Deep Learning Crash Course



- Single Layer Perceptrons
- Multiple Layer Perceptrons
- Convolutional Neural Nets

Linear Binary Classification



Two classes shown as different colors:

- The label $y \in \{-1,1\}$ or $y \in \{0,1\}$.
- The samples with label 1 are called positive samples.
- The samples with label -1 or 0 are called negative samples.

Signed Distance



Signed Distance in 3D



 $\mathbf{x} \in R^3, 0 = ax + by + cz + d$ $\tilde{\mathbf{x}} \in R^4, \tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}} = 0$

Signed distance $h = \tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}}$ if $w_1^2 + w_2^2 + w_3^2 = 1$.

Signed Distance in N Dimensions



Binary Classification in N Dimensions

Hyperplane: $\mathbf{x} \in \mathbb{R}^N$, $\tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}} = 0$, with $\tilde{\mathbf{x}} = [1 | \mathbf{x}]$.

Signed distance: $\tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}}$, with $\tilde{\mathbf{w}} = [w_0 | \mathbf{w}]$ and $||\mathbf{w}|| = 1$.

Problem statement: Find $\tilde{\mathbf{w}}$ such that

- for all or most positive samples $\tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}} > 0$,
- for all or most negative samples $\tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}} < 0$.

Logistic Regression $y(\mathbf{x}; \tilde{\mathbf{w}}) = \sigma(\tilde{\mathbf{w}} \cdot \tilde{\mathbf{x}})$

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- When the noise is Gaussian, this is the maximum likelihood solution.
- $y(\mathbf{x}; \tilde{\mathbf{w}})$ can be interpreted at the probability that \mathbf{x} belongs to positive class.

Non Separable Distribution



- Logistic regression can handle a few outliers but not a complex nonlinear boundary.
- How can we learn a function y such that $y(\mathbf{x}; \tilde{\mathbf{w}})$ is close to 1 for positive samples and close to 0 or -1 for negative ones?

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—> Use LOTS of hyperplanes.

Reformulating Logistic Regression



$$y(\mathbf{x}) = \sigma(\mathbf{w} \cdot \mathbf{x} + b)$$

$$\mathbf{x} = \begin{bmatrix} x_1, x_2, \dots, x_n \end{bmatrix}^T$$

$$\mathbf{w} = \begin{bmatrix} w_1, w_2, \dots, w_n \end{bmatrix}^T$$



Repeating the Process



$$h_1 = \sigma(\mathbf{w}_1 \cdot \mathbf{x} + b_1)$$
$$\mathbf{w}_1 = \begin{bmatrix} w_{11}, w_{12}, w_{13}, w_{14} \end{bmatrix}^T$$

$$h_2 = \sigma(\mathbf{w}_2 \cdot \mathbf{x} + b_2)$$
$$\mathbf{w}_2 = \left[w_{21}, w_{22}, w_{23}, w_{24}\right]^T$$

•

$$h_{H} = \sigma(\mathbf{w}_{H} \cdot \mathbf{x} + b_{H})$$
$$\mathbf{w}_{H} = \left[w_{H1}, w_{H2}, w_{H3}, w_{H4}\right]^{T}$$

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Repeating the Process









Multi-Layer Perceptron



• The process can be repeated several times to create a vector **h**.





Multi-Layer Perceptron



$$\mathbf{h} = \sigma_1 (\mathbf{W}_1 \mathbf{x} + \mathbf{b}_1)$$
$$\mathbf{y} = \sigma_2 (\mathbf{W}_2 \mathbf{h} + \mathbf{b}_2)$$

- The process can be repeated several times to create a vector **h**.
- It can then be done again to produce an output **y**.

—> This output is a **differentiable** function of the weights.



ReLU



$\sigma(\mathbf{x}) = \max(0, \mathbf{x})$

- Each node defines a hyperplane.
- The resulting function is piecewise linear affine and continuous.



ReLu Behavior



 $\mathbf{h} = \operatorname{ReLu}(\mathbf{W}_1 \mathbf{x})$ $\mathbf{y} = \mathbf{w}_2^T \mathbf{h} + b_2$





Binary Case

- Let the training set be $\{(\mathbf{x}_n, t_n)_{1 \le n \le N}\}$ where $t_n \in \{0, 1\}$ is the class label and let us consider a neural net with a 1D output.
- We write In this case w2 is vector. $y_n = \sigma(\mathbf{w}_2(\sigma(\mathbf{W}_1\mathbf{x}_n + \mathbf{b}_1)) + \mathbf{b}_2) \in [0.1]$
- We want to minimize the binary cross entropy $E(\mathbf{W}_1, \mathbf{w}_2, \mathbf{b}_1, \mathbf{b}_2) = \frac{1}{N} \sum_{n=1}^N E_n(\mathbf{W}_1, \mathbf{w}_2, \mathbf{b}_1, \mathbf{b}_2) ,$ $E_n(\mathbf{W}_1, \mathbf{w}_2, \mathbf{b}_1, \mathbf{b}_2) = -(t_n \ln(y_n) + (1 - t_n) \ln(1 - y_n)) ,$

with respect to the coefficients of W_1 , w_2 , b_1 , and b_2 .

• E can be minimized using a gradient-based technique.





Multi-Class Case

Let the training set be $\{(\mathbf{x}_n, [t_n^1, ..., t_n^K])_{1 \le n \le N}\}$ where $t_n^k \in \{0, 1\}$ is the probability that sample \mathbf{x}_n belongs to class k.

• We write

$$\mathbf{y}_{n} = \sigma(\mathbf{W}_{2}(\sigma(\mathbf{W}_{1}\mathbf{x}_{n} + \mathbf{b}_{1})) + \mathbf{b}_{2}) \in \mathbb{R}^{K}$$
$$p_{n}^{k} = \frac{\exp(\mathbf{y}_{n}[k])}{\sum_{j} \exp(\mathbf{y}_{n}[j])}$$

• We minimize the cross entropy $E(\mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = \frac{1}{N} \sum_{n=1}^N E_n(\mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) ,$ $E_n(\mathbf{W}_1, \mathbf{W}_2, \mathbf{b}_1, \mathbf{b}_2) = -\sum_{n=1}^N t_n^k \ln(p_n^k) ,$

with respect to the coefficients of W_1 , W_2 , b_1 , and b_2 .



Sigmoid and Tanh



- Each node defines a hyperplane.
- The resulting function is continuously differentiable.

Non-Linear Binary Classification



Problem statement: Find **w** such that

- for all or most positive samples $y(\tilde{\mathbf{x}}; \tilde{\mathbf{w}}) > 0.5$,
- for all or most negative samples $y(\tilde{\mathbf{x}}; \tilde{\mathbf{w}}) < 0.5$.





Regression



Problem statement: Find $\tilde{\mathbf{w}}$ such that $y(\mathbf{x}; \tilde{\mathbf{w}}) \approx 100(x_2 - x_1^2)^2 + (1 - x_1)^2$



From Classification to Regression



Minimize $\sum_{i} [t_i \log(\text{sigm}(f(x_i, y_i))) + (1 - t_i) \log(1 - \text{sigm}(f(x_i, y_i)))]$ with respect to $\mathbf{W}_1, \mathbf{w}_2, b_x, b_y, b_z$.



Minimize $\sum_{i} (z_i - f(x_i, y_i))^2$, with respect to $\mathbf{W}_1, \mathbf{w}_2, b_x, b_y, b_z$.





Approximating a Surface





Minimize $\sum_{i} (z_i - f(x_i, y_i))^2$, with respect to $\mathbf{W}_1, \mathbf{w}_2, b_x, b_y, b_z$.



Interpolating a Surface







Interpolating a Surface





Adding more Nodes





Adding more Nodes







More Complex Surface



$$I = f(x, y)$$







More Complex Surface

50 L

125 nodes -> loss 2.40e-01



$$I = f(x, y)$$











Universal Approximation Theorem

A feedforward network with a linear output layer and at least one hidden layer with any 'squashing' activation function (e.g. logistic sigmoid) can approximate any Borel measurable function (from one finite-dimensional space to another) with any desired nonzero error.

Any continuous function on a closed and bounded set of R^n is Borel-measurable.

—> In theory, any reasonable function can be approximated by a onehidden layer network as long as it is continuous.





In Practice



- It may take an exponentially large number of parameters for a good approximation.
- The optimization problem becomes increasingly difficult.
- —> The one hidden layer perceptron may not converge to the best solution!



MNIST



- The network takes as input 28x28 images represented as 784D vectors.
- The output is a 10D vector giving the probability of the image representing any of the 10 digits.
- There are 50'000 training pairs of images and the corresponding label, 10'000 validation pairs, and 5'000 testing pairs.



MNIST Results



nIn = 784nOut = 1020 < hidden layer size < 120

- MLPs have **many** parameters.
- This has long been a major problem.
- —> Was eventually solved by using GPUs.

SVM: 98.6

Knn: 96.8

- Around 2005, SVMs were often felt to be superior to neural nets.
- This is no longer the case



Deep Learning



- The descriptive power of the net increases with the number of layers.
- In the case of a 1D signal, it is roughly proportional to $\prod_{n} W_{n}$ where W_{n} is the width of layer n.



Second Layer for Approximation



I = f(x, y)

1 Layer: 125 nodes -> loss 2.40e-01 2 Layers: 20 nodes -> loss 8.31e-02 501 weights in both cases





Adding a Third Layer



$$I = f(x, y)$$

2 Layers: 20 nodes -> loss 8.31e-02 3 Layers: 14 nodes -> loss 7.55e-02 501 weights 477 weights





Adding a Third Layer



$$I = f(x, y)$$

3 Layers: 15 nodes -> loss 5.93e-02 3 Layers: 19 nodes -> loss 4.38e-02 541 weights 837 weights




Multi Layer Perceptrons



0.05 L

ΈΡΞΙ

Number of weights



MLP to ResNet



Further improvements in the convergence properties have been obtained by adding a bypass, which allows the final layers to only compute residuals.





Improving the Network



Digital Images



 $\begin{array}{c} 136 \ 134 \ 161 \ 159 \ 163 \ 168 \ 171 \ 173 \ 173 \ 171 \ 166 \ 159 \ 157 \ 155 \\ 152 \ 145 \ 136 \ 130 \ 151 \ 149 \ 151 \ 154 \ 158 \ 161 \ 163 \ 163 \ 159 \ 151 \\ 145 \ 149 \ 149 \ 145 \ 140 \ 133 \ 145 \ 143 \ 145 \ 145 \ 145 \ 146 \ 148 \ 148 \\ 148 \ 143 \ 141 \ 145 \ 145 \ 145 \ 141 \ 136 \ 136 \ 135 \ 135 \ 136 \ 135 \ 133 \\ 131 \ 131 \ 129 \ 129 \ 133 \ 136 \ 140 \ 142 \ 142 \ 138 \ 130 \ 128 \ 126 \ 120 \\ 115 \ 111 \ 108 \ 106 \ 106 \ 110 \ 120 \ 130 \ 137 \ 142 \ 144 \ 141 \ 129 \ 123 \\ 117 \ 109 \ 098 \ 094 \ 094 \ 094 \ 100 \ 110 \ 125 \ 136 \ 141 \ 147 \ 147 \ 145 \\ 136 \ 124 \ 116 \ 105 \ 096 \ 096 \ 100 \ 107 \ 116 \ 131 \ 141 \ 147 \ 147 \ 145 \\ 136 \ 124 \ 116 \ 105 \ 096 \ 096 \ 100 \ 107 \ 116 \ 131 \ 141 \ 147 \ 150 \ 152 \\ 152 \ 152 \ 157 \ 157 \ 159 \ 135 \ 121 \ 120 \ 120 \ 121 \ 127 \ 136 \ 147 \ 158 \ 163 \\ 165 \ 165 \ 163 \ 163 \ 163 \ 166 \ 136 \ 131 \ 135 \ 138 \ 140 \ 145 \ 154 \ 163 \\ 166 \ 168 \ 170 \ 168 \ 166 \ 168 \ 170 \ 173 \ 177 \ 178 \ 151 \ 151 \ 153 \ 156 \\ 161 \ 170 \ 176 \ 177 \ 177 \ 179 \ 176 \ 177 \ 179 \ 155 \ 157 \\ 161 \ 162 \ 168 \ 176 \ 180 \ 180 \ 180 \ 180 \ 180 \ 180 \ 180 \ 175 \ 175 \ 178 \ 180 \ 180 \\ 180 \$



- A MxN image can be represented as an MN vector, in which case neighborhood relationships are lost.
- By contrast, treating it as a 2D array preserves neighborhood relationships.





Image Specificities



- In a typical image, the values of neighboring pixels tend to be more highly correlated than those of distant ones.
- An image filter should be translation invariant.

—> These two properties can be exploited to drastically reduce the number of weights required by CNNs using so-called convolutional layers.



Fully Connected Layers



- The descriptive power of the net increases with the number of layers.
- In the case of a 1D signal, it is roughly proportional to $\prod_n W_n$ where W_n represents the width of a layer.



Convolutional Layer

input neurons

000000000000000000000000000000000000000	first hidden layer

$$\sigma\left(b + \sum_{x=0}^{n_x} \sum_{y=0}^{n_y} w_{x,y} a_{i+x,j+y}\right)$$





Feature Maps







Filters









d/dx2 g(x,y)





d/dy g(x,y)

d/dx3 g(x,y)









Derivatives

Learned filters





Pooling Layer

hidden neurons (output from feature map)

000000000000000000000000000000000000000	max-pooling units
00000000000000000000000000000000000000	00000000000
000000000000000000000000000000000000000	
000000000000000000000000000000000000000	0000000000000
000000000000000000000000000000000000000	000000000000000000000000000000000000000
000000000000000000000000000000000000000	000000000000000000000000000000000000000
000000000000000000000000000000000000000	0000000000000
000000000000000000000000000000000000000	000000000000000000000000000000000000000
000000000000000000000000000000000000000	000000000000000000000000000000000000000
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- Reduces the number of inputs by replacing all activations in a neighborhood by a single one.
- Can be thought as asking if a particular feature is present in that neighborhood while ignoring the exact location.





Adding the Pooling Layers



The output size is reduced by the pooling layers.





Adding a Fully Connected Layer



- Each neutron in the final fully connected layer is connected to all neurons in the preceding one.
- Deep architecture with many parameters to learn but still far fewer than an equivalent multilayer perceptron.



LeNet (1989-1999)









Lenet Results







AlexNet (2012)





Task: Image classification

Training images: Large Scale Visual Recognition Challenge 2010 Training time: 2 weeks on 2 GPUs

Major Breakthrough: Training large networks has now been shown to be practical!!



Krizhevsky, NIPS'12



AlexNet Results

mite leopard container ship motor scooter container ship mite motor scooter leopard go-kart black widow jaguar lifeboat amphibian moped cheetah cockroach bumper car snow leopard tick fireboat drilling platform golfcart Egyptian cat starfish grille mushroom cherry Madagascar cat squirrel monkey convertible agaric dalmatian grape spider monkey grille mushroom pickup jelly fungus elderberry titi beach wagon gill fungus ffordshire bullterrier indri fire engine dead-man's-fingers howler monkey currant

ImageNet Large Scale Visual Recognition Challenge Accuracy



- At the 2012 ImageNet Large Scale Visual Recognition Challenge, AlexNet achieved a top-5 error of 15.3%, more than 10.8% lower than the runner up.
- Since 2015, networks outperform humans on this task.



Feature Maps





First convolutional layer

er Second convolutional layer

- Some of the convolutional masks are very similar to oriented Gaussian or Gabor filters.
- The trained neural nets compute oriented derivatives, which the brain is also **believed** to do.



Filter Banks





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Derivatives of order 0, 1, and 2.

Learned



Bigger and Deeper



VGG19, 3 weeks of training.

"It was demonstrated that the representation depth is beneficial for the classification accuracy, and that state-of-the-art performance on the ImageNet challenge dataset can be achieved using a conventional ConvNet architecture."

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Simonyan & Zisserman, ICLR'15



GoogleLeNet.



—> Add skip connection to produce an output of the same size as the input.



Regression



$$\min_{\mathbf{W}_l,\mathbf{B}_l} \sum_i ||\mathbf{F}(\mathbf{x}_i,\mathbf{W}_1,\ldots,\mathbf{W}_L,\mathbf{b}_1,\ldots,\mathbf{b}_L) - \mathbf{y}_i||^2$$

using

- stochastic gradient descent on mini-batches,
- dropout,
- hard example mining,

Hand Pose Estimation (2015)



Input: Depth image.

Output: 3D pose vector.





Roc Hunting



DeepFace Taigman et al. 2014



Deep Edge Detection Shen et al. 2015



Deeper and Deeper



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He et al., CVPR'16

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Image Classification Taxonomy



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Monocular Pose Estimation





Recurrent Auto Encoder





Remelli et al., ArXiv'19

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Hand Pose Estimation from Video (2019)



- This is considerably more difficult than estimating from range images.
- It requires a large training database.



Alpha Go



EPE

- Uses Deep Nets to find the most promising locations to focus on.
- Performs Tree based search when possible.
- Relies on reinforcement learning and other ML techniques to train.
- —> Beat the world champion in 2017.



Visual Cortex



Recognize And Classify: Animal /No Animal

Subjects must raise their hand if they see an animal:

- 60 images
- 1 image per second
- \rightarrow Measure their reaction time.



Simon Thorpe, Nature, 1996





Reminder: Recurrent Pathways



"Shape stimuli are optimally reinforcing each other when separated in time by ~60 ms, suggesting an underlying recurrent circuit with a time constant (feedforward + feedback) of 60 ms."



Drewes et al., Journal of Neuroscience, 2016



Adversarial Images







XKCD's View On The Matter







Deep Nets in Short

- Deep Neural Networks can handle huge training databases.
- When the objective function can be minimized, the results are outstanding.
- There are failure cases and performance is hard to predict.

—> Many questions are still open and there is much theoretical work left to do.

