Low-power radio design for the IoT
Exercise 1 (25.02.21)

Christian Enz

Swiss Federal Institute of Technology (EPFL), Lausanne, Switzerland

Problem 1 L Impedance Matching Network

For the L impedance matching network shown in Fig. 1, address the following points.

- Determine L and C.
- Indicate the main disadvantage of this network and a possible solution.

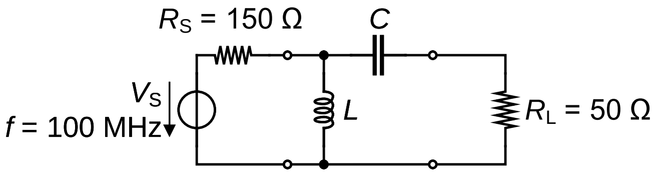

Figure 1: Matching network

Problem 2 Π Impedance Matching Network

Part I: Consider the example for the Π impedance matching network shown in the slides 24 and 25 (Chapter 2). In this example, to calculate the value of R_{virt}, R_{max} is taken equal to R_S ($3000\,\Omega$). Repeat the same example by taking R_L ($50\,\Omega$) as the resistance value used to calculate R_{virt}. In this example, Q is equal to 10 and the resonant frequency, f, is 1 MHz.

- Find the new values for X_{c1} and C_1, X_{c2} and C_2, X_L and L.
- Compare the values for L, C_1 and C_2 with the ones obtained in the example shown in the slides.

Part II: Design the Π filter shown in Fig. 2 for a Q of 5 and a value of f equal to 27 MHz.

- Calculate two values for R_{virt}, one by using R_S and the other by using R_L. Reject one of the two values and motivate the choice.
- Determine X_L, L, X_{c1}, C_1, X_{c2}, C_2.

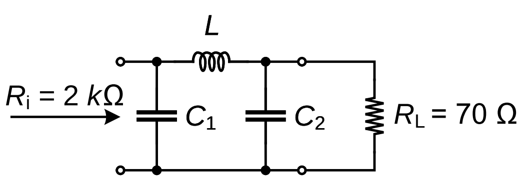

Figure 2: Π filter

Problem 3 Tapped-capacitor Impedance Matching Network

The circuit in Fig. 3 is a tapped-capacitor impedance matching network. Assume that this network operates at $f = 1\,\text{MHz}$ and that $C_1 = 200\,\text{pF}$, $C_2 = 1000\,\text{pF}$ and $R_L = 200\,\Omega$.

- Determine the Q, R_{in} and L for the network in Fig. 3. (Hint: Convert R_L and C_2 to a series equivalent.)

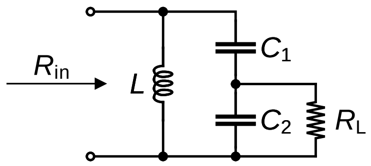

Figure 3: Tapped-capacitor impedance matching network

Solutions to Exercise 1 (25.02.21)

Problem 1 L Impedance Matching Network

- Determine L and C.

The matching network in Fig. 1 is a L impedance matching network where R_L is lower than R_S. The quality factor, Q, is

$$Q = \sqrt{\frac{R_{in}}{R_L} - 1} = \sqrt{\frac{150}{50} - 1} = 1.414 \tag{1}$$

The reactances are given by

$$X_C = Q \cdot R_L = 1.414 \cdot 50 = 70.7\,\Omega \tag{2}$$

$$X_L = \frac{R_{in}}{Q} = \frac{150\,\Omega}{1.414} = 106\,\Omega. \tag{3}$$

Then, the values of the inductor and capacitor are given by

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \cdot 100\,\text{MHz} \cdot 70.7\,\Omega} = 22.51\,\text{pF} \tag{4}$$

$$L = \frac{X_L}{2\pi f} = \frac{106\,\Omega}{2\pi \cdot 100\,\text{MHz}} = 168.7\,\text{nH}. \tag{5}$$

- Indicate the main disadvantage of this network and a possible solution.

The main disadvantage of this L matching network is that once the source and load resistance are specified, the Q of the circuit and hence its selectivity is defined. There are not enough degrees of freedom to choose Q independently. This drawback can be alleviated by adding one component leading to the Π or T impedance matching network.

Problem 2 Π Impedance Matching Network

The values for L, C_1 and C_2 found in the example shown in the slides are 51.18 µH, 530.5 pF and 2.63 nF, respectively.

- Part I: Find the new values for X_{c1} and C_1, X_{c2} and C_2, X_L and L.

If we repeat the exercise by using R_L instead of R_S to calculate R_{virt}, we obtain

$$R_{virt} = \frac{R_L}{1+Q^2} = \frac{50\,\Omega}{1+10^2} = \frac{50\,\Omega}{101} = 0.49\,\Omega. \tag{6}$$

The reactances of the input section are then

$$X_{c2} = \frac{R_L}{Q} = \frac{50\,\Omega}{10} = 5\,\Omega \tag{7}$$

$$X_{L2} = Q \cdot R_{virt} = 10 \cdot 0.49\,\Omega = 4.9\,\Omega. \tag{8}$$

The Q of the first section is equal to

$$Q_1 = \sqrt{\frac{R_S}{R_{virt}} - 1} = \sqrt{\frac{3000}{0.49} - 1} = 78.24 \tag{9}$$

So, the reactances of the first section are calculated

$$X_{c1} = \frac{R_S}{Q_1} = \frac{3000\,\Omega}{78.24} = 38.34\,\Omega \tag{10}$$

$$X_{L1} = Q_1 \cdot R_{virt} = 78.24 \cdot 0.49\,\Omega = 38.33\,\Omega. \tag{11}$$

Finally, the values for the capacitors and the inductor are

$$L = \frac{X_{L1}}{2\pi f} + \frac{X_{L2}}{2\pi f} = \frac{38.33\,\Omega}{2\pi \cdot 1\,\mathrm{MHz}} + \frac{4.9\,\Omega}{2\pi \cdot 1\,\mathrm{MHz}} = 6.88\,\mu\mathrm{H} \tag{12}$$

$$C_1 = \frac{1}{2\pi f X_{c1}} = \frac{1}{2\pi \cdot 1\,\mathrm{MHz} \cdot 38.34\,\Omega} = 4.15\,\mathrm{nF} \tag{13}$$

$$C_2 = \frac{1}{2\pi f X_{c2}} = \frac{1}{2\pi \cdot 1\,\mathrm{MHz} \cdot 5\,\Omega} = 31.83\,\mathrm{nF}. \tag{14}$$

- Compare the values for L, C_1 and C_2 with the ones obtained in the example shown in the slides.

In this example, R_{virt} has a value 60 times lower than the one in the slides, while the value found for Q_1 is around 100 times higher than the value found in the slides for Q_2. This reduces the reactance of the inductor (from $321.58\,\Omega$ to $43.23\,\Omega$), hence a smaller value for the inductor L is needed (from $51.18\,\mu\mathrm{H}$ to $6.88\,\mu\mathrm{H}$). The reactances for the two capacitors, C_1 and C_2, change from $300\,\Omega$ to $38.34\,\Omega$ in one case (X_{c1}) and from $60.49\,\Omega$ to $5\,\Omega$ in the other case (X_{c2}). The impact on the capacitors is an higher value of C_1 (from $530.5\,\mathrm{pF}$ to $4.15\,\mathrm{nF}$) and also for C_2 (from $2.63\,\mathrm{nF}$ to $31.83\,\mathrm{nF}$). In this specific case both solutions are possible. The main difference between the two is the final values of the needed components which leads to different requirements in terms of area and technology. In general, depending on the values of Q, R_S and R_L, one of the two solutions can be easier to implement than the other.

- Part II: Calculate two values for R_{virt}, one by using R_S and the other with R_L. Reject one of the two values and motivate the choice.

In one case for the virtual resistance, R_{virt}, we have

$$R_{virt} = \frac{R_L}{1+Q^2} = \frac{70\,\Omega}{1+5^2} = \frac{70\,\Omega}{26} = 2.69\,\Omega, \tag{15}$$

while in the other, we obtain

$$R_{virt} = \frac{R_S}{1+Q^2} = \frac{2000\,\Omega}{1+5^2} = \frac{2000\,\Omega}{26} = 76.9\,\Omega. \tag{16}$$

In the second case, the value of R_{virt} is not lower than R_L. If we choose the latter then, one of the two Q factors will not be real. For this reason we use the first value of R_{virt}, calculated by using R_L, and we assume Q_2 to be equal to the given value of Q.

- Determine X_L, L, X_{c1}, C_1, X_{c2}, C_2.

The reactances of the output L section are then given by

$$X_{c2} = \frac{R_L}{Q_2} = \frac{70\,\Omega}{5} = 14\,\Omega \tag{17}$$

$$X_{L2} = Q_2 \cdot R_{virt} = 5 \cdot 2.69\,\Omega = 13.45\,\Omega. \tag{18}$$

The Q_1 of the first L section is equal to

$$Q_1 = \sqrt{\frac{R_S}{R_{virt}} - 1} = \sqrt{\frac{2000\,\Omega}{2.69\,\Omega} - 1} = 27.24 \tag{19}$$

The reactances of the output L section are then given by

$$X_{c1} = \frac{R_S}{Q_1} = \frac{2000\,\Omega}{27.24\,\Omega} = 73.42\,\Omega \tag{20}$$

$$X_{L1} = Q_1 \cdot R_{virt} = 27.24 \cdot 2.69\,\Omega = 73.27\,\Omega. \tag{21}$$

Then X_L is equal to

$$X_L = X_{L1} + X_{L2} = 13.45\,\Omega + 73.27\,\Omega = 86.72\,\Omega. \tag{22}$$

Finally, the values for the capacitors and the inductor are

$$L = \frac{X_L}{2\pi f} = \frac{86.72\,\Omega}{2\pi \cdot 27\,\text{MHz}} = 511\,\text{nH} \tag{23}$$

$$C_1 = \frac{1}{2\pi f X_{c1}} = \frac{1}{2\pi \cdot 27\,\text{MHz} \cdot 73.42\,\Omega} = 80\,\text{pF} \tag{24}$$

$$C_2 = \frac{1}{2\pi f X_{c2}} = \frac{1}{2\pi \cdot 27\,\text{MHz} \cdot 14\,\Omega} = 0.42\,\text{nF}. \tag{25}$$

Problem 3 Tapped-capacitor Impedance Matching Network

- Determine the Q, R_{in} and L for the network in Fig. 3. (Hint: Convert R_L and C_2 to a series equivalent.)

In the equivalent parallel RLC network of the tapped capacitor, the values of the capacitor and the resistor are

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{200\,\text{pF} \cdot 1000\,\text{pF}}{200\,\text{pF} + 1000\,\text{pF}} = 166.7\,\text{pF} \tag{26}$$

$$R = n^2 \cdot R_L = \left(1 + \frac{C_2}{C_1}\right)^2 \cdot R_L = \left(1 + \frac{1000\,\text{pF}}{200\,\text{pF}}\right)^2 \cdot 200\,\Omega = 7200\,\Omega \tag{27}$$

Then, $R_{in} = n^2 \cdot R_L$ is then equal to $7200\,\Omega$.

Finally, the values of L and Q are given by

$$L = \frac{1}{\omega_0^2 \cdot C} = \frac{1}{(2\pi \cdot 1\,\text{MHz})^2 \cdot 166.7\,\text{pF}} = 152\,\mu\text{H} \tag{28}$$

$$Q = \frac{R}{\omega_0 \cdot L} = \frac{7200\,\Omega}{(2\pi \cdot 1\,\text{MHz}) \cdot 152\,\mu\text{H}} = 7.54 \tag{29}$$