Homework 11 (due Friday, December 13)

Exercise 1. Let X, Y be two Markov chains with following same transition probabilities:

$$p_{ij} = \begin{cases} 1/2 & \text{if } j = i \\ 1/4 & \text{if } j = i \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

That is, X and Y are two versions of the symmetric lazy random walk on $\mathbb{Z}$. Let us assume now that $X_0 = 0$ and $Y_0 = 1$.

a) Using a random mapping representation, describe a coupling of X and Y such that these two chains meet with the highest probability after one step only. What is the value of this probability?

b) Using a random mapping representation, describe *two* different couplings of X and Y such that these two chains *never* meet.

Variant: Consider the same questions, but now with the initial conditions $X_0 = 0$ and $Y_0 = 2$.