Name / first name

Exam
Neural Networks and Biological Modeling

- Write your name in readable letters on top of this page
- The exam lasts 160 minutes
- **All** responses must be on these exam sheets
- Except for one paper A4 of **handwritten** notes, no documentation is allowed.
- You may use a pocket calculator, but not a programmable computer
- The total number of points that can be achieved is 48

Evaluation

Section 1: ....../6 pts

Section 2: ....../10 pts

Section 3: ....../11 pts

Section 4: ....../7 pts

Section 5: ....../7 pts

Section 6: ....../7 pts

————————————–

....................../48

The exam has 8 pages, the back of the pages is also used!

Question 1: Ion channel (6 points)

We consider the following model of a ion channel

$$I_{ion} = g_0\, r^4\, s^1 (u - E)$$

where u is the membrane potential and $g_0 = 1$[arbitrary units] and $E = 0$ are two constants. The variables r and s are given by

$$\frac{dr}{dt} = -\frac{r - r_0(u)}{\tau_r(u)}$$

$$\frac{ds}{dt} = -\frac{s - s_0(u)}{\tau_s(u)}$$

We assume

$r_0(u) = 0$ for $u < 20$mV and r_0=1 for $u \geq 20$mv.

$s_0(u) = 1$ for $u < 50$mV and s_0=0.1 for $u \geq 50$mv.

$\tau_r(u) = 0.1$ms for $u < 20$mV and $\tau_r(u) = 2 * (u - 20mV)ms/mV$ for $20 \leq u \leq 50$mv and $\tau_r(u) = 60ms - (u - 50mV)ms/mV$ for $50 \leq u \leq 100$mv

$\tau_s(u) = 10$ms independent of u.

(a) What is the meaning of the variable r? .... /1 point

(b) Under voltage clamp, the voltage had been held constant for a long time at $u = 0$ and was switched at $t = 0$ to $u = 40mV$ and is held there for 1 second. The current can be measured.

What is the value of the current at $t = 1000ms$? I = ... /1 point

Use the space below to sketch the time course of the current that you expect for $0 \leq t \leq 100ms$. **Pay particular attention to the area around $t \approx 0$.**

/2 points

(c) At $t = 1000ms$, the voltage is switched from 40 mV to 80 mV. Use the space below to sketch the time course of the current for $990ms < t < 1100ms$.

/2 points

QUESTION 2: PHASE PLANE ANALYSIS (10 points)

An integrate-and-fire neuron model with adaptation is described by the two differential equations

$$\frac{du}{dt} = F(u) - w + I \tag{1}$$

$$\tau \frac{dw}{dt} = -w + a\,(u-1) \tag{2}$$

If $u > 5$ the variable u is reset to $u = 0$. The variable w is increased by an amount 2 during reset.

We take

$$F(u) = -(u-1) \quad \text{for } u \leq 2 \tag{3}$$

$$F(u) = 4u - 9 \quad \text{for } u > 2 \tag{4}$$

(a) Plot the nullclines in the phase plane (u, w) for $I = 0$ and $a = 0.2$ using the space here:

/2 points

(b) In the same graph, add representative arrows indicating qualitatively the flow in different regions of the phase plane (you may assume $\tau = 2$). /2 points

(c) In the same graph, indicate a trajectory in the phase plane, after a stimulus $I(t) = 0.5\delta(t)$ has been applied [δ denotes the Dirac delta function]. /1 point

(d) In the same graph, indicate a trajectory in the phase plane, after a stimulus $I(t) = 2\delta(t)$ has been applied [δ denotes the Dirac delta function]. /2 points

(e) Under the assumption of a CONSTANT current $I = I_0$, what is the minimum current I_C so that for $I > I_C$ the neuron shows regular firing?

$I_C =$ .... /1 point

(f) assume that $\tau \ll 1$ (e.g. $\tau = 0.01$) and approximate the system of two equations by a single equation. Give this equation

.... /1 point

(g) interpret your result in (f) in the regime $u < 1$ and $a = 1$. .... /1 point

Question 3: Biological Modeling (11 points)

Two students who have taken the class in Neural Networks and Biological Modeling discuss the equation

$$D\frac{dx(t)}{dt} = -x(t) + G(a\,x(t) - b) + y(t) \tag{5}$$

where G is some **nonlinear** function with $a > 0$, and x a real variable and $D > 0$ a parameter.

(a) Student A says: This is the equation of a non-linear integrate-and-fire neuron under current injection. Do you agree?

Mark your choice by a cross and give the argument

[ ] Yes, I agree because I can choose $G(z) =$ ....
and give the following interpretation:
D is ....
a is ....
b is ....
x is ....
y is ....
Furthermore, I have to assume ....
....
[ ] No, this cannot be the equation of a nonlinear integrate-and-fire neuron because
....
....
....

/3 points

(a) student B says: This is the equation of a population of neurons in cortex. Do you agree?

Mark your choice by a cross and give the argument

[ ] Yes, I agree because I can choose $G(z) =$ ....
and give the following interpretation:
D is ....
a is ....
b is ....
x is ....
y is ....
Furthermore, I have to assume ....
....
[ ] No, this cannot be the equation of a cortical population because
....
....
....

/3 points

(c) Three other students with backgrounds in physics, electrical engineering, or life sciences who have not taken the class, join the discussion and give a different interpretation.

Can you think of an example of a valid interpretation of the above equation in a field outside neural networks?

....
....
....

/1 point

(d) Take $G(z) = z^2$ and $a = b = 1$. We consider $y = c$ as a constant under our control.

Calculate the fixed points (if there are any) for $c = 0$ and $c = 1$ and $c = 3$. The fixed point(s) is/are

for $c = 0$ ....

for $c = 1$ ....

for $c = 3$ ....

/2 points

(e) Consider the stability of the fixed point(s) for $c = 1$ and state your result (why, give the reason).

....
....

/2 points

Question 4: Stochastic Neuron Model (7 points)

A stochastic neuron model described by a Poisson process fires at a rate $r = 10\text{Hz}$ if $u > u_0$ and does not fire if $u < u_0$.

The voltage u is given by

$$\tau \frac{du}{dt} = -u + RI(t) \tag{6}$$

with $\tau = 0.01$ [units are in seconds]. At $t = 0$, the voltage has an initial value $u = 0$.

Each experimental trial lasts for 20 seconds. During the first 10 seconds of a trial, the stimulus $RI(t)$ has a value $RI(t) = 2u_0$. During the last ten seconds it vanishes.

(a) Calculate $u(t)$ for $0 < t < 20$.

$u(t) =$ ....

....

/1 point

(b) Show that $u(t)$ stays exactly 10 seconds above u_0.

....
....
....

/2 points

(c) How many spikes is the neuron expected to fire during 1 trial?

....

/1 point

(d) Suppose we compare two trials. What percentage of spikes are expected to coincide between the two trials? We count spikes in the two trials as coincident if a spike in trial 1 occurs within $\pm 10ms(0.01\text{seconds})$ of a spike in trial 2.

....
....
....

/2 points

(e) Without calculation, what is the expected percentage of coincidences if we replace the Poisson Neuron by a standard leaky integrate-and-fire neuron?

....

/1 point

Question 5: Stochastic Modeling (7 points)

(i) Continuity equation. In a population of integrate-and-fire neurons the distribution of membrane potentials $p(u)$ evolves according to

$$\tau \frac{d}{dt} p(u,t) = -\frac{d}{du} J(u,t) + r(t)\,\delta(u) \tag{7}$$

(a) What is the meaning of the term $r(t)$, why do we need this term?
....
....

/1 point

(b) In a specific model we have a flux term
$J(u,t) = p(u,t)\,\{-u(t) + u_0 + u_1 \exp[\beta\, u(t)] + u_2 \sin(\omega\, t)\}$. What can you say about each term in particular with respect to the
the neuron model ....
....
the noise ....
the input ....

/2 points

(ii) We now consider a linear neuron model under stochastic spike arrival. The neuron has two input with weights w_1 and w_2 which are not the same. Each input causes a postsynaptic potential $\alpha(s) = 1$ for $0 < s < 1$ and zero elsewhere. The total membrane potential is

$$u(t) = \sum_{t_1^f} w_1 \alpha(t - t_1^f) + \sum_{t_2^f} w_2 \alpha(t - t_1^f) \tag{8}$$

The sums are over all spike times arriving at the first and second synapse, respectively. Spikes are generated by two independent homogeneous Poisson processes and arrive stochastically at a rate r_1 at synapse 1 and r_2 at synapse 2.

(c) What is the mean membrane potential?
$\langle u \rangle = \bar{u} =$ ....
....

/2 points

(d) What is the variance of the membrane potential?
$\langle (u - \bar{u})^2 \rangle =$ ....
....

/2 points

QUESTION 6: REINFORCEMENT LEARNING (7 points)

We study the SARSA algorithm in the following problem.

From a start state S the rat can go left to a state L. If it goes left again it receives a reward -1. If it goes right from L it receives a reward +1.

Starting from S it can also go to the right state R. If it goes right again it receives a reward -1. If it goes left from r it receives a reward +0.5.

Each time the rat receives a reward, it restarts from S.

(a) In a table of Q values, how many entries do you need? .... /1 point

(b) Initialize all Q values at $Q = 1$. The choice of actions is deterministic, but with a slight left-bias in case of a tie. More specifically, in an arbitrary state s the rat takes action left if $Q(s, left) \geq Q(s, right)$ and else the action right.

Update your Q values with the SARSA rule $\Delta Q(s,a) = 0.5\,[r - (Q(s,a) - Q(s',a')]$.

Give the set of Q-values after the first trial.

....
....
....
....

/1 point

(c) Give the set of Q-values after the second trial.

....
....
....
....

/1 point

(c) Give the set of Q-values after the third trial.

....
....
....
....

/1 point

(d) What is the best trajectory? Will the rat eventually find the best trajectory? How long will it take? Why?

....
.... /1 point

(d) Now assume that we use a Sarsa algorithm with eligibility trace [SARSA(λ)] with $0 < \lambda < 1$. Will the rat eventually find the best trajectory? How long will it take? Why?

....
....
....
.... /2 points