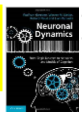
Biological Modeling of Neural Networks

Week 8 – Continuum models: Cortical fields and perception

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Reading for week 8:
NEURONAL DYNAMICS
Ch. 18 +
+Ch. 12.3.7+Ch 15.1-15.2.3
Cambridge Univ. Press



- **8.1 Transients**
  - sharp or slow
- **8.2 Spatial continuum**
  - model connectivity
  - cortical connectivity
- **8.3 Solution types**
  - uniform solution
  - bump solution
- **8.4. Perception**
- **8.5. Head direction cells**

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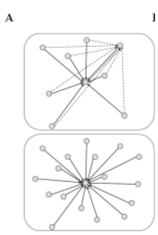
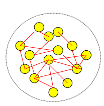
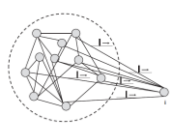
review from Week 7: mean-field arguments



Single population full connectivity



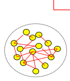


All neurons receive the same total input current ('mean field')

Review from Week 7: stationary state/asynchronous activity

Homogeneous network
All neurons are identical,
Single neuron rate = population rate

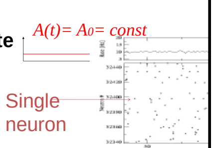
$$\nu = g(I_0) = A_0$$



constant input

$$I_0 = c$$

Gain function at appropriate noise level



Single neuron

frequency (single neuron) $\nu = 1/\langle s \rangle$ rate=1/(meanInterval)

Review from week7: mean-field arguments

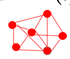
All neurons receive the same total input current ('mean field')

$$I_0 = [J_0 q A_0 + I_0^{ext}]$$

assume asynchronous state

$$I_i(t) = J_0 \int \alpha(s) A(t-s) ds + I^{ext}(t)$$

Index i disappears



fully connected

$$w_{ij} = \frac{J_0}{N}$$

All spikes, all neurons

$$I^{net}(t) = \sum_j \sum_f w_{ij} \alpha(t - t_j^f) + I^{ext}$$

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Reading for week 8.1:
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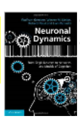
Ch 15.1-15.2.3

Cambridge Univ. Press



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8.5. Head direction cells

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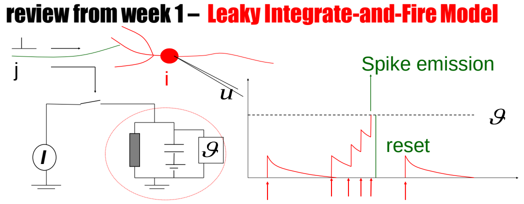
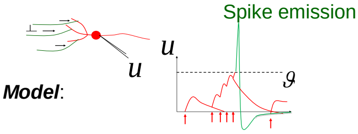
review from week 1 – Leaky Integrate-and-Fire Model



$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$ linear

$u(t) = \vartheta \Rightarrow$ Fire+reset $u \rightarrow u_r$ threshold

review from week 1 – Leaky Integrate-and-Fire type Model



***Leaky Integate-and-Fire Model**:*
passive membrane
+ threshold
+ reset

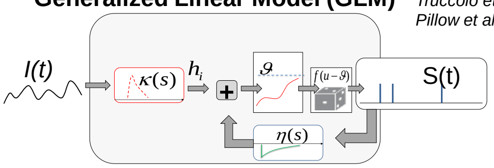
Input spike causes an EPSP
= excitatory postsynaptic potential

-output spikes are events
-generated at threshold
-after spike: reset/refractoriness

$\eta(s)$

Spike Response Model (SRM) Generalized Linear Model (GLM)

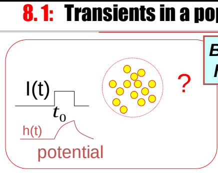
Gerstner et al., 1992,2000
Truccolo et al., 2005
Pillow et al. 2008



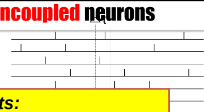
potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

8.1: Transients in a population of uncoupled neurons

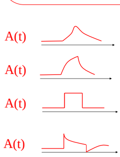


***Blackboard:** h(t)*



***Students:** Which would you choose?*

population activity $A(t) = \frac{n(t-\frac{\Delta t}{2}; t+\frac{\Delta t}{2})}{N \Delta t}$

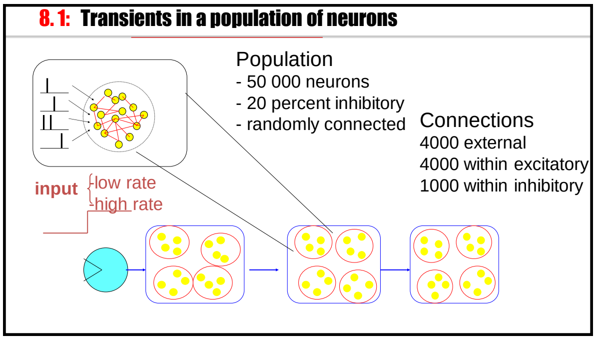


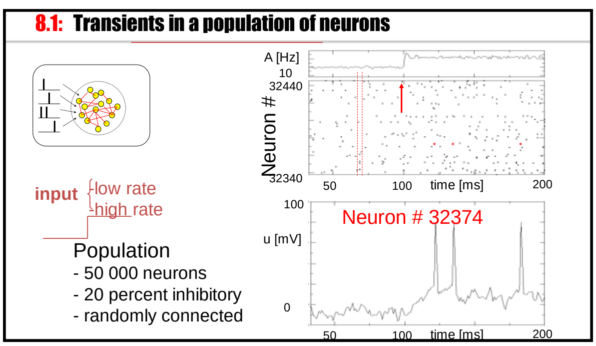
$$\tau \frac{d}{dt} A(t) = -A(t) + F(h(t))$$

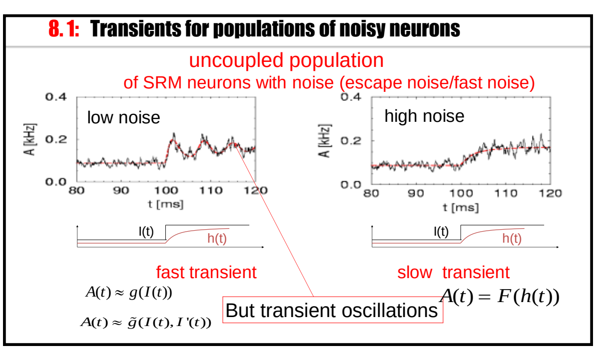
$$A(t) = F(h(t)) = F(\int \kappa(s) I(t-s) ds)$$

$$A(t) = g(I(t))$$

$$A(t) = \tilde{g}(I(t), I'(t))$$


8.1: Transients in a population of neurons
Population
- 50 000 neurons
- 20 percent inhibitory
- randomly connected
Connections
4000 external
4000 within excitatory
1000 within inhibitory
input
low rate
high rate


8.1: Transients in a population of neurons
input
low rate
high rate
Population
- 50 000 neurons
- 20 percent inhibitory
- randomly connected
A [Hz]
10
Neuron #
32440
32340
50
100
time [ms]
200
100
u [mV]
0
Neuron # 32374


8.1: Transients for populations of noisy neurons
uncoupled population
of SRM neurons with noise (escape noise/fast noise)
low noise
high noise
A [kHz]
0.4
0.2
0.0
80
90
100
110
120
t [ms]
I(t)
h(t)
fast transient
slow transient
A(t) ≈ g(I(t))
A(t) ≈ g̃(I(t), I'(t))
But transient oscillations
A(t) = F(h(t))

8.1: High-noise activity equation

blackboard

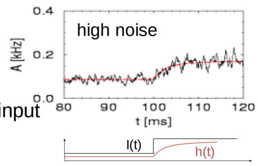
In the limit of **high noise**,
Population activity

$$A(t) = F(h(t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(t) = -h(t) + R\, I(t)$$

noise model A
(escape noise/fast noise)



slow transient

$$A(t) = F(h(t))$$

8.1: High-noise activity equation

Population activity

$$A(t) = F(h(t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(t) = -h(t) + R\, I(t)$$

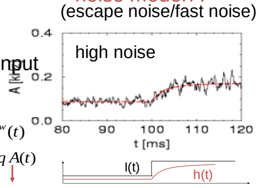
$$I(t) = I^{ext}(t) + I^{netw}(t)$$

$$I(t) = I^{ext}(t) + J_0\, q\, A(t)$$

$$I(t) = I^{ext}(t) + J_0\, q\, F(h(t))$$

$$\tau \frac{d}{dt} h(t) = -h(t) + R\, I^{ext}(t) + \gamma\, F(h(t))$$

noise model A
(escape noise/fast noise)



slow transient

$$A(t) = F(h(t))$$

1 population = 1 differential equation

Review from week 7: mean-field also works for random coupling

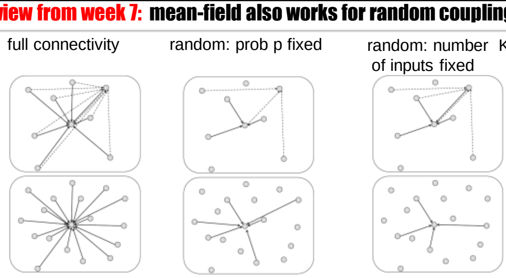


Image: Gerstner et al. Neuronal Dynamics (2014)

Quiz 1, now

Population equations

[ ] A single cortical model population can exhibit transient oscillations
[ ] Transients are always sharp
[ ] Transients are always slow
[ ] in a certain limit transients can be slow
[ ] An escape noise model in the high-noise limit has transients which are always slow
[ ] A single population described by a single first-order differential equation (no integrals/no delays) can exhibit transient oscillations

Biological Modeling of Neural Networks

Week 8 – Continuum models: Cortical fields and perception

Wulfram Gerstner
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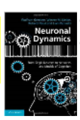
Reading for week 8.2:
NEURONAL DYNAMICS
Ch 12.3.7+18.1

Cambridge Univ. Press



8.1 Transients
- sharp or slow

8.2 Spatial continuum
- model connectivity
- cortical connectivity

8.3 Solution types
- uniform solution
- bump solution

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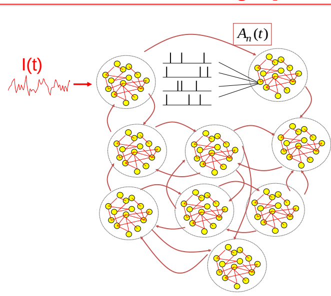
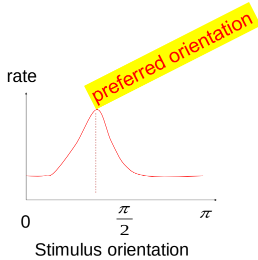
8.5. Head direction cells

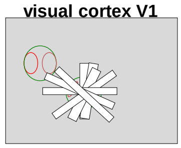
Review from week 7: Interacting Populations



7.1: Review from week7: Receptive fields with Orientation Tuning



Receptive fields:
visual cortex V1



Orientation selective

8.2: Orientation Map

population of neighboring neurons: similar orientations
As we move along cortical surface: orientation changes

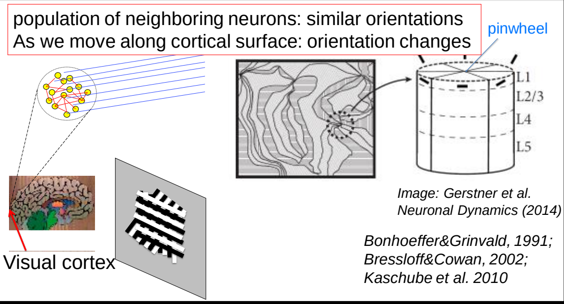
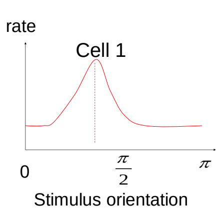


Image: Gerstner et al. Neuronal Dynamics (2014)

Bonhoeffer&Grinvald, 1991; Bressloff&Cowan, 2002; Kaschube et al. 2010

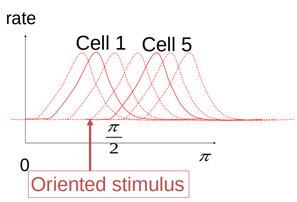
7.1: Do Orientation Columns exist? Do identical cells exist?



8.2: Do Orienation colums exist? Do identical cells exist?

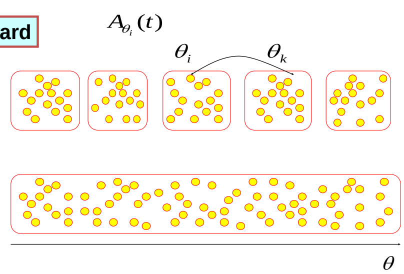
Course coding

Many cells
(from different columns)
respond to a single
stimulus with different rate



8.2: multiple populations → continuum

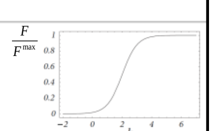
Blackboard



8.2: Field equation (continuum model)

Population activity

$$A(x,t) = F(h(x,t))$$



Membrane potential caused by input

$$\tau \tfrac{d}{dt} h(x,t) = -h(x,t) + R\, I(x,t)$$

$$I(x,t) = I^{ext}(x,t) + I^{netw}(x,t)$$

$$I^{netw}(x,t) = d \int w(x-x',t)\, A(x',t)\, dx'$$

$$\tau \tfrac{d}{dt} h(x,t) = -h(x,t) + R\, I^{ext}(x,t) + d \int w(x-x') F(h(x',t))\, dx'$$

1 field = 1 integro-differential equation

Exercise 1.1 now (stationary solution)

Consider a continuum model,
Find analytical solutions:

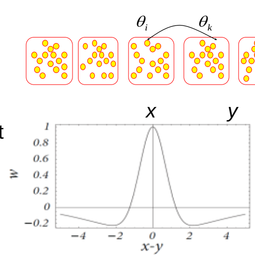
- spatially uniform solution $A(x,t)= A_0$

Next lecture at 10:50

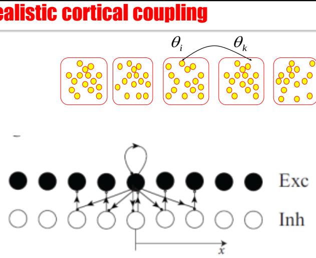
If done: start with Exercise 1.2 now (spatial stability)

8.2: coupling across continuum

Mexican hat



8.2: more realistic cortical coupling



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Week 8 – Continuum models: Cortical fields and perception

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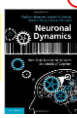
EPFL, Lausanne, Switzerland

Reading for week 8.3-8.5
NEURONAL DYNAMICS
Ch. 18.



Cambridge Univ. Press

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8.3: Solution types (ring model)

Coupling:

Input-driven regime

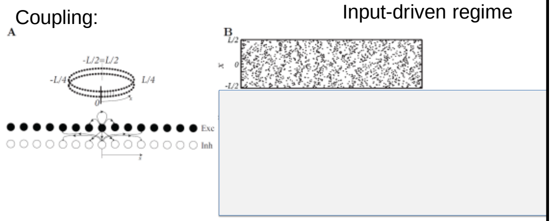
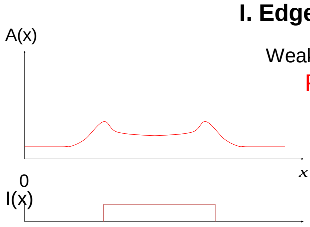


Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014),

8.3: Solution types: input driven regime

Field Equations:
Wilson and Cowan, 1972

I. Edge enhancement



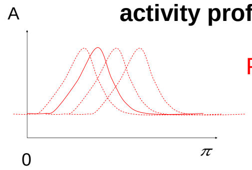
Weaker lateral connectivity

Possible interpretation of visual cortex cells:
(see later this week)

8.3: Solution types: bump solution

Field Equations:
Wilson and Cowan, 1972

II: Bump formation:
activity profile in the absence of input
strong lateral connectivity

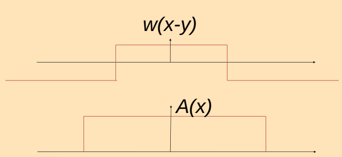


Possible interpretation
of head direction cells:
→ (see later today)

Exercise 2.1+2.2 now (stationary bump solution)

Next lecture at 11:30

Consider a **discretized** continuum model
Find analytically the bump solutions

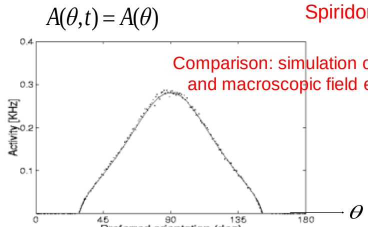


8.3: Solution types: bump solution

$$A(\theta,t)=A(\theta)$$

Spiridon&Gerstn

Comparison: simulation of neurons
and macroscopic field equation



Continuum: stationary profile

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014),

8.3: Solution types (continuum model)

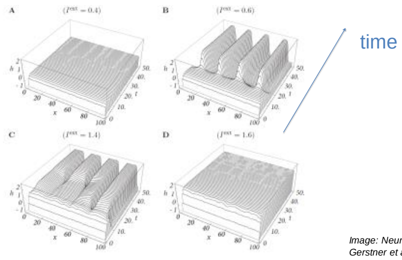


Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014),

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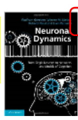
Ch. 18.



Cambridge Univ. Press

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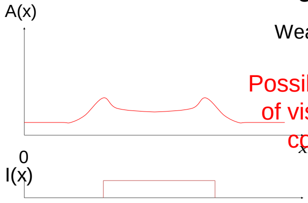
8.4: uniform/input driven solution

Field Equations:
Wilson and Cowan, 1972

Basic phenomenology **I. Edge enhancement**



Weaker lateral connectivity

Possible interpretation
of visual cortex cells:
contrast enhancement in
- orientation
- location

8.4. Perception -grid illusion

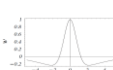
Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014),

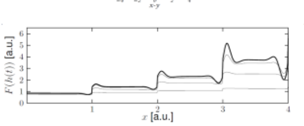
8.4. Perception – Mach bands

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014),

8.4. Mach bands in a continuum model

Mexican-hat coupling

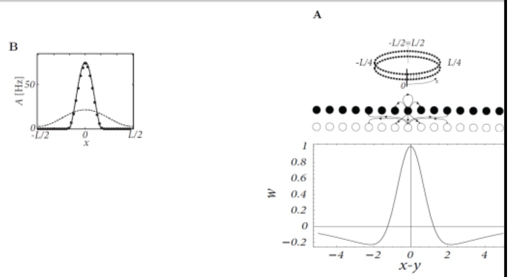




ig. 18.9: **A.** Mach bands in a field model with mexican hat

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014),

8.4: Field models and Perception



8.4: Field models and Perception

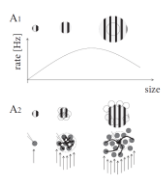


Fig. 18.12: Surround suppression.

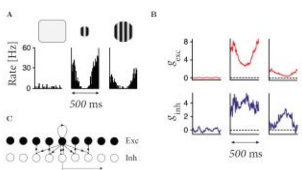


Fig. 18.13: Network stabilized by local inhibition. The schematic model could potentially explain why larger gratings lead not only to less excitatory input g_{exc}, but also to less inhibitory input g_{inh}. A. The firing rate as a function of the phase of the moving grating for the three stimulus conditions (blank screen, small and large grating). B.Top: Excitatory input into the cell. Bottom: Inhibitory input into the same cell. As in A, left, middle and right correspond to a blank screen, a small grating and or a large grating. Note that the larger grating leads to a reduction of both excitation and inhibition; adapted from (Ozeki et al., 2009). C. Network model with long range excitation and local inhibition. Excitatory neurons within a local population excite themselves (feedback arrow), and also send excitatory input to inhibitory cells (downward arrows). Inhibitory neurons project to local excitatory neurons.

Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014),

Biological Modeling of Neural Networks

Week 8 – Continuum models: Cortical fields and perception

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Reading for week 8.3-8.5
NEURONAL DYNAMICS
Ch. 18.

Cambridge Univ. Press

8.1 Transients
- sharp or slow

8.2 Spatial continuum
- model connectivity
- cortical connectivity

8.3 Solution types
- uniform solution
- bump solution

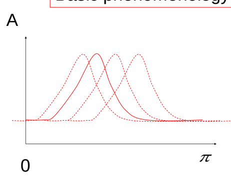
8.4. Perception

8.5. Head direction cells

8. 5: Bump solution

Basic phenomenology

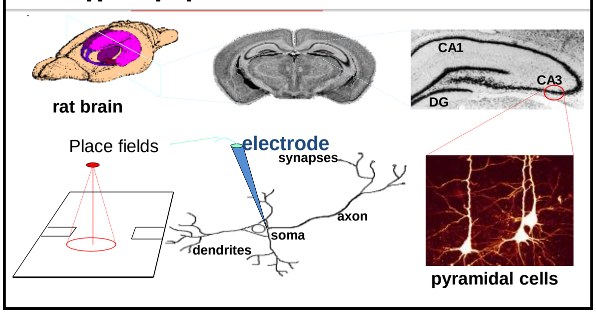
II: Bump formation



strong lateral connectivity

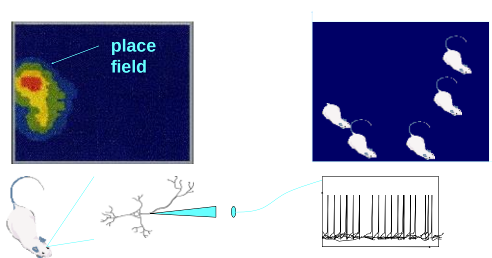
Possible interpretation
of head direction cells:
always some cells active
→ indicate current orientation

8. 5: Hippocampal place cells



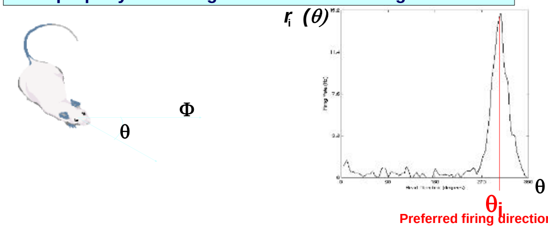
8.5: Hippocampal place cells

Main property: encoding the animal's location



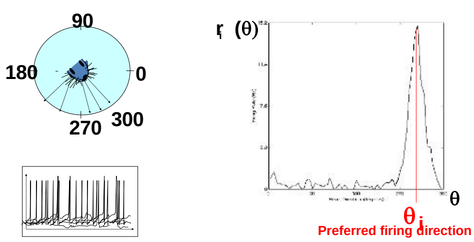
8.5: Head direction cells

Main property: encoding the animal 's heading



8. 5: Head direction cells

Main property: encoding the animal 's allocentric heading



8. 5: Head direction cells

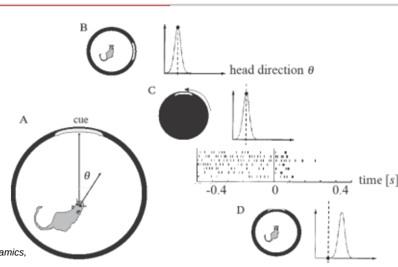


Image: Neuronal Dynamics, Gerstner et al., Cambridge Univ. Press (2014), Adapted from Zugaro et al. (2003), J. Neurosci. 23:3478-3482