Neural Networks and Biological Modeling

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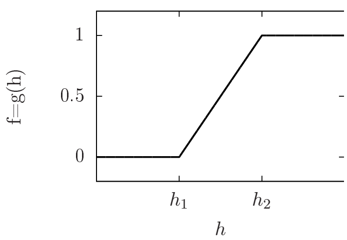
Question set 7

Exercise 1: Mean field model

Consider a network of N neurons with all-to-all connectivity and scaled synaptic weights $w_{ij} = J_0/N$. The transfer function (rate as a function of input potential) of the neurons is piecewise linear:



$$f = g(h) = \begin{cases} 0 & , \quad h < h_1 \\ \frac{h-h_1}{h_2-h_1} & , \quad h_1 \le h \le h_2 \\ 1 & , \quad h_2 < h \end{cases} \tag{1}$$

The dynamics of the input potential for neuron i is:

$$\tau \frac{dh_i}{dt} = -h_i + RI_i(t) \tag{2}$$

where

$$I_i(t) = I^{\text{ext}}(t) + \sum_j \sum_f w_{ij} \alpha(t - t_j^f) \tag{3}$$

where α denotes the shape of a the input current caused by a single spike. We assume a constant external current I^{ext} and are interested in the stationary solutions.

1.1 Find graphically the value of stationary activity $A(t) = A_0$ in the asynchronous state. You may assume that N is large ($N \to \infty$).

1.2 How does the solution change if you change the coupling constant J_0? Choose $h_1 = 1$ and $h_2 = 2$ and consider $J_0 = 1$ and $J_0 = 3$. You can assume $R = 1$ and $I^{\text{ext}}(t) = 0$. What happens at $J_0 = 2$? How does the solution change if we additionally have $I^{\text{ext}}(t) \neq 0$?

1.3 Find the solutions analytically (for arbitrary J_0, h_1, h_2).

Exercise 2: Randomly connected network: Fixed number of inputs

We consider a homogeneous network of N neurons. Each neuron receives input from K presynaptic neurons (see figure 1). When a spike arrives it generates a postsynaptic current pulse $\alpha(t - t_k^f)$. The current to neuron i is therefore:

$$I_i = \sum_{k,f} w_{ik} \alpha(t - t_k^f) \quad (4)$$

Assume the weights are $w_{ik} = \frac{w_0}{K}$ and the network activity is constant: $A(t) = A_0$.

2.1 Give an intuitive or mathematical argument for the following relationship:

$$I_i \approx w_0 A_0 \int_0^\infty \alpha(s) ds \quad (5)$$

2.2 What happens if N increases? Does the current increase? How about fluctuations?

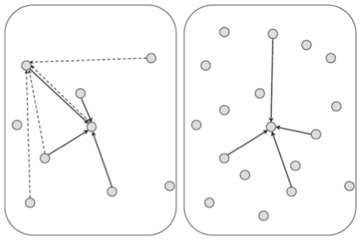


Figure 1: Two randomly connected networks of different size N. The number of inputs per neuron is fixed. (Left: inputs to two representative neurons. Right: inputs to one representative neuron).