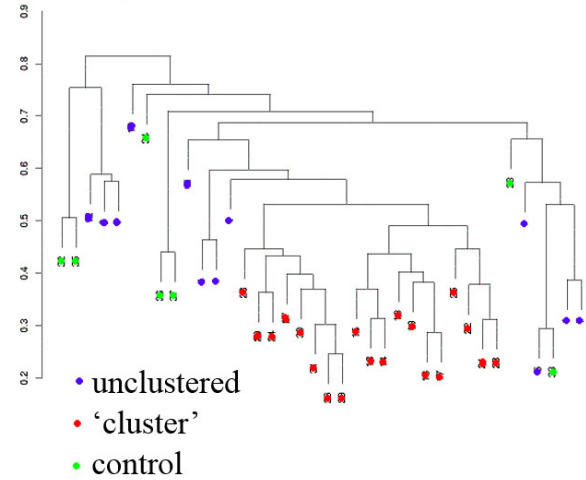


Different methods, different samples

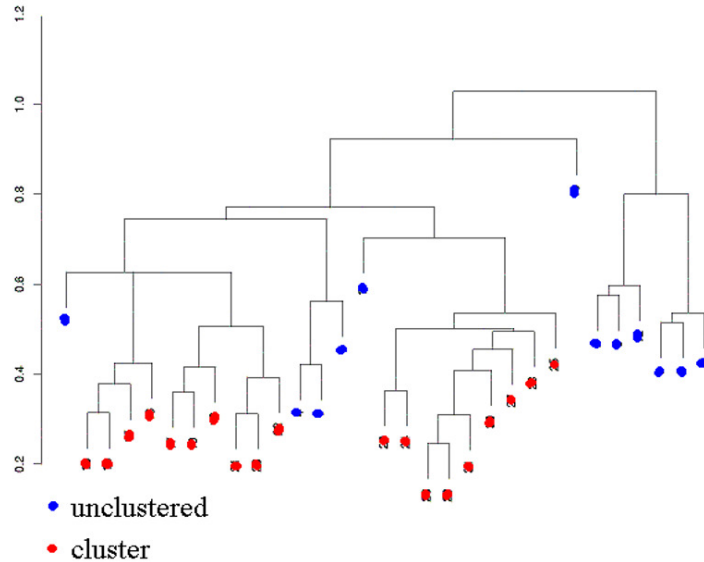
Average linkage, *melanoma only*



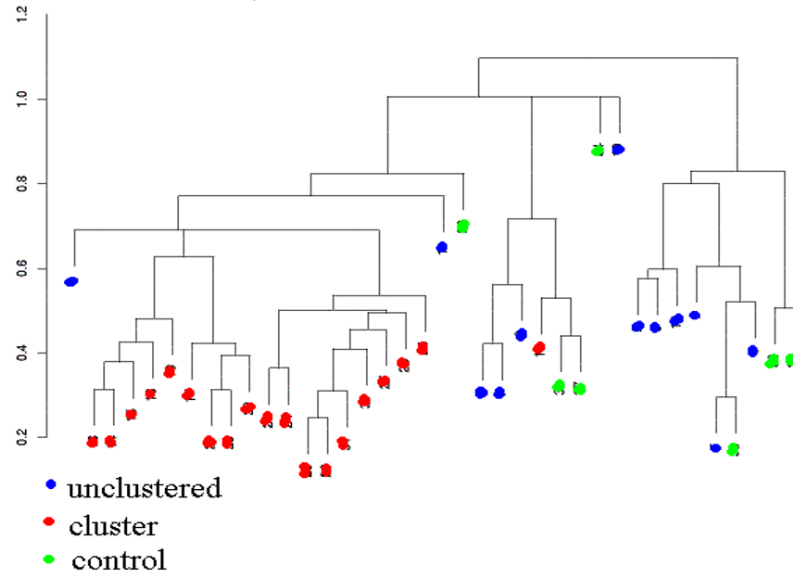
Avg linkage, *melanoma & controls*



Divisive clustering, *melanoma only*



Divisive, *melanoma & controls*



How many clusters K ?

- Many suggestions for how to decide this!
- Milligan and Cooper (Psychometrika 50:159-179, 1985) studied 30 methods
- A number of new methods, including GAP (Tibshirani) and clest (Fridlyand and Dudoit)
- Applying several methods yielded estimates of $K = 2$ (largest cluster has 27 members) to $K = 8$ (largest cluster has 19 members)

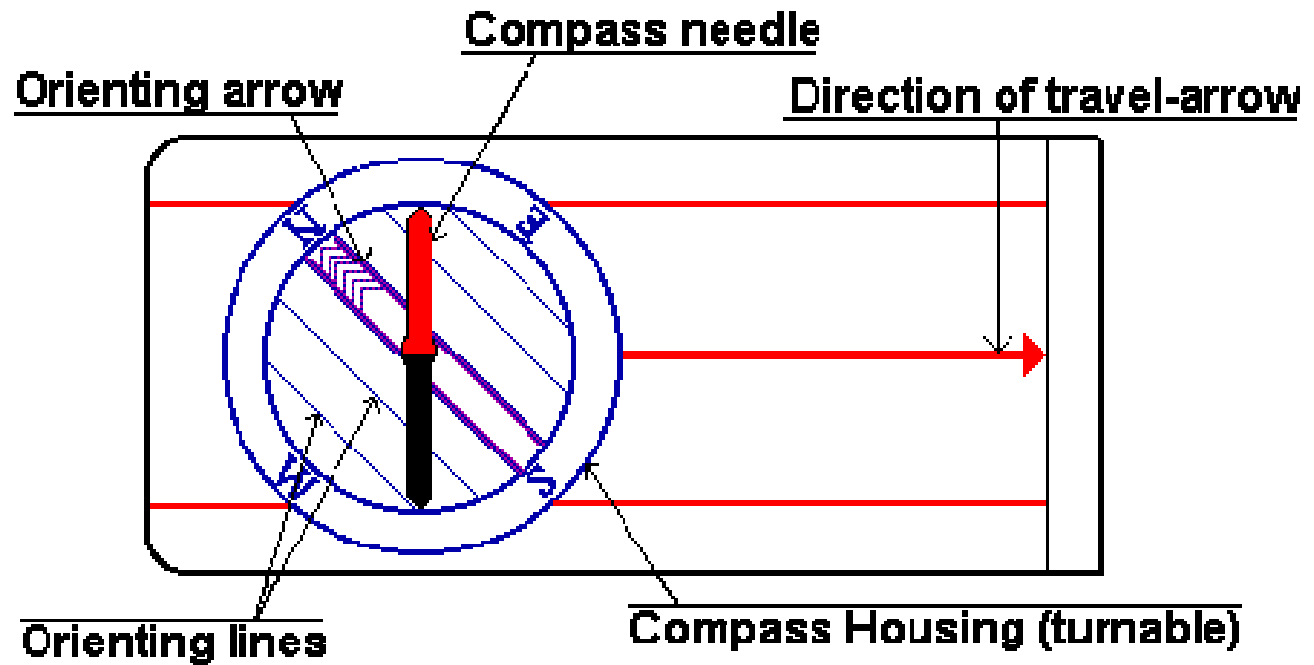
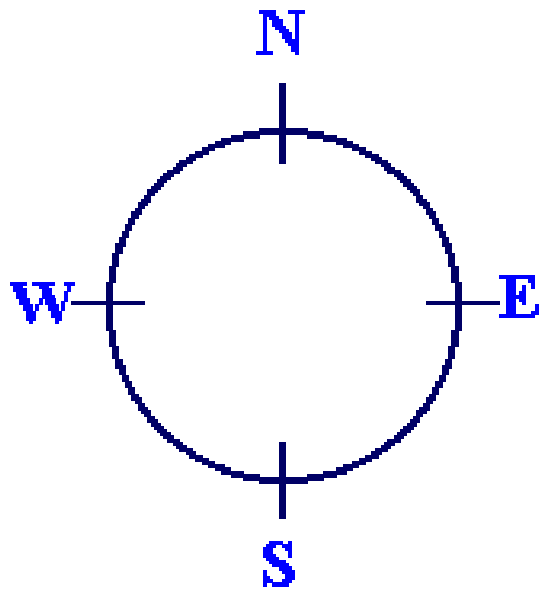
Summary

- Buyer beware – results of cluster analysis should be treated with **GREAT CAUTION** and **ATTENTION TO SPECIFICS**, because...
- Many things can vary in a cluster analysis
- If covariates/group labels are known, then clustering is usually inefficient

Locating a point in the plane

- We can describe the location of a point in the plane by saying how much we move in the horizontal (X) direction, then how much we move in the vertical (Y) direction
- As an example, think of describing how to get to some particular place from where you are (for example, how to get to CE 105 from MA 11)
- One way to do this is to say how far you go NORTH, then how far you go EAST

Directions: North = 1st?



Variance-Covariance matrix

- Consider a data set consisting of p variables measured on n cases
- How the variables change together is summarized by the variance-covariance matrix (or by the correlation matrix)
- For a simple example (just 2 variables):

```
> cov(head) | > cor(head)
      [,1] [,2] |      [,1] [,2]
[1, ] 96.95061 54.48939 | [1, ] 1.0000 .7859
[2, ] 54.48939 49.57918 | [2, ] 0.7859 1.0000
```

Principal Component Analysis (PCA)

- One aim of principal component analysis (PCA) is to *reduce the dimensionality* from p variables
- Try to explain the variance-covariance structure through *linear combinations (principal components)* of the (original) variables
- Another aim is to interpret the first few principal components in terms of the original variables to give greater insight into the data structure

More on PCA

- Each principal component (PC) accounts for a certain amount of the variation in the data
- The 1st PC is the linear combination that accounts for ('explains') the *most variation*
- Subsequent PCs account for as much as possible of the remaining variation, while being *uncorrelated* with earlier PCs
- *Aubergine*
- Where do these come from?

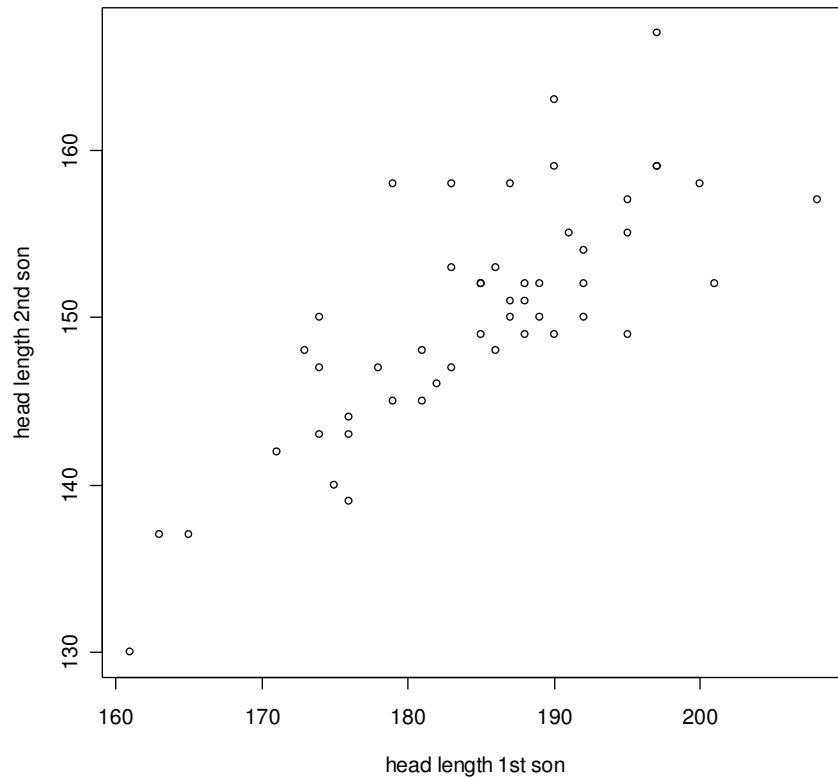
What does this have to do with PCA?

- Consider the variance-covariance matrix A
- The eigenvectors of A provide sets of coefficients defining p linear functions of the original variables
- ***These functions are the PCs***
- If A has eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_p$ then the PCs have variances $\lambda_1, \lambda_2, \dots, \lambda_p$ and zero covariances

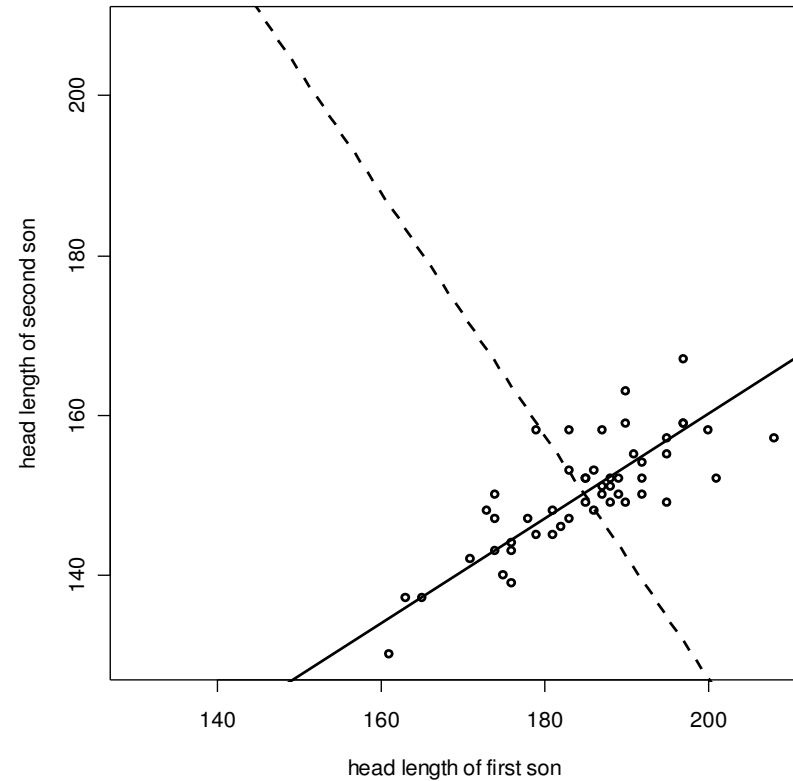
Cautions

- Sometimes used as a method for *simplifying data* because PCs associated with smaller eigenvalues have smaller variances and might therefore be ‘ignored’
- *This assumption requires caution*
- When variables are on *different scales*, it is customary to use the correlation matrix (rather than the covariance matrix)
- *These two formulations give different results* : the eigenvalues for the two matrices are not related in a simple way
- Theory not simple for correlation-based PCA

Original axes



Principal axes



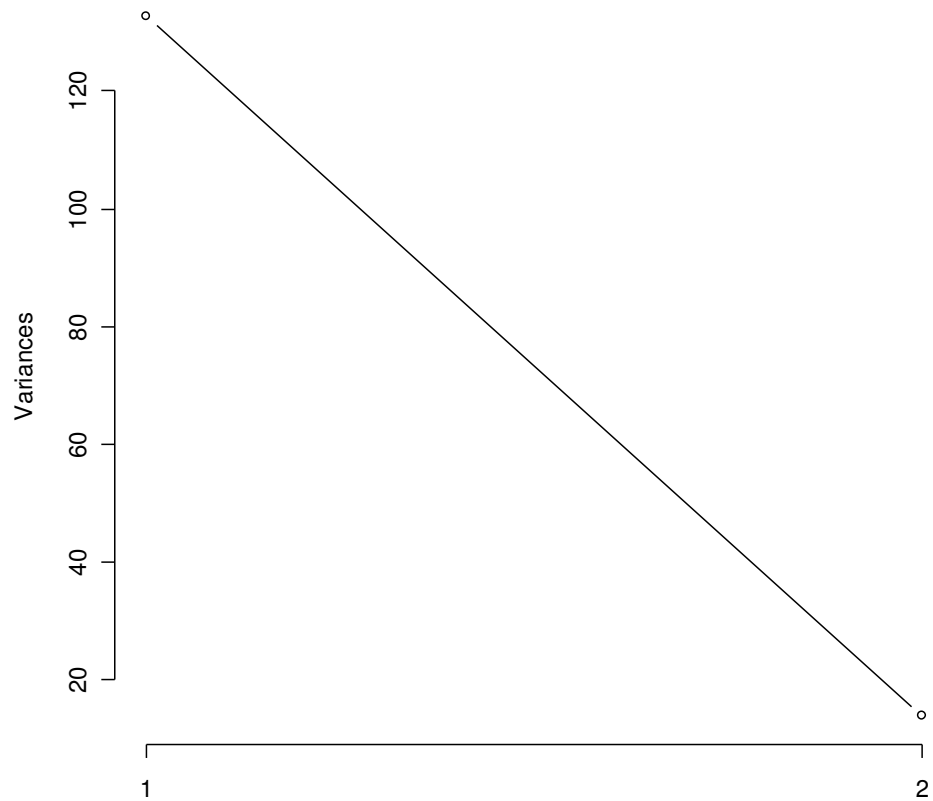
- Head length (in mm) for each of the first two adult sons in 50 families

How many PCs?

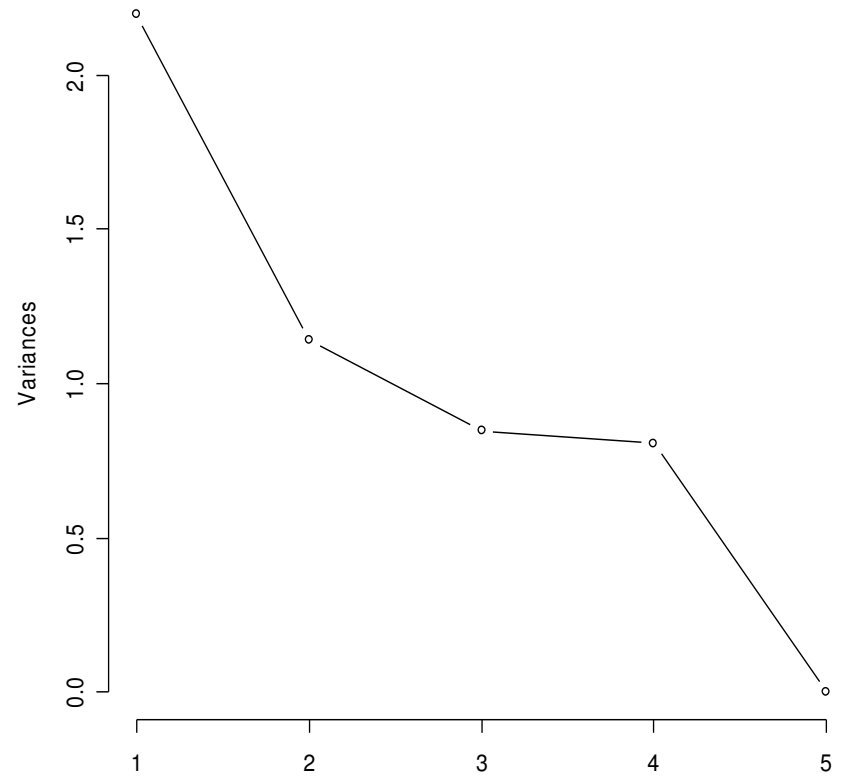
- There are a few ways to decide how many PCs to retain
- Some common methods are:
 - retain the number required to explain some percentage of the total variation (e.g. 90%)
 - number of eigenvalues $>$ average (1 if correlation matrix is used)
 - look for ‘elbow’ in scree plot
 - compromise between these
- The scree plot shows proportion of variance (or just variance) explained by each component

R: scree plots

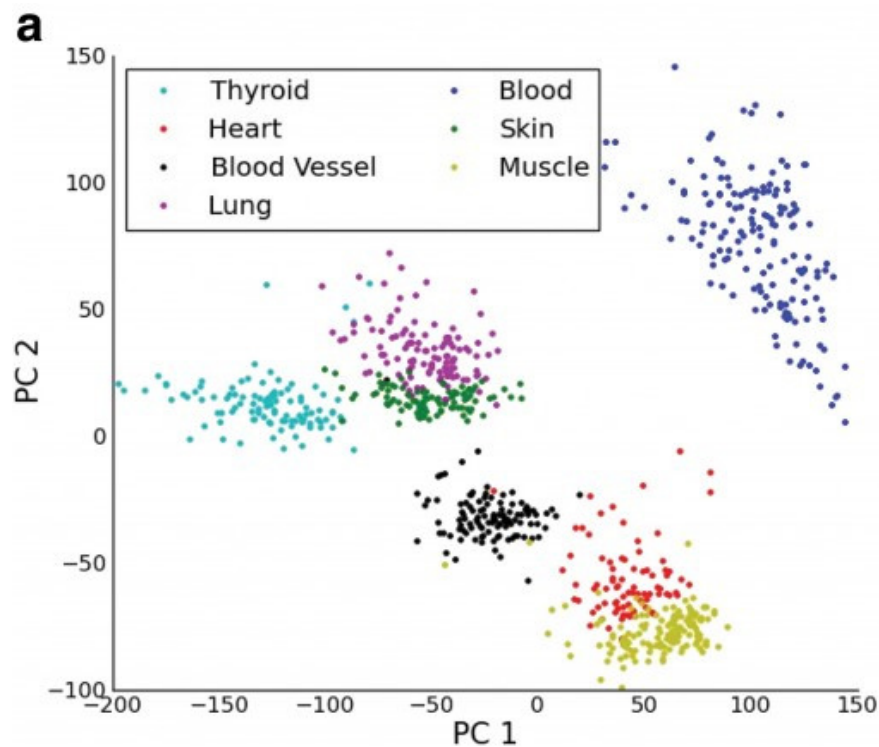
head.pc



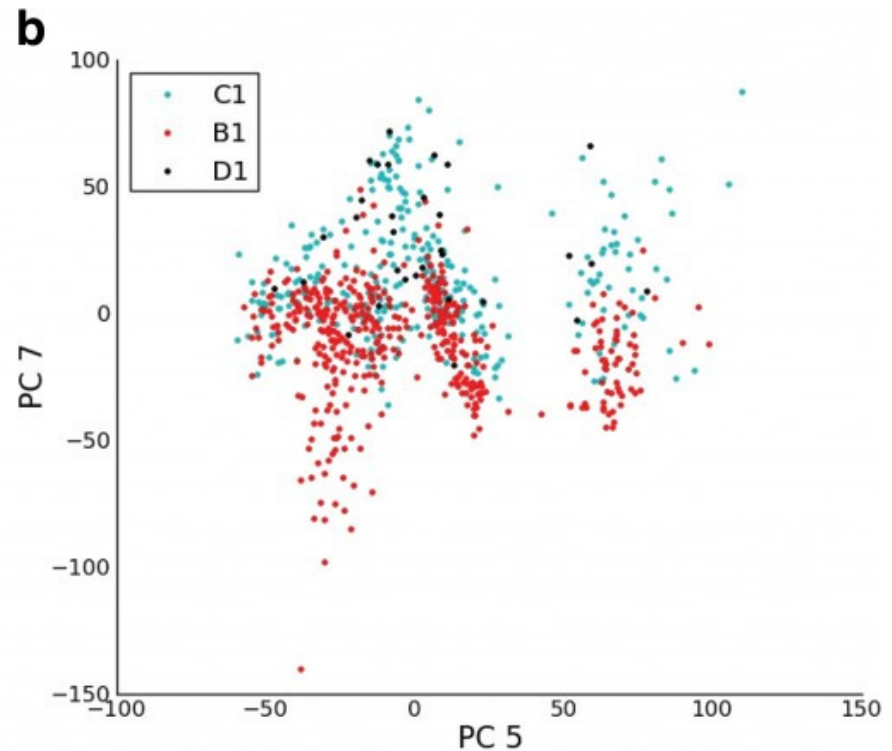
food.pc



PCA to assess data quality



(a) RNA-seq data projected onto PCs 1&2, where spot corresponds to a sample and color to tissue type. Samples from the same tissue cluster together.



(b) RNA-seq data projected onto PCs 5&7, now colored by enrollment center (C1, B1, D1). There is an obvious relation between PC 7 and center.