### Biological Modeling of Neural Networks:



### **Week 9 – Decision models: Competitive dynamics**

- Wulfram Gerstner
- EPFL, Lausanne, Switzerland

#### **9.1 Introduction**

- decision making
- **9.2 Perceptual decision making**
	- V5/MT
	- Decision dynamics: Area LIP

#### **9.3 Theory of decision dynamics**

- competition via shared inhibition
- effective 2-dim model

#### **9.4. Solutions**

- symmetric case
- -biased case

#### **9.5. Simulations and Experiments**

- simulations and theory
- simulations and experiments

#### **9.6. Decisions, actions, volition**

- the problem of free will

#### *Reading for week 9*: NEURONAL DYNAMICS Ch. 16 (except 16.4.2)

Cambridge Univ. Press



#### 9.1. How do I decide?

#### We take decisions all the time - Coffee before class or not?



#### - Vote for candidate A or B?



#### - Turn left or right at the crossing?

#### 9.1. How do I decide?



#### 9.1. Decision making







#### Attention:

 - valid for high noise only, else transients might be wrong - valid for high noise only, else spontaneous oscillations may arise

### 9.1. Review of week 8: High-noise activity equation

## Population activity<br> $A(t) = F(h(t))$

Membrane potential caused by input  $\tau \frac{d}{dt} h(t) = -h(t) + R I(t)$ 

$$
\tau \frac{d}{dt} h(t) = -h(t) + R I^{ext}(t) + w_{ee} F(h(t))
$$

I(t)



#### 9.1. Review: microscopic vs. macroscopic

#### Input indicating 'left'

 $A_{e,1}(t)$ 



#### Input indicating 'right'

 $A_{e,2}(t)$ 

#### 9.1. Competition between two populations

#### 9. 1. How do YOU decide?

#### 30CHF tomorrow / 100 CHF May first next year

#### 90CHF tomorrow / 100 CHF May first next year

'Neuro-economics'

#### As selected EPFL student, pick your money at EPFL:

#### 'Is the middle bar shifted to the left or to the right?'





#### 9.1. Perceptual decision making?

#### **Bisection task:**

#### 9.1. decision making - aims

#### Decisions are everywhere

Perceptual Decision task





Model: populations of neurons

Model feature Competition

Experimental data

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#### 'Is the middle bar shifted to the left or to the right?'



![](_page_11_Picture_4.jpeg)

![](_page_11_Picture_5.jpeg)

#### 9.2. Perceptual decision making?

e.g., Herzog lab, EPFL

#### **Bisection task:**

### **9.2. Detour: receptive fields in V5/MT**

![](_page_12_Picture_1.jpeg)

**IMAGE** Nature Reviews | Neuroscience

- 1) Cells in visual cortex MT/V5 respond to motion stimuli
- 2) Neighboring cells in visual cortex MT/V5 respond to motion in similar direction cortical columns

![](_page_12_Picture_5.jpeg)

![](_page_12_Picture_6.jpeg)

## *Albright, Desimone,Gross, J. Neurophysiol, 1985*

#### Recordings from a single neuron in V5/MT

![](_page_13_Figure_2.jpeg)

![](_page_13_Picture_3.jpeg)

Receptive Fields depend on direction of motion

#### 9. 2. Detour: receptive fields in V5/MT

*Random moving dot stimuli: e.g.Salzman, Britten, Newsome, 1990 Roitman and Shadlen, 2002 Gold and Shadlen 2007*

### 9.2. Detour: receptive fields in V5/MT

![](_page_14_Picture_1.jpeg)

![](_page_14_Picture_2.jpeg)

Receptive Fields depend on direction of motion:  $β = perfectred$  direction = P

*Image: Gerstner et al. (2014), Neuronal Dynamics*

![](_page_15_Figure_3.jpeg)

#### 9.2. Experiment of Salzman et al. 1990

![](_page_15_Figure_1.jpeg)

![](_page_16_Figure_1.jpeg)

![](_page_16_Figure_2.jpeg)

### 9. 2: Experiment of Salzman et al. 1990

excites this group of neurons

![](_page_18_Picture_4.jpeg)

### coherence  $0.5 =$ 50%

![](_page_18_Figure_1.jpeg)

© 1990 Nature I

![](_page_18_Picture_3.jpeg)

![](_page_18_Picture_7.jpeg)

#### 9.2: Experiment of Salzman et al. 1990

#### Behavior: psychophysics

![](_page_19_Figure_2.jpeg)

#### With stimulation

#### 9. 2. Experiment of Salzman et al. 1990

*Image: Gerstner et al. (2014), Neuronal Dynamics; Redrawn after Salzman et al, 1990*

### **9.1 Review: Population dynamics**

#### - competition

#### **9.2 Perceptual decision making**

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#### **9.3 Theory of decision dynamics**

- shared inhibition
- effective 2-dim model

#### **9.4. Decisions in connected pops.**

- unbiased case
- biased input

#### **9.5. Decisions, actions, volition**

- the problem of free will

#### 9. 2. Perceptual Decision Making

![](_page_20_Picture_1.jpeg)

**Nature Reviews | Neuroscience** 

![](_page_20_Picture_3.jpeg)

![](_page_21_Picture_3.jpeg)

coherence 0%

### coherence 50%

![](_page_21_Picture_5.jpeg)

![](_page_21_Figure_2.jpeg)

faster if signal is stronger

*Roitman and Shadlen 2002*

### 9.2. Experiment of Roitman and Shadlen in LIP (2002)

![](_page_21_Figure_1.jpeg)

Neurons in LIP: -selective to target of saccade -increases faster if signal is stronger - activity is noisy

LIP is somewhere between MT (movement detection) and Frontal Eye Field (saccade control)

### 9. 2: Experiment of Roitman and Shadlen in LIP (2002)

![](_page_22_Figure_1.jpeg)

Response of an LIP neuron during the RT-direction-Figure  $\overline{4}$ .

#### **Neurons in LIP**:

- Selective to target of saccade - Activity increases faster if signal is stronger
- 
- Activity is noisy
- Located in the signal processing stream between sensory areas and saccade control - I do not claim that these neurons 'take the
	-
- decision'
- Interesting correlations with decision outcome

### 2. Experiment of Roitman and Shadlen in LIP (2002)

Quiz 1, now

Receptive field in LIP [] related to the target of a saccade [ ] depends on movement of random dots

### Biological Modeling of Neural Networks:

![](_page_25_Picture_1.jpeg)

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Cambridge Univ. Press

![](_page_25_Picture_8.jpeg)

population activity

## activity equations  $F(h_1(t)) + w_{ei} F(h_{inh}(t))$

 $F(h_2(t)) + w_{ei} F(h_{inh}(t))$ 

 $W_{ee}$ 

 $A_{inh}(t)$ 

 $W_{ei}$ 

 $W_{ei}$ 

#### Input indicating right moveme

Membrane potential caused by input

$$
\tau \frac{d}{dt} h_1(t) = -h_1(t) + R I_1^{ext}(t) + w_{ee}
$$

$$
\tau \frac{d}{dt} h_2(t) = -h_2(t) + R I_2^{ext}(t) + w_{ee}.
$$

 $W_{ee}$ 

 $W_{ie}$ 

Input indicating left movement

![](_page_26_Figure_6.jpeg)

 $A_{e,2}(t)$ Blackboard: reduction from 3 to 2 equations

#### 9.3. Theory of decision dynamics

$$
A_n(t) = F(h_n(t))
$$

#### activity equations

$$
F(h) = h \text{ for } 0.2 < h < 0.8
$$
  
F(0) = 0.1  
F(1) = 0.9

## Inhibitory  $P_{A_{i...k}}(t) = P(h_{i...k}(t)) = h_{i...k}(t) = w_{ie}(A_{e,1}(t) + A_{e,2}(t))$

![](_page_28_Figure_1.jpeg)

### Blackboard: Linearized inhibition

#### 9.3. Theory of decision dynamics

Membrane potential caused by input

 $\left|\tau \frac{d}{dt} h_1(t) \right| = -h_1(t) + h_1^{ext}(t) + (w_{ee} - \alpha) F(h_1(t)) - \alpha F(h_2(t))$ 

$$
\tau \frac{d}{dt} h_2(t) = -h_2(t) + h_2^{ext}(t) + (w_{ee} - \alpha)F
$$

 $W_{ee}-\alpha$ 

 $W_{ie}$ 

 $W_{ei}$ 

population activity

![](_page_29_Figure_8.jpeg)

Input indicating left movement

 $A_{e,1}(t)$ 

#### 9.3. Effective 2-dim. model

$$
A_n(t) = F(h_n(t))
$$

![](_page_30_Figure_0.jpeg)

## $g(h) = h$  for  $0.2 < h < 0.8$  $g(0) = 0.1$  $g(0.9) = 0.85$  $g(1) = 0.9$

$\frac{d}{dt}h_2 = 0$	$h_2$	$g(h_1)$	$h_1$
		1.0	
		0.8	
0.2	0.0		

### Biological Modeling of Neural Networks:

![](_page_32_Picture_1.jpeg)

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#### *Reading for week 9*: NEURONAL DYNAMICS Ch. 16 (except 16.4.2)

Cambridge Univ. Press

![](_page_32_Picture_8.jpeg)

#### Phase plane, strong external input

![](_page_33_Figure_2.jpeg)

![](_page_33_Picture_3.jpeg)

$$
= 0.8 = h_2^{ext}
$$

$$
g(h) = h
$$
 for  $0.2 < h < 0.8$   
\n $g(0) = 0.1$   
\n $g(1) = 0.9$ 

#### 9.4. Theory of decision dynamics

#### Quiz 2, take 5 minutes now

Continue with Exercise 1, but change the external inputs. A Keep the input to population 1, but reduce the input to population 2 from 0.8 to 0.2 [ ] The nullcline for dh2/dt shifts vertically downward [ ] The nullcline for dh2/dt shifts horizontally leftward [] The nullcline for dh1/dt shifts vertically downward [ ] The number of fixed points changes B In addition, you now also reduce the input to population 1, 0.8 to 0.2

[ ] The nullcline for dh2/dt shifts vertically downward [] The nullcline for dh2/dt shifts horizontally leftward [ ] The nullcline for dh1/dt shifts vertically downward

#### Phase plane – biased input:

 $h_2^{ext} = 0.2$ 

![](_page_35_Figure_1.jpeg)

#### 9.4. Theory of decision dynamics: biased input

![](_page_36_Figure_1.jpeg)

$$
h_1^{ext} = 0.2 = h_2^{ext}
$$

#### Weak external input: Stable fixed point

$$
\frac{d}{dt}h_2=0
$$

#### 9.4. Theory of decision dynamics: unbiased weak

![](_page_37_Figure_2.jpeg)

Symmetric, but strong input

#### 9.4. decision dynamics: unbiased strong to biased

![](_page_37_Figure_1.jpeg)

## unbiased strong input = 2 stable fixed points

![](_page_38_Figure_1.jpeg)

![](_page_38_Figure_2.jpeg)

$$
h_1^{ext} = 0.8;
$$
  

$$
h_2^{ext} = 0.2
$$

#### Biased input = stable fixed point decision reflects bias

### 9.4. Theory of decision dynamics: biased strong

![](_page_39_Figure_1.jpeg)

$$
h_1^{ext} = 0.8 = h_2^{ext}
$$

#### Homogeneous solution = saddle point decision must be taken

$$
\frac{d}{dt}h_2=0
$$

#### 9.4. Theory of decision dynamics: unbiased strong

### Biological Modeling of Neural Networks:

![](_page_40_Picture_1.jpeg)

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![](_page_40_Picture_8.jpeg)

![](_page_41_Figure_3.jpeg)

#### Simulation of 3 populations of spiking neurons, unbiased strong input 9.5. Decisions in populations of neurons: simulation

![](_page_41_Figure_1.jpeg)

![](_page_41_Picture_2.jpeg)

#### 9.5: Comparison: Simulation and Theory

#### Exercise 2 at home: stability of symmetric solution

#### 1) Before stimulus is given: symmetric but small input

![](_page_42_Figure_2.jpeg)

$$
h_1^{ext} = 0.2 = h_2^{ext}
$$

Weak unbiased input: Stable fixed point □ no decision

$$
h_2^2=0
$$

![](_page_43_Figure_2.jpeg)

$$
\frac{\mathbf{1} \mathbf{e}}{h_1^{ext}} = 0.8 = h_2^{ext}
$$

#### Homogeneous solution = saddle point decision must be taken

$$
\frac{d}{dt}h_2=0
$$

#### 9.5: Comparison: Simulation and Theory

#### 2) When stimulus is given: symmetric but strong input

![](_page_44_Picture_2.jpeg)

![](_page_44_Figure_1.jpeg)

Prediction by theory - for input potential

![](_page_45_Figure_2.jpeg)

 $h_1(t)$ - population activity  $A(t) = F(h(t))$ 

$$
e^{ext}_{1} = 0.8;
$$
  

$$
e^{ext}_{2} = 0.2
$$

#### Biased input = stable fixed point  $\Box$  decision reflects bias

$$
^{-}\textit{h}_{_{1}}\left( t\right)
$$

#### 9.5. Comparison with experiment: biased strong input

#### *Roitman and Shadlen 2002*

![](_page_46_Figure_0.jpeg)

Figure 7. Time course of LIP activity in the RT-direction-discrimination task. A, Average response from 54 LIP neurons. Responses are grouped by motion strength and choice as indicated by color and line type. The responses are aligned to two events in the trial. On the left, responses are aligned to the onset of stimulus motion. Response averages in this portion of the graph are drawn to the median RT for each motion strength and exclude any activity within 100 msec of eye movement initiation. On the right, responses are aligned to initiation of the eye movement response. Response averages in this portion of the graph show the buildup and decline in activity at the end of the decision process. They exclude any activity within 200 msec of motion onset. The average firing rate was smoothed using a 60 msec running mean. Arrows indicate the epochs used to compare spike rate as a function

![](_page_46_Figure_4.jpeg)

### 9.5. Decisions in populations of neurons: simulations and data

simulation of competing populations shares properties with LIP data: - faster increase for strong bias - suppression for opposite saccade

BUT: there is no claim that decision is taken in LIP

LIP is somewhere in the processing stream from input to saccades

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![](_page_48_Picture_1.jpeg)

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Cambridge Univ. Press

![](_page_48_Picture_8.jpeg)

#### **9.6. Decision: risky vs. safe**

![](_page_49_Picture_1.jpeg)

#### **9.6. Decision: risky vs. safe**

![](_page_50_Picture_1.jpeg)

![](_page_51_Picture_7.jpeg)

x a r h q e f g y t u

Preparation

![](_page_51_Picture_4.jpeg)

- -Subject decides spontaneously to move left or right hand - report when they made their decision
- *Libet, Behav. Brain Sci., 1985 Soon et al., Nat. Neurosci., 2008*

### 9.6. fMRI variant of Libet experiment: volition and free will

x a r h q e f g y t u

![](_page_51_Picture_2.jpeg)

What decides? Who decides? -Your experiences are memorized in your brain -Your values are memorized in your brain -Your decisions are reflected in brain activities '*Your brain decides what you want or what you prefer … '* ' … but your brain – this is you!!!' '*We don't do what we want, but we want what we do' (W. Prinz)*

# goal<br>The problem of goal The problem of Free Will (see e.g. Wikipedia article)

**St** 

art and the state

#### 9.6. Decision: risky vs. safe

![](_page_53_Picture_4.jpeg)

- decisions are taken in the brain - competition between populations is a transparent model - relevant decisions involve personal values and experiences

- 
- 
- Suggested Reading:  *Salzman et al. Nature 1990 - Roitman and Shadlen, J. Neurosci. 2002*
	- *Abbott, Fusi, Miller: Theoretical Approaches to Neurosci.*
	- *X.-J. Wang, Neuron 2002*
	- *Libet, Behav. Brain Sci., 1985*
	- *Soon et al., Nat. Neurosci., 2008*
	- *free will, Wikipedia*

#### Chapter 16, *Neuronal Dynamics*, Gerstner et al. Cambridge 2014

### 9.6. Selected References: Decision Making

#### Exercise 2.1 now: stability of homogeneous solution

 $A_n(t) = g(h_n(t))$ Membrane potential caused by input  $\left|\tau \frac{d}{dt} h_1(t) \right| = -h_1(t) + b + (w_{ee} - \alpha)g(h_1(t)) - \alpha g(h_2(t))$ 

$$
\tau \frac{d}{dt} h_2(t) = -h_2(t) + b + (w_{ee} - \alpha)
$$

Assume: 
$$
h_1^{ext} = h_2^{ext} = b
$$

a) Calculate homogeneous fixed point  $h_1 = h_2 = h^*(b)$ 

 $|g(h_2(t)) - \alpha g(h_1(t))|$ 

# b) Analyze stability of the fixed point h(b) as a function of b