Tidal and wave power

Learning objectives

- Explain the origin of tidal power, how we can exploit it, its advantages and its (real) potential
- Compare tidal turbines with wind turbines
- Explain the origin of wave power, how it is exploited, and its (real) potential

Tidal power

Tides result from the sum of gravitational pulls of the Moon and Sun on the surface of the spinning Earth.

The shape of the shore and adjacent seafloor affects the **tidal range** (difference between high and low tides) along specific coast-lines.



The origin of tides

Earth orbits around its axis in 24h, while the Moon orbits around Earth in 28 days. Someone standing at the coast at not very high latitudes therefore sees 2 high tides and 2 low tides during one day (once every $6\frac{1}{4}$ h).



Explanations

- when Sun and Moon align to 'pull' together on the Oceans' masses, tides are heighest and called 'spring' (New Moon, Full Moon)
- when Sun and Moon misalign 'pulling' on the Oceans' masses at 90° angle (1st & 3rd quarter), tides are lowest and called 'neap'.
- Earth orbits around its axis in 24 h, while the Moon orbits around Earth in 28 days; i.e. for 1 terrestrial day, the Moon is almost 'stationary' (it moves on 1/28th of its orbit, or by 360/28 = 13°)
- ⇒ a person standing at the coast at not very high latitudes sees 2 high tides and 2 low tides during one day (once every ≈6¼ h).

(it's $\approx 61/_4$ h and not 6h because the Moon is not 'stationary': for the person to find the Moon in the same position he has to execute 1 rotation (2π rad) + x rad where

$$t[day] = \frac{x[rad]}{\omega_{Moon}[rad/day]} = \frac{(2\pi + x[rad])}{\omega_{Earth}[rad/day]} \implies t = \frac{28}{27} day = 24.89h$$

$$\Rightarrow 1 tide = \frac{24.89}{4}h = 6.22h = 6h13m20s$$

ME460 Renewable Energy

Tide mechanism



'Tides' = twice-daily rises and falls of water level relative to land



The **Coriolis** force makes the tidal range larger at **West** banks than at East banks (i.e. higher at the European Atlantic Coast than at the American Atlantic Coast).

Gravitational forces of Moon and Sun

Newton's Law of gravitation : F = 0

$$F = G \frac{M_1 M_2}{d^2}$$

G	= 6.674 x 10 ⁻¹¹ N m ² kg ⁻² , universal gravitational constant
M _{Sun}	= 1.989 x 10 ³⁰ kg (333'000 x Earth's mass)
M _{Earth}	= 5.9736 x 10 ²⁴ kg
M _{Moon}	= 7.3477 x 10 ²² kg (1.2% of Earth's mass)
d _{Earth-Sun}	= 1.496 x 10 ¹¹ m (150 million km)
d _{Earth-Moon}	= 3.843 x 10 ⁸ m (0.384 million km)

\rightarrow F _{Sun-Earth}	= 3.54 x 10 ²² N
$ ightarrow F_{Moon-Earth}$	= 1.98 x 10 ²⁰ N

Ratio of the forces of gravity : 'Sun-Earth' = 179 * 'Moon-Earth'

It's the <u>difference</u> of gravity between 'near' and 'far' side of the Earth that counts

Earth diameter = 12.756 10⁶ m Near side of Earth is 12756 km closer to both the Sun and the Moon than its far side. $d_{Earth-Sun} = 1.496 \times 10^{11} \text{ m}$ $\rightarrow \Delta d_{near-far} = 0.00856 \% \rightarrow \Delta F_{near-far} = 0.0173\%$ (square of distance dependence) $d_{Earth-Moon} = 3.843 \times 10^8 \text{ m}$ $\rightarrow \Delta d_{near-far} = 3.3\% \rightarrow \Delta F_{near-far} = 7\%$



The acceleration force due to gravity of the Sun is 179 times that of the Moon. The **Sun's tidal effect** (the difference in force between far and near side of the Earth) is then 179 * 0.0173% = 3.1%, compared to the **Moon's (7%)**.

Hence total tidal force is for 2/3rd due to the Moon, and for 1/3rd due to the Sun.

Exploitation of tidal power

1. Based on height difference:

- dams built in estuaries or coastal bays with important tidal range
- so-called 'lagoons' built out in the sea

$$P = \rho_{H_2O} \cdot V g \Delta h \longleftarrow$$

- 2. Based on the moving water stream :
 - different tidal stream turbine designs
 - Iike 'underwater' wind turbines

$$P = \frac{1}{2} \rho_{H_2O} . C_P . Av_{H_2O}^3$$

20 to 50%, depending on design

typically 1-2 m/s (ocean current)

Height difference: dams in bays

To be a practical source for electricity generation, the **tidal** range in a coastal area must typically be at least 4 m.

Only a handful of suitable tidal power station locations have been identified worldwide.



N.B.



TIDE COMING IN





One-way / two-way exploitation

The variation of the sea level resulting from the tide movements can be exploited to produce electricity in two different ways: **simple effect** and **double effect**

Simple effect: for middle tides or neap tides



Double effect

For important tides, the use of the sea and estuary variation levels is based on a double effect cycle with 6 transitions per cycle.



Double effect:

- A Inverse turbining
- **B** Open gates
- **C** Pumping
- **D** Direct turbining

Example : La Rance (F)



Largest and oldest commercial tidal power plant in operation

Construction:	1960 - 1967					
Dam:	330 m long					
Tide pool reservoir:	22 km ²					
Average tidal range:	8 m (max 13.5 m)					
24 Bulb turbines (5.4 m diameter), rated at 10 MW _e each (240 MW_e total),						
connected to the 225 kV French transmission network						

La Rance (F) tidal power plant

The axial flow turbines allow generation on both floods and ebbs of the tide (**double effect**); they are also designed to **pump** water into the basin.

Number of units: 24 Installed power: 240 MW_e Net power: 544 GWh/yr → avg power: 60 MW_e (taking year-round operation)

Turbines: horizontal Kaplan 4 blades, blade inclination -5° to + 35°

Alternators: rated power: 10 MW/unit Output voltage: 3.5 kV



For comparison: Swiss electricity ~ 57 TWh ≈ 100 x La Rance

Example : UK

Other plants than La Rance (F) are operated in Canada and Russia, and others could be installed in certain other areas, e.g. the UK (Severn Barrage).



spring tide: 11 m => max **14.3** GW_e neap tide: 6 m => max **3.9** GW_e

artist's impression of hypothetical Severn Barrage, UK



ME460 Renewable Energy



Tidal power 'on-stream'

Tidal stream generators draw energy from underwater **currents** in much the same way that **wind generators** are powered by the wind.

The much higher **density** of water vs. air gives significant power to a single generator.

The technology is at the early stage of development and requires significantly more research.



Tidal power 'on-stream' : Swan-turbine

Particular type of tidal stream turbine

Blades are connected directly to the electrical generator, without a gearbox.

A "gravity base" large concrete block holds it to the seabed.

The blades are **fixed pitch** (more reliable), rather than actively controlled.



Tidal power 'on-stream' : sea-flow propeller

Several prototypes have shown promise; in the UK, a 300 kW Seaflow marine current propeller type turbine was first tested in 2003, followed by a 1.2 MW SeaGen unit in Strangford Lough (Northern Ireland) in 2008.

'Blue Energy' (CAN) plans to install large array tidal current devices in 'tidal fences', based on a vertical axis turbine design



http://www.youtube.com/watch?v=lzc9-V9DSew&feature=related



Tidal 'fences'



'Darrieus' tidal turbine

Four fixed vertical blades connect to a rotor shaft that drives an integrated gearbox/generator.

The turbine sits in a concrete marine casing; the casing directs water through the turbine and supports the generator and other machinery.

Water flowing either way spins the turbine, letting the unit generate power during a tide's ebb and flood.



Darrieus-type device

'Gorlov' helical tidal turbine

The turbine rotates regardless of the direction of the water current.

The Gorlov turbine captures 35% (=C_p) of the water energy, compared to 23% for a straight Darrieus turbine and 20% for a conventional turbine.







Relation to wind turbines



Estimate of tidal power (UK)



D MacKay, 'Sustainable Energy'

We want to estimate the power extractable from an underwater turbine farm that occupies a certain (sub)surface of the ocean.

In 1st approximation, we use the formula

$$P\left[\frac{W}{m_{Turbine_area}^2}\right] = \frac{1}{2}\rho_{H_2O}.C_P.Av_{H_2O}^3$$

assuming Cp \approx 35% and where v can be 0.5 m/s to 5 m/s (typically : 1 m/s). <u>Remark</u> : 1 "knot" = 0.5 m/s

The turbines should be spaced apart by minimum 5 diameters to their neighbours, i.e. each turbine would occupy an area of $25 \cdot D^2$ = 100 \cdot A / \pi (since A = \pi \cdot D^2 / 4) This leads to the simplified expression:

$$P\left[\frac{W}{m_{ocean-subsurface}^2}\right] = \frac{0.5 \cdot 1000 \cdot 0.35 \cdot A \cdot v^3}{\frac{100 \cdot A}{\pi}} = 5\pi \cdot 0.35 \cdot v^3 \approx 5.5v^3$$

This is **substantial power**.

Placing a sea-turbine farm over an area of 1 km² (e.g. 100 rows by 100 columns of turbines (10'000 turbines!) of 4 m², D=2 m, spaced 5D=10 m apart) in a location where **ocean current** is 2 m/s (7 km/h), will generate 44 MW_e of electrical power (4.4 kW_e per turbine). (The equivalent **wind** turbine of 4 m², even at avg v³ = 1000 m³/s³, would generate <1 kW_e, i.e. **5 times less**)

It is moreover argued that tidal current in shallow water (i.e with depth *d* of the same magnitude as the length of the wave (λ), typically 100 m) can be derived to be (*cf. « Wave-power »* chapter):



ME460 Renewable Energy

Week	13	-	May	19	2020

Speed v (m/s)	Power W / m²
0.5	0.7
1	5.5
2	44
3	150
4	350
5	687



Tidal stream farm

The sea-turbine power values (W per m² of seafloor) even with only 1-2 m/s ocean current are only **peak** values.

D

neaps





Due to the sinusoidal back-and-forth rocking of tides every 6h, the average effective current is ≈ 0.4 peak current. At neap tide, power is further reduced.

(a)

15

max spring power = $4 \times \max \operatorname{neap} \operatorname{power}$ average max power = $(\operatorname{spring}_{\max} + \operatorname{neap}_{\max}) / 2$ average power $(W/m^2) = 0.4 \cdot \operatorname{avg} \max \operatorname{power}$

D MacKay, 'Sustainable Energy'

5

šorinas

speed (m/s)

1.5

1

0

n

0.5

-0.5 -1 -1.5

Advantages of tidal power

- entirely predictable; reliable; year-round operation
- known hardware; but with challenges:
 - salt corrosion
 - silt accumulation
 - entanglement of floating matter
 - **•**
- in particular when comparing tidal turbines vs. wind turbines:
 - no visual impact !
 - low turbine safety factor needed

(there are no underwater tidal 'storms' as opposed to wind storms)

• smaller turbines for the same power, due to $\rho_{H,O} >> \rho_{air}$

Wave power

Wave power: origin and characteristics

Ocean surface wave motion is **3rd generation sun power**: sun heat creates the winds, and **winds create the waves**.

The wavelength λ (m) and period T (s) depend on the windspeed. The height h of the wave depends on the time the wind has been blowing! Like with wind, you can extract power only <u>once</u>.

The potential is especially interesting on the Atlantic (direction west \rightarrow east); the feasibility of wave power is therefore especially investigated in the UK.

Power Generators :

- = either coupled to *floating* devices (oscillating buoys)
- = or turned by *air*, *displaced* by waves in a hollow concrete structure

Numerous practical problems exist

Energy stored in ocean surface waves: calculation

Two components:

- *kinetic* (horizontal: water flow)
- *potential* (vertical: level difference between crest and trough of the wave)



The energy that can be extracted from a wave corresponds roughly to the **potential energy liberated by the collapse of the wave** crest, i.e. the energy obtained when the mass M of the "half-wave" falls from a **height 2** y_c , where y_c represents the mass centre of the "half-wave".



Position
$$y_c$$
 of the mass centre = ?:

$$M_z = \frac{\rho y_0 \lambda}{\pi}$$

$$y_c = \frac{\rho}{\frac{\lambda/2}{9}} \int_{0}^{\sqrt{2}} \frac{y_c \sin(2\pi x/\lambda)}{\sqrt{9}} = \frac{\rho}{\frac{1}{9}} \int_{0}^{\sqrt{2}} \frac{y_c \sin(2\pi x/\lambda)}{\sqrt{9}} = \frac{\rho}{\sqrt{9}} \int_{0}^{\sqrt{2}} \frac{y_c^2}{2} \Big]_{0}^{y_c \sin(2\pi x/\lambda)} dx$$

$$y_c = \frac{\pi}{y_0 \lambda} \int_{0}^{\lambda/2} \left(\frac{y_0^2}{2}\right) \sin^2(2\pi x/\lambda) dx = \frac{\pi y_0}{2\lambda} \int_{0}^{\lambda/2} \frac{\sin^2(2\pi x/\lambda) dx}{\sqrt{9}}$$

$$\frac{\int \sin^2(x) dx = \frac{1}{2} \int (1 - \cos(2x)) dx \quad \text{because:} \quad \cos(2x) = \cos^2(x) - \sin^2(x) \\ \cos^2(x) + \sin^2(x) = 1$$

$$\int \sin^2(\frac{2\pi x}{\lambda}) dx = \frac{1}{2} \int \left(1 - \cos(\frac{4\pi x}{\lambda})\right) dx$$
Substitute: $t = \frac{4\pi x}{\lambda}, \quad dt = \frac{4\pi}{\lambda} dx, \quad dx = \frac{\lambda}{4\pi} dt$

$$\frac{1}{2} \int dx - \frac{1}{2} \int \frac{\lambda}{4\pi} \cos(t) dt = \frac{x}{2} - \frac{\lambda}{8\pi} \int \cos(t) dt \\ \left(\frac{x}{2} - \frac{\lambda}{8\pi} \sin(t)\right) \Big|_{0}^{\lambda/2} = \left(\frac{x}{2} - \frac{\lambda}{8\pi} \sin(\frac{4\pi x}{\lambda})\right) \Big|_{0}^{\lambda/2} = \frac{\lambda}{4} - 0 - \frac{\lambda}{8\pi} (\sin 2\pi - \sin 0) = \frac{\lambda}{4}$$

$$y_c = \frac{\pi y_0}{8} \\ \approx 0.39 y_0$$

Energy stored in ocean surface waves: result

Total energy W (=m.g.h) *per unit length (=Joule/m)* of the wave *front (=z)* due to the collapse of the wave, generated during time of passage of the wave (period T):

$$W_{\text{wave}} = M_z \cdot g \cdot 2y_c = 2 \frac{\rho \cdot y_0 \cdot \lambda}{\pi} \cdot g \cdot \frac{\pi \cdot y_0}{8} = \frac{\rho \cdot g \cdot y_0^2 \cdot \lambda}{4}$$

Wave characteristics, e.g.:
$$\lambda = 100 \text{ m}$$
$$y_0 = 1 \text{ m}$$
$$W = 250 \text{ kJ / m}$$

How to compute the *power* in a wave? (=W_{wave} / T) (T=period)

- The water particles move themselves (= kinetic energy) but at much lower speed, v, than the wave, C₀ (as demonstrated by the slow movement of objects floating on water, 'under' which the waves travel faster).
- ⇒ we need to know this **speed v** with which the water itself moves (**≠ wave speed c**₀ = λ / T !), in order to find an expression for T
- \Rightarrow a relation between the water velocity v and the wavelength λ can be physically derived (next slide)

Correlation between water velocity v and wavelength λ :

Consider a reference frame in movement with the wave: cylindrical surfaces, tangential to the trough and crest of the wave respectively, rotating with the angular velocity $\mathbf{v} = \mathbf{c}_0/\lambda = 1/T = \mathbf{v} / (2\pi y_0)$ (=> speed $\mathbf{v} = 2\pi y_0 \mathbf{v}$) (The speed ratio of 'wave' vs. 'water' = $c_0/\mathbf{v} = \lambda/2\pi y_0$ = wavelength vs. $\approx (\pi \cdot \text{waveheight})$) We then calculate the difference in **kinetic** energy of the water particles at the wave crest and in the trough: E_{kin} (out – in) = 2.v



Kinetic & potential energy of the water in the wave:

Variation of the kinetic energy of the water particles:

$$c_{0}/\lambda \quad \text{speed } v = 2\pi y_{0}v$$

$$\Delta w_{\text{kin, water}} = m \cdot c_{0} \cdot 2v = m \cdot c_{0} \cdot 2 \cdot 2\pi \cdot y_{0} \cdot v = (4 \cdot m \cdot c_{0}^{2} \cdot \pi \cdot y_{0})/\lambda$$

$$E_{kin, wat} = \left[\frac{1}{2}m(c_{0} + v)^{2} - \frac{1}{2}m(c_{0} - v)^{2}\right] = \frac{1}{2}m(2c_{0}v - (-2c_{0}v)) = 2mc_{0}v$$

$$cf. \text{ Euler equation}: \text{ mass energy w (J/kg)} = v_{e} \cdot \Delta v_{t}$$

$$= \text{ entrainment velocity * (tangential velocity difference out-in)} = c_{0} * 2v$$
Corresponding variation of the **potential** energy
of the water particles:
$$\Delta w_{\text{pot, water}} = m \cdot g \cdot 2 \cdot y_{0}$$
Since
$$\Delta w_{\text{kin, water}} = \Delta w_{\text{pot, water}} \text{ (like for all oscillating movement),}$$
we can derive:

$$\frac{2\pi}{g}c_{0} = \frac{\lambda}{c_{0}} = T = 0.64 c_{0}$$

$$\frac{4mc_{0}^{2}\pi y_{0}}{\lambda} = 2mgy_{0} \Rightarrow c_{0}^{2} = \frac{g\lambda}{2\pi} \Rightarrow c_{0} = \left(\frac{g\lambda}{2\pi}\right)^{1/2}$$

$$wave \text{ period T (s)}$$

$$T = \frac{\lambda}{c_{0}} = \frac{\lambda}{\sqrt{g\lambda'/2\pi}} = \left(\frac{2\pi\lambda}{g}\right)^{1/2} = \sqrt{0.64 \cdot \lambda}$$

=> power P in the wave (W_{wave} / T) : result

The energy per unit length of the wave front (J/m) was found before as:

$$W_{\text{wave}} = \frac{\rho \cdot g \cdot y_0^2 \cdot \lambda}{4}$$

The *power* per unit length of the wave front (W/m) is then this wave *energy* (J/m) divided by the period T (s), found on the previous slide:

$$P_{wave} = \frac{W_{wave}[J]}{T[s]} = \frac{\rho g y_0^2 \lambda_{4}}{\sqrt{\frac{2\pi\lambda}{g}}} = \frac{\rho g^{\frac{3}{2}}}{\sqrt{\sqrt{2\pi}}} y_0^2 \sqrt{\lambda} \approx 3064 \cdot y_0^2 \sqrt{\lambda}$$

$$rac{const: 3064}{const: 3064}$$

$$P_{wave} = \frac{W_{wave}[J]}{T[s]} = \frac{\rho g y_0^2 \lambda_{4}}{\sqrt{\frac{2\pi\lambda}{g}}} = \frac{\rho g^{\frac{3}{2}}}{\sqrt{\sqrt{2\pi}}} y_0^2 \sqrt{\lambda} \approx 3064 \cdot y_0^2 \sqrt{\lambda}$$

=> numerical application:

 $P[W/m] = 3064 \cdot y_0^2 \cdot \lambda^{1/2} \quad (y_0, \lambda \text{ in } m)$



Wave characteristics: $\lambda = 100 \text{ m}$ $y_0 = 1 \, \text{m}$ waveheight = 2 m T = 8 s, v = 1 / 8 $c_0 = 12.5 \text{ m/s}$ v = 0.8 m/s

 $P = 30 \, \text{kW} / \text{m}$



The wavelength λ (m) and period T (s) depend on the windspeed. The height h (= $2y_0$) of the wave depends on the time the wind is blowing

Week 13 - May 19, 2020

wave speed (m/s)

Wave power inspires ingenuity...

The oscillating up-down motion of waves is captured by different approaches, in a rotary or piston motion to drive a generator.



'Nodding duck' oscillating device



In shallow water (approaching the shore), the wave energy is lost in sea-bottom friction: from 100 m depth to 15 m depth, 70% is lost.



 \rightarrow a 30 kW/m wave becomes a 10 kW/m wave

Buoy piston oscillating device

http://www.youtube.com/watch?v=90AcxxwoPu0&feature=fvsr http://www.youtube.com/watch?v=tt_lqFyc6Co&feature=related

In the permanent magnet linear generator buoy, waves cause the coil to move up & down relative to the fixed magnetic shaft. Voltage is induced and electricity generated. A buoy could potentially generate 250 kW power.



'Pelamis' - device

Semi-submerged, articulated structure composed of cylindrical sections linked by hinged joints.

The wave-induced motion of the joints is resisted by hydraulic rams, which pump high-pressure oil through hydraulic motors via smoothing accumulators.

The hydraulic motors drive electrical generators to produce electricity. Power is fed down a single umbilical cable to a junction on the sea bed.





'Pelamis'-device

http://www.youtube.com/watch?v=F0mzrbfzUpM



Application example

1 Pelamis-snake : **130 m long**; **3.5***m* diameter D. 750 kW max wave power (i.e. it can take up to 200 kW/m waves!); 350 ton steel (**500 kg/kW**!); *vastly overdimensioned* !



1 km² area = 400 m long x **2.5 km wide incoming wave**, 3 rows of 40 Pelamis (120 Pelamis). Assume **30 kW / m** waves ($c_0 = 12.5 \text{ m/s}$, $y_0 = 1 \text{ m}$) $\rightarrow 2.5 \text{ km}$ wide equals 75 MW_e

1 every 60 m in each row, 1 every 20 m in the wave front, staggered from row 1 to row 3

 \rightarrow effective wave-front use = **3.5m** / 20m = 17%

Incoming power used =

 $3.5 \text{ m} * 30 \text{ kW/m} (1 \text{ Pelamis}) * 120 = 12.6 \text{ Mw}_{e}$

- \rightarrow suppose 50% efficiency : 6.3 MW_e
- → 55 GWh_e per year gross electricity, assuming full load 8760 h/yr (460 MWh_e/yr for 1 snake)

The same power with same wind (12.5 m/s), at full load (8760h/yr), could be achieved with one 12 m diameter 50 kW_e wind turbine for 1 Pelamis! Week 13 - May 19, 2020





Real potential...?

- assume the UK can 'block' 10% of its coast with wave generators ...
- the effective wave front use could be $\approx 17\%$
- assume conversion efficiency $\approx 70\%$

→ 0.1 * 0.17 * 0.7 ≈ 1.2%

- a net real power of 360 MW_e would result
 (≈ of the same magnitude as the La Rance
 __tidal scheme alone(!), 240 MW_e)
- again one notices the discrepancy between *potentiality* and *reality*, and the huge installation scales involved.
- wave power is marginal, yet interesting for islands