Low-power radio design for the IoT
Exercise 5 (25.03.2021)

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Problem 1 Common Source Analysis

The circuit in the Figure below is a single MOSFET Common Source (CS) Amplifier.

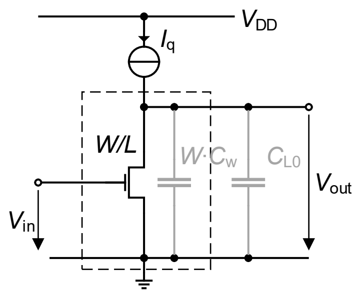

Figure 1: Common Source Amplifier.

- Draw the small-signal equivalent circuit of the CS. Consider the bias current source to be ideal. (*Hint*: split the gate capacitance, C_G, into the intrinsic and the extrinsic parts, C_{Gi} and C_{Ge}, and the load capacitance, C_L, into a constant capacitance and a self-loading capacitance per unit width, C_{L0} and C_w, respectively).
- Derive the expression of the transit frequency, ω_t, by using the small signal equivalent circuit.
- Express ω_t in terms of the inversion coefficient, IC. (*Hint*: introduce the specific frequency, $\omega_{spec} \triangleq \frac{2\mu U_T}{L^2}$, and neglect the velocity saturation ($\lambda_c = 0$)).
- Derive ω_{tspec}, the transit frequency for $IC = 1$ and WI.
- Derive the expression for the unity-gain frequency, ω_u. (*Hint*: neglect the output conductance of the transistor ($g_{ds} = 0$)).
- Express ω_u in terms of the normalized gain-bandwidth (GBW) for a squared transistor biased in WI, $\omega_L \triangleq \frac{I_{spec\square}}{nU_T C_L}$, and the normalized source transconductance, g_{ms}.

Problem 2 Common Source Design

Once the self loading capacitance per unit width is taken into account, the formulas for the I_D and W/L normalized to $\Omega \triangleq \frac{\omega_u}{\omega_L}$ are

$$\begin{aligned} i_b &\triangleq \frac{I_D}{I_{spec\square}} \cdot \frac{1}{\Omega} = \frac{IC}{g_{ms} - \Theta}; \\ AR &\triangleq \frac{W}{L} \cdot \frac{1}{\Omega} = \frac{1}{g_{ms} - \Theta}, \end{aligned} \tag{1}$$

where Θ is equal to $\frac{C_w L}{C_{L0}} \cdot \frac{\omega_u}{\omega_L} = \frac{\omega_u}{\omega_{tspec}}$. Design the CS amplifier, shown in Fig. 1, for the following specifications at room temperature:

$$f_u = 18\,\text{MHz} \quad C_{L0} = 60\,\text{fF} \quad V_{DD} = 1.8\,\text{V} \quad L = 40\,\text{nm} \quad C_w = 0.450\,\text{fF/nm}, \tag{2}$$

and by assuming the following values for the technology parameters

$$I_{spec\square} = 950\,\text{nA} \quad n = 1.5 \quad V_{T0} = 455\,\text{mV} \quad \lambda_c = 0.4875 \quad L_{sat} = 19.5\,\text{nm}. \tag{3}$$

- Find the IC_{opt}, the value of the inversion coefficient for which the bias current is minimum. Assume no velocity saturation.
- Based on the IC_{opt}, find the values of the bias current, I_q, and the transistor aspect ratio, W/L, to achieve the specified gian-bandwidth, w_u.

Solutions to Exercise 5 (25.03.2021)

Problem 1 Common Source Analysis

- Draw the small-signal equivalent circuit of the CS.

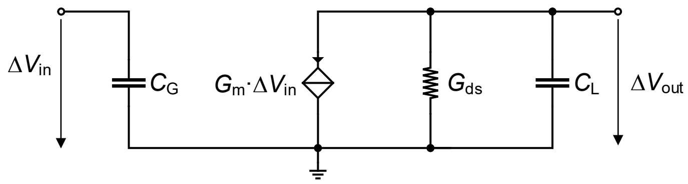

Figure 1: Common Source Amplifier Small Signal Equivalent Circuit.

where

$$C_G \triangleq C_{Gi} + C_{Ge};$$
$$C_L \triangleq C_{L0} + C_w \cdot W. \tag{1}$$

- Derive the expression of the transit frequency, ω_t, by using the small signal equivalent circuit.

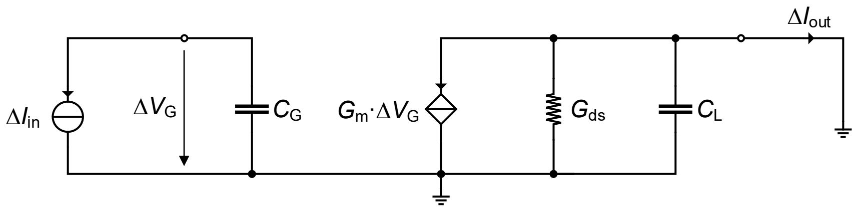

Figure 2: Small Signal Equivalent Circuit to derive ω_t.

So,

$$\Delta I_{in} = sC_G V_G;$$
$$\Delta I_{out} = G_m V_G, \tag{2}$$

which leads to

$$\frac{\Delta I_{out}}{\Delta I_{in}} = \frac{G_m}{sC_G}. \tag{3}$$

By imposing $|\frac{\Delta I_{out}}{\Delta I_{in}}| = 1$, we have

$$\omega_t = \frac{G_m}{C_G}. \tag{4}$$

- Express ω_t in terms of the inversion coefficient, IC.

The term G_m can be written as

$$G_m = \frac{G_{ms}}{n} = \frac{G_{spec}}{n} g_{ms}. \tag{5}$$

while C_G is

$$C_G = C_{Gi} + C_{Ge} = W \cdot L \cdot C_{ox} \left(c_{Gi} + \frac{C_{Ge}}{W \cdot L \cdot C_{ox}} \right), \tag{6}$$

hence

$$\omega_t = \frac{\frac{G_{spec}}{n}}{W \cdot L \cdot C_{ox}} \cdot \frac{g_{ms}}{\left(c_{Gi} + \frac{C_{Ge}}{W \cdot L \cdot C_{ox}}\right)}. \tag{7}$$

Since $G_{spec} = \frac{I_{spec\square}}{U_T} \cdot \frac{W}{L}$ and $I_{spec\square} = 2n\mu C_{ox} U_T^2$, we obtain beginequation

$$\frac{\frac{G_{spec}}{n}}{W \cdot L \cdot C_{ox}} = \frac{2\mu U_T}{L^2} = \omega_{spec}, \tag{8}$$

and so

$$\omega_t = \omega_{spec} \cdot \frac{g_{ms}}{c_{Gi} + \frac{C_{Ge}}{W \cdot L \cdot C_{ox}}}. \tag{9}$$

By replacing g_{ms} we obtain

$$\omega_t = \omega_{spec} \cdot \frac{\frac{1}{2}(\sqrt{4IC+1} - 1)}{c_{Gi} + \frac{C_{Ge}}{W \cdot L \cdot C_{ox}}}. \tag{10}$$

- Derive ω_{tspec}, the transit frequency for $IC = 1$ and WI.

For bias in WI

$$g_{ms} = IC, \tag{11}$$

hence, for $IC = 1$ we have

$$\omega_{tspec} = \frac{\omega_{spec}}{c_{Gi} + \frac{C_{Ge}}{W \cdot L \cdot C_{ox}}}. \tag{12}$$

- Derive the expression for the unity-gain frequency, ω_u.

For the CS in Fig. 1, we have

$$A_v = \frac{\Delta V_{out}}{\Delta V_{in}} = -\frac{G_m}{sC_L}. \tag{13}$$

By imposing $|A_v| = 1$, we obtain ω_u

$$\omega_u = \frac{G_m}{C_L}. \tag{14}$$

- Express ω_u in terms of the normalized gain-bandwidth (GBW) for a squared transistor biased in WI, $\omega_L \triangleq \frac{I_{spec\square}}{nU_T C_L}$, and the normalized source transconductance, g_{ms}.

$$\omega_u = \frac{G_m}{C_L} = \frac{I_{spec\square}}{nU_T C_L} \cdot \frac{W}{L} \cdot g_{ms} = \omega_L \cdot \frac{W}{L} \cdot g_{ms}. \tag{15}$$

Problem 2 Common Source Design

- Find the IC_{opt}, the value of the inversion coefficient for which the bias current is minimum. Assume no velocity saturation.

$$\Theta = \frac{C_w L}{C_{L0}} \cdot \frac{\omega_u}{\omega_L} = \frac{0.450\,\mathrm{fFnm^{-1}} \cdot 40\,\mathrm{nm}}{60\,\mathrm{fF}} \cdot \frac{2\pi \cdot 18\,\mathrm{MHz}}{\frac{950\,\mathrm{nA}}{1.5 \cdot 26\,\mathrm{mV} \cdot (60\,\mathrm{fF} + 0.450\,\mathrm{fF/nm} \cdot 40\,\mathrm{nm})}} = 0.1086 \tag{16}$$

Hence,

$$IC_{opt} \approxeq 2\Theta + sqrt(\Theta) = 0.547 \tag{17}$$

- Based on the IC_{opt}, find the values of the bias current, I_q, and the transistor aspect ratio, W/L, to achieve the specified gian-bandwidth, w_u.

By using the formula reported in the text, we obtain

$$\begin{aligned} i_b &= \frac{0.547}{0.3927 - 0.1086} = 1.98254; \\ AR &= \frac{1}{0.3927 - 0.1086} = 3.5205, \end{aligned} \tag{18}$$

and finally

$$\begin{aligned} I_q &= i_b \cdot I_{spec\square} \cdot \frac{\omega_u}{\omega_L} = 662\,\text{nA}; \\ \frac{W}{L} &= AR \cdot \frac{\omega_u}{\omega_L} = 1.275 \rightarrow W = 51\,\text{nm}. \end{aligned} \tag{19}$$