Last Name............ First Name..................

# Neural Networks and Biological Modelling: Graded exercise

The team of Professor and TAs wants everybody to work on the following exercise. The exercise will be part of the graded miniprojects 2. The coding part of miniproject 2 will be announced soon. It will contain phase plane analysis (theory here), mean-field analysis (topic covered in 2 weeks), and simulation of epidemics in networks. You should submit the theory exercise below as part of miniproject 2. In total miniprojects 2 will have N points. One fifth of the points can be gained by this theory exercise. In other words, if you do all the rest of the miniproject, but not this exercise, your maximum grade is 5.0.

## 1 Phase Plane Analysis and Epidemics (25 points)

The mathematical tools of phase plane analysis can be used for the SIR model of epidemics that we have seen in class. In this exercise we guide you through the analysis step by step. A very similar model has been used in an exam some years ago – and we give this exercise in the format that is typically used in the exams.

Here are the two equations that we will study:

$$\frac{dx}{dt} = f(x, y) \tag{1}$$

$$\frac{dy}{dt} = g(x, y) \tag{2}$$

(a) Take the equations from the epidemics lecture in class. Introduce the parameter $R_0$ and two variables: $x = S/N$ and $y = I/N$.

What are the two functions $f(x, y)$ and $g(x, y)$?

$f(x, y) =$.......................... $g(x, y) =$..........................

number of points: 1

(b) Assume $R_0 = 2.0$. Calculate the two nullclines.

Hint: Each of the two nullclines has two branches (corresponding to two different solutions).

Answer:
The two branches of the nullcline $dx/dt = 0$ are

(i) .............

(ii) ...........

The two branches of the nullcline $dy/dt = 0$ are

(i) .............

(ii) ...........

number of points: 2

(c) **Plot the two nullclines in the empty space on the next page. Annotate your lines by writing e.g., $x$-nullcline or $y$-nullcline.**

number of points: 2

(d) In the same figure, plot the curve $y = 1 - x$

number of points: 2

Space for your phase plane graphics

(e) In the above graph, add a flow arrow indicating the direction of flow at the point $(x = 0.5, y = 0.5)$. Make the length of the arrow correspond to a suitable time step $\Delta t$ such that $\Delta x = \Delta t(dx/dt)$ is in the order of 0.1 ... 0.2.

number of points: 1

(f) Determine the direction of arrows along the line $y = 1 - x$ and give the solution in the form of a ratio $\Delta y/\Delta x = dir(x)$

Answer:

$dir(x) =$ ......................

number of points: 3

(g) Plot the result of (e) and (f) in the phase plane (pick 4 locations where you put the flow arrows).

number of points: 2

(h) Calculate the flow arrows $(\Delta x, \Delta y)$ at the following points (assume that $\epsilon$ is a small number).

$(0.2, \epsilon) \to (\Delta x, \Delta y) =$ .............

$(0.4, \epsilon) \to (\Delta x, \Delta y) =$ .............

$(0.6, \epsilon) \to (\Delta x, \Delta y) =$ .............

$(0.8, \epsilon) \to (\Delta x, \Delta y) =$ .............

number of points: 2

(i) Add the flow arrows at the four points calculated in (h) in your graph.

number of points: 2

(j) Add flow arrows on all non-horizontal nullclines in your graph.

number of points: 2

(k) Construct the trajectory starting at $(0.6, \epsilon)$.

number of points: 1

(l) Construct the trajectory starting at $(1 - \epsilon, \epsilon)$.

number of points: 1

(m) Construct the trajectory starting at $(0.45, \epsilon)$

number of points: 1

(n) Start a new graphics, but now with $R_0 = 0.8$. Repeat the construction of three trajectories starting at $(0.45, \epsilon)$, $(0.6, \epsilon)$ and $(1 - \epsilon, \epsilon)$.

number of points: 3

Space for your phase plane graphics