Biological Modeling of Neural Networks

Wulfram Gerstner

EPFL, Lausanne, Switzerland

TAs in 2020:
Martin Barry (head)
Valentin Schmutz
Georgios Iatropoulos
Christos Sourmpis

COURSE WEBPAGE:
Moodle

Week 1: **A first simple neuron model/ neurons and mathematics**
Week 2: **Hodgkin-Huxley models and biophysical modeling**
Week 3: **Two-dimensional models and phase plane analysis**
Week 4: **Two-dimensional models, type I and type II models**
Week 5,6: **Associative Memory, Hebb rule, Hopfield**
Week 7-10: **Networks, cognition, learning**
Week 11,12: **Noise models, noisy neurons and coding**
Week 13: **Estimating neuron models for coding and decoding: GLM**
Week x: **Online video: Dendrites/Biophysics**
Week xx: **Density equations**

LEARNING OUTCOMES

- Solve linear one-dimensional differential equations
- Analyze two-dimensional models in the phase plane
- Develop a simplified model by separation of time scales
- Analyze connected networks in the mean-field limit
- Formulate stochastic models of biological phenomena
- Formalize biological facts into mathematical models
- Prove stability and convergence
- Apply model concepts in simulations
- Predict outcome of dynamics
- Describe neuronal phenomena

Look at samples of past exams

Transversal skills

- Plan and carry out activities in a way which makes optimal use of available time and other resources.
- Collect data.
- Write a scientific or technical report.

Use a textbook, (Use video lectures) don't use slides (only)

miniproject

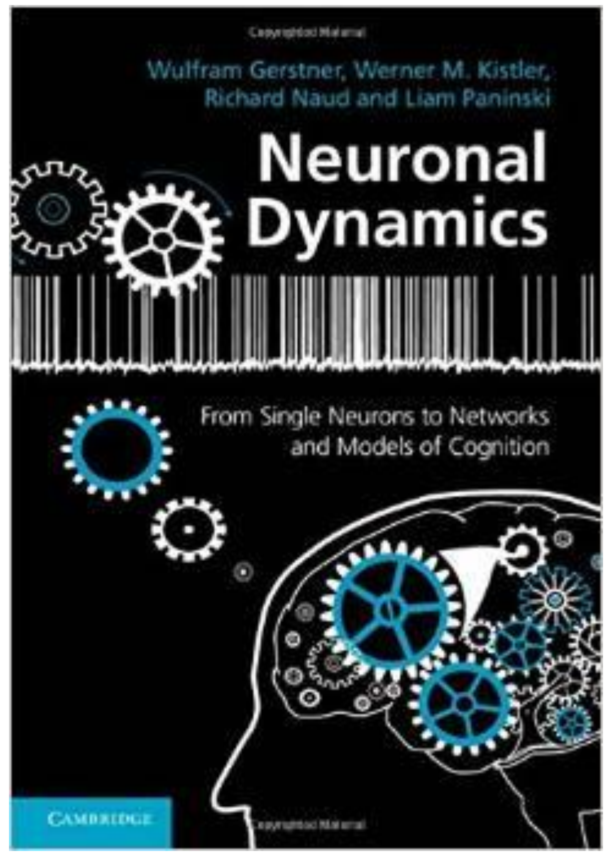
Biological Modeling of Neural Networks

Written Exam (70%)
+ miniproject (30%)

Miniproject consists of
3 extended computer exercises,
of which you have to hand in 2
(first one is easier, recommended)

Textbook:

http://neuronaldynamics.epfl.ch/



Video:

https://lcnwww.epfl.ch/gerstner/NeuronalDynamics-MOOC1.html

https://lcnwww.epfl.ch/gerstner/NeuronalDynamics-MOOC2.html

Welcome back to EPFL!!

Today: Course in BS 170

BS 170 for the first week, but
INM 200 for the rest of the semester

Biological Modeling of Neural Networks

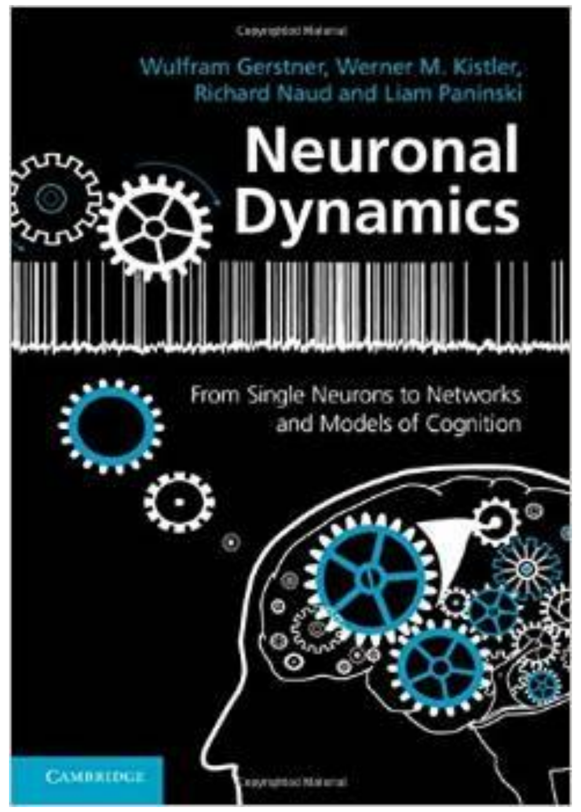
ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Week 1 – neurons and mathematics: a first simple neuron model

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Reading for week 1:
NEURONAL DYNAMICS
- Ch. 1 (without 1.3.6 and 1.4)
- Ch. 5 (without 5.3.1)



Cambridge Univ. Press

1.1 **Neurons and Synapses:**
Overview

1.2 **The Passive Membrane**
- Linear circuit
- Dirac delta-function

1.3 **Leaky Integrate-and-Fire Model**

1.4 **Generalized Integrate-and-Fire Model**

1.5. **Quality of Integrate-and-Fire Models**

Biological Modeling of Neural Networks

1.1 **Neurons and Synapses:**
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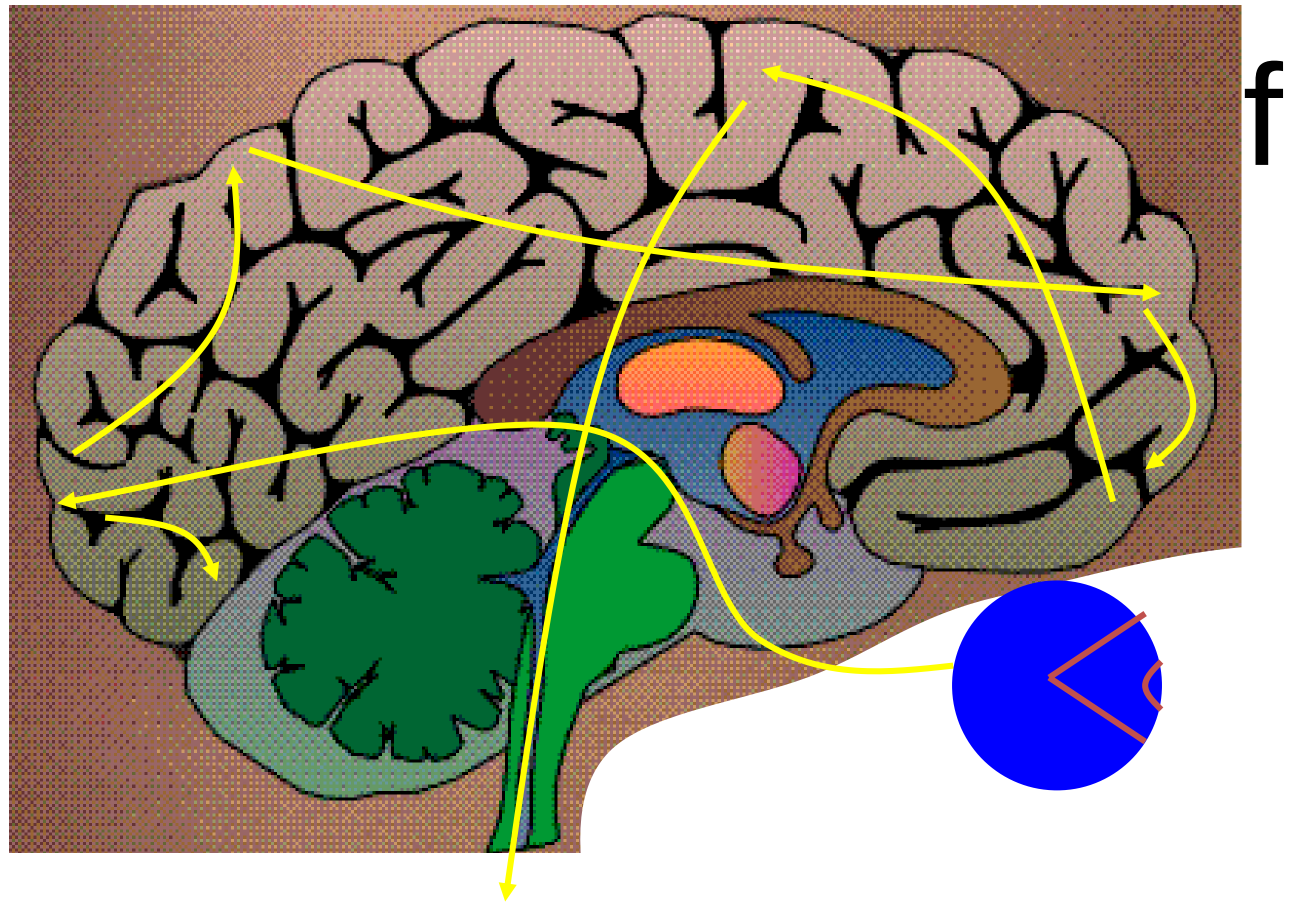
1.4 **Generalized Integrate-and-Fire Model**

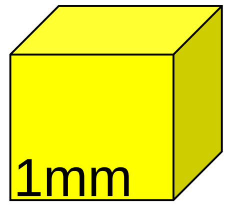
1.5. **Quality of Integrate-and-Fire Models**

Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

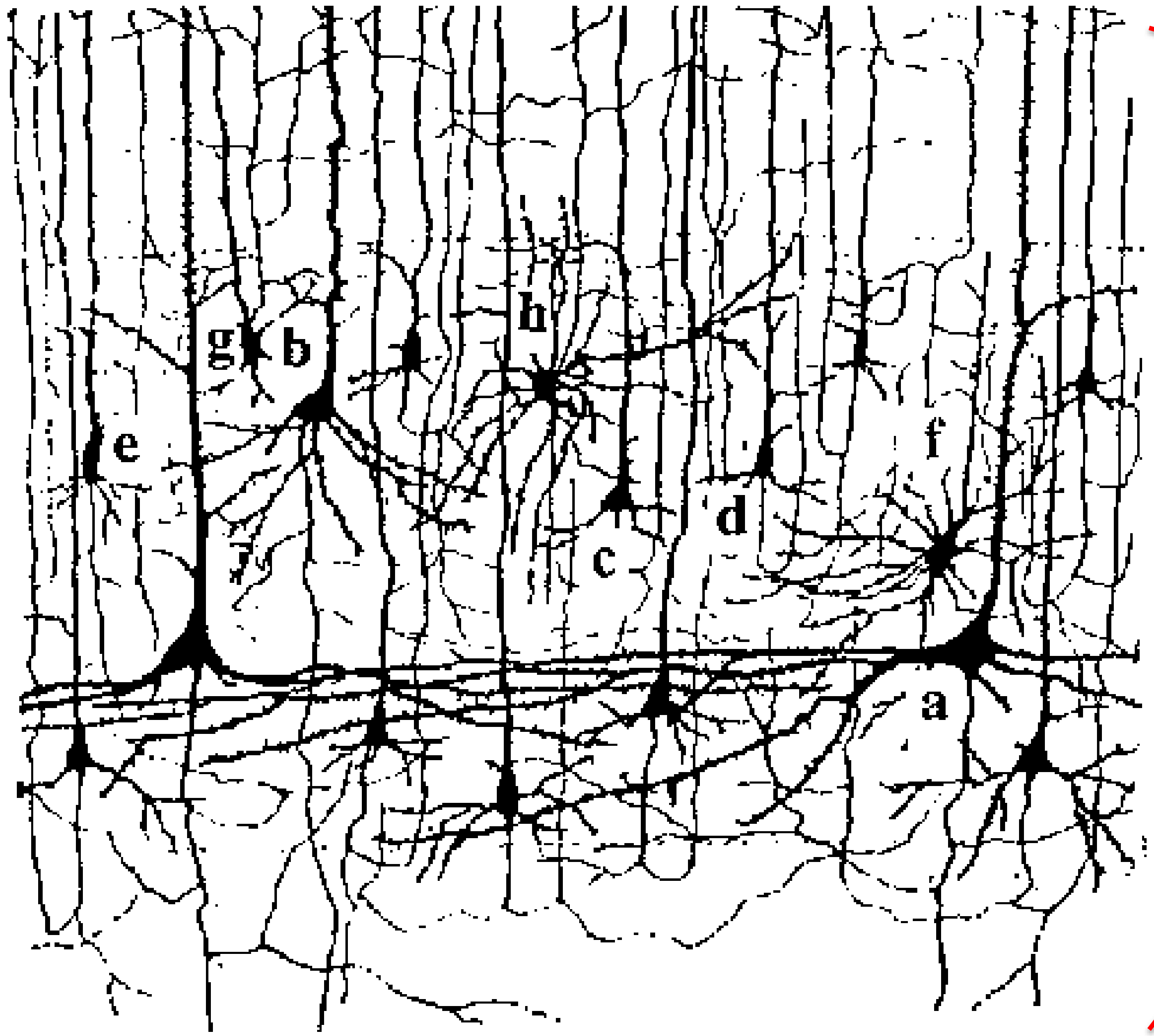
How do we recognize things?
Models of cognition
Weeks 5-10

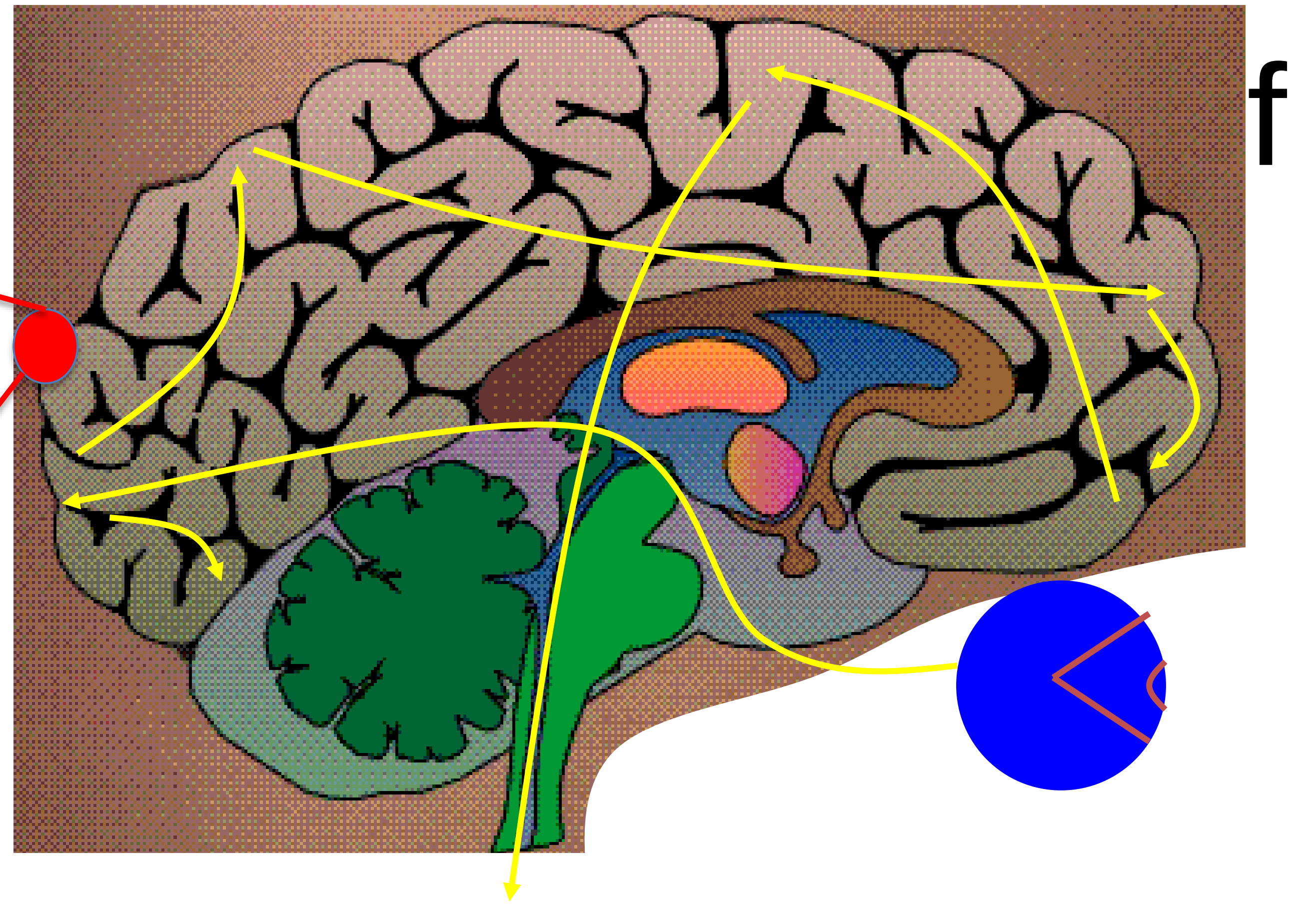


Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

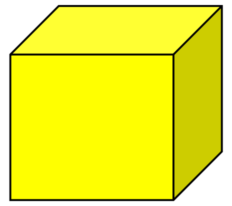


10 000 neurons
3 km of wire



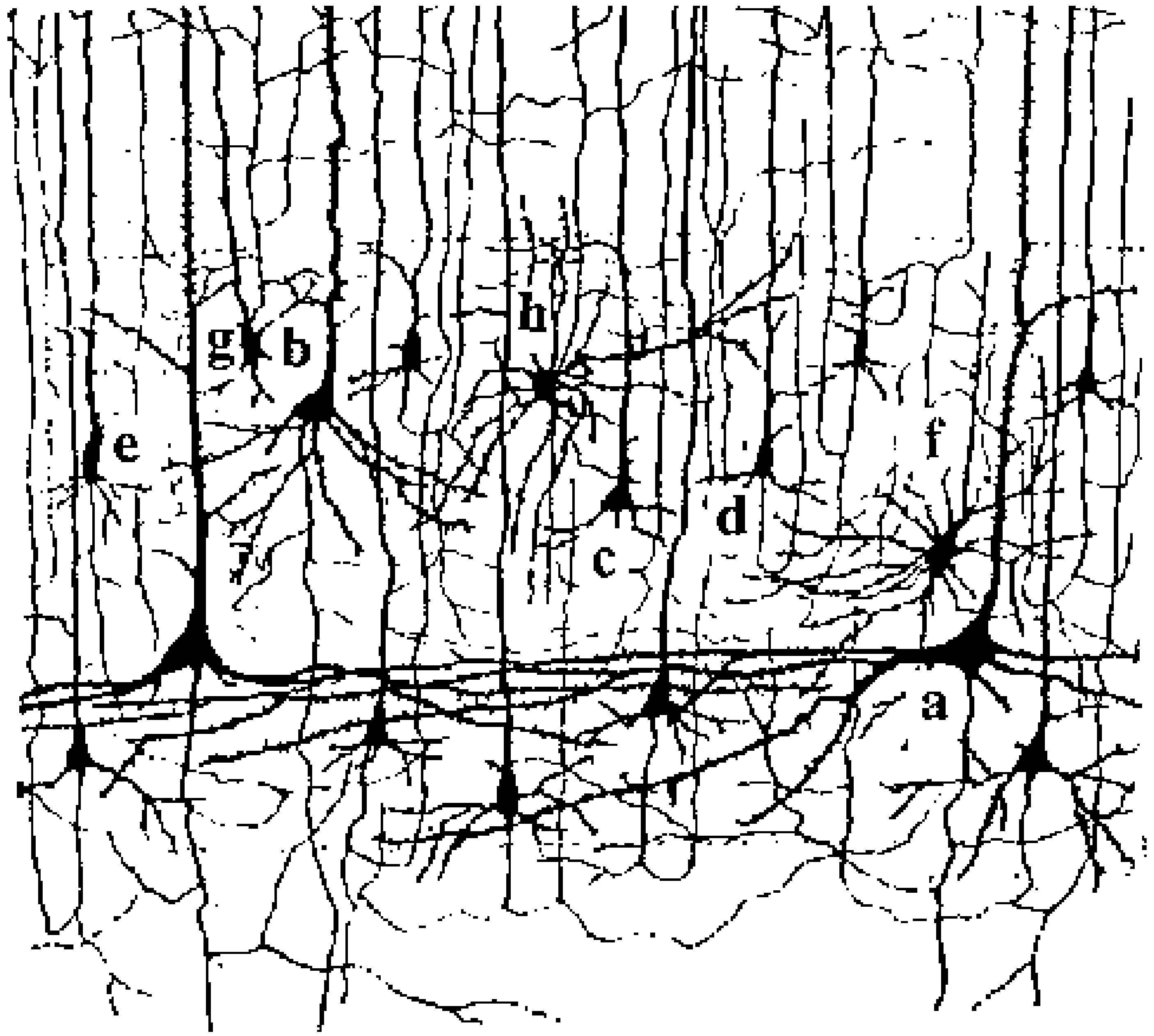


Neuronal Dynamics – 1.1. Neurons and Synapses/Overview



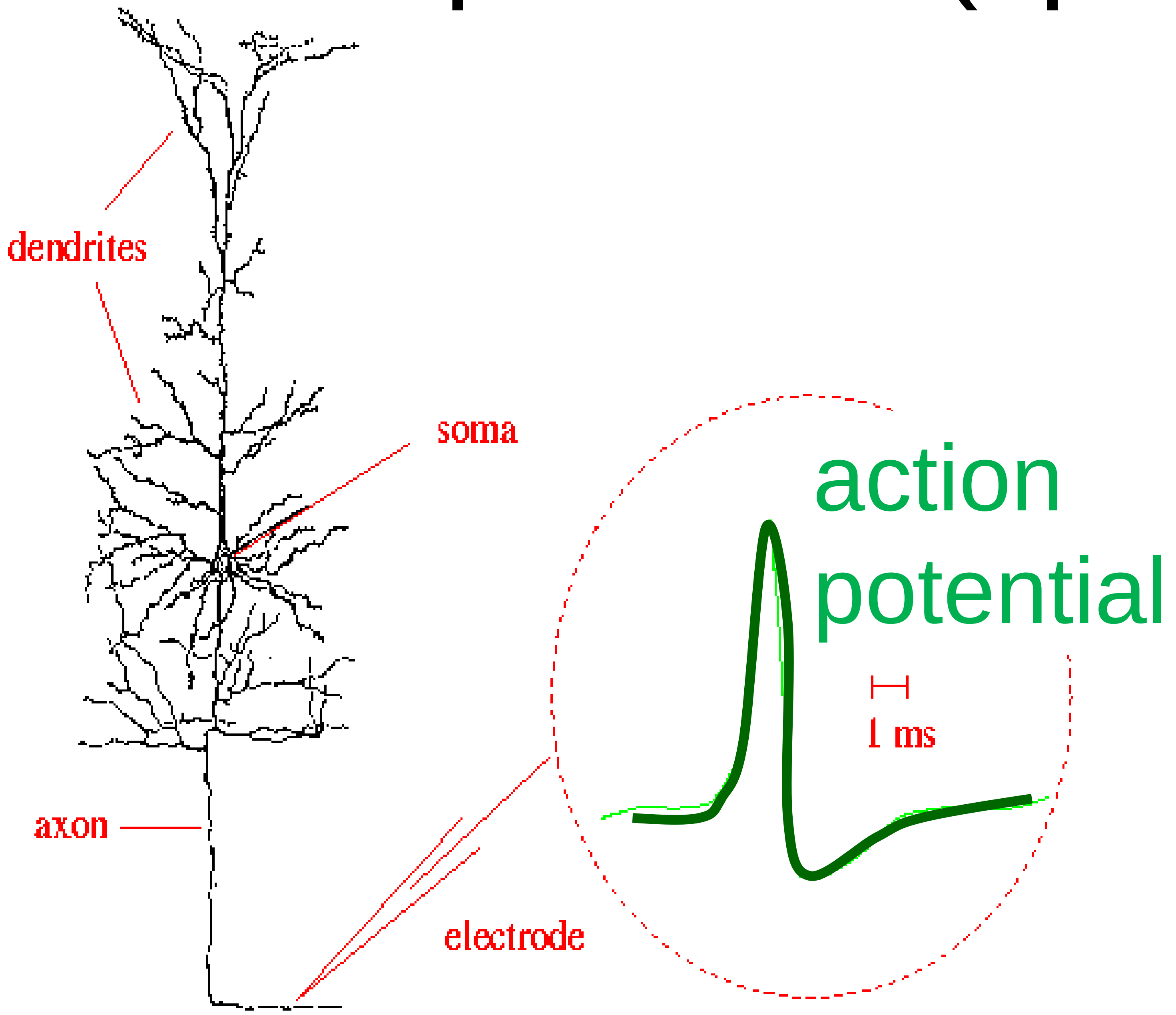
1mm

10 000 neurons
3 km of wire



Ramon y Cajal

Signal:
action potential (spike)



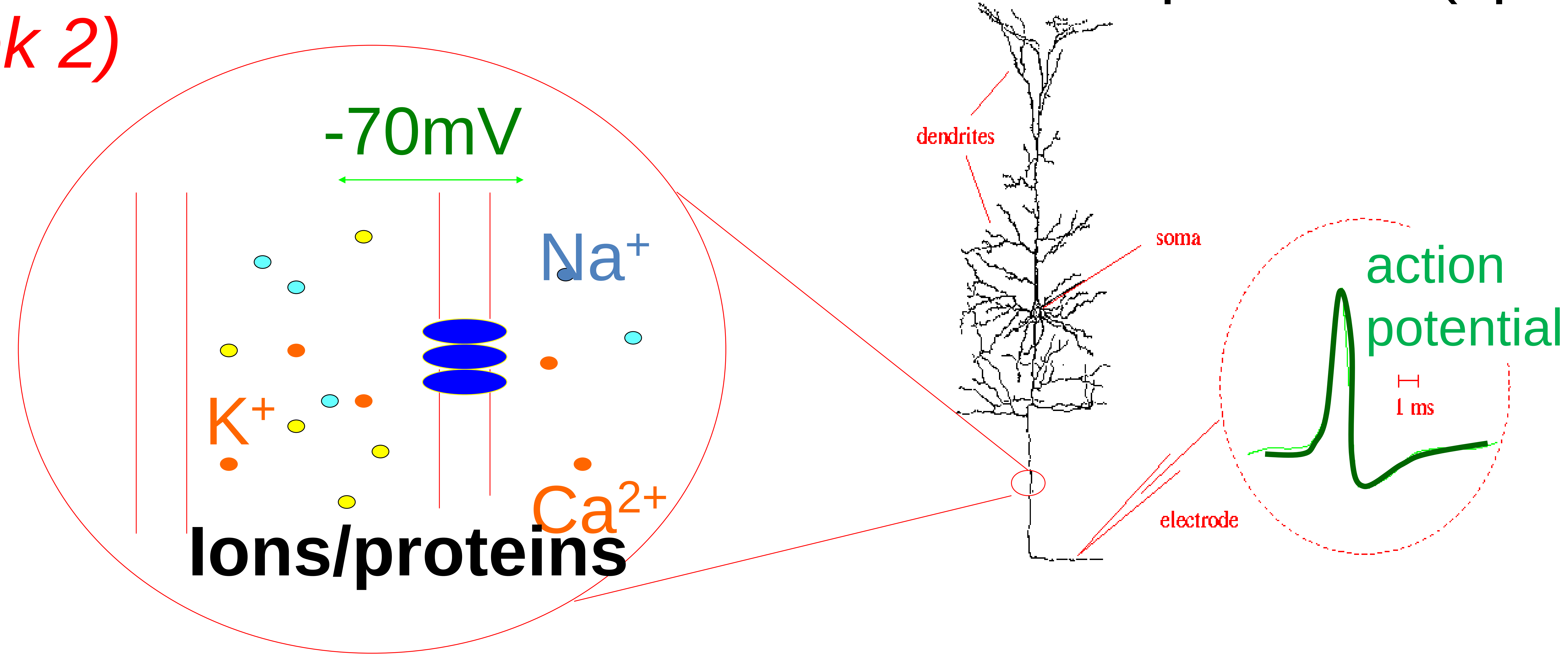
Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

Hodgkin-Huxley type models:

Biophysics, molecules, ions

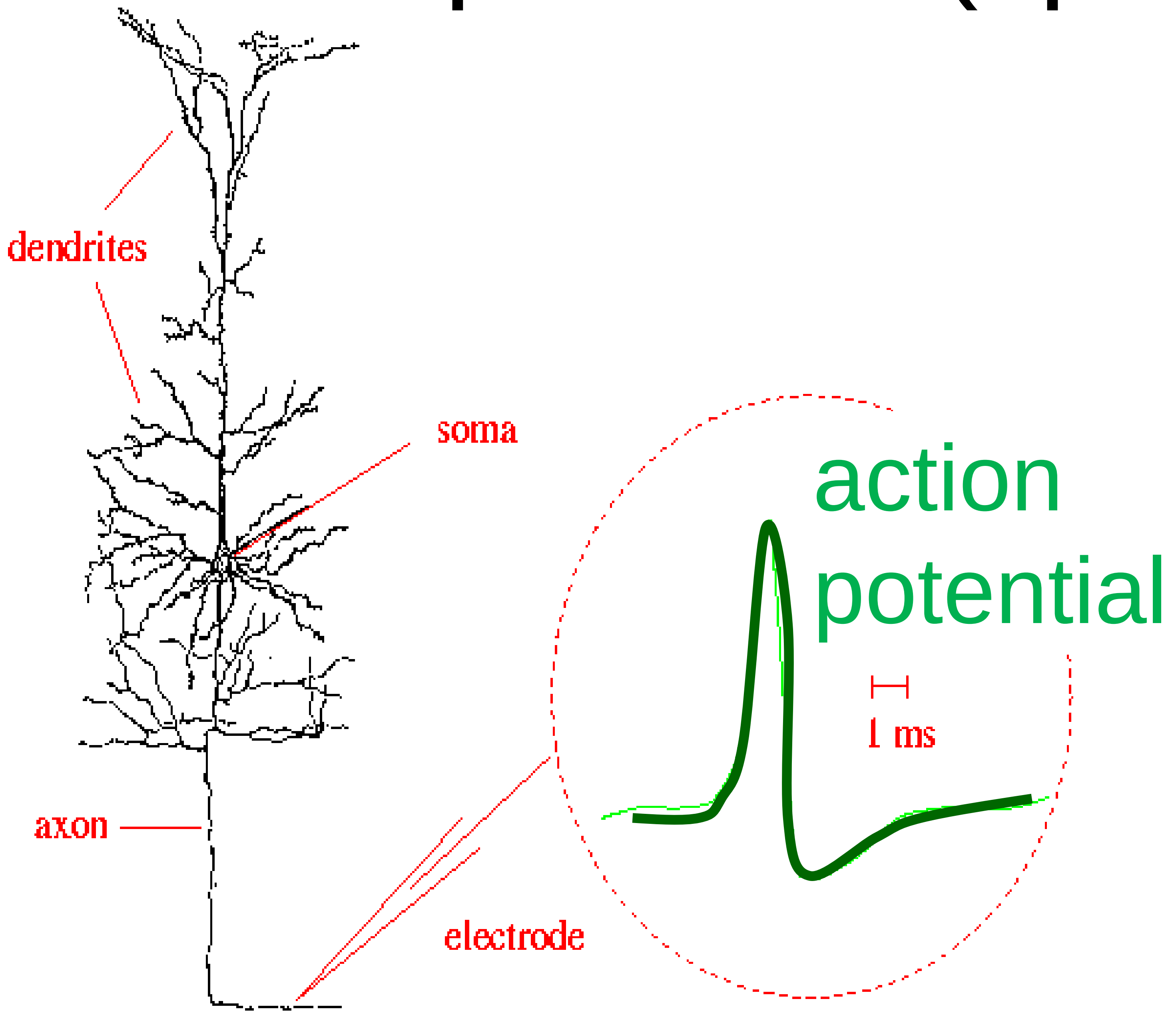
(week 2)

Signal:
action potential (spike)



Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

Signal:
action potential (spike)

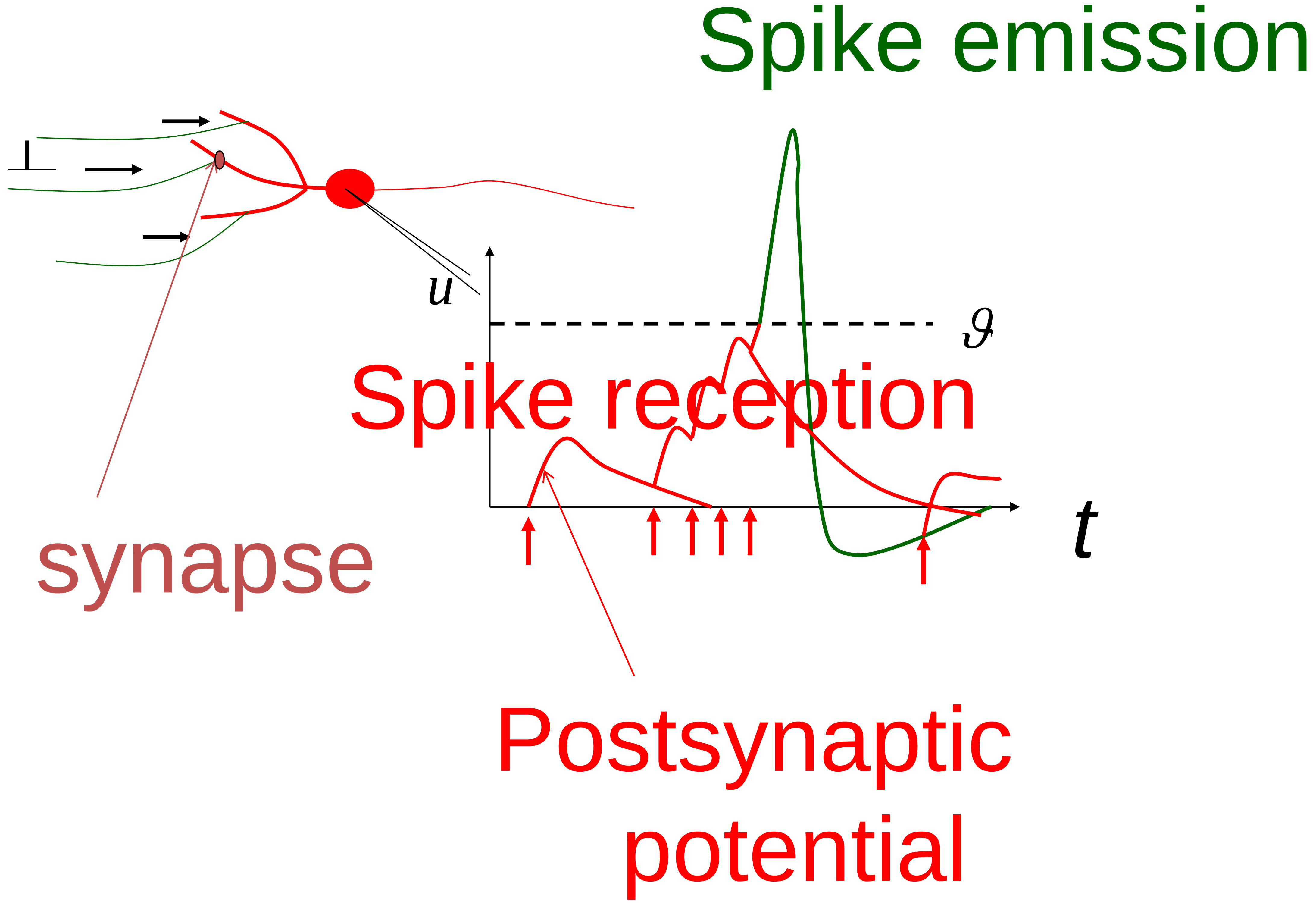


Neuronal Dynamics – 1.1. Neurons and Synapses/Overview

Integrate-and-fire models:

Formal/phenomenological

(week 1 and week 7-9)

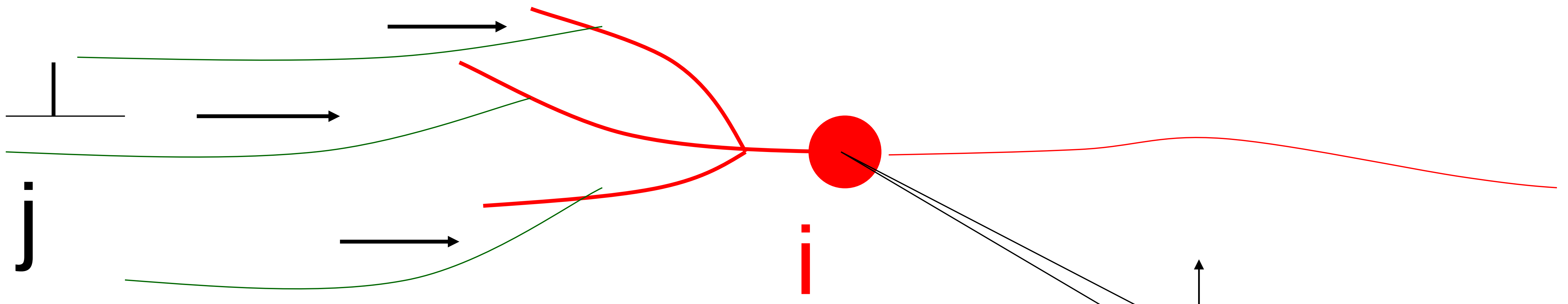


-spikes are events

-triggered at threshold

-spike/reset/refractoriness

Noise and variability in integrate-and-fire models



Output

-spikes are **rare** events

-triggered at threshold

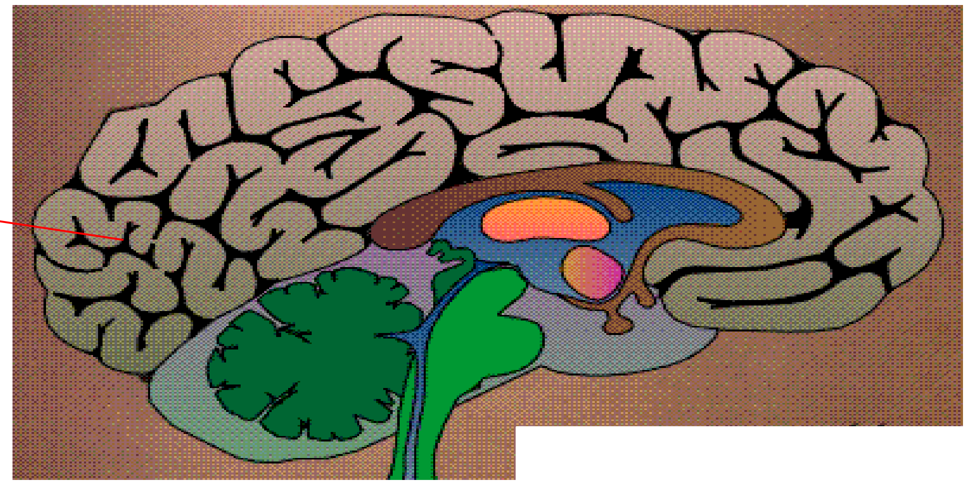
Subthreshold regime:

-trajectory of potential shows fluctuations

Neuronal Dynamics – membrane potential fluctuations

Spontaneous activity *in vivo*

electrode



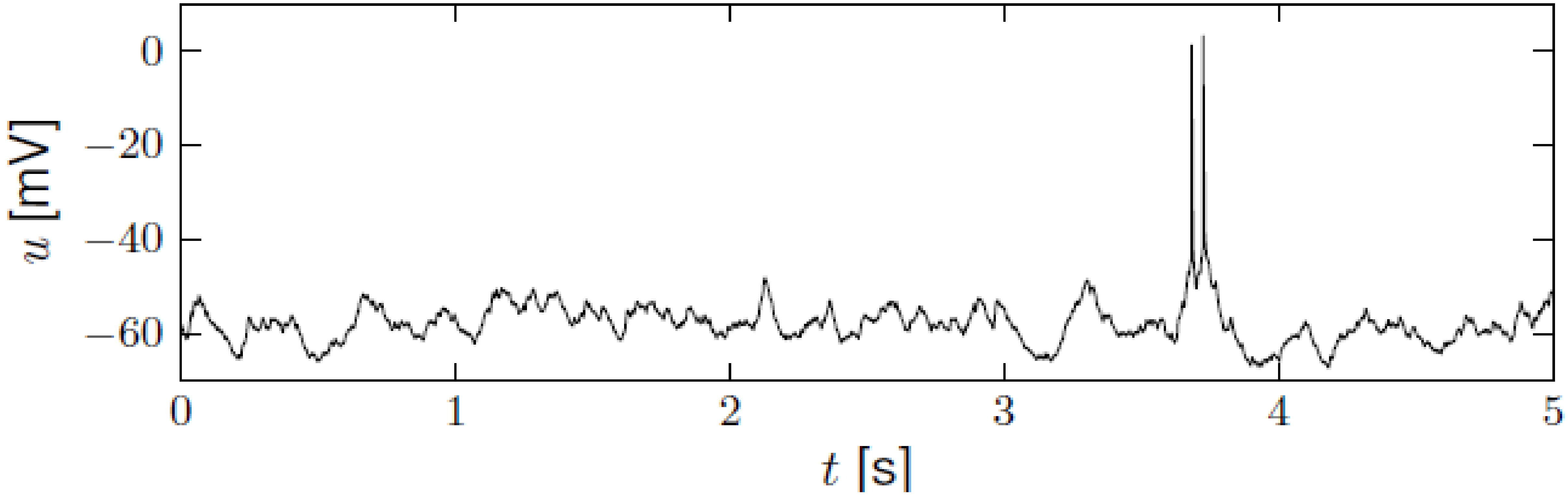
Brain

What is noise?

What is the neural code?

(week 11-13)

awake mouse, cortex, freely whisking,



Lab of Prof. C. Petersen, EPFL

Crochet et al., 2011

Biological Modeling of Neural Networks – Quiz 1.1

A cortical neuron sends out signals which are called:
- [ ] action potentials
- [ ] spikes
- [ ] postsynaptic potential

The dendrite is a part of the neuron
- [ ] where synapses are located
- [ ] which collects signals from other neurons
- [ ] along which spikes are sent to other neurons

In an integrate-and-fire model, when the voltage hits the threshold:
- [ ] the neuron fires a spike
- [ ] the neuron can enter a state of refractoriness
- [ ] the voltage is reset
- [ ] the neuron explodes

In vivo, a typical cortical neuron exhibits
- [ ] rare output spikes
- [ ] regular firing activity
- [ ] a fluctuating membrane potential

Multiple answers possible!

Biological Modeling of Neural Networks

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Biological modeling of Neural Networks

Course: Monday : 9:15-13:00

A typical Monday:

1st lecture 9:15-9:50

1st exercise 9:50-10:00 ← *paper and pencil*

2nd lecture 10:15-10:35

2nd exercise 10:35-11:00 ← *paper and pencil*

3rd lecture 11:15 – 11:40 ← *paper and pencil OR interactive toy examples on computer*

3rd exercise 11:45-12:40

have your laptop with you

Course of 4 credits = *6 hours of work per week*
4 ‘contact’ + 2 homework

moodle.epfl.ch

Week 1 – part 2: The Passive Membrane

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Biological Modeling of Neural Networks

Week 1 – neurons and mathematics: a first simple neuron model

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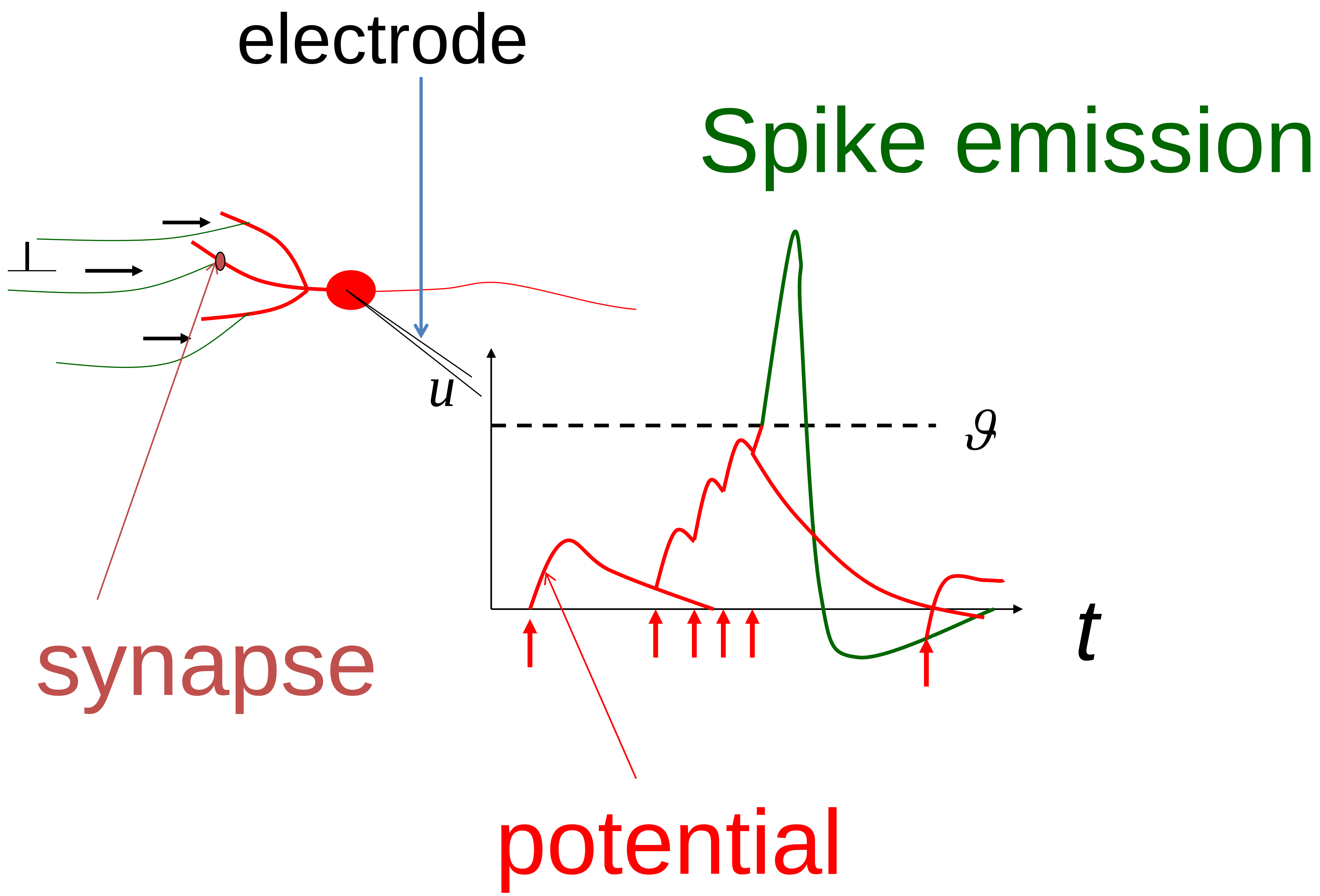
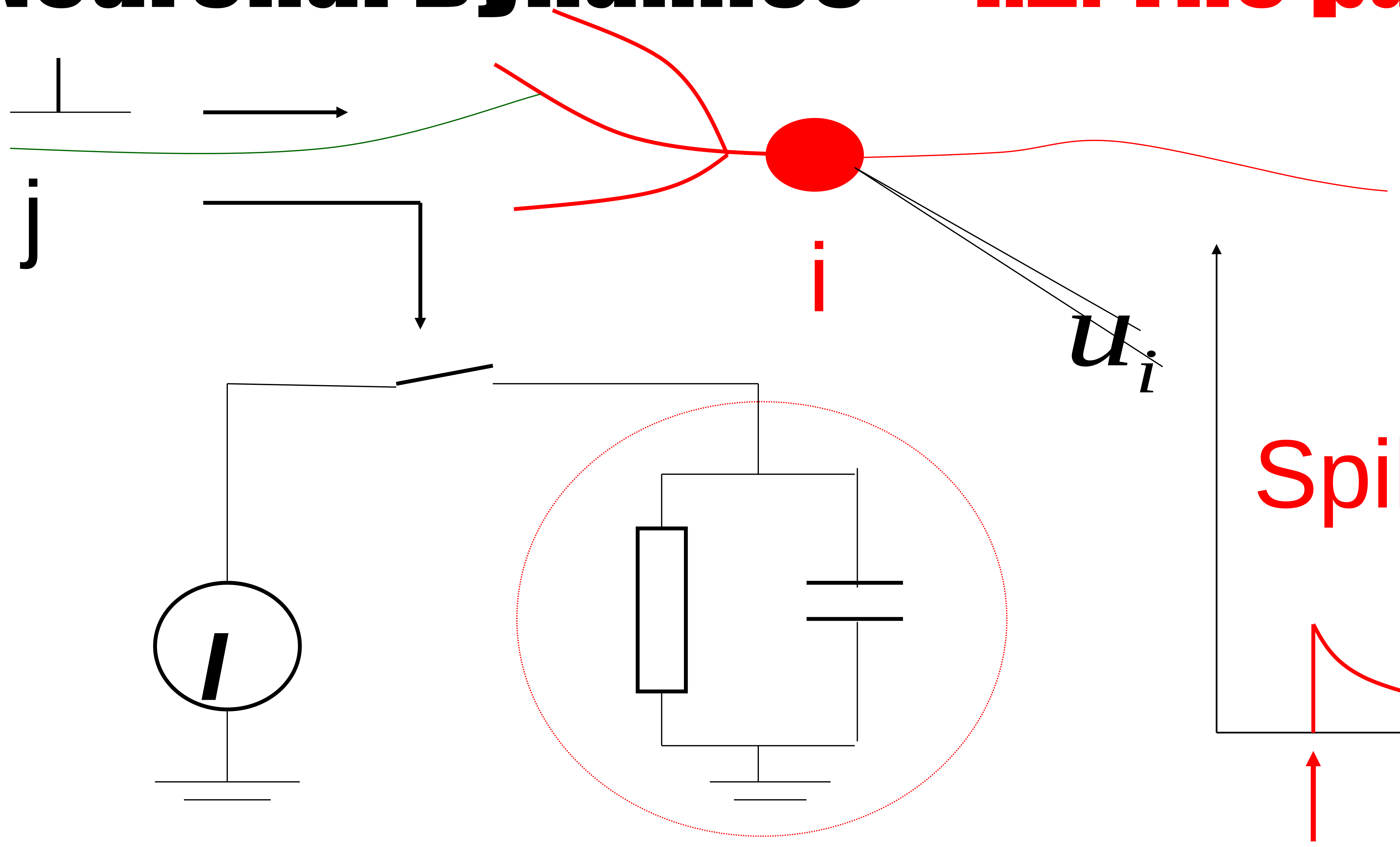
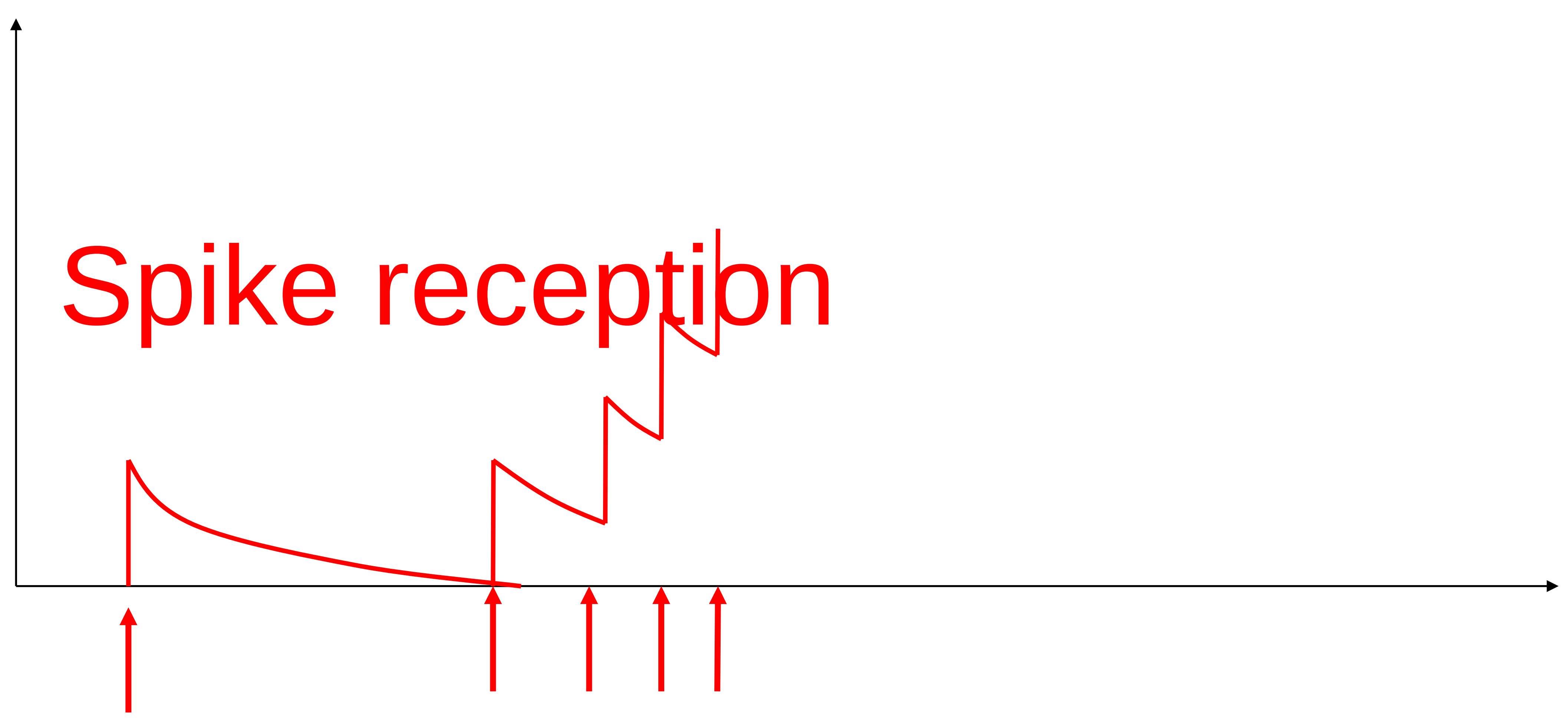
Neuronal Dynamics – 1.2. The passive membrane



Integrate-and-fire model

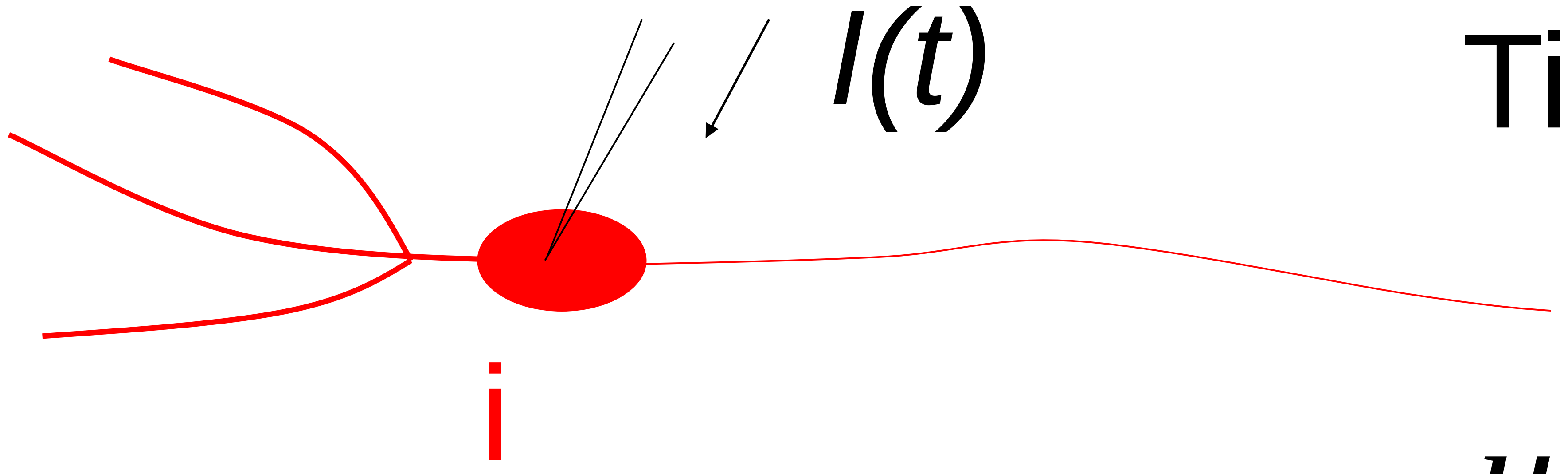
Neuronal Dynamics – 1.2. The passive membrane



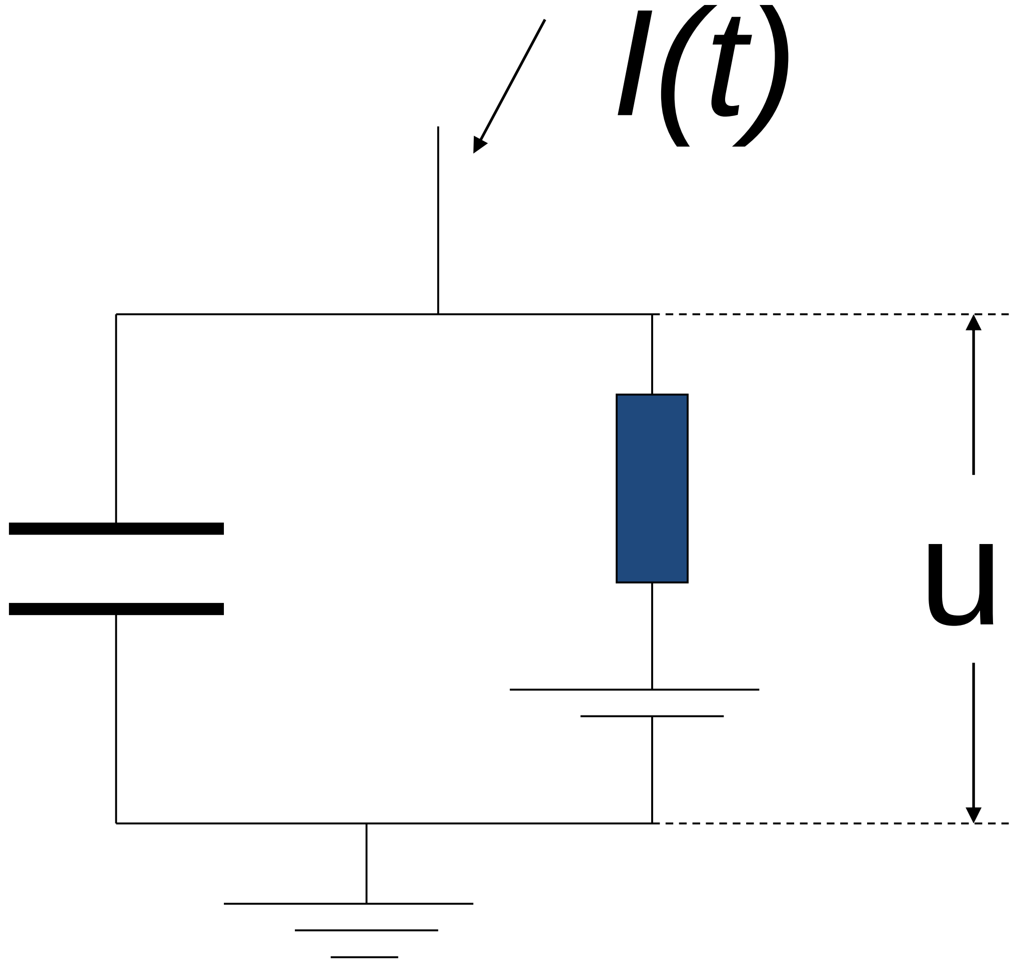


Subthreshold regime
- linear
- passive membrane
- RC circuit

Neuronal Dynamics – 1.2. The passive membrane

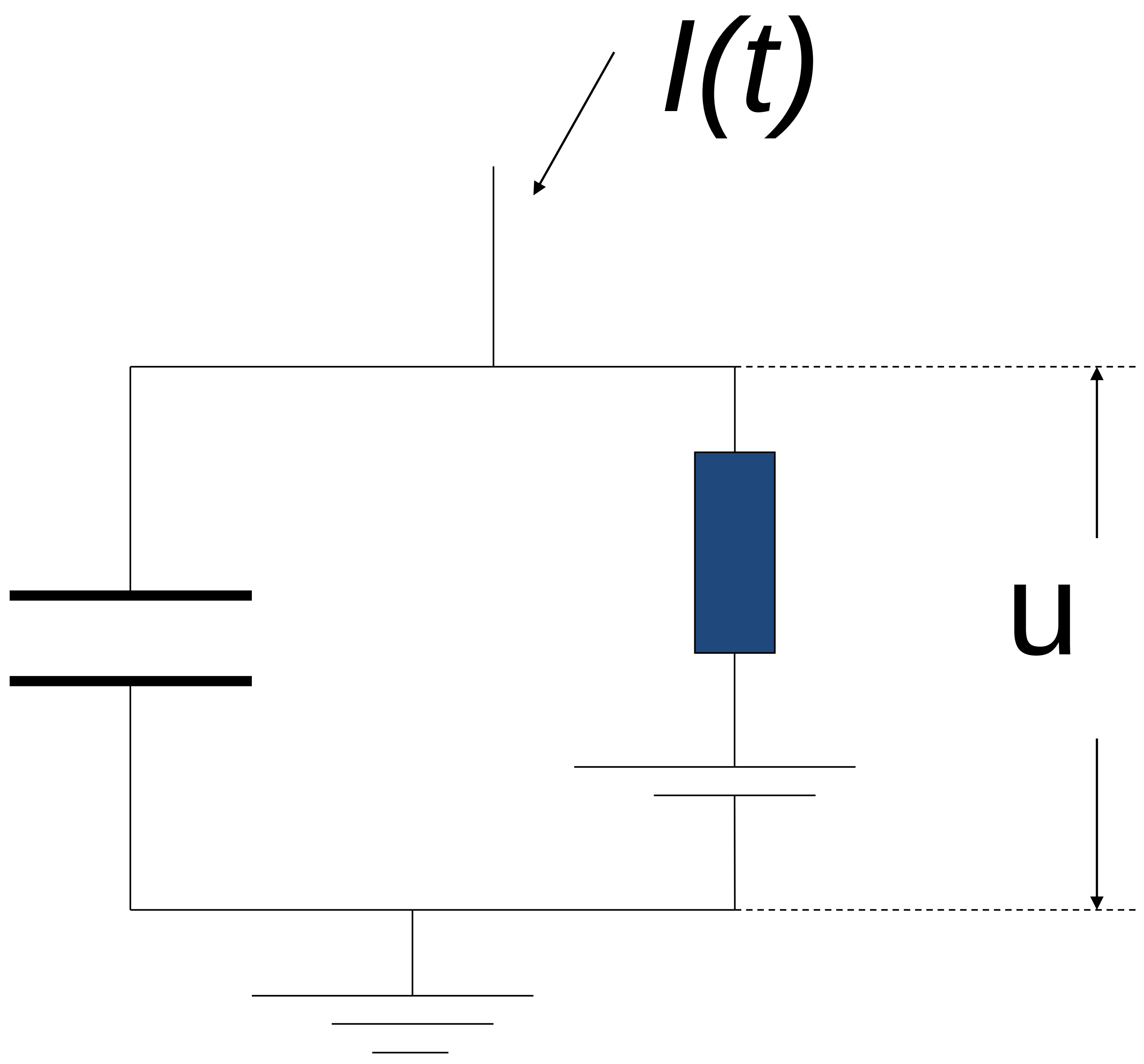


Time-dependent input

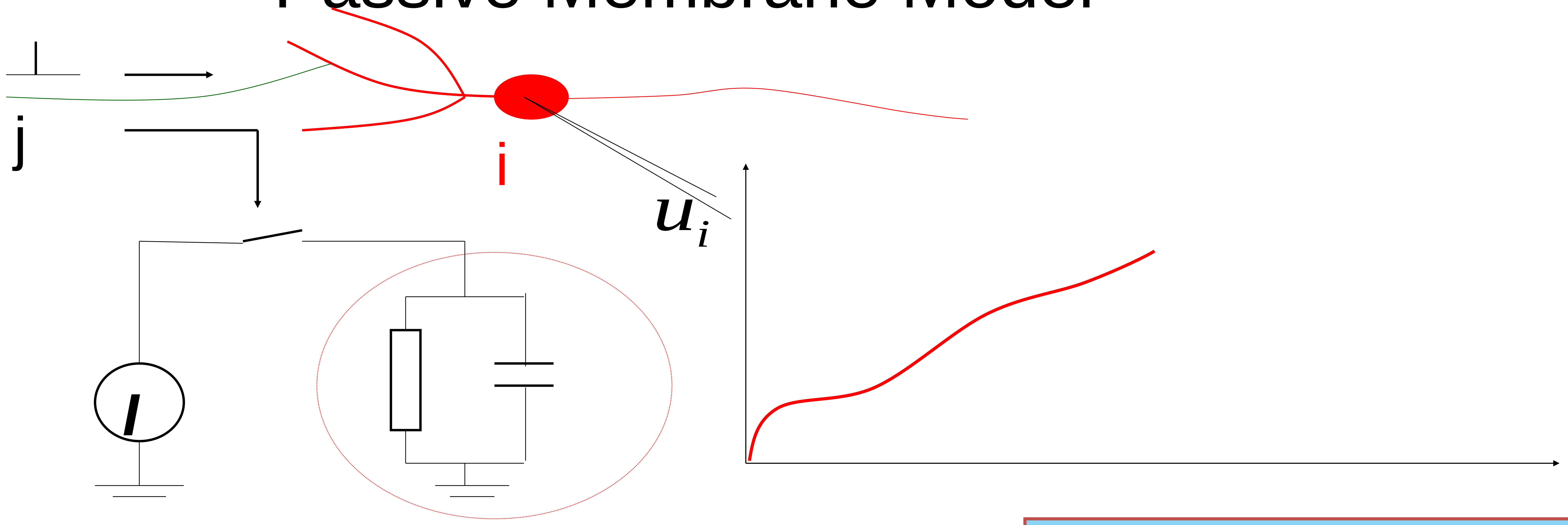


Math development:
Derive equation
(Blackboard)

Passive Membrane Model



Passive Membrane Model



$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$\tau \cdot \frac{d}{dt} V = -V + RI(t); \quad V = (u - u_{rest})$$

Math Development: Voltage rescaling (blackboard)

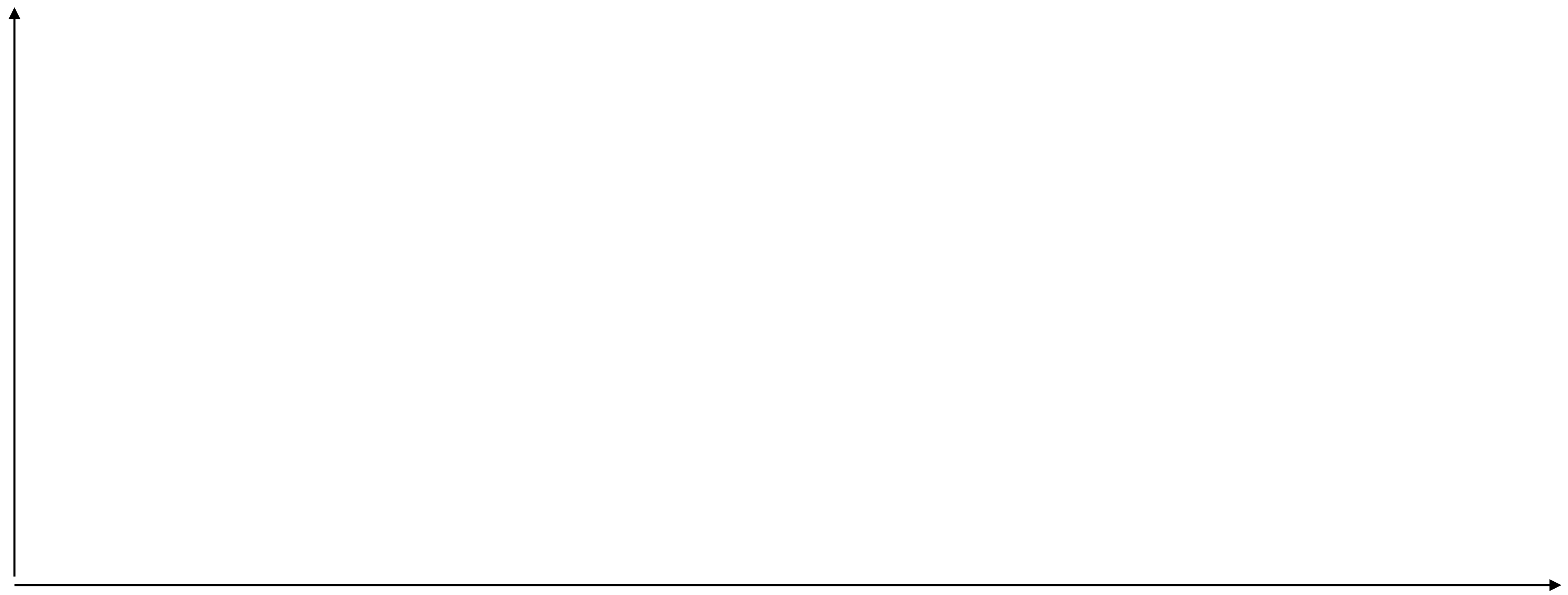
Passive Membrane Model

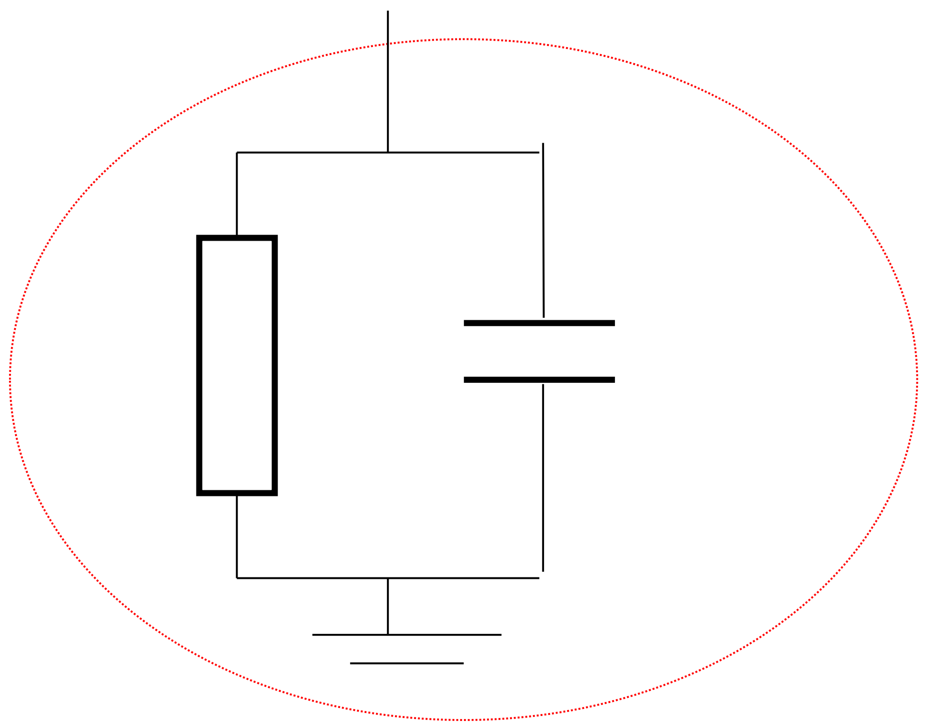
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$\tau \cdot \frac{d}{dt} V = -V + RI(t); \qquad V = (u - u_{rest})$$

Passive Membrane Model/Linear differential equation

$$\tau \cdot \frac{d}{dt} V = -V + \cancel{RI(t)};$$

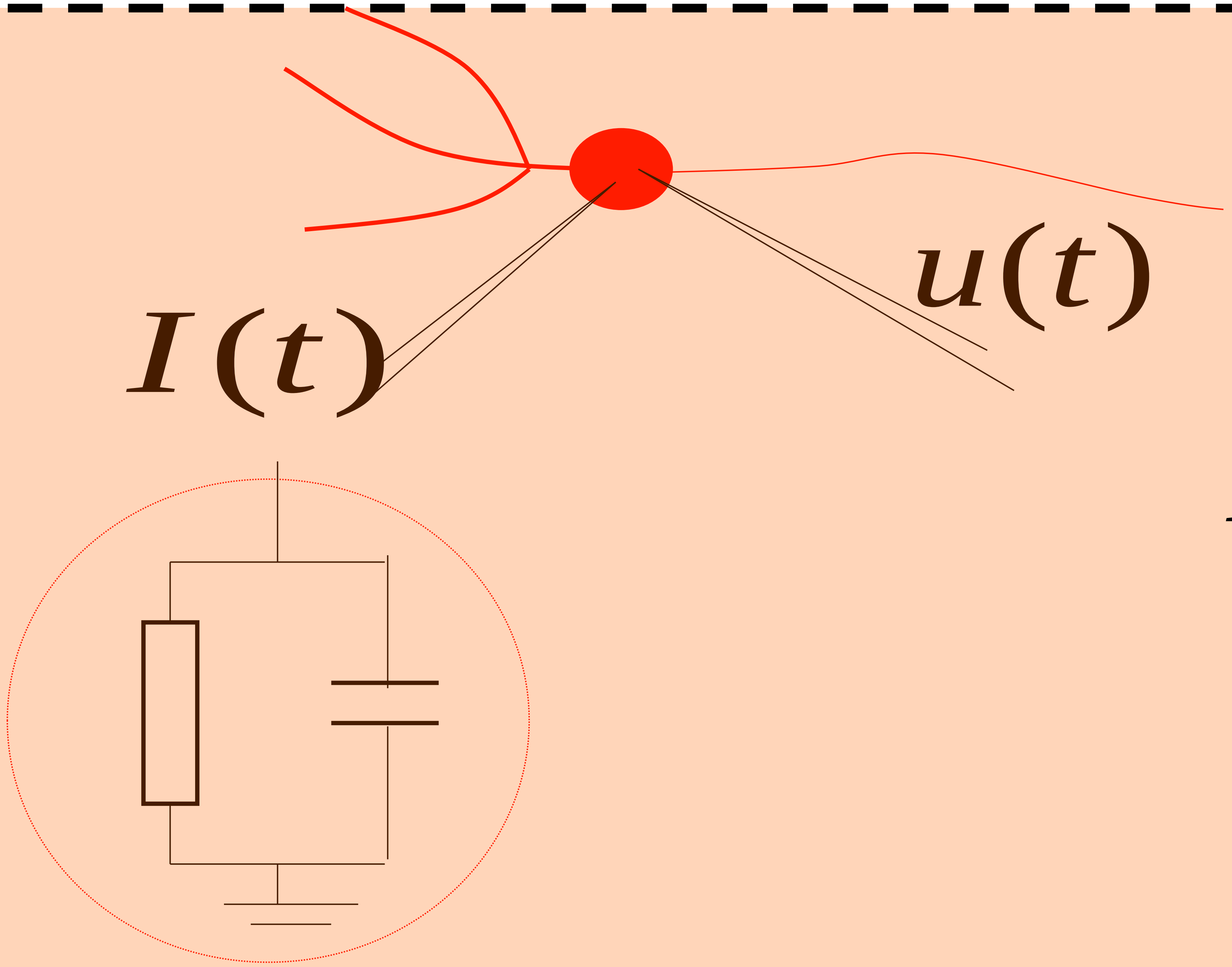




Free solution:
exponential decay

Neuronal Dynamics – Exercises NOW

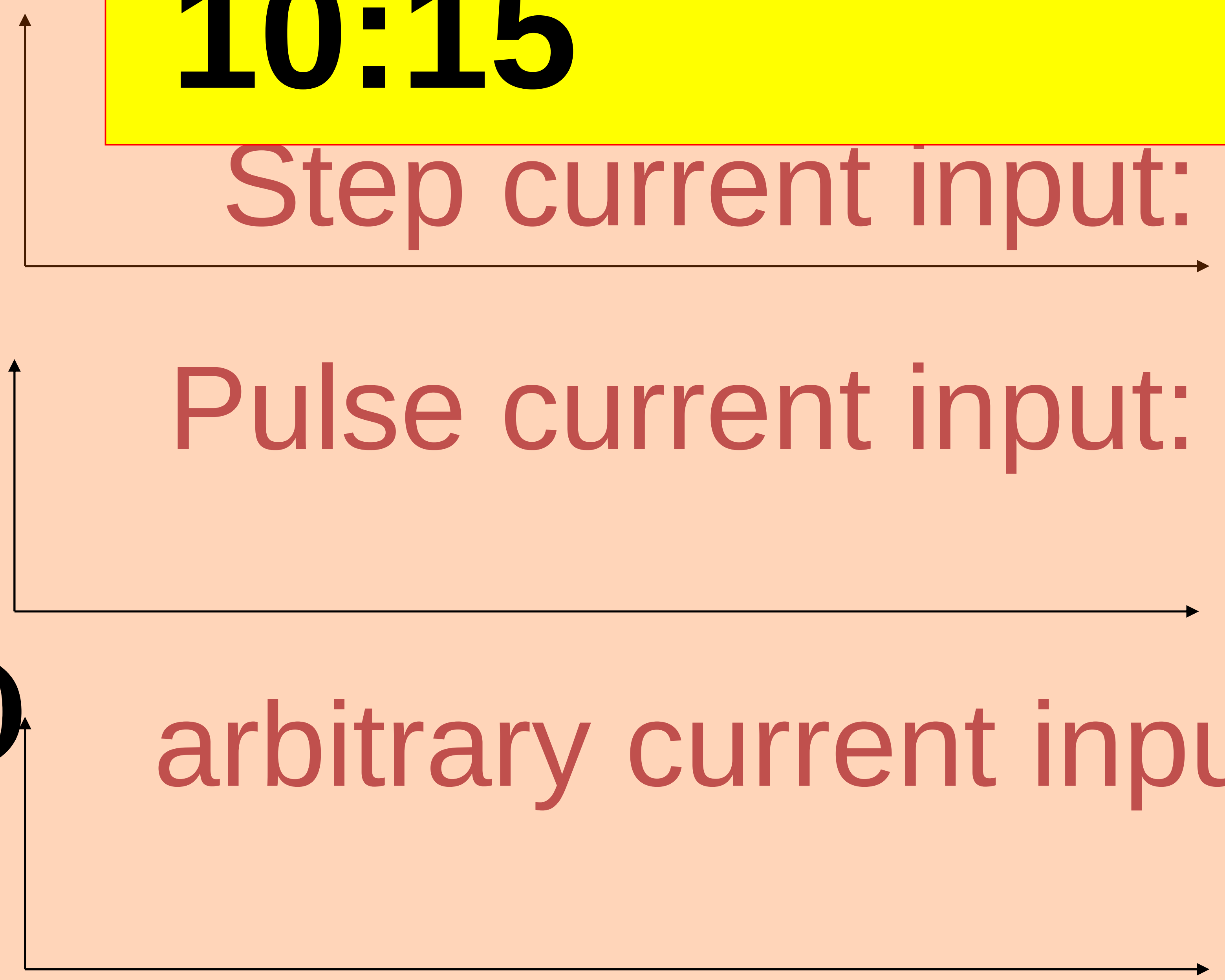
Start Exerc. at 9:47.
Next lecture at
10:15



$I_1(t)$ Step current input:

$I_2(t)$ Pulse current input:

$I_3(t)$ arbitrary current input:



$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$\tau \cdot \frac{d}{dt} V = -V + RI(t); \quad V = (u - u_{rest})$$

Calculate the voltage, for the 3 input currents

Exercise 1: Passive Membrane

The voltage across a passive membrane can be described by the equation

$$\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + R\, I(t)\,. \tag{1}$$

1.1 Step current

Consider a current $I(t) = 0$ for $t < t_0$ and $I(t) = I_0$ for $t > t_0$. Calculate the voltage $u(t)$, given that the neuron is at rest at time t_0. (Hint: Instead of solving the differential equation explicitly, try to construct the response to the step current along the lines: What is the value of $u(t)$ for $t \le t_0$? What is the asymptotic value of $u(t)$ for $t \gg t_0$? What is the functional form and time scale of the transition?)

1.2 Pulse current

Consider a current pulse

$$I(t) = \begin{cases} 0 & \text{for } t < t_0 \text{ and } t > t_0 + \Delta \\ q/\Delta & \text{for } t \ge t_0 \text{ and } t < t_0 + \Delta\,, \end{cases} \tag{2}$$

where Δ is a short time and q is the total electrical charge.

Consider first $\Delta = 0.1\tau$, and then $\Delta = 0.05\tau$, $\Delta = 0.025\tau$. Sketch the input current pulse and the voltage response. What happens in the limit $\Delta \to 0$? (Hint: Use $e^{-x} \approx 1 - x$ for $x \ll 1$.)

1.3 Delta function

The Dirac delta function can be defined by the limit of a short pulse:

$$\delta(t - t_0) = \lim_{\Delta \to 0} f_\Delta(t) \quad \text{where} \quad f_\Delta(t) = \begin{cases} 1/\Delta & \text{for } t_0 \le t < t_0 + \Delta \\ 0 & \text{otherwise}\,. \end{cases} \tag{3}$$

Convince yourself that the integral $\int_{t_1}^{t_2} \delta(t - t_0)\, dt$ is equal to one if $t_1 \le t_0 < t_2$ and vanishes otherwise.

Express $I(t)$ in Eq. 1 using the δ-function for the case that an extremely short current pulse arrives at time t^f. Pay attention to the units!

1.4 General solution

Assuming that before a given time t_0 the current is null and the membrane potential is at rest, derive the general solution to Eq. (1) for arbitrary $I(t)$.

The voltage across a passive membrane can be described by the equation

$$\tau \frac{du}{dt} = -(u - u_{\text{rest}}) + R\, I(t)\,. \tag{1}$$

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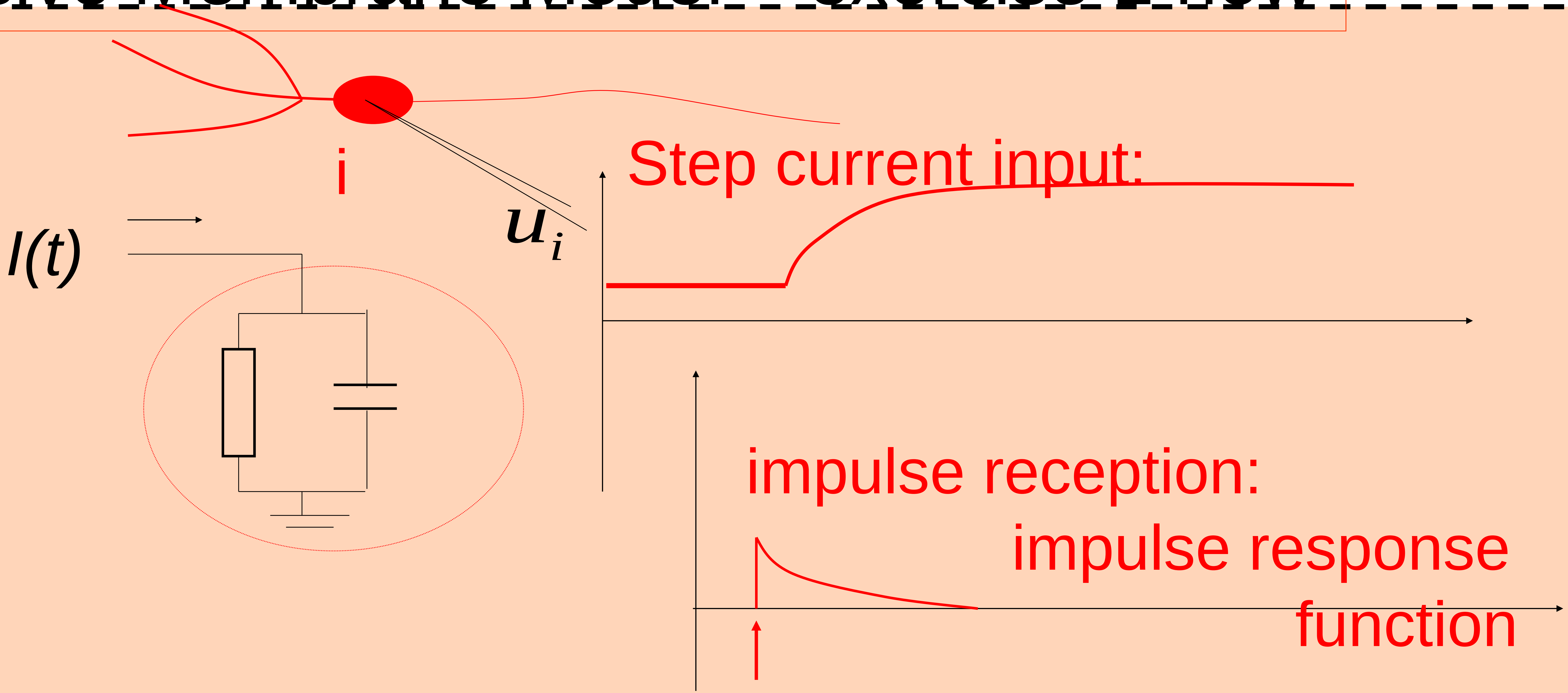
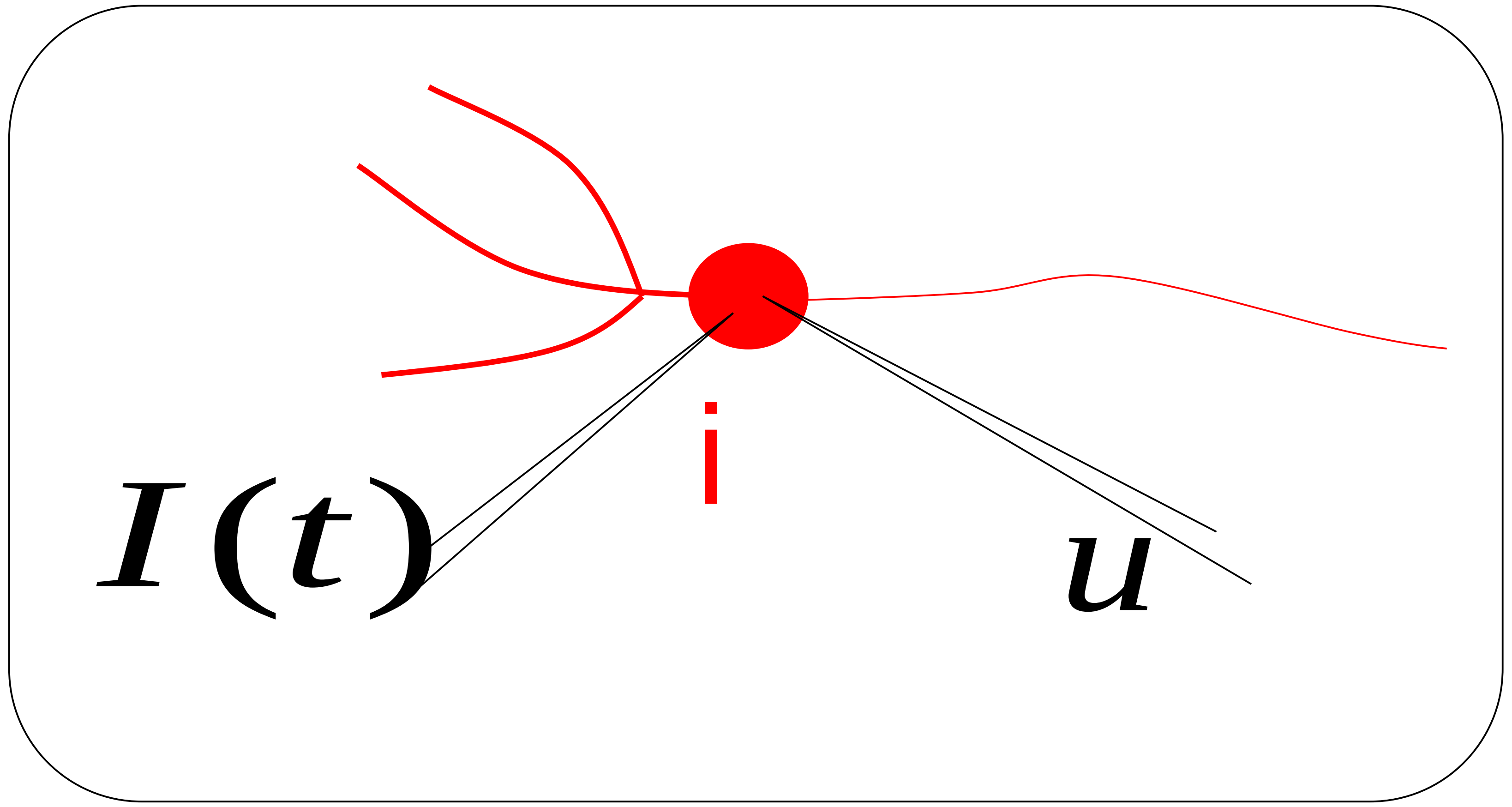
Passive Membrane Model – exercise 1 now

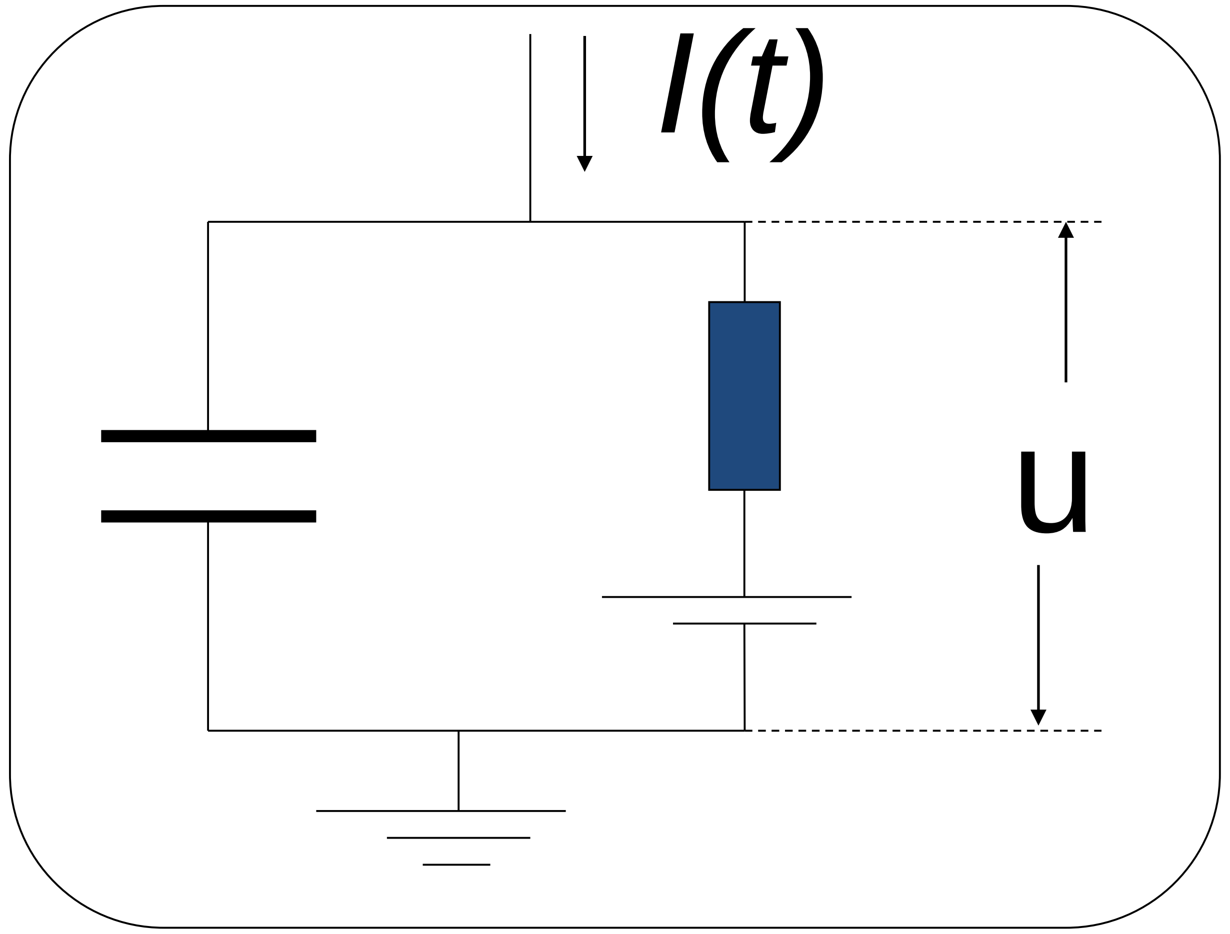


Linear equation

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

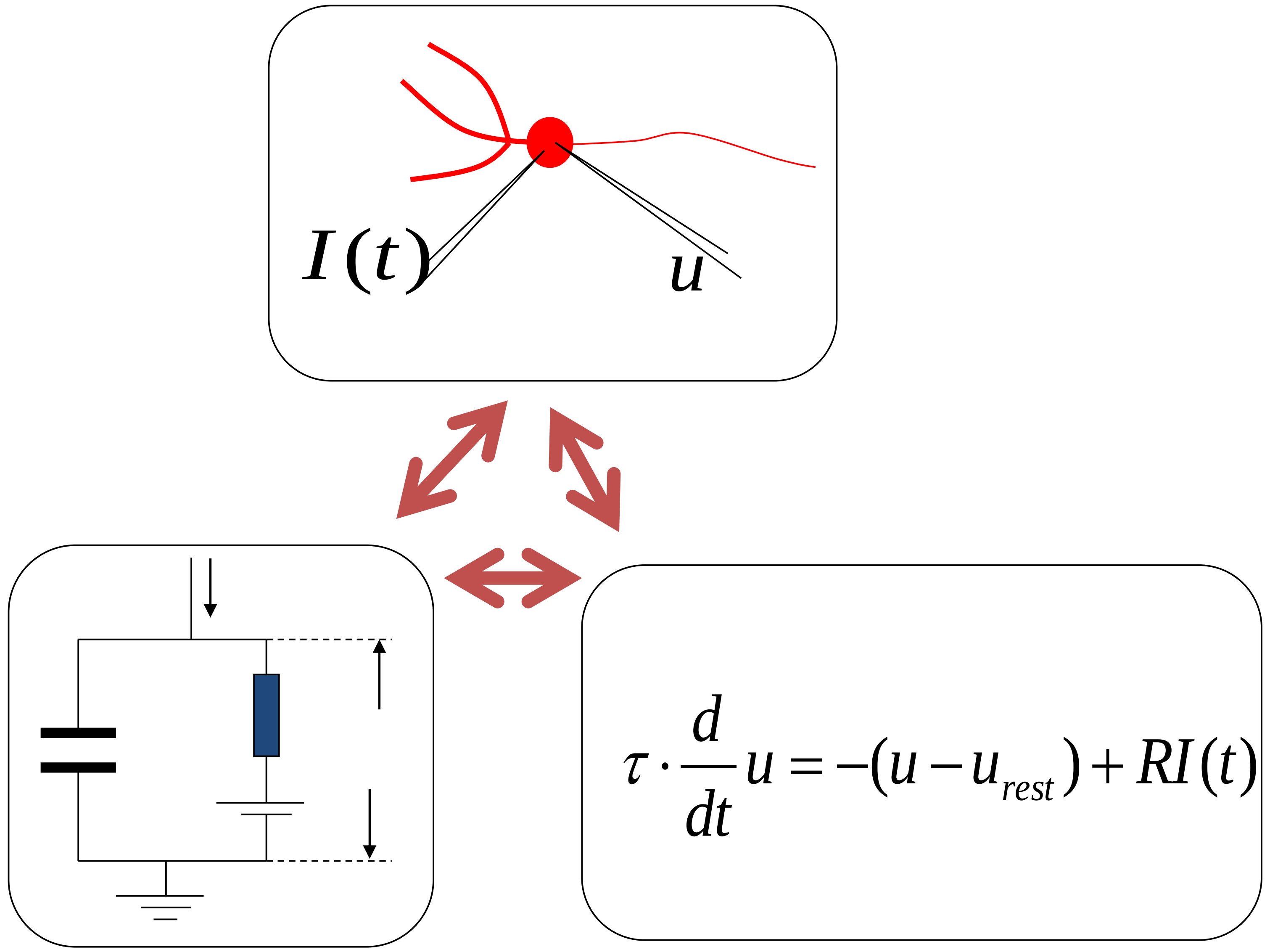
Triangle: neuron – electricity - math

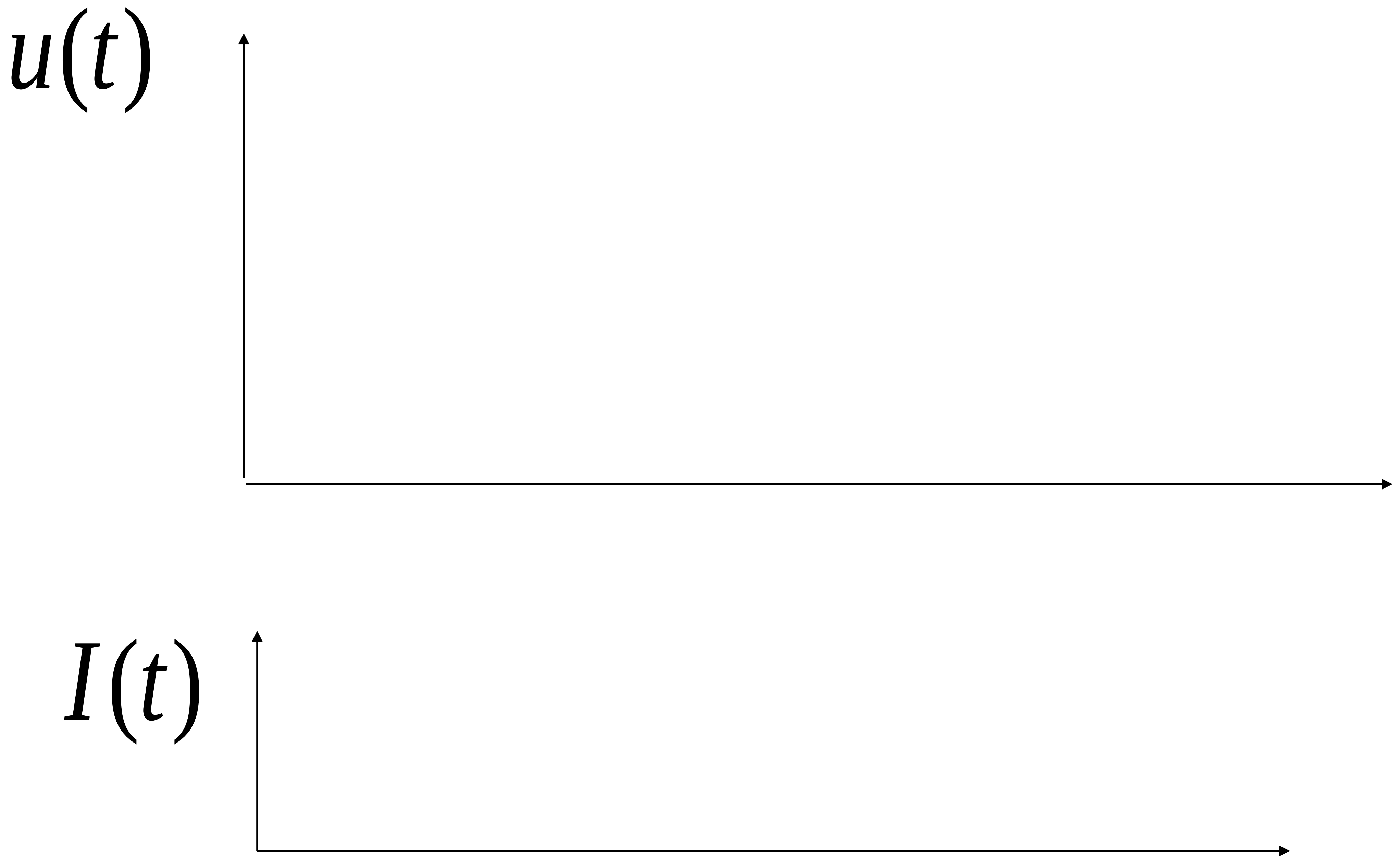




$$\tau \cdot \frac{d}{dt}u = -(u - u_{rest}) + RI(t)$$

Pulse input – charge – delta-function

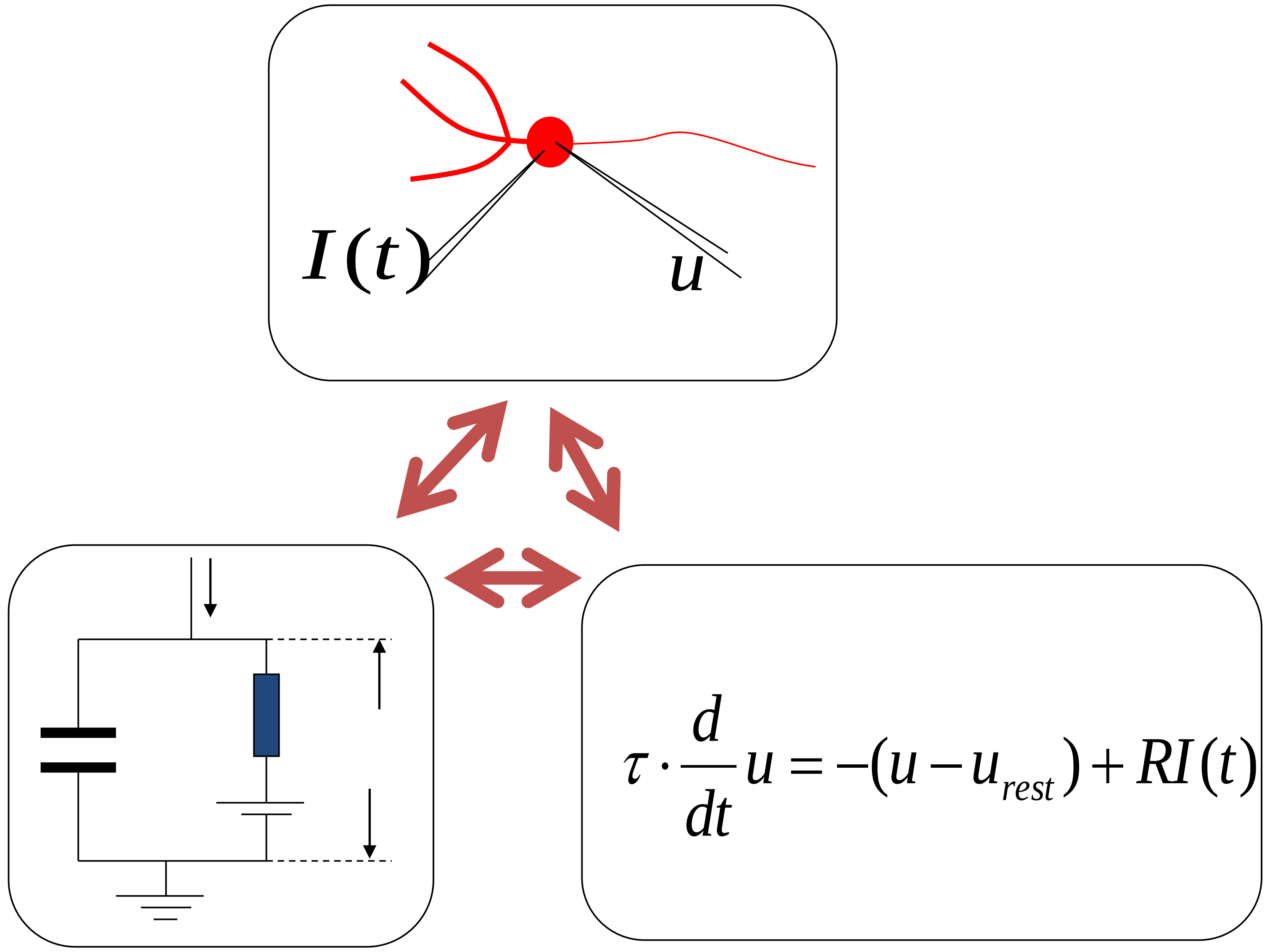




$$I(t) = q \cdot \delta(t - t_0)$$

Pulse current input

Dirac delta-function



$$I(t) = q \cdot \delta(t - t_0)$$

$$1 = \int_{t_0-a}^{t_0+a} \delta(t - t_0)\, dt$$

$$f(t_0) = \int_{t_0-a}^{t_0+a} f(t)\, \delta(t - t_0)\, dt$$

Neuronal Dynamics – Solution of Ex. 1 – arbitrary input

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Arbitrary input

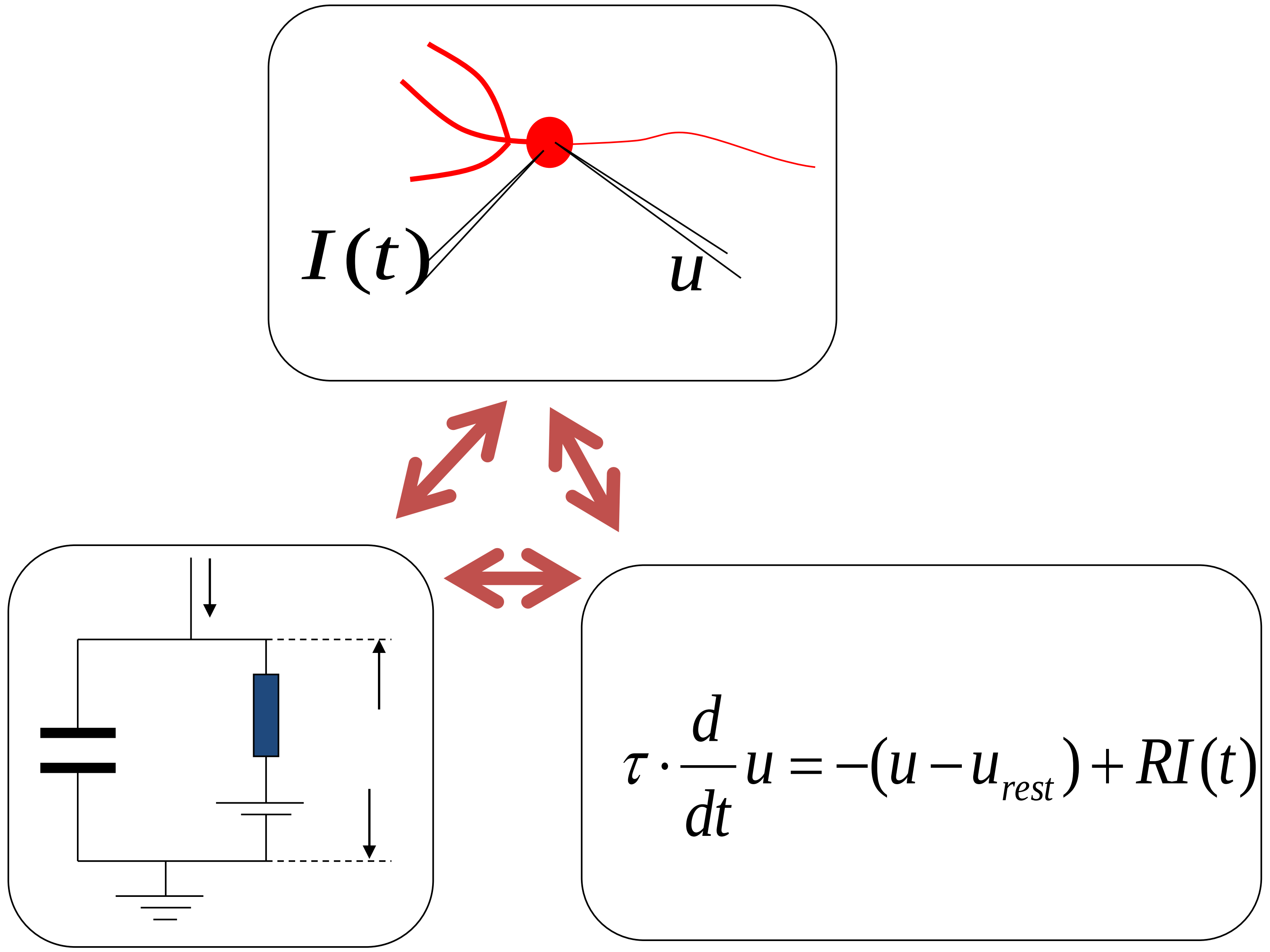
$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{c} e^{-(t-t')/\tau} I(t')dt'$$

Single pulse

$$\Delta u(t) = \frac{q}{c} e^{-(t-t_0)/\tau}$$

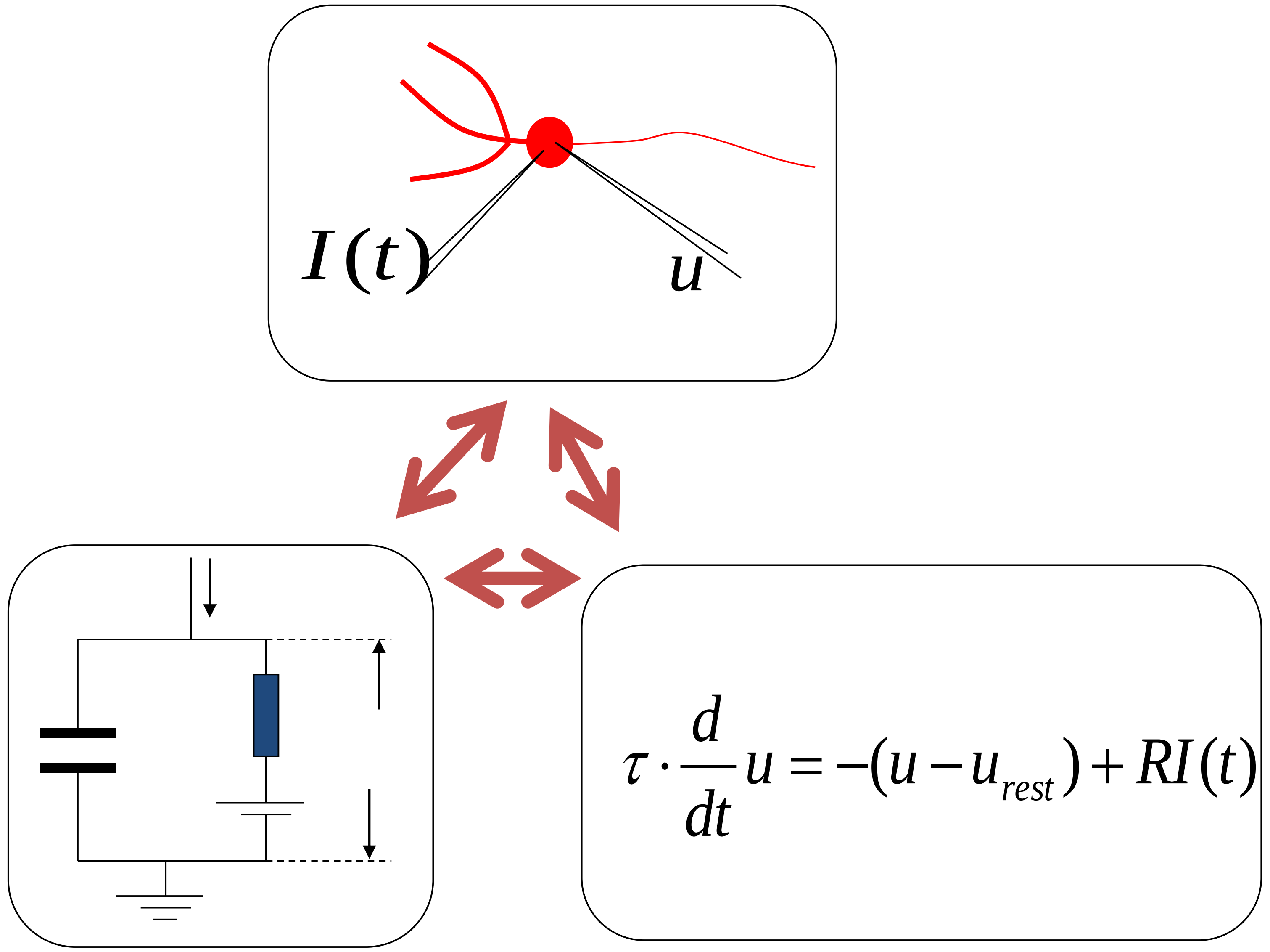
you need to know the solutions of linear differential equations!

Passive membrane, linear differential equation



Passive membrane, linear differential equation

If you have difficulties, watch lecture 1.2detour.



Three prerequisits:
-Analysis 1-3
-Probability/Statistics
-Differential Equations or Physics 1-3 or Electrical Circuits

https://lcnwww.epfl.ch/gerstner/NeuronalDynamics-MOOC1.html

LEARNING OUTCOMES

- Solve linear one-dimensional differential equations
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miniproject

Biological Modeling of Neural Networks

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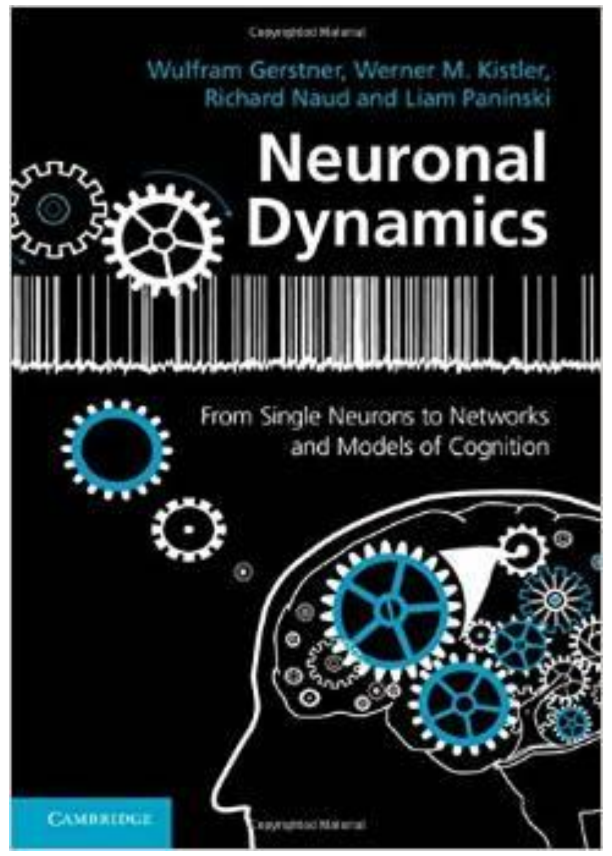

Video:

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https://lcnwww.epfl.ch/gerstner/NeuronalDynamics-MOOC2.html

Biological Modeling of Neural Networks

Questions?

This is **not** a course on
Deep learning or **Artificial neural networks**
→ Deep Learning, master EE, (*Fleuret*)
→ Artificial NN, master CS, (*Gerstner*)

Summary of Section 1.1 and 1.2.

Neurons emit spikes (action potentials) which are short standardized events in the form of voltage pulses.

Spikes are emitted at the firing threshold.

Below the threshold the electrical behavior is often well characterized by a linear differential equation (math) corresponding to an RC circuit (electricity) or to a passive membrane (biology). We will often in this class walk along the triangle that connects math with electricity and biology.

This class has a strong focus on mathematical modeling of the biological phenomena. Differential equations, Dirac-delta pulses, and their link to biology are important concepts.

The time constant of an RC circuit is τ=RC.

Week 1 – part 3: Leaky Integrate-and-Fire Model

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks

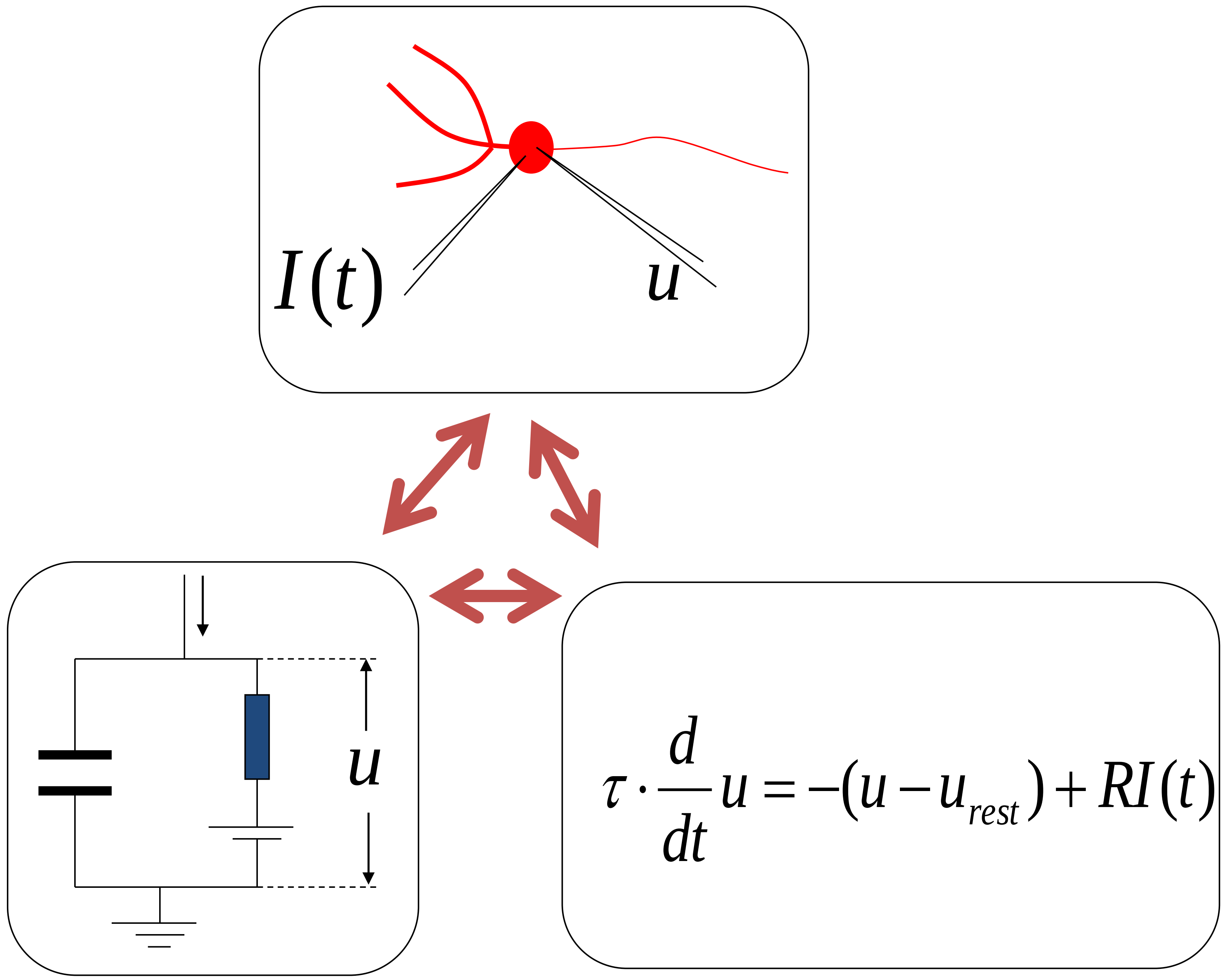
Week 1 – neurons and mathematics: a first simple neuron model

Wulfram Gerstner

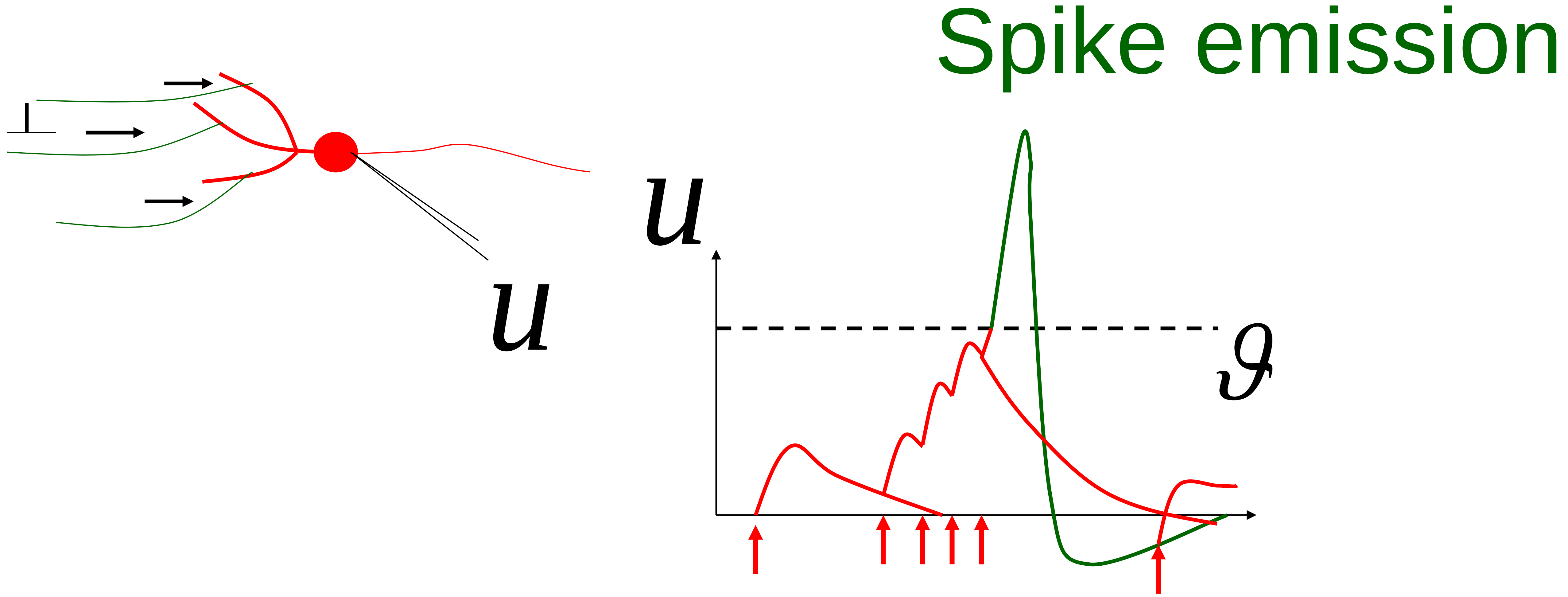
EPFL, Lausanne, Switzerland

Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$



Neuronal Dynamics – Integrate-and-Fire type Models



Input spike causes an EPSP
= excitatory postsynaptic potential

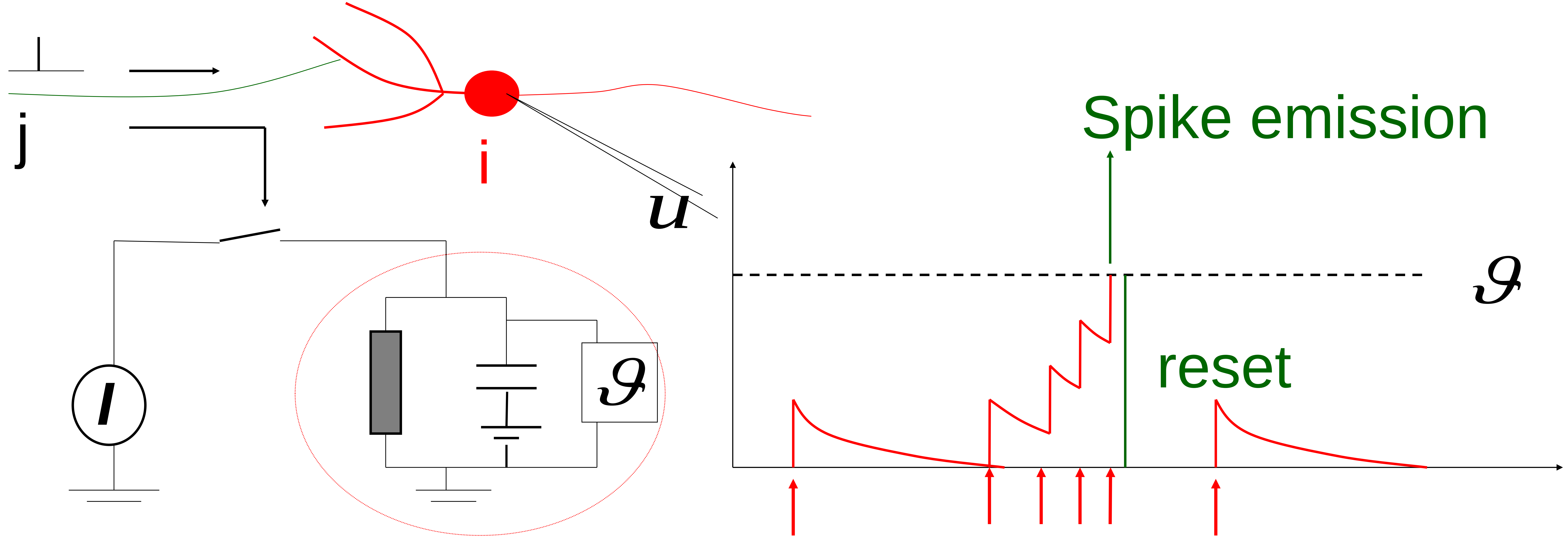
Simple
Integate-and-Fire Model:

passive membrane
+ threshold

-output spikes are events
-generated at threshold
-after spike: reset/refractoriness

Leaky Integrate-and-Fire Model

Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model



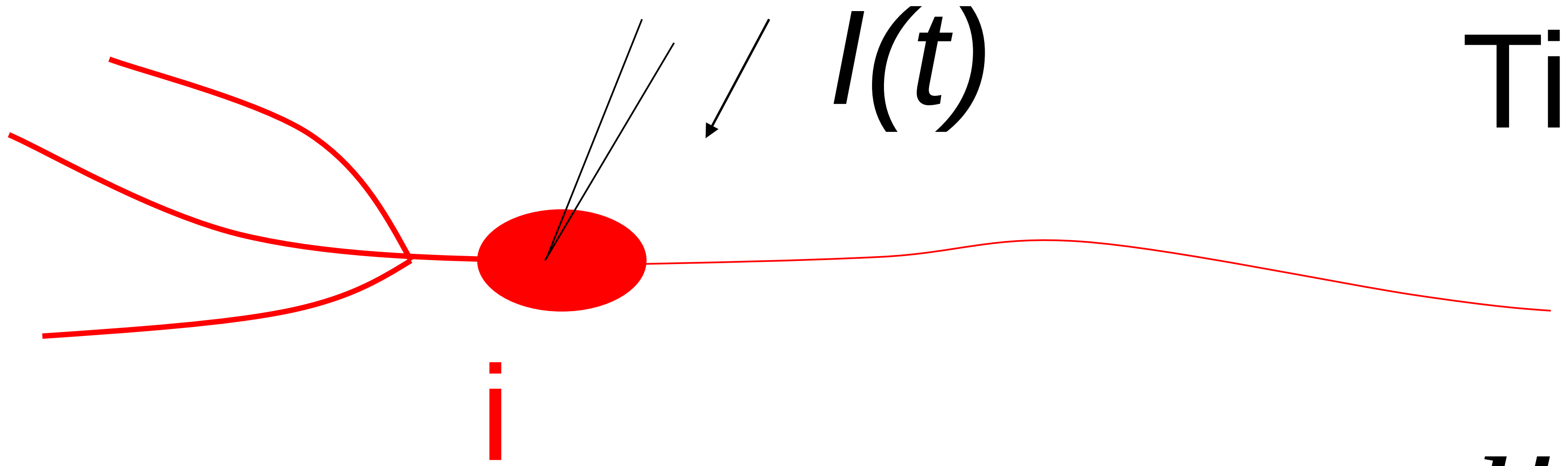
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

linear

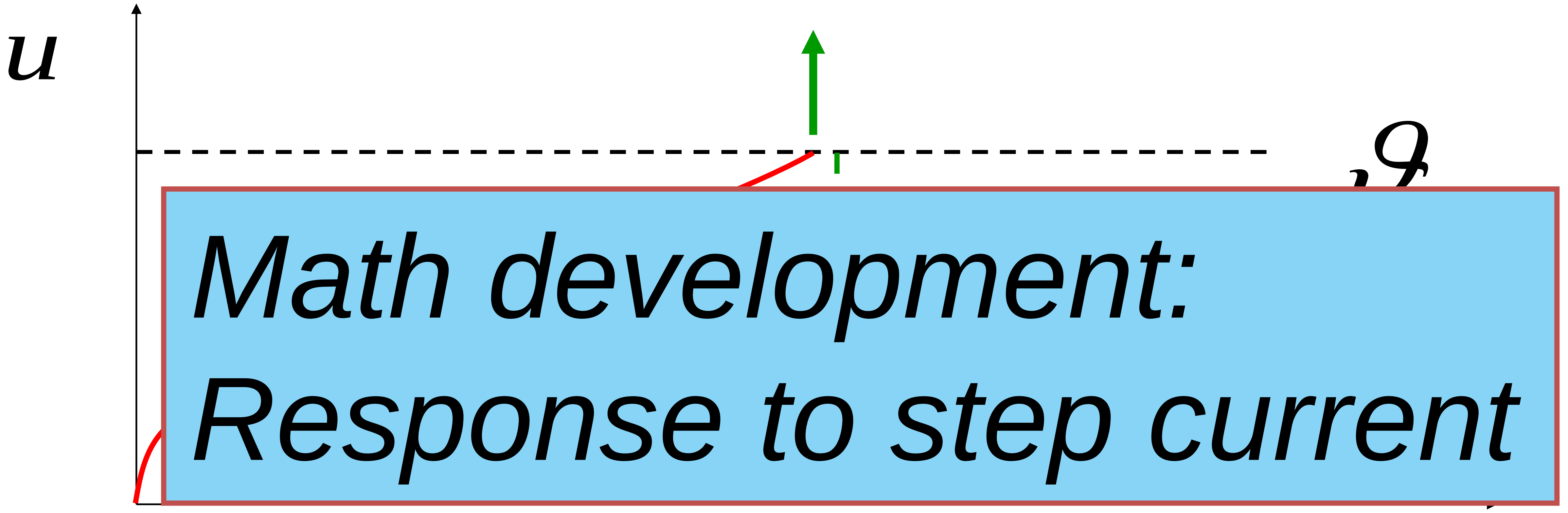
$$u(t) = \vartheta \Rightarrow \text{ Fire+reset } \; u \rightarrow u_r$$

threshold

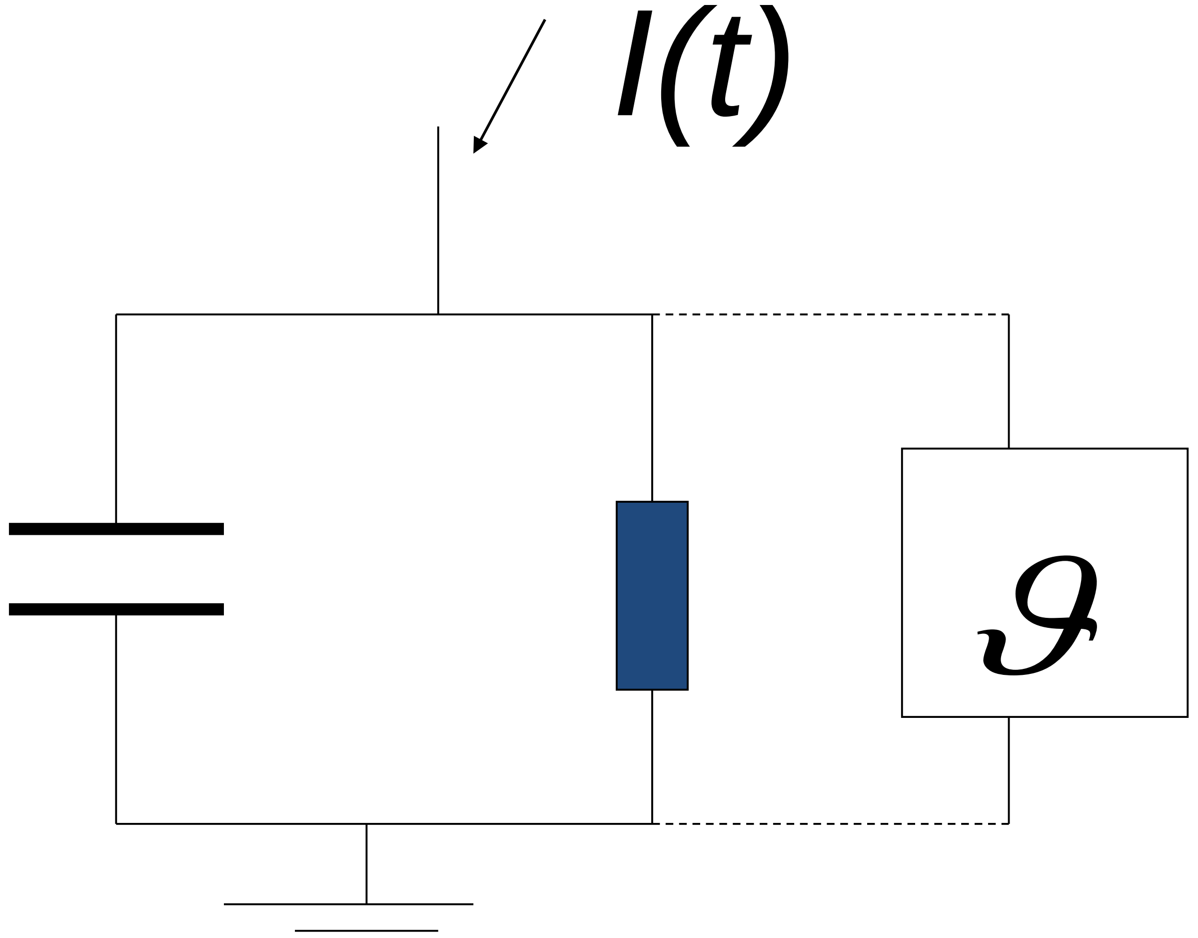
Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model



Time-dependent input



Math development:
Response to step current



-spikes are events
-triggered at threshold
-spike/reset/refractoriness

Week1 – Quiz 2.

Take 90 seconds:

Consider the linear differential equation $\tau \cdot \frac{d}{dt} x = -x + x_c$

with initial condition $at\, t = 0 : x = 0$

The solution for t>0 is

(i) $x(t) = x_c \exp(t/\tau)$

(ii) $x(t) = x_c \exp(-t/\tau)$

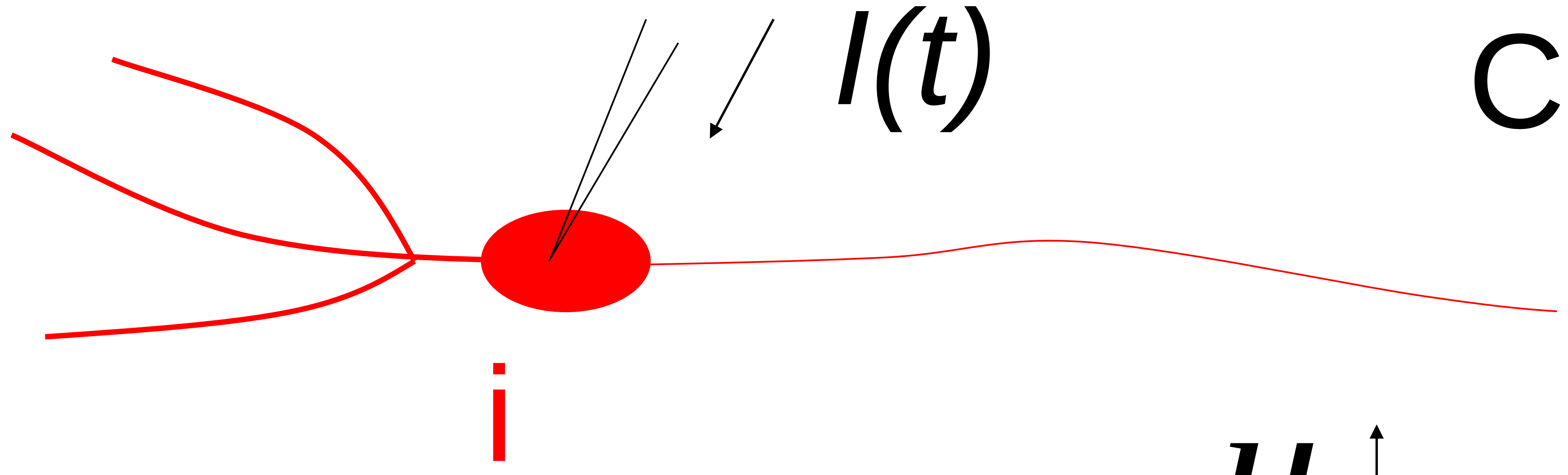
(iii) $x(t) = x_c[1 - \exp(-t/\tau)]$

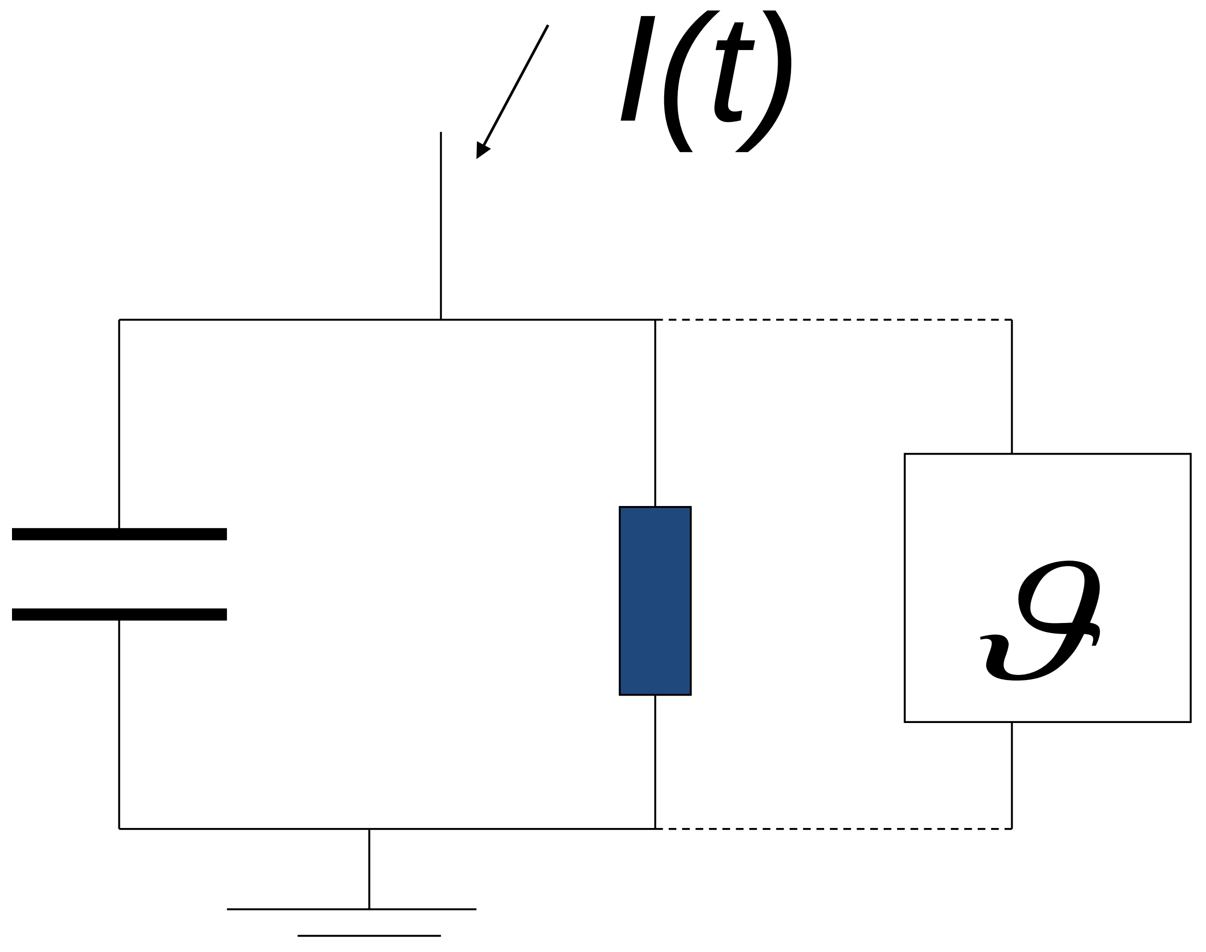
(iv) $x(t) = 0.5x_c[1 + \exp(-t/\tau)]$

You will have to use the results: response to constant input/step input again and again

Neuronal Dynamics – 1.3 Leaky Integrate-and-Fire Model

CONSTANT input/step input





Leaky Integrate-and-Fire Model (LIF)

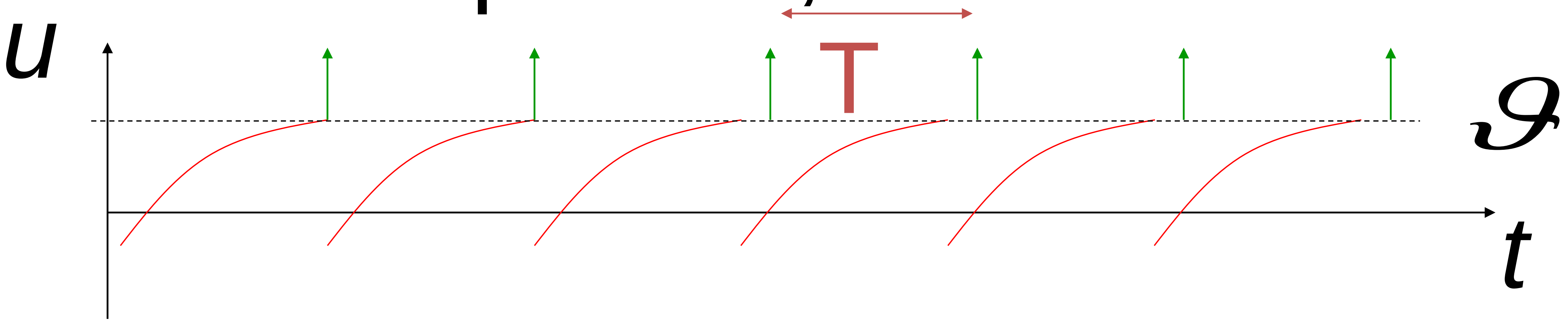
$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI_0$$

LIF

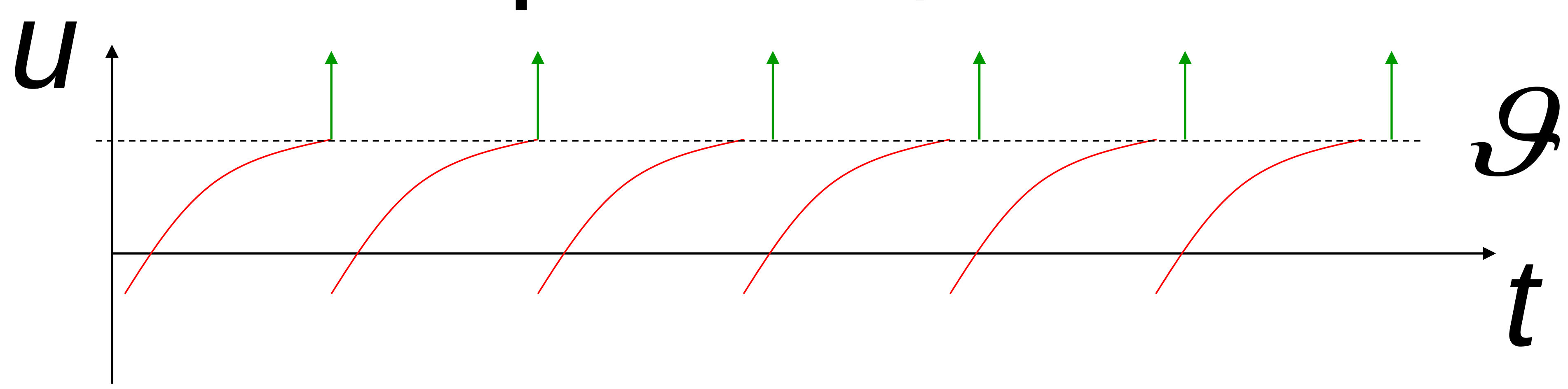
If $u(t) = \vartheta \Rightarrow \quad u \rightarrow u_r$

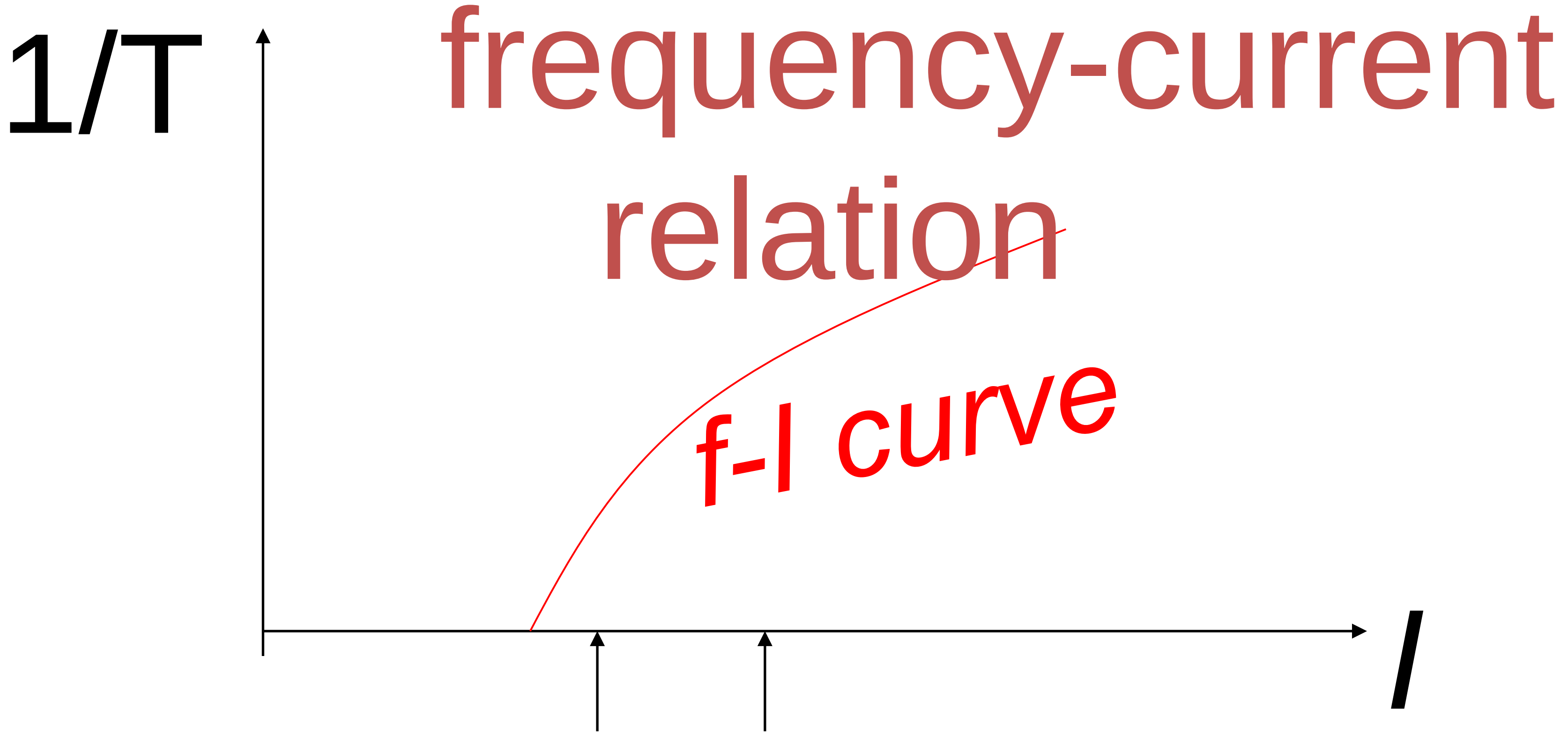
'Firing'

Repetitive, current I_0



Repetitive, current $I_1 > I_0$

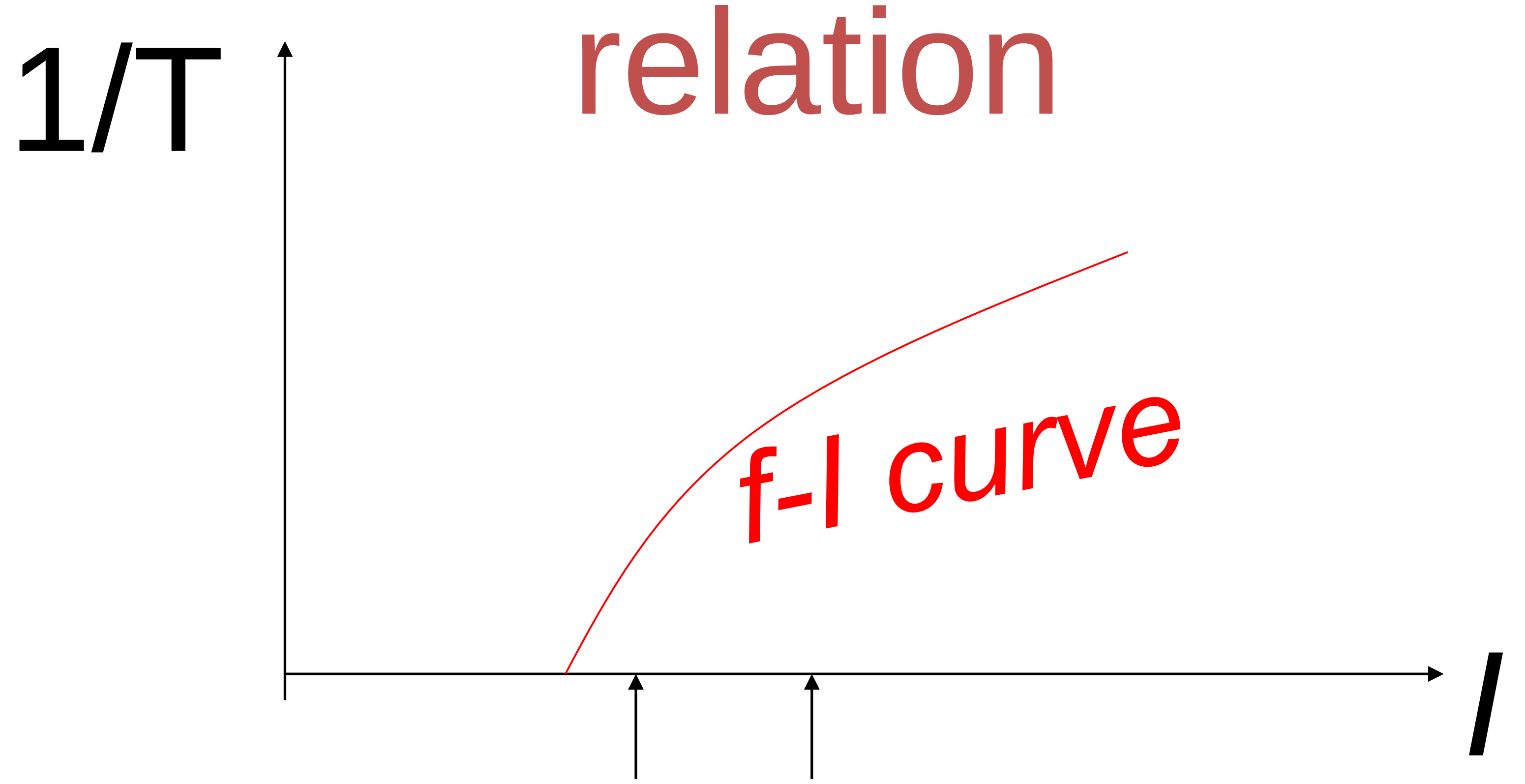




Neuronal Dynamics – First week, Exercise 2

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

frequency-current relation

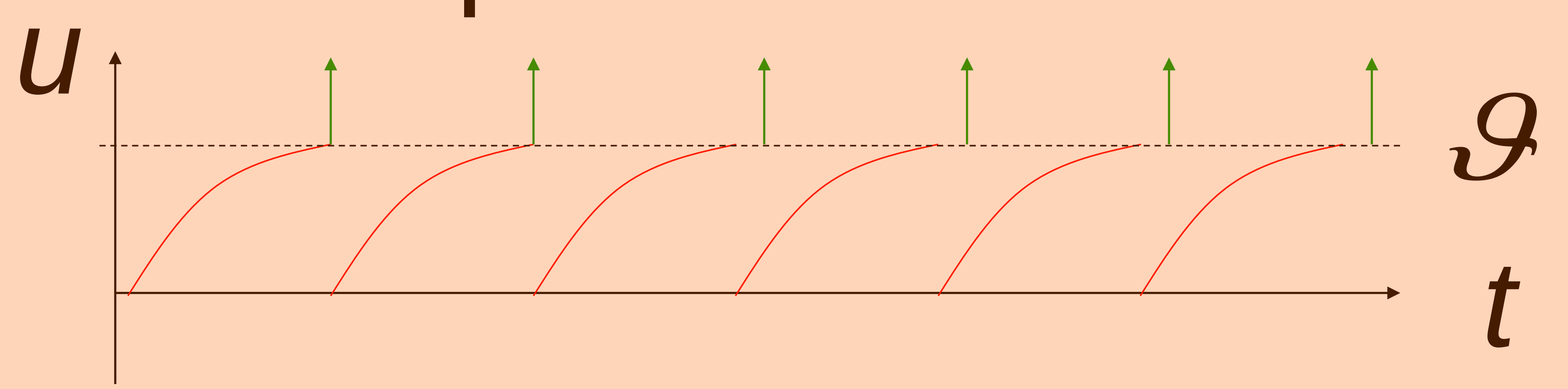


EXERCISE 2 NOW: Leaky Integrate-and-fire Model (LIF)

LIF $\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI_0$ If firing: $u \rightarrow u_r$

$assume: \ u_r = u_{rest}$

repetitive



Exercise!
Calculate the interspike interval *T* for constant input *I*.
Firing rate is *f=1/T*.
Write *f* as a function of *I*.
What is the frequency-current curve *f=g(I)* of the LIF?

Start Exerc. at 10:53.
Next lecture at 11:15

Exercise 2: Integrate-and-fire model

Consider the model of Eq. (1) with a threshold at u $= \vartheta > u_{\text{rest}}$. If the membrane potential reaches the threshold, the neuron is said to fire and the membrane potential is reset to u_{rest}. The injected current is a step of magnitude I_0:

$$I(t) = \begin{cases} 0 & t \leq t_0 \\ I_0 & t > t_0 \end{cases}$$

2.1 What is the minimal current to reach the threshold, assuming $u(t=0) = u_{\text{rest}}$?

2.2 At what time will the voltage first reach the threshold?

2.3 Calculate the firing frequency f as a function of I_0.

The function $g(I_0)$ which gives the firing frequency as a function of the constant applied current is called gain function.

Summary of Section 1.3.
The leaky integrate-and-fire neuron model is the combination of a passive membrane (linear differential equation) with a threshold. The moment when the potential hits the threshold defines the firing time of a spike. Immediately after firing the voltage is reset to a lower value (not necessarily to the resting potential).

For the leaky integrate-and-fire model, the gain function (frequency of firing for constant input, as a function of input strength) can be calculated analytically. The firing frequency decreases with decreasing input. If the constant input is below a critical value no firing can occur. This value defines the rheobase current threshold. For the leaky integrate-and-fire model the rheobase current threshold can be predicted from the voltage threshold and the model parameters R and C.

Week 1 – part 4: Generalized Integrate-and-Fire Model

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks

Week 1 – neurons and mathematics:
a first simple neuron model

Wulfram Gerstner
EPFL, Lausanne, Switzerland

√ 1.1 **Neurons and Synapses:**
Overview

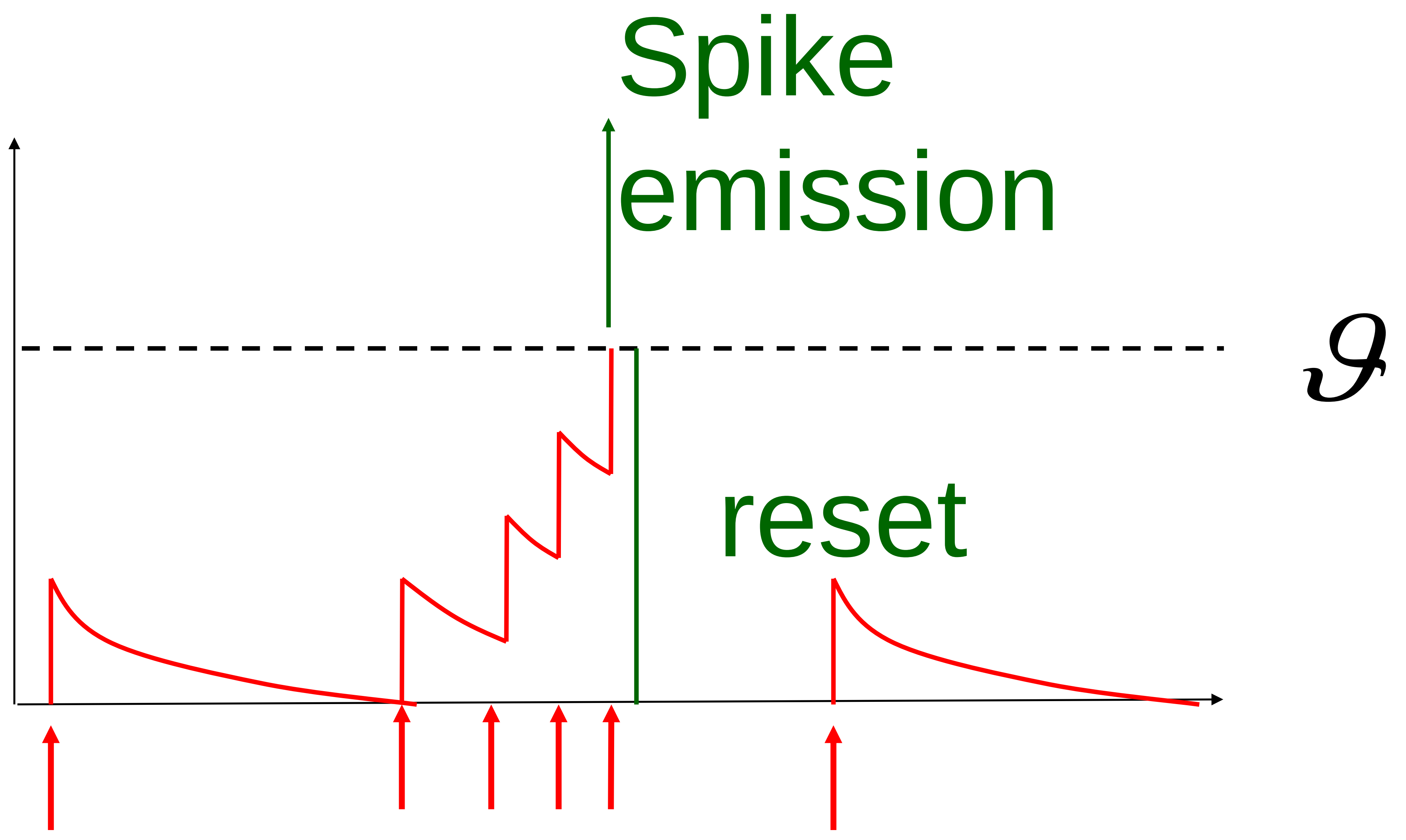
√ 1.2 **The Passive Membrane**
- Linear circuit
- Dirac delta-function

√ 1.3 **Leaky Integrate-and-Fire Model**

1.4 **Generalized Integrate-and-Fire Model**

1.5. **Quality of Integrate-and-Fire Models**

Neuronal Dynamics – 1.4. Generalized Integrate-and Fire



Integrate-and-fire model

LIF: linear + threshold

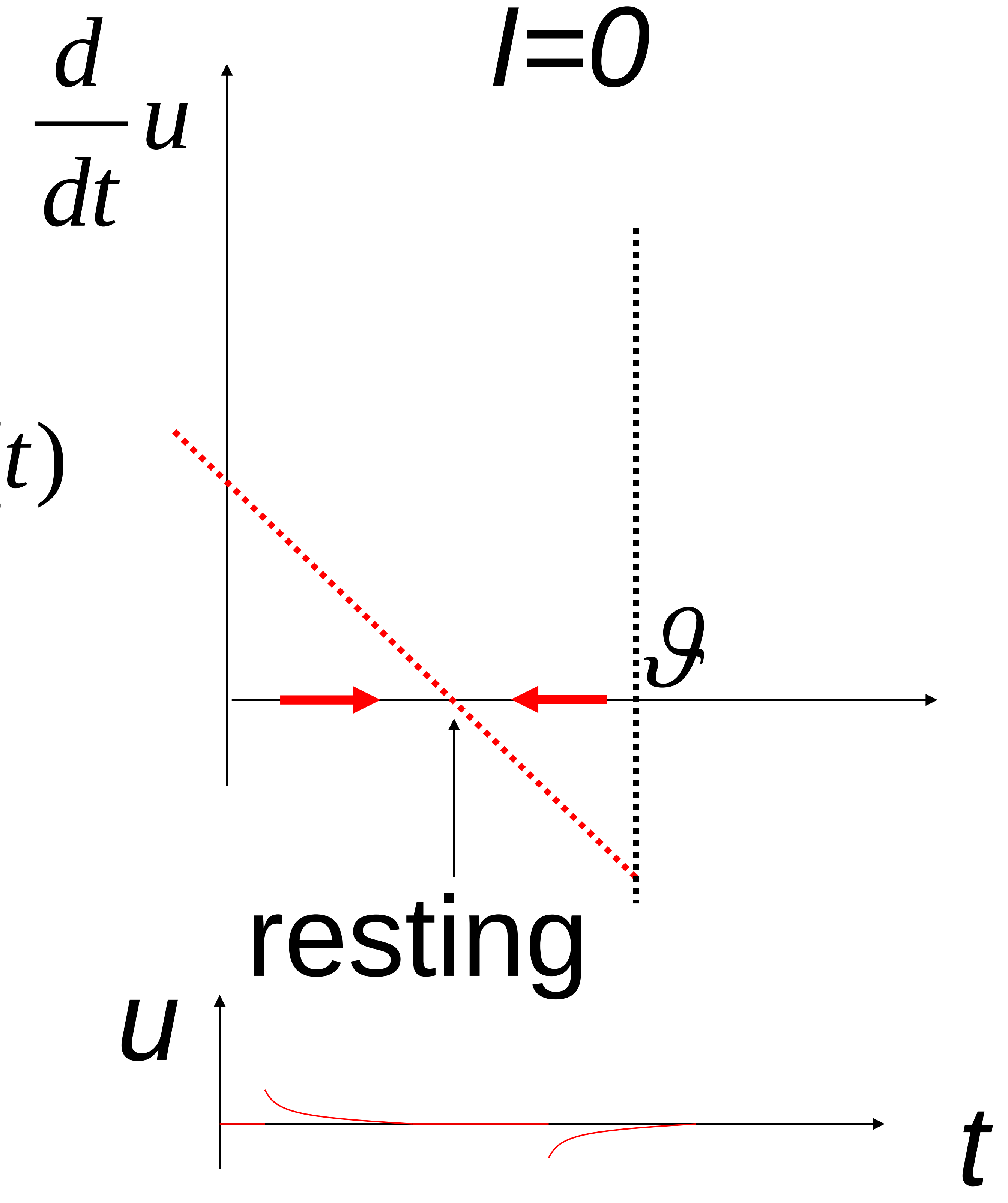
Neuronal Dynamics – 1.4. Leaky Integrate-and Fire revisited

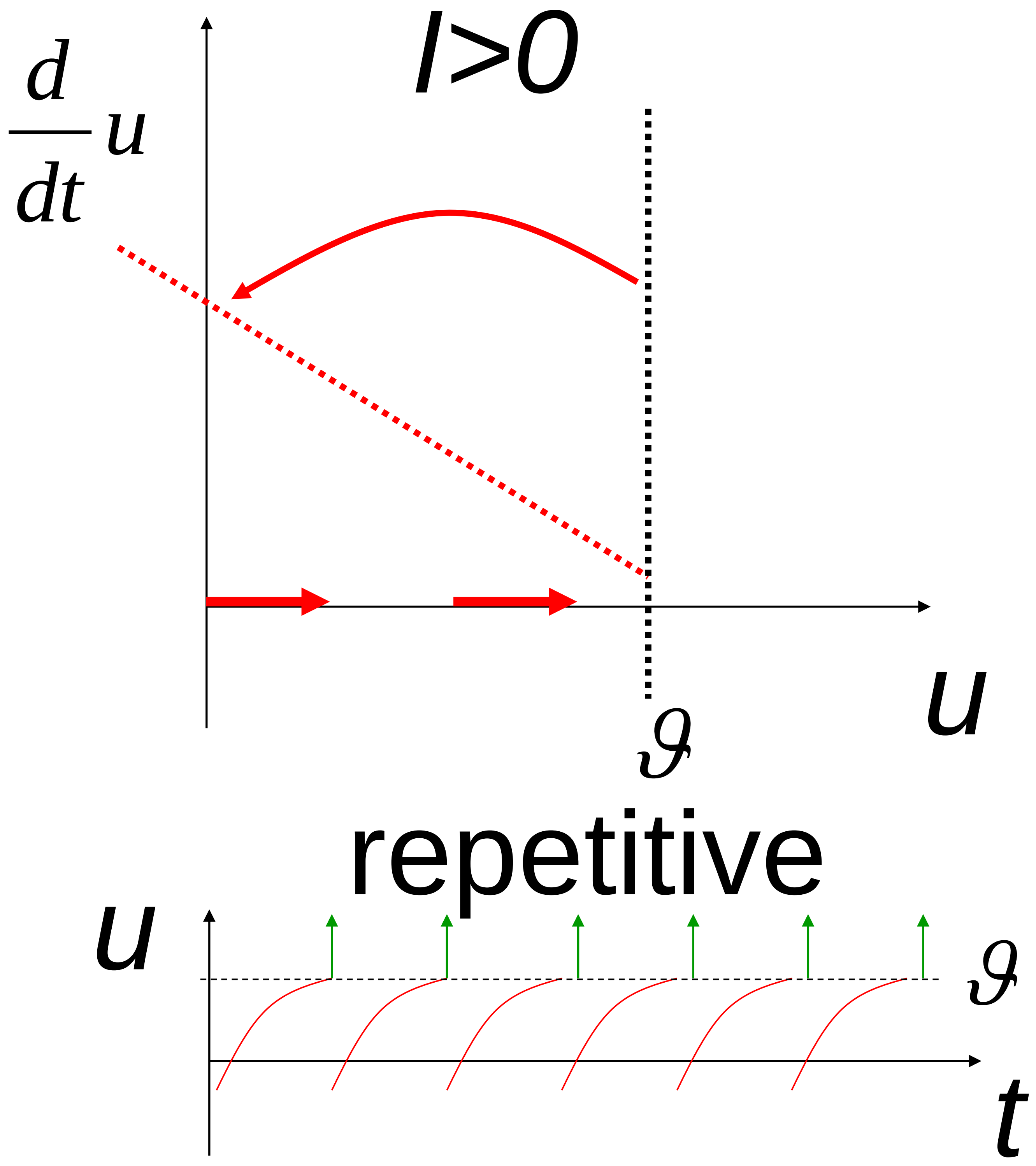
LIF

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

If firing:

$$u \to u_r$$





Neuronal Dynamics – 1.4. Nonlinear Integrate-and Fire

LIF

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

NLIF

$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

If firing:

$$u \to u_{reset}$$

Neuronal Dynamics – 1.4. Nonlinear Integrate-and Fire

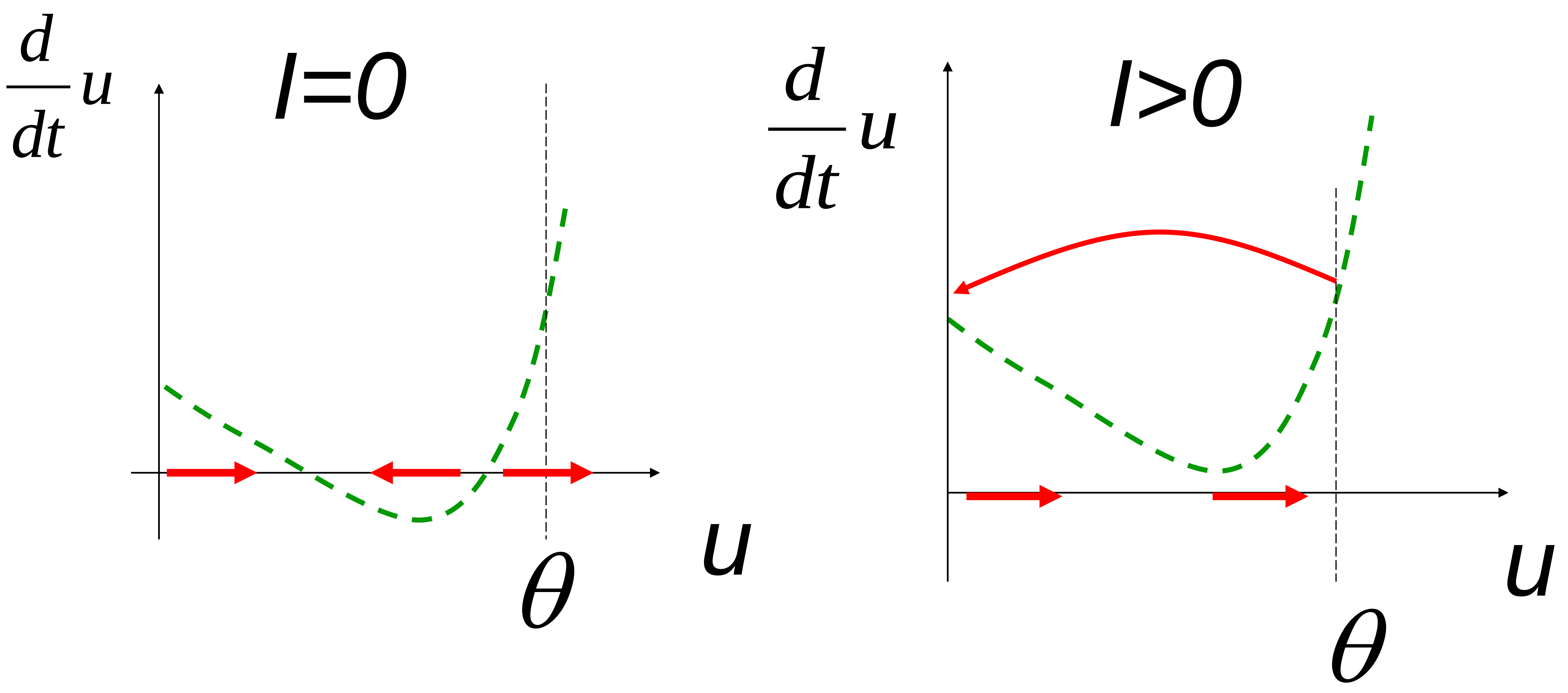
Nonlinear Integrate-and-Fire

NLIF

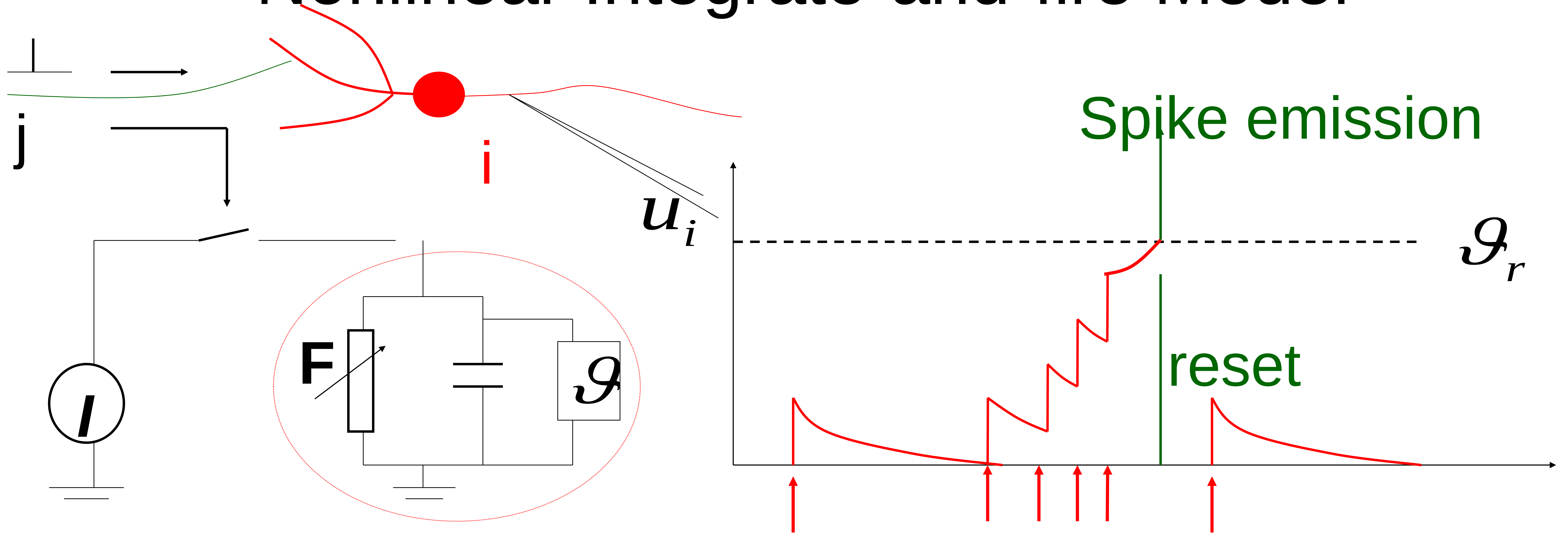
$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$

firing: $u(t) = \theta \Rightarrow$

$$u \rightarrow u_r$$



Nonlinear Integrate-and-fire Model



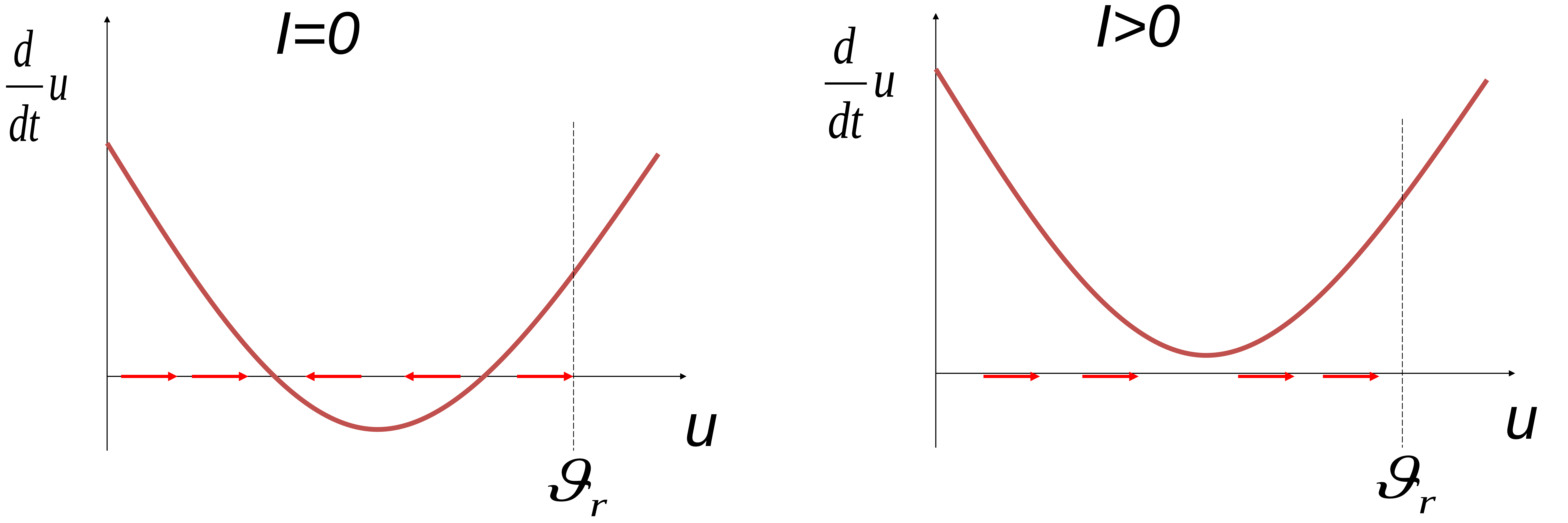
$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

NONlinear

$$u(t) = \vartheta_r \Rightarrow$$

Fire+reset threshold

Nonlinear Integrate-and-fire Model



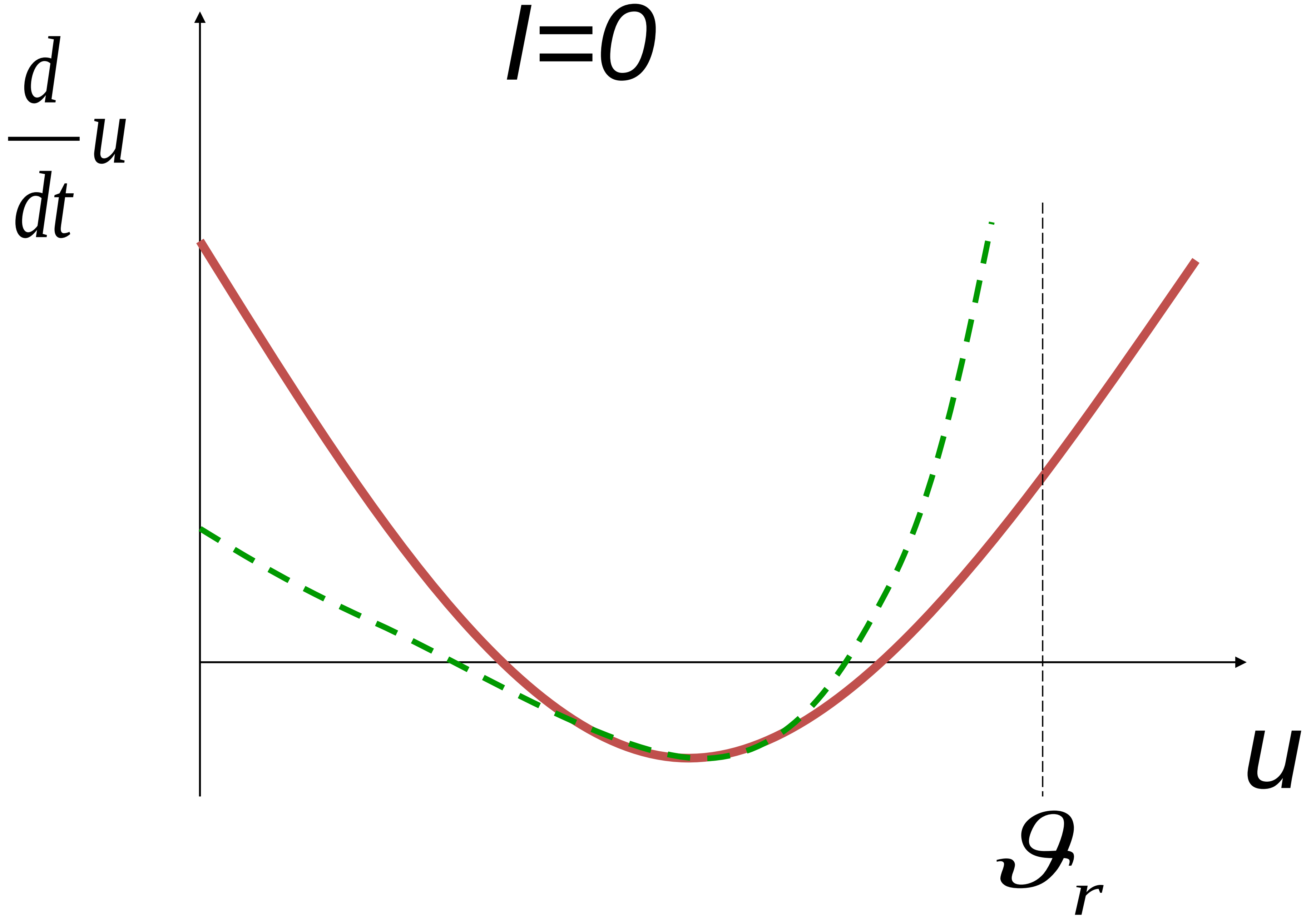
$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$ NONlinear

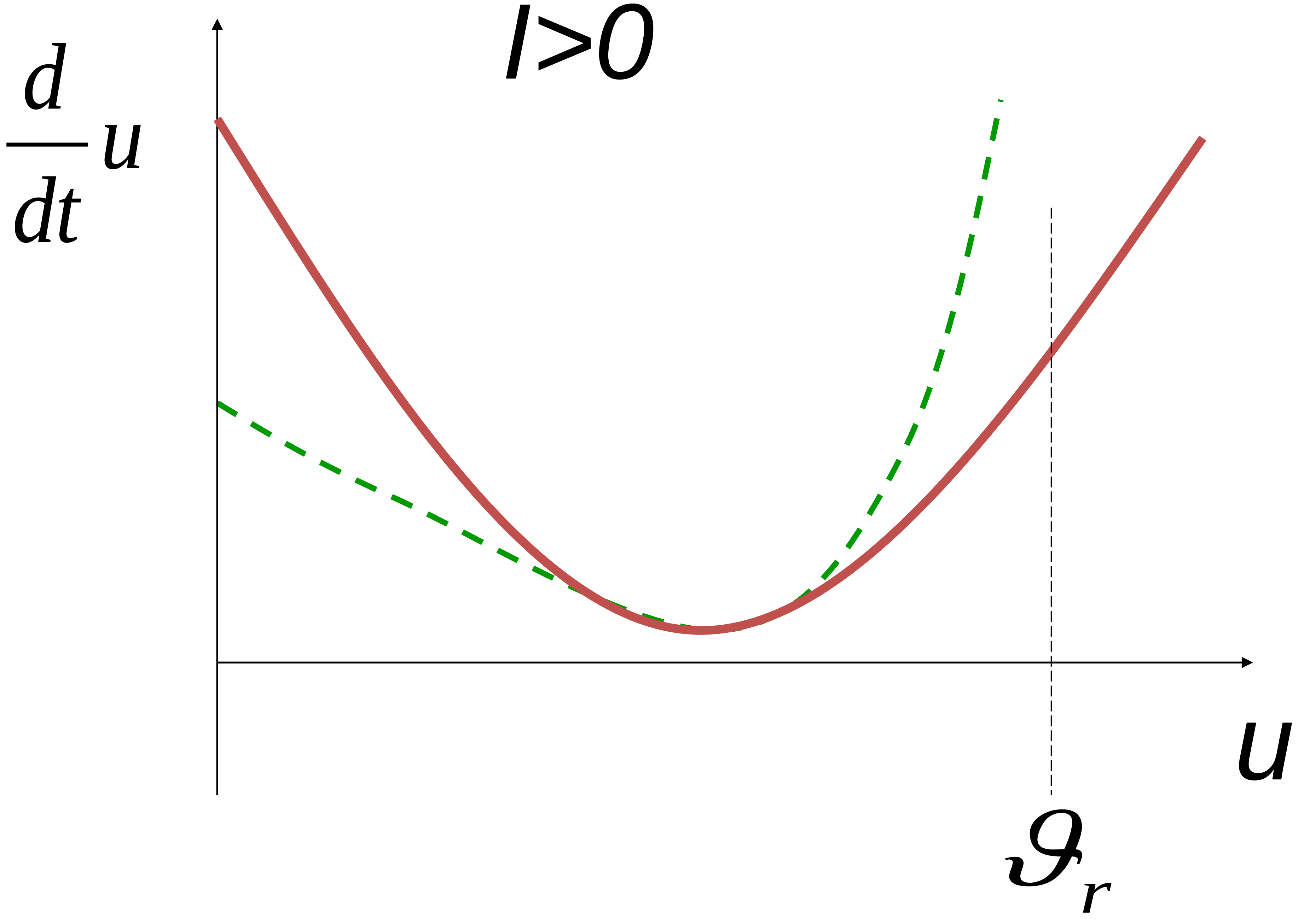
$u(t) = \vartheta_r \Rightarrow$ Fire+reset threshold

Quadratic I&F:

$$F(u) = c_2 (u - c_1)^2 + c_0$$

Nonlinear Integrate-and-fire Model





$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

$$u(t) = \vartheta_r \Rightarrow \text{ Fire+reset}$$

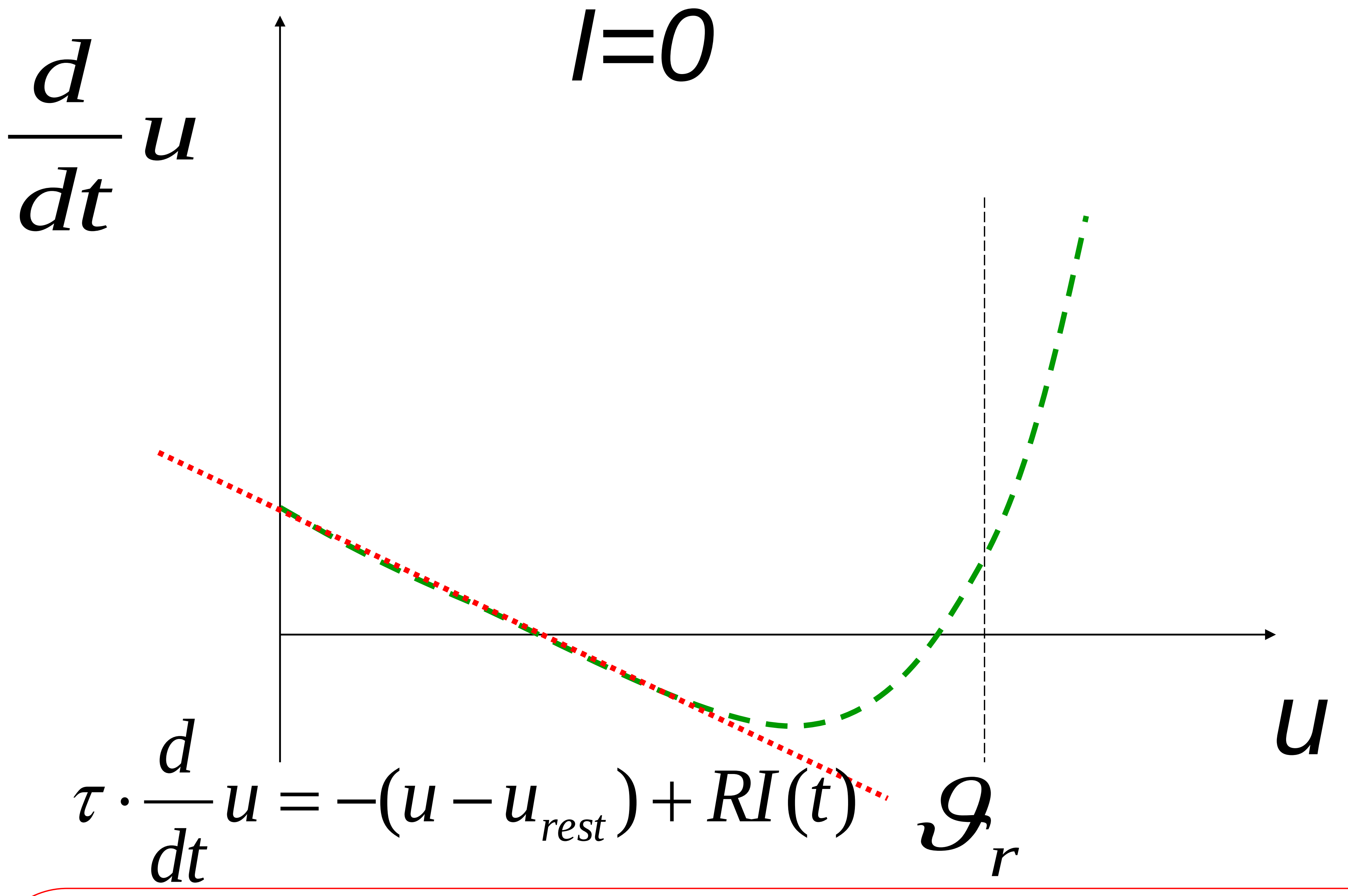
Quadratic I&F:

$$F(u) = c_2 (u - c_1)^2 + c_0$$

exponential I&F:

$$F(u) = -(u - u_{rest}) + c_0 \exp(u - \vartheta)$$

Nonlinear Integrate-and-fire Model



$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$ NONlinear

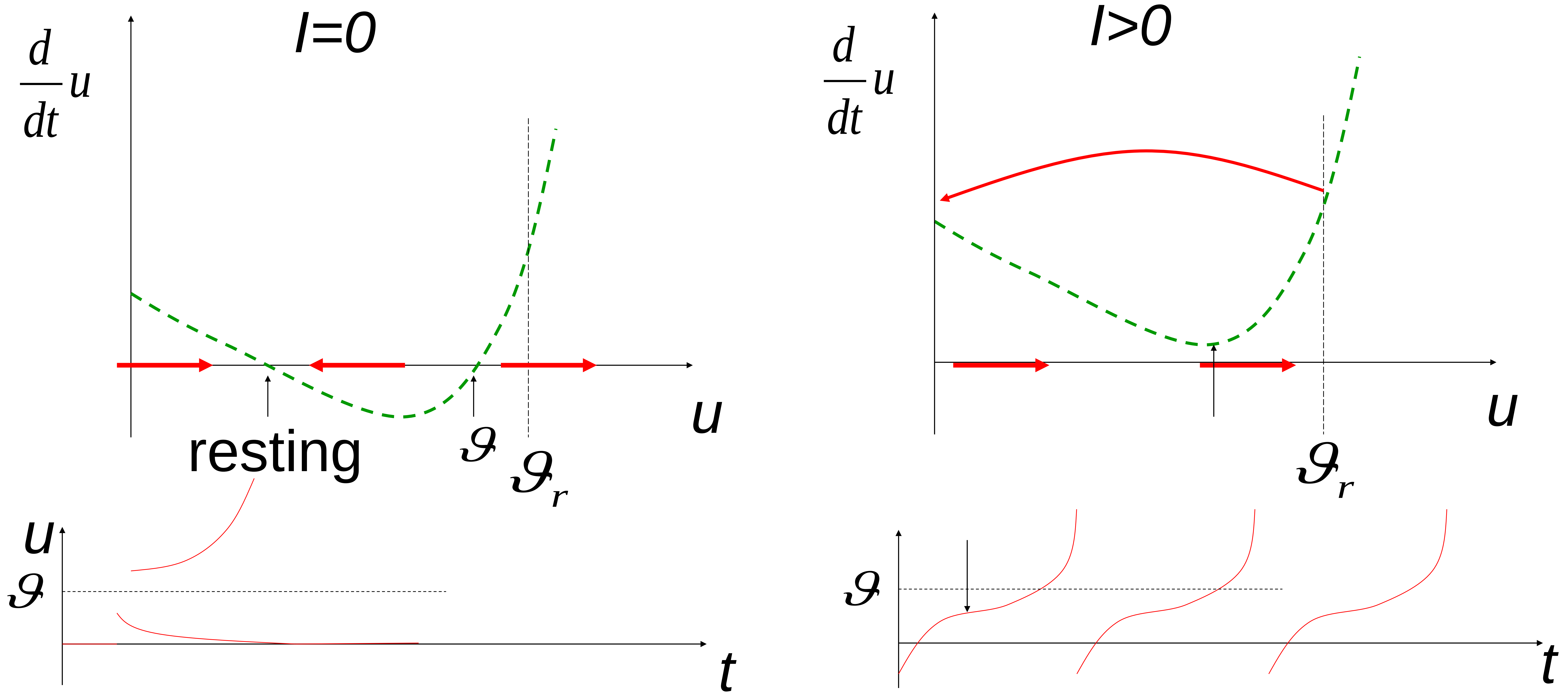
$$u(t) = \vartheta_r \Rightarrow$$ Fire+reset threshold

exponential I&F:

$$F(u) = -(u - u_{rest}) + c_0 \exp(u - \vartheta)$$

Nonlinear Integrate-and-fire Model

Where is the firing threshold?



$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$

Summary of Section 1.4.

Nonlinear integrate-and-fire models with a nonlinear function $f(u)$ of the voltage u are excellent models of spiking behavior. The exponential integrate-and-fire (EIF) model is linear for low voltage, but combines it with an exponential nonlinearity for large voltages. The quadratic integrate-and-fire (QIF) is symmetric for low and large voltages. The LIF has a linear function $f(u) = u - u_{rest}$. All generalized integrate-and-fire models have a voltage reset after a spike. For constant input, the zero-crossings of the function f(u) define stationary states.

We distinguish two important stimulation paradigms: (i) stimulation with short current pulses (Dirac delta pulses) – such pulses correspond to an initial condition on the voltage axis; (ii) stimulation with constant current – such stimuli correspond to a vertical shift of the function *f(u)*. We find in both EIF and QIF that the minimal current for spike initiation depends on the stimulation protocol.

Week 1 – part 5: How good are Integrate-and-Fire Model?

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks

Week 1 – neurons and mathematics: a first simple neuron model

Wulfram Gerstner

EPFL, Lausanne, Switzerland

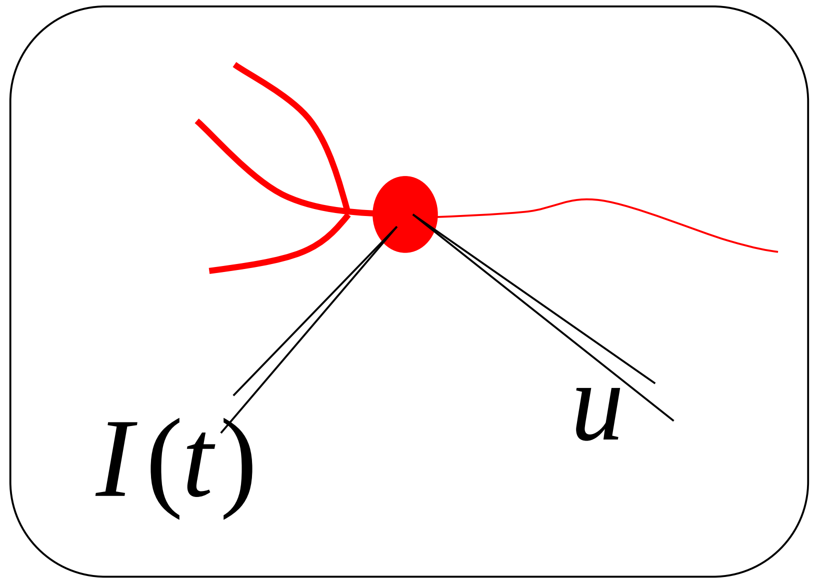
√ 1.1 **Neurons and Synapses:**
Overview

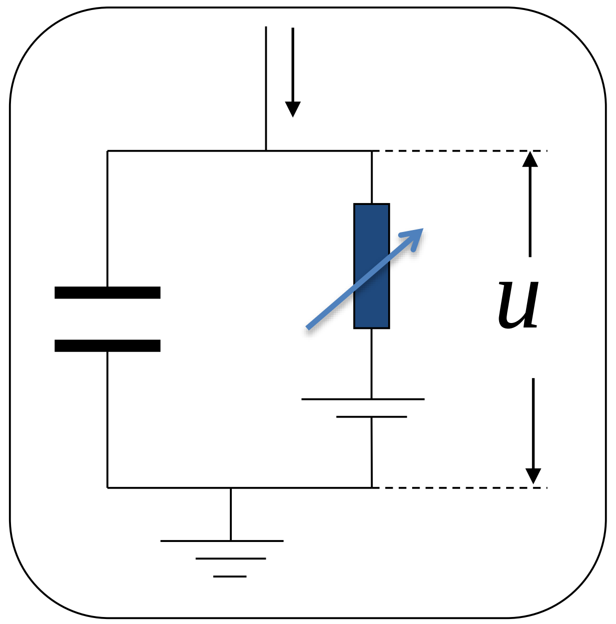
1.2 **The Passive Membrane**
- Linear circuit
- Dirac delta-function

√ 1.3 **Leaky Integrate-and-Fire Model**

√ 1.4 **Generalized Integrate-and-Fire Model**
- where is the firing threshold?

1.5. **Quality of Integrate-and-Fire Models**
- Neuron models and experiments



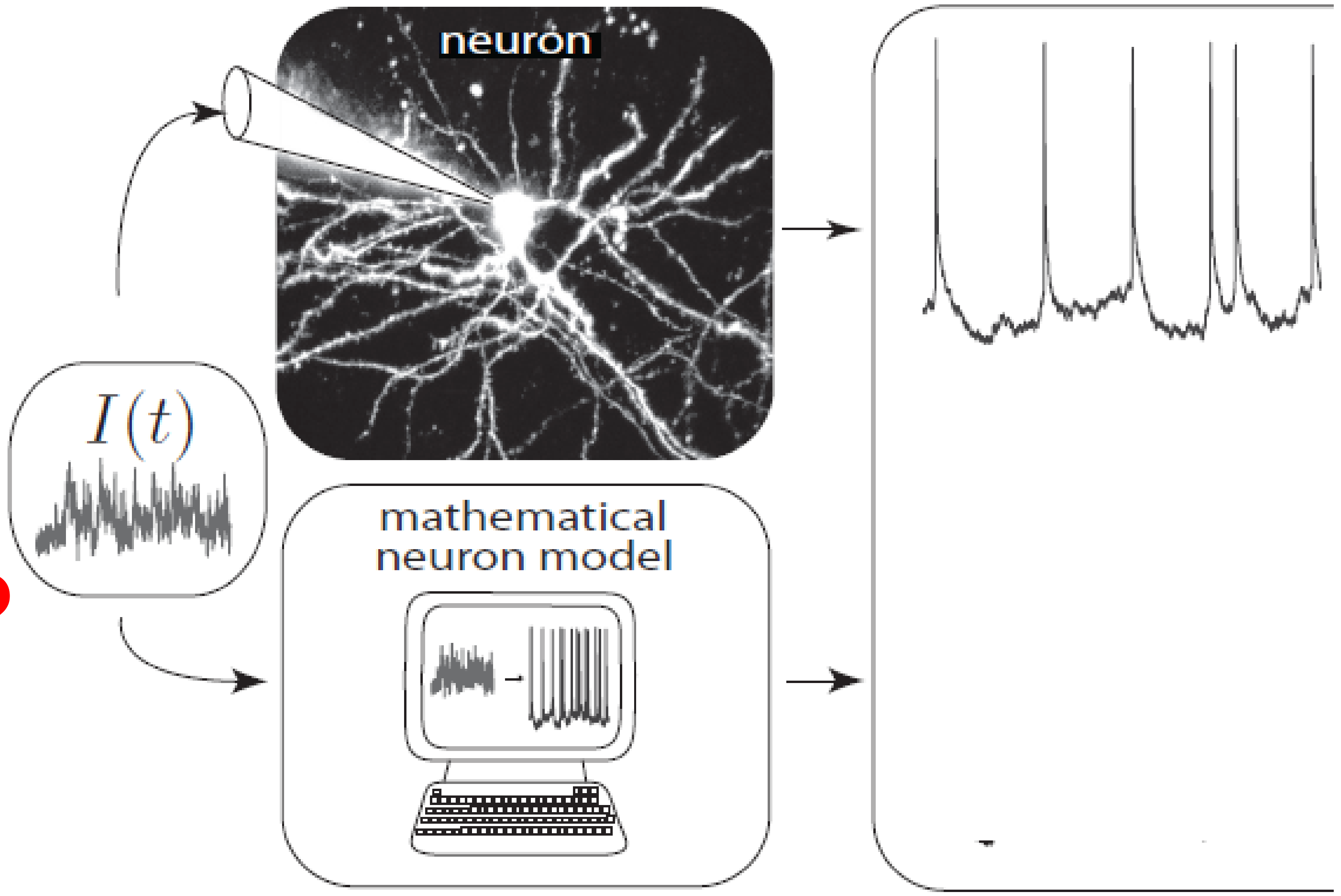


$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

$$\text{if } u = \vartheta_r \text{ then } u \to u_r$$

Can we compare neuron models
with experimental data?

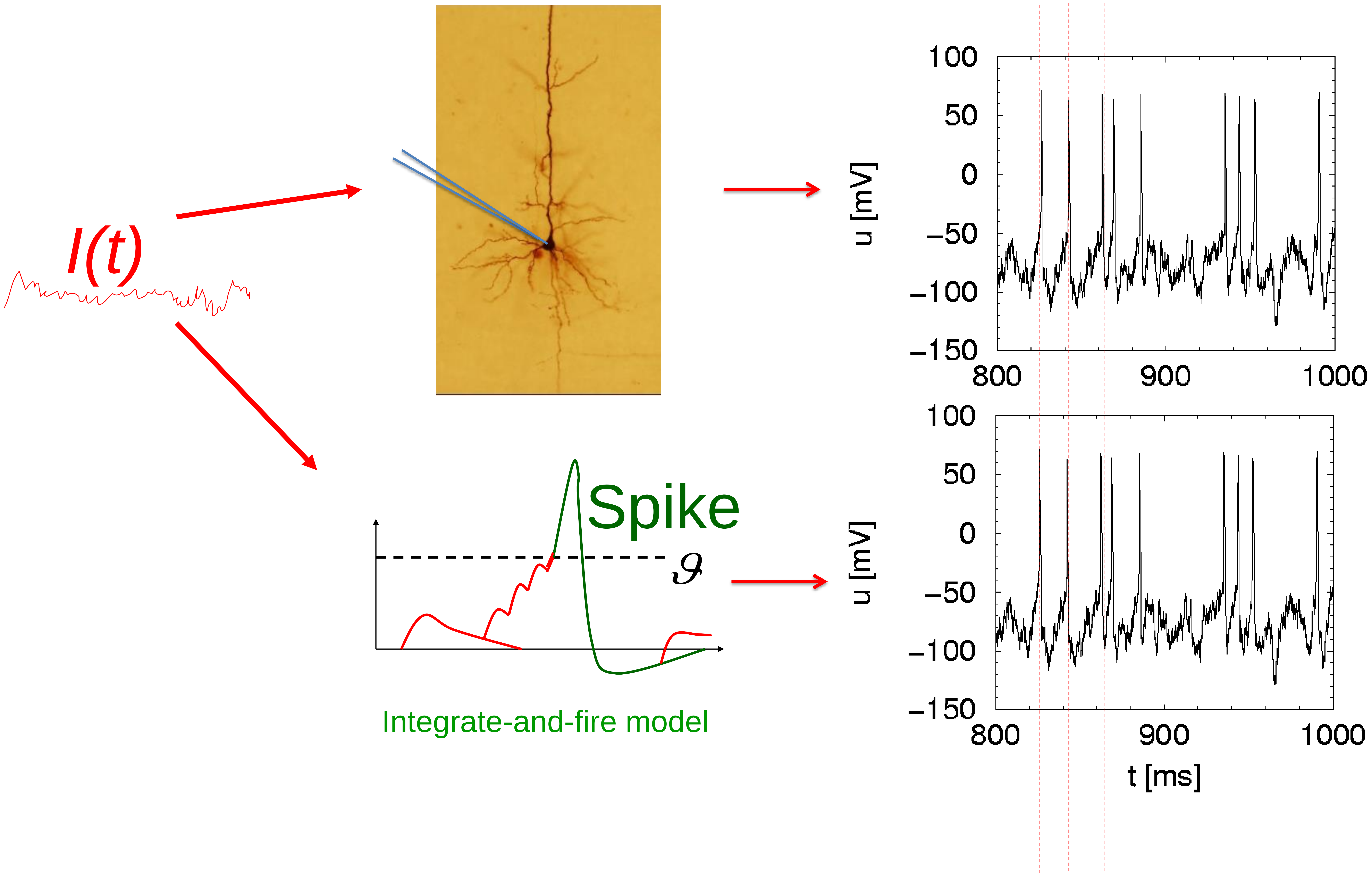
Neuronal Dynamics – 1.5.How good are integrate-and-fire models?



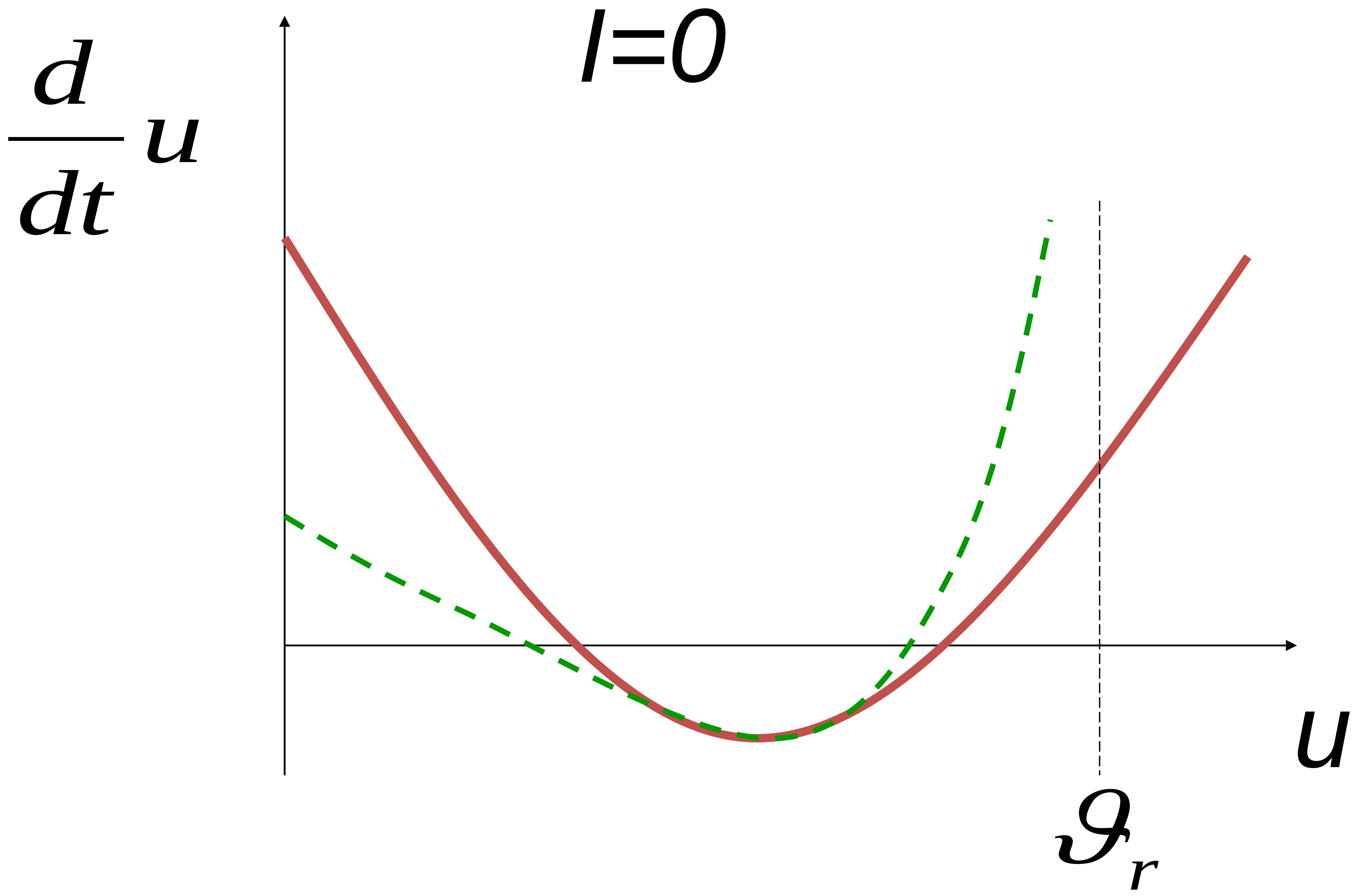
What is a good neuron model?

Can we compare neuron models with experimental data?

Neuronal Dynamics – 1.5.How good are integrate-and-fire models?



Nonlinear Integrate-and-fire Model



Can we measure the function *F(u)*?

$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$

$$u(t) = \vartheta_r \Rightarrow \text{ Fire+reset}$$

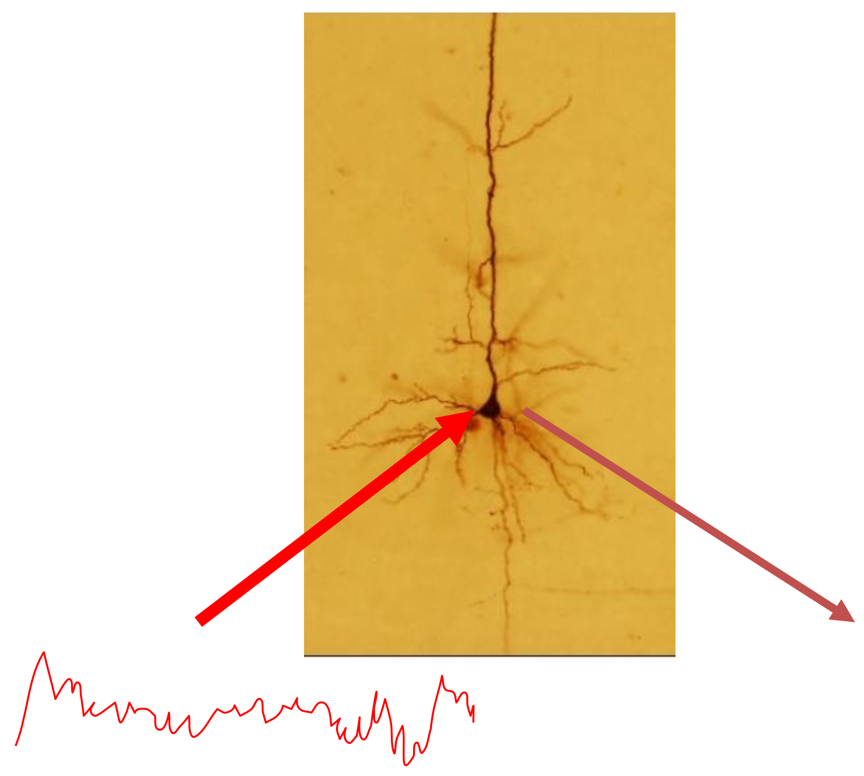
Quadratic I&F:

$$F(u) = c_2(u - c_1)^2 + c_0$$

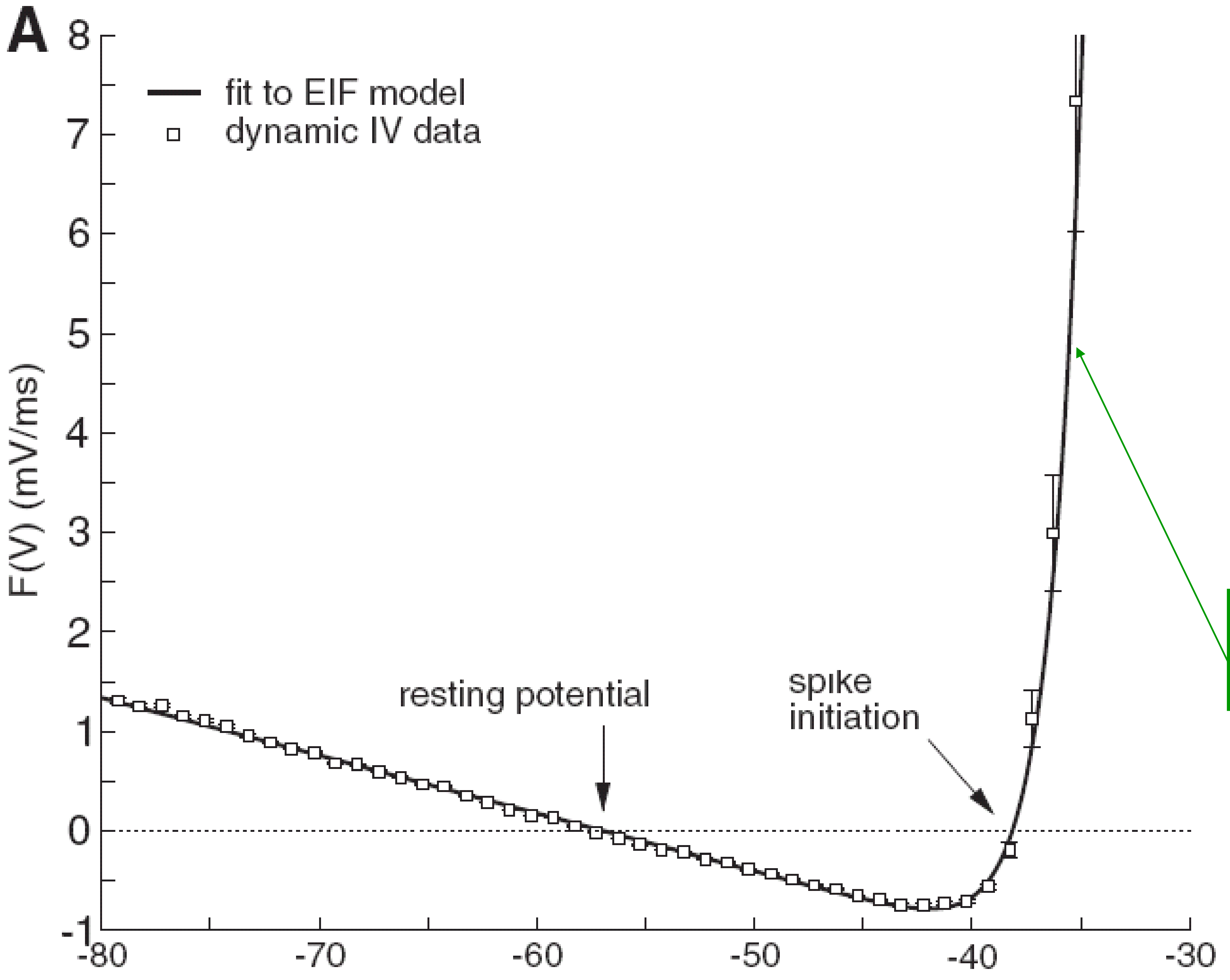
exponential I&F:

$$F(u) = -(u - u_{rest}) + c_0 \exp(u - \vartheta)$$

Neuronal Dynamics – 1.5.How good are integrate-and-fire models?



I(t)



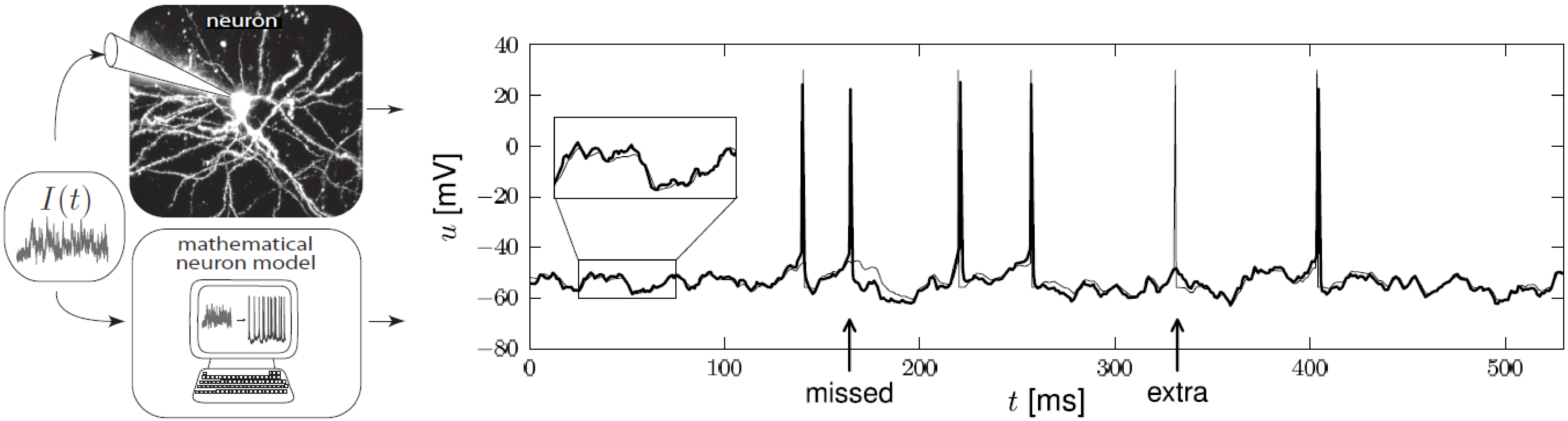
$$F(u) = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right)$$

u [mV]

$$\frac{du}{dt} - \frac{1}{C} I(t) = F(u)\frac{1}{\tau}$$

Badel et al., J. Neurophysiology 2008

Neuronal Dynamics – 1.5.How good are integrate-and-fire models?



Nonlinear integrate-and-fire models are good

Mathematical description → prediction

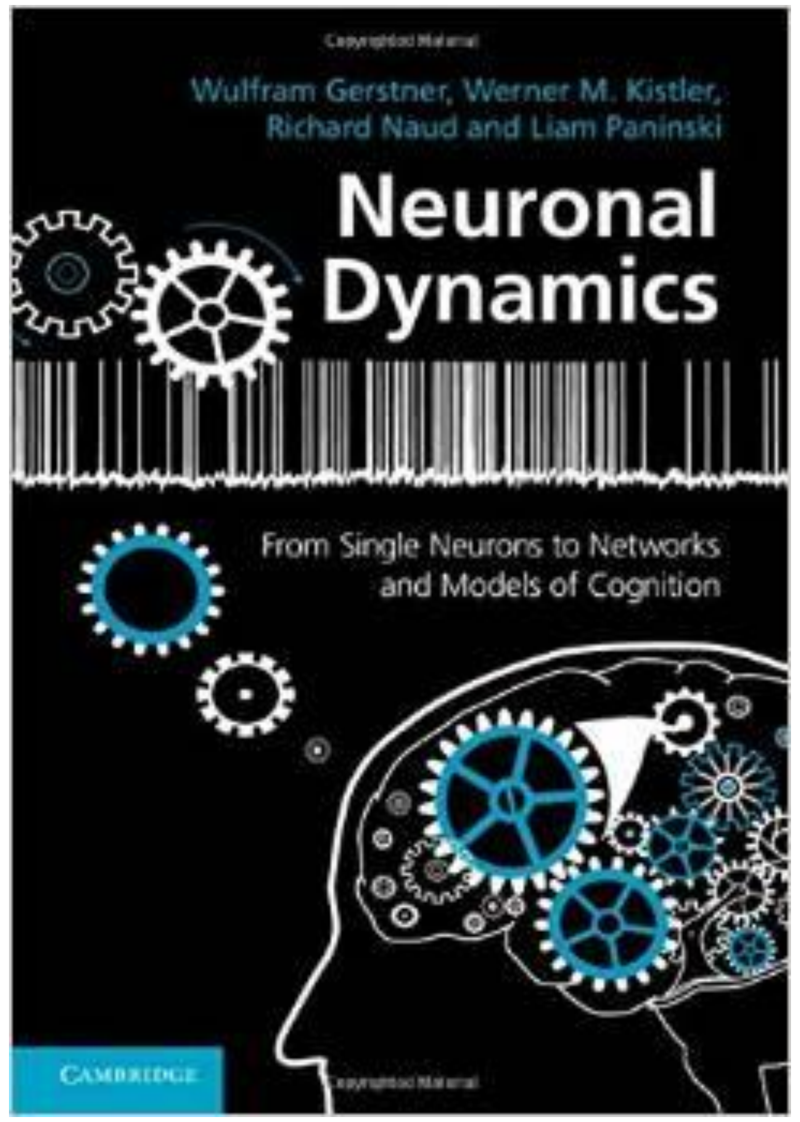
Need to add
- adaptation
- noise
- dendrites/synapses

Computer exercises:
Python

Biological Modeling of Neural Networks

http://neuronaldynamics.epfl.ch/

Textbook:

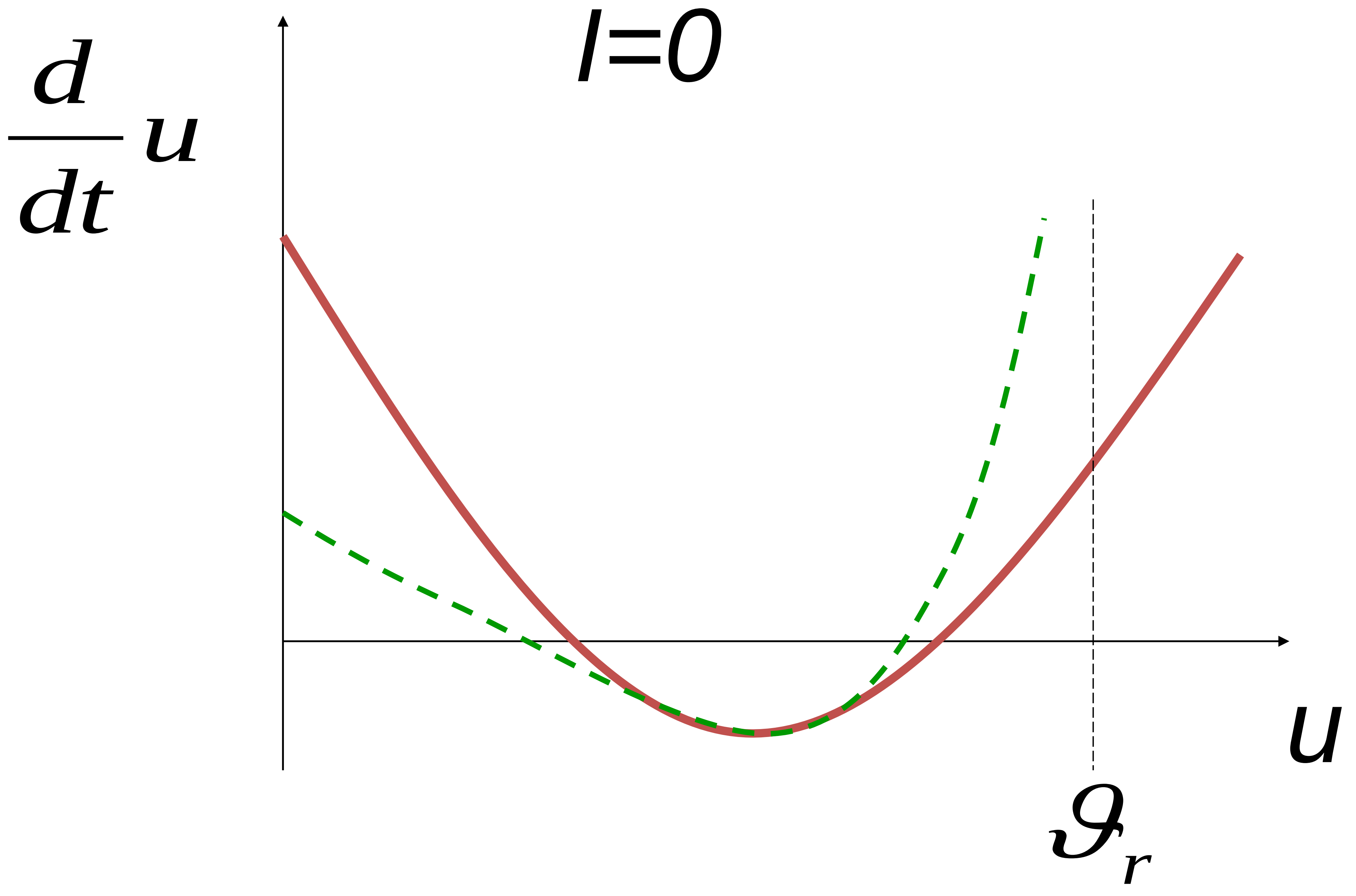


Lecture today:
-Chapter 1
-Chapter 5

Exercises today:
-Install PYTHON for Computer Exercises
-Exercise 3, on sheet

Videos (for today: 'week 1'):

Biological Modeling of Neural Networks – week1/Exercise 3



$$\tau \cdot \frac{d}{dt}u = F(u) + RI(t)$$

Homework!

Exercise 3: Integrate-and-fire models

The general form of an integrate-and-fire model is

$$\frac{du}{dt} = F(u) + \frac{RI(t)}{\tau} \tag{4}$$

where $F(u)$ is an appropriate function and $I(t)$ is the injected current. Three popular choices for the function F are the following (see Fig.1);

Leaky integrate-and-fire $F(u) = -\dfrac{u - u_{\text{rest}}}{\tau}$

Quadratic integrate-and-fire $F(u) = k\dfrac{(u - u_{\text{rest}})(u - u_{\text{th}})}{\tau}$

Exponential integrate-and-fire $F(u) = \dfrac{-(u - u_{rest}) + \Delta e^{\frac{u - u_{th}}{\Delta}}}{\tau}$.

3.1 Identify the resting potential u_{rest} and the spike threshold u_{th} in Fig. 1.

3.2 Consider three different values u_1, u_2 and u_3 for the voltage such that (i) u_1 is below u_{rest} (the resting potential), (ii) u_2 is between u_{rest} and u_{th} (the spike threshold), and (iii) u_3 is above u_{th} (see Fig. 1). For the three models described above, determine qualitatively the evolution of $u(t)$ when started at u_1, u_2, and u_3, assuming that the external input $I(t) \equiv 0$.

- For $u(t = 0) = u_1$, the voltage increases/decreases slowly/rapidly.
- For $u(t = 0) = u_2$, ...
- For $u(t = 0) = u_3$, ...

3.3 Why is u_{rest} called the resting potential? What is the role of u_{th}?

3.4 Consider the two voltage traces shown in Fig. 2(b) (top) in response to a step current (bottom). Using the graphs in Fig. 2(a), determine which of the two models was used to generate each trace.

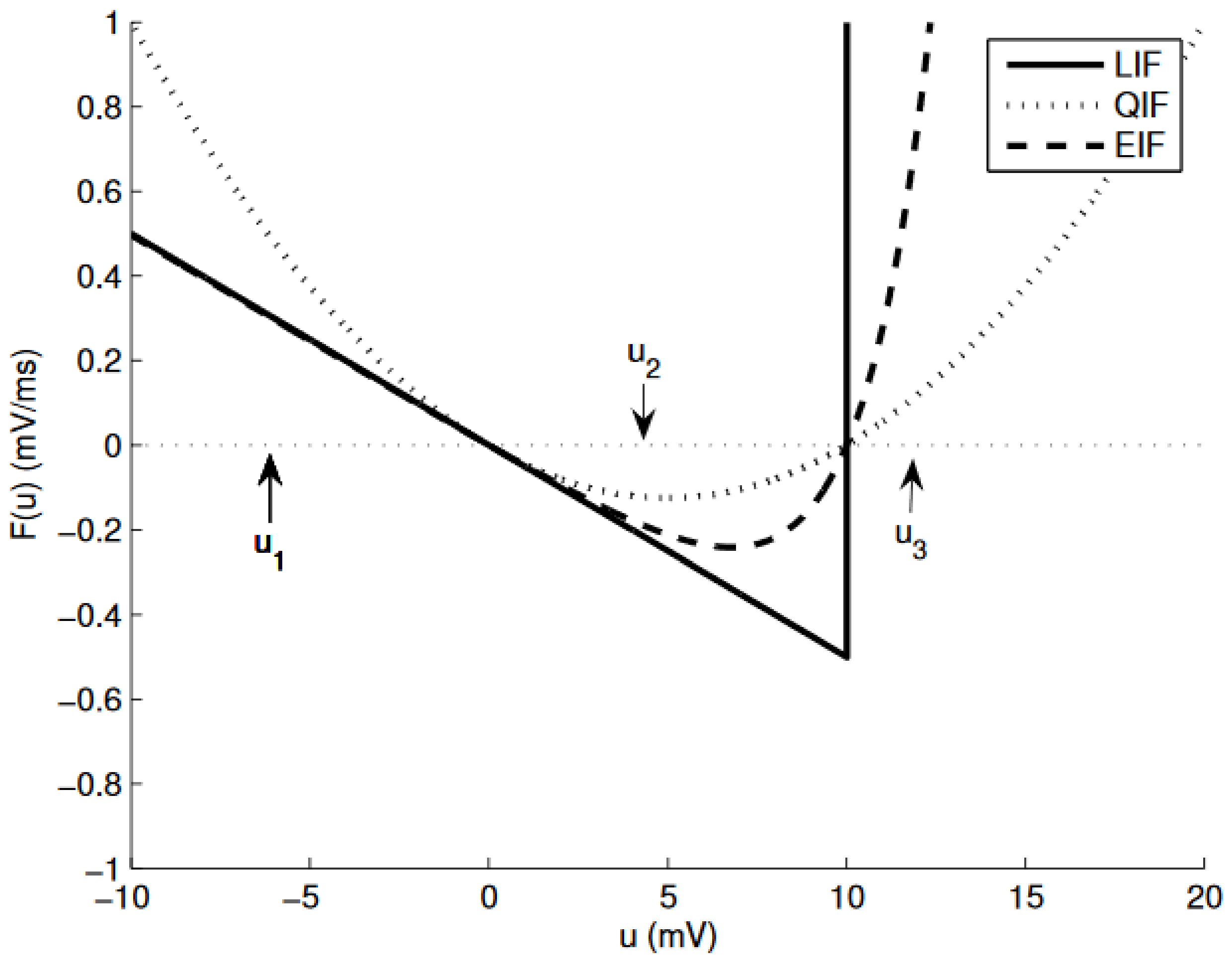
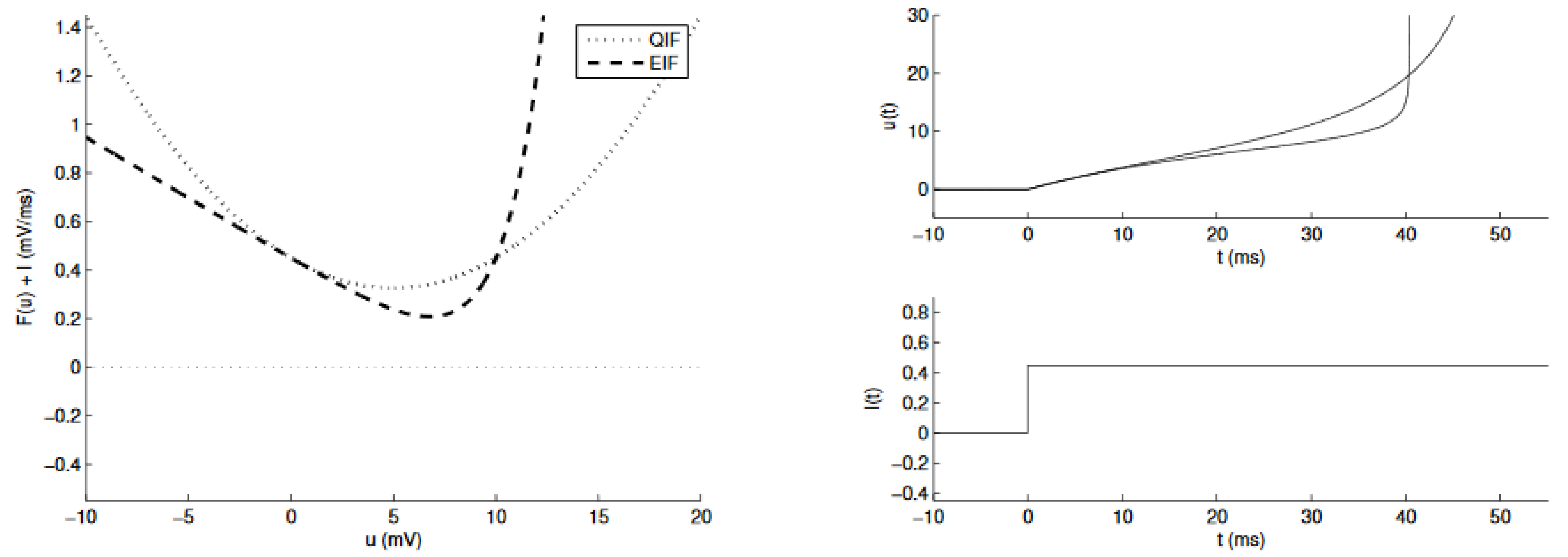


Figure 1: Sketch of the function $F(u)$ for three popular integrate-and-fire models.



(a) (b)

Figure 2: Left: Right-hand side of Eq. 4 for the quadratic and exponential integrate-and-fire models if a constant input current $I(t) > 0$ is applied. Lower right: Trace of the injected current. Upper right: Voltage trace of the two models (EIF and QIF).

Summary of Section 1.5.

The exponential integrate-and-fire (EIF) model is an excellent model of spike initiation in neurons. [- And better than the QIF which is ‘too symmetric’].

The quality of a neuron model can be measured by predicting spikes for new input that was used to optimize the model parameters.

Further improvements of generalized integrate-and-fire models are possible and involve (i) noise in the spike generation process and (ii) adaptation.

First week – References and Suggested Reading

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski,
Neuronal Dynamics: from single neurons to networks and models of cognition. Chapter 1*: Introduction*. Cambridge Univ. Press, 2014

Selected references to linear and nonlinear integrate-and-fire models

- Lapicque, L. (1907). *Recherches quantitatives sur l'excitation electrique des nerfs traitee comme une polarization.* J. Physiol. Pathol. Gen., 9:620-635.

-Stein, R. B. (1965). A theoretical analysis of neuronal variability. Biophys. J., 5:173-194.

-Ermentrout, G. B. (1996). *Type I membranes, phase resetting curves, and synchrony*. Neural Computation, 8(5):979-1001.

-Fourcaud-Trocme, N., Hansel, D., van Vreeswijk, C., and Brunel, N. (2003). *How spike generation mechanisms determine the neuronal response to fluctuating input.* J. Neuroscience, 23:11628-11640.

-Badel, L., Lefort, S., Berger, T., Petersen, C., Gerstner, W., and Richardson, M. (2008). Biological Cybernetics, 99(4-5):361-370.

- Latham, P. E., Richmond, B., Nelson, P., and Nirenberg, S. (2000). *Intrinsic dynamics in neuronal networks. I. Theory.* J. Neurophysiology, 83:808-827.

First week

THE END (of main lecture)

MATH DETOUR SLIDES

(for online VIDEO)

Week 1 – part 2: Detour/Linear differential equation

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Neuronal Dynamics:
Computational Neuroscience of Single Neurons

Week 1 – neurons and mathematics: a first simple neuron model

Wulfram Gerstner
EPFL, Lausanne, Switzerland

√ 1.1 **Neurons and Synapses:**
Overview

√ 1.2 **The Passive Membrane**
- Linear circuit
- Dirac delta-function
- $\boxed{\text{Detour: solution of 1-dim linear differential equation}}$

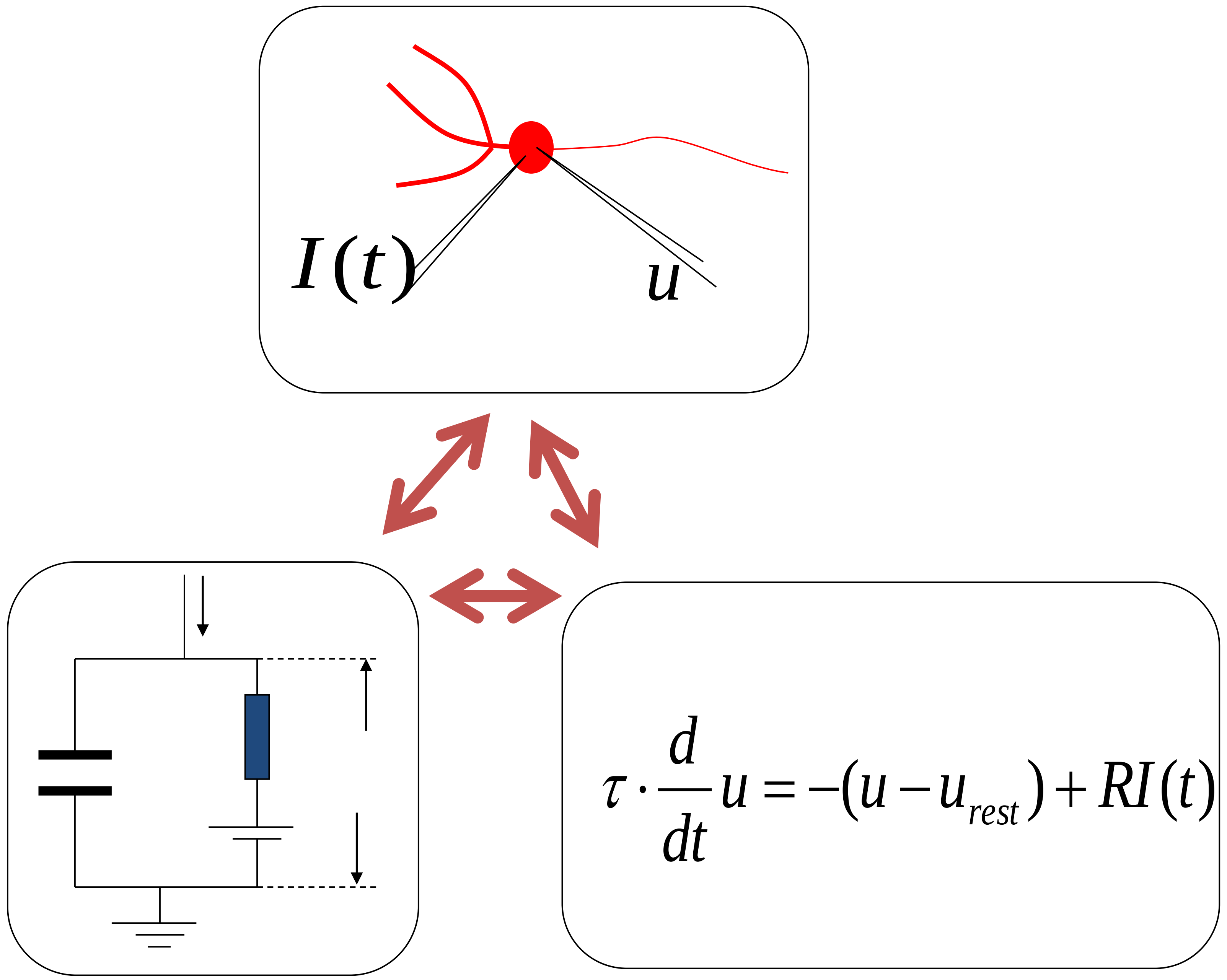
1.3 **Leaky Integrate-and-Fire Model**

1.4 **Generalized Integrate-and-Fire Model**

1.5. **Quality of Integrate-and-Fire Models**

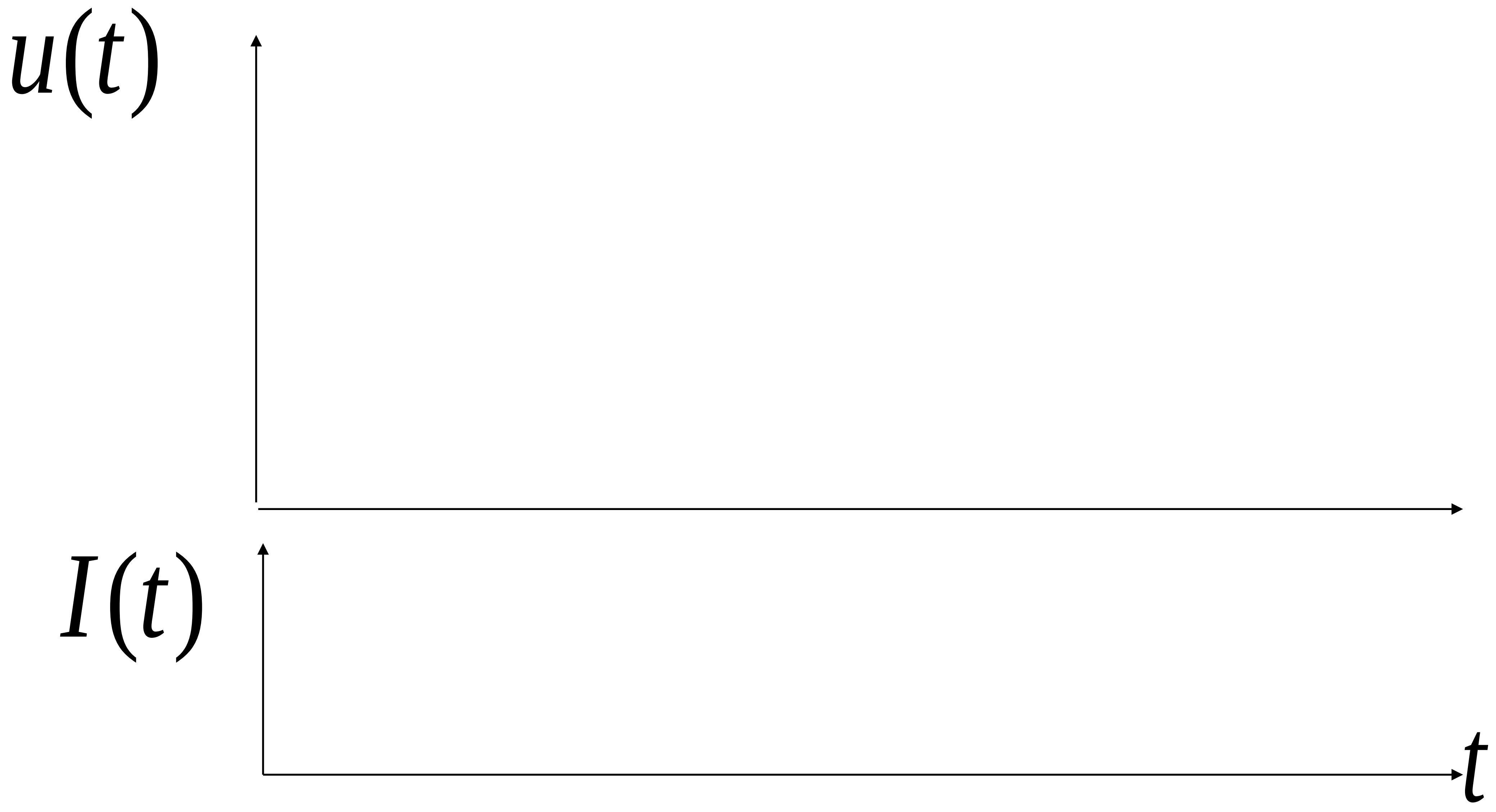
Neuronal Dynamics – 1.2Detour – Linear Differential Eq.

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

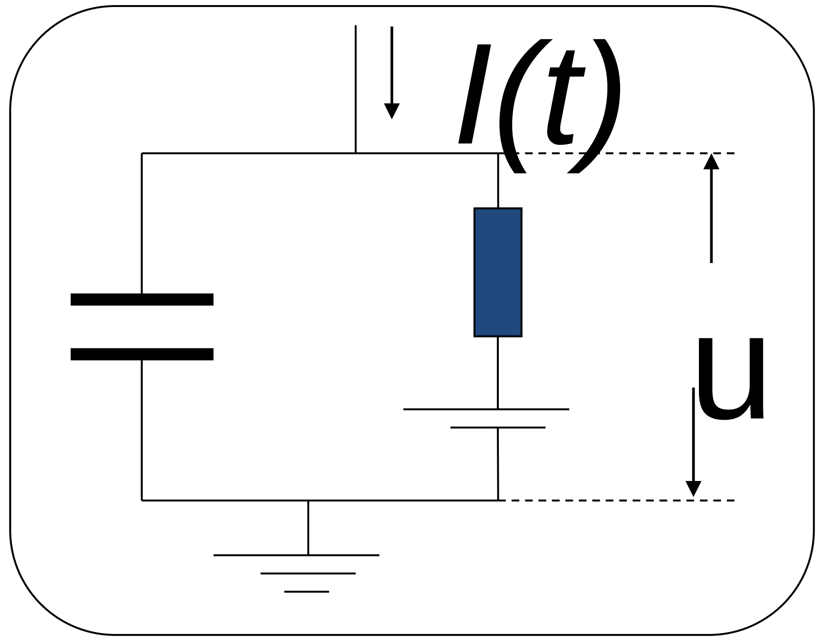


Neuronal Dynamics – 1.2Detour – Linear Differential Eq.

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

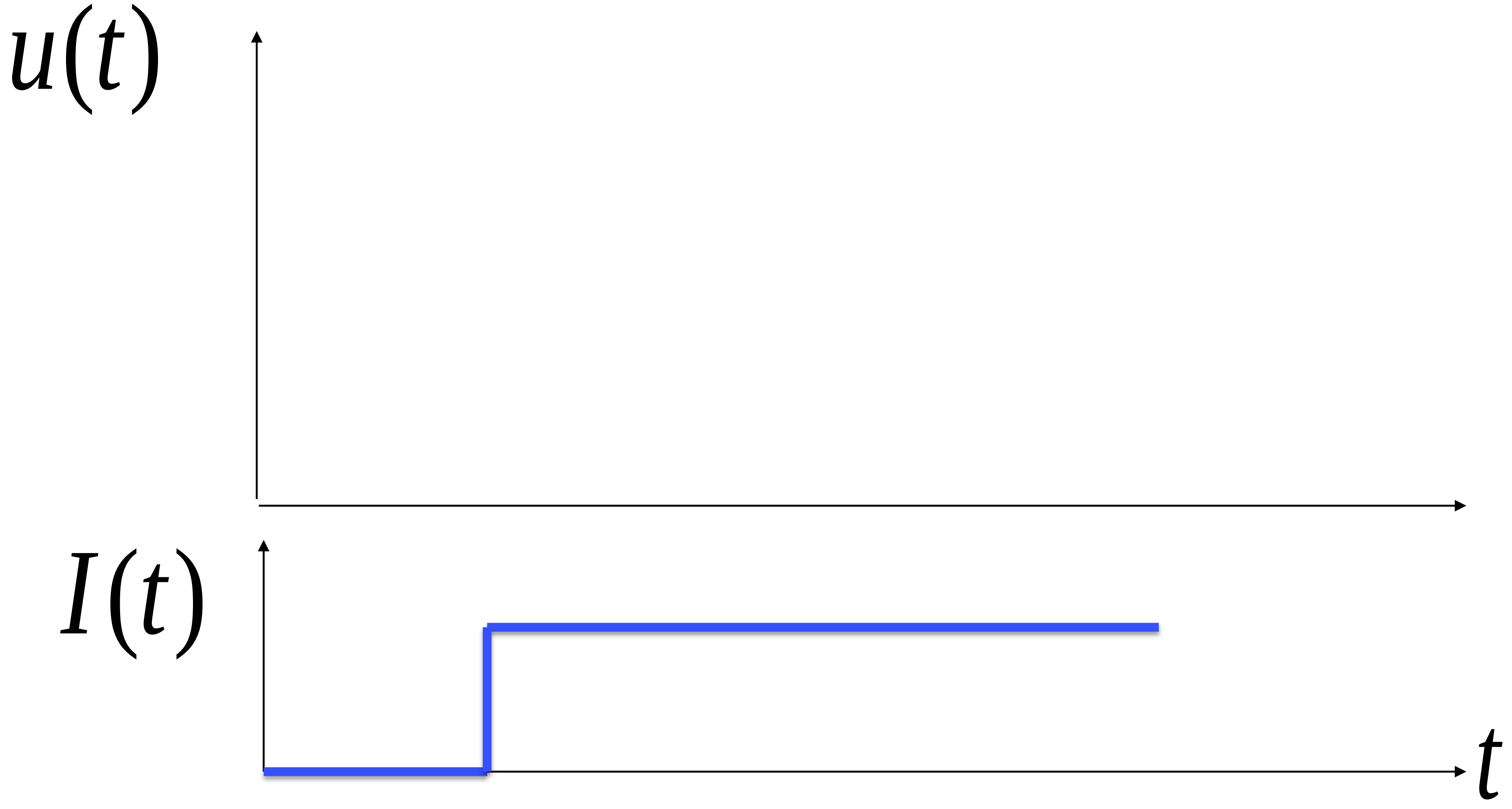


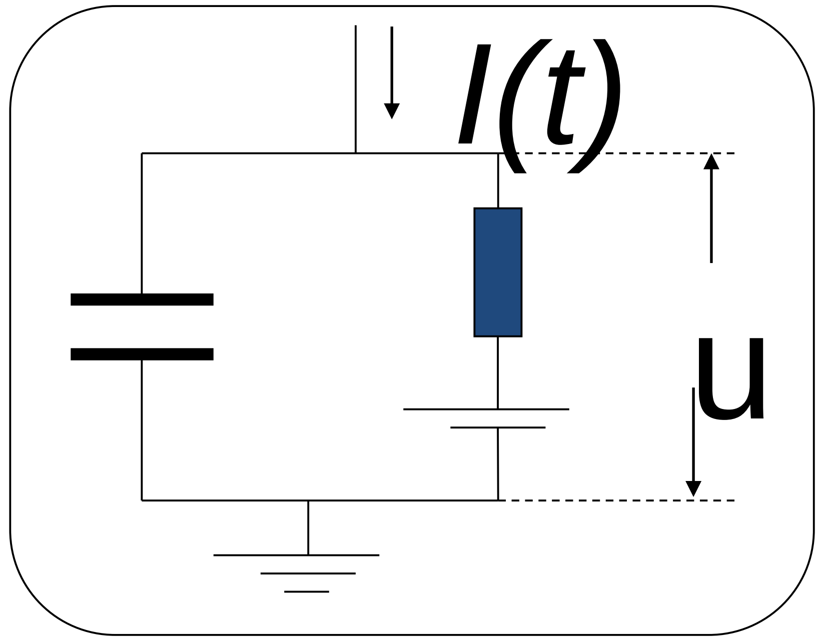
Math development:
Response to step current



Neuronal Dynamics – 1.2Detour – Step current input

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$





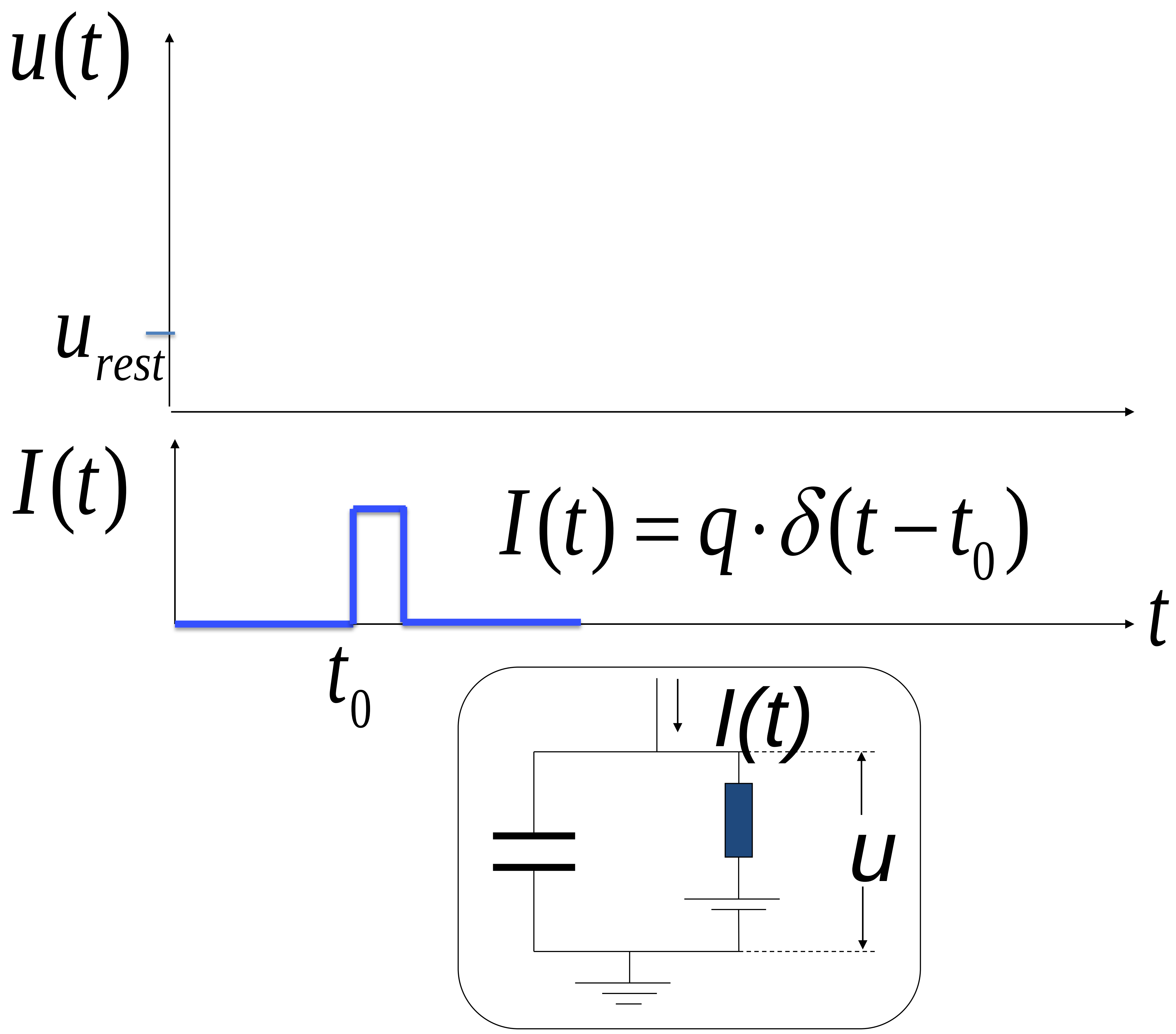
Neuronal Dynamics – 1.2Detour – Short pulse input

$$u(t) = u_{rest} + RI_0\left[1 - e^{-(t-t_0)/\tau}\right]$$

$$\tau \cdot \frac{d}{dt}u = -(u - u_{rest}) + RI(t)$$

short pulse: $(t - t_0) << \tau$

Math development: Response to short current pulse



Neuronal Dynamics – 1.2Detour – Short pulse input

$$u(t) = u_{rest} + RI_0\left[1 - e^{-(t-t_0)/\tau}\right]$$

$$\tau \cdot \frac{d}{dt}u = -(u - u_{rest}) + RI(t)$$

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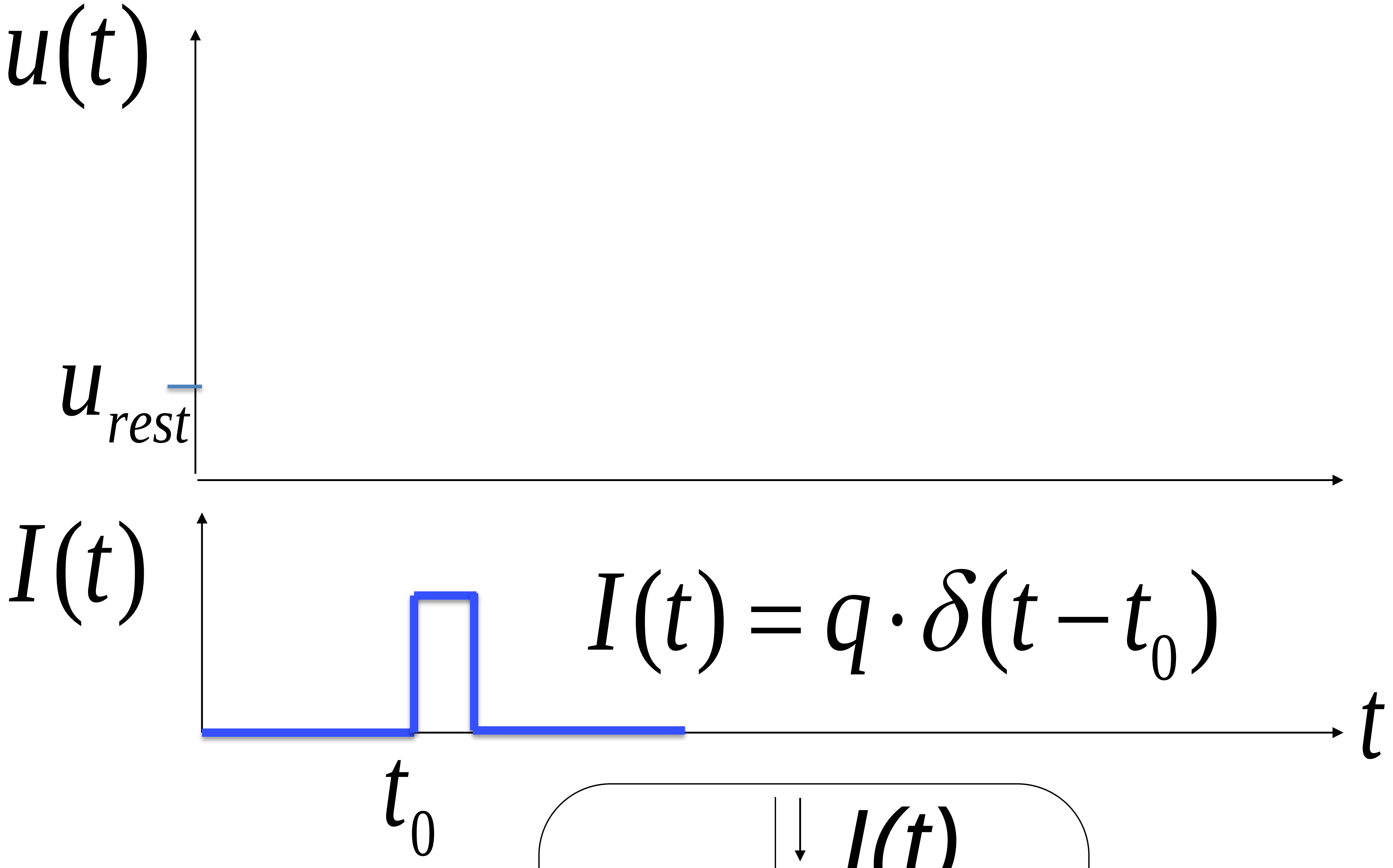
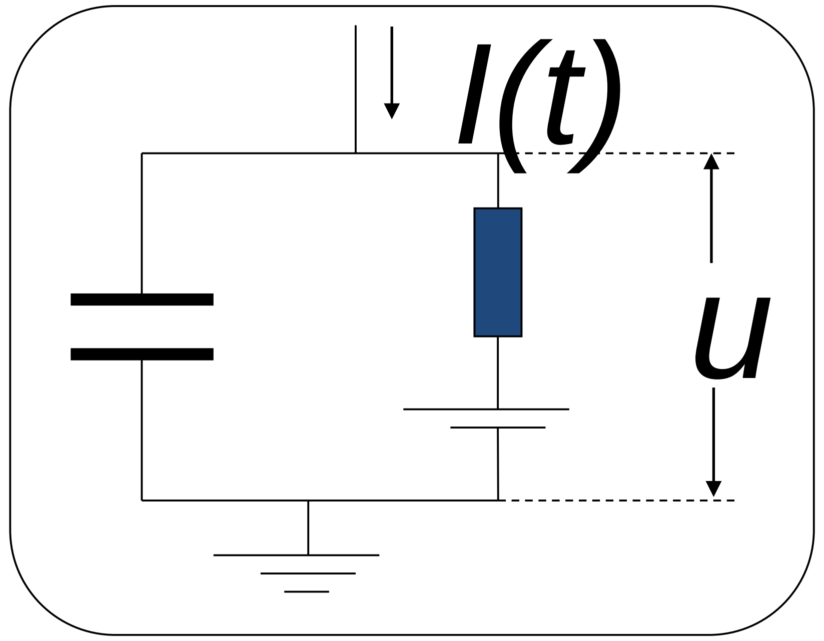




$$u(t) = u_{rest} + \frac{q}{C}e^{-(t-t_0)/\tau}$$

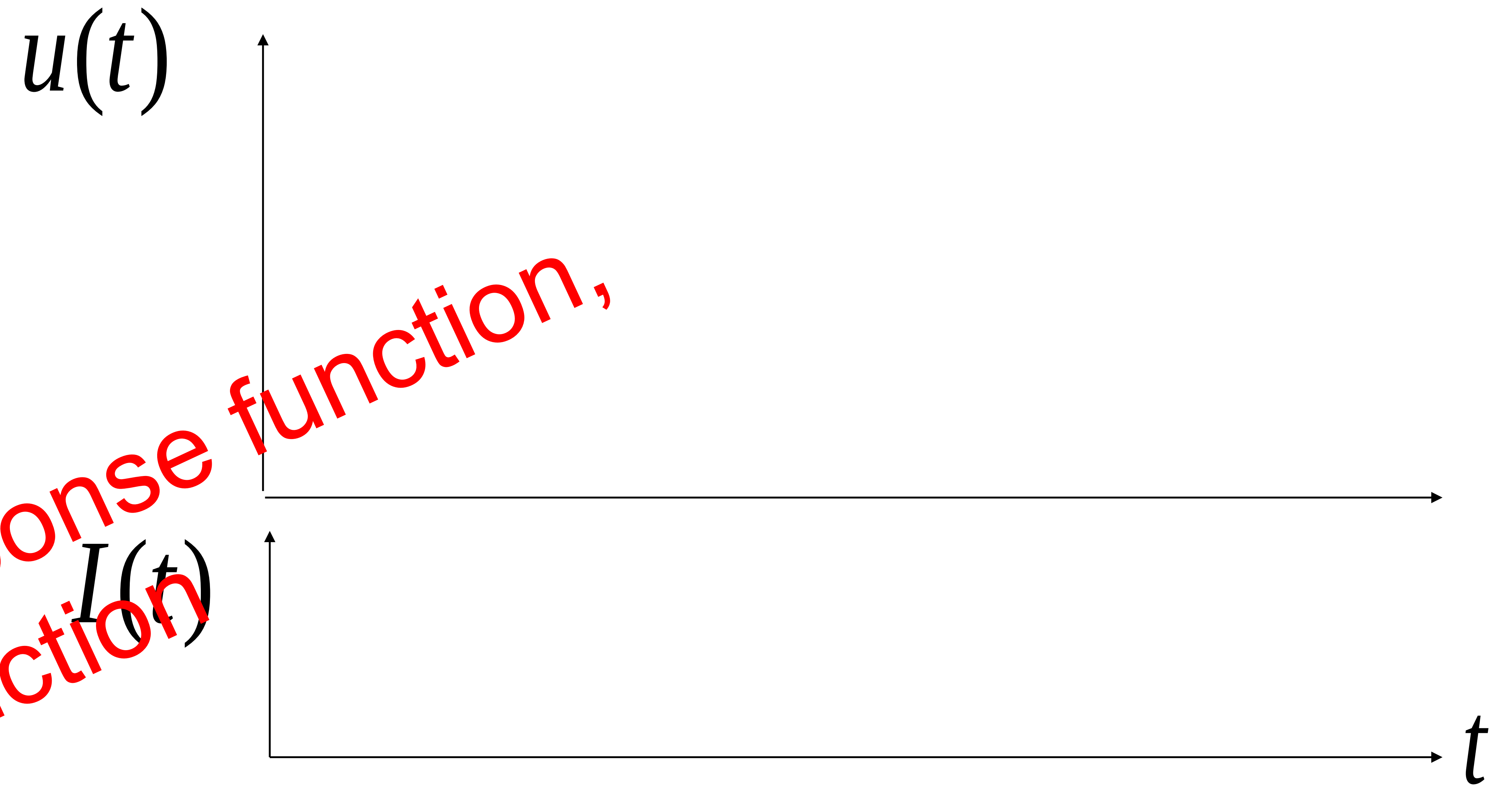
Neuronal Dynamics – 1.2Detour – arbitrary input

Single pulse

$$u(t) = u_{rest} + \frac{q}{C} e^{-(t-t_0)/\tau}$$

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

Multiple pulses:



Impulse response function,
Green's function

$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t')dt'$$

Neuronal Dynamics – 1.2Detour – Greens function

Single pulse

$$\Delta u(t) = q\frac{1}{C}e^{-(t-t_0)/\tau}$$

$$\tau \cdot \frac{d}{dt}u = -(u - u_{rest}) + RI(t)$$

Multiple pulses:

$$u(t) = u_{rest} + [u(t_0) - u_{rest}] + \int_{t_0}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t')dt'$$

Impulse response function,
Green's function

$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t')dt'$$

Neuronal Dynamics – 1.2Detour – arbitrary input

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

*If you don't feel at ease yet, spend **10 minutes** on these mathematical exercises*
And quiz 2 in week 1.

Arbitrary input

$$u(t) = u_{rest} + \int_{-\infty}^{t} \frac{1}{C} e^{-(t-t')/\tau} I(t')dt'$$

Single pulse

$$\Delta u(t) = \frac{q}{C} e^{-(t-t_0)/\tau}$$

you need to know the solutions of linear differential equations!