Low-power radio design for the IoT
Exercise 6 (22.04.2021)

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Problem 1 Noise Parameters of Common-Source Stage

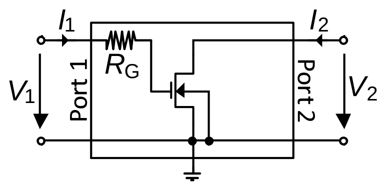

Figure 1: Two-port network corresponding to a common-source amplifier (bias not shown)

Fig. 1 shows the two-port network corresponding to a source-gate stage including the gate resistance, but without the details of the bias network.

1.1 Small-Signal Equivalent Schematic

Draw the small-signal schematic (only account for the gate-to-source capacitance C_{GS}, no other capacitances)

1.2 Two-Port Noise Parameters

Calculate the noise parameters of the noisy two-port accounting for the channel noise (drain noise) and the noise of the gate resistance. Ignore the induced gate noise and the correlation between drain and gate noise (parameter $c_g = 0$)

- First calculate the parameters R_v, G_i, G_c and B_c
- Then derive the noise parameters G_{opt}, B_{opt} and F_{min}

1.3 Effective Noise Factor

Calculate the effective noise factor by the following two different equivalent methods.

- Use the expressions for the noise parameters R_v, G_i, G_c and B_c
- Directly by calculating the total input referred noise voltage (v_{neq})

1.4 F versus IC

Sketch the noise factor F as a function of the inversion coefficient IC in the following conditions:

- Assume long-channel transistor. Hint: γ_{nD} constant.
- Assume short-channel transistor. Hint: $\gamma_{\mathrm{nD}} = 1 + \alpha \cdot IC$.
- Comment on the differences between the two plots and the optimum design for noise in both cases.

Solutions to Exercise 6 (22.04.2021)

Problem 1 Noise Factor of Common-Source Amplifier

1.1 Small-signal equivalent schematic (in saturation)

Fig. 1 shows the two-port network corresponding to a common-source stage including the gate resistance, but without the details of the bias network.The simplified small-signal schematic corresponding to the two-port network shown in Fig. 1 is presented in Fig. 2. In order to keep the derivation simple, it only includes the gate-to-source capacitance C_{GS}, neglecting all other capacitances and access resistances except the gate resistance R_G. It includes the channel noise source ΔI_{nD} and the gate resistance noise ΔV_{nrg}, but neglects the induced gate noise (IGN) source ΔI_{nG}.

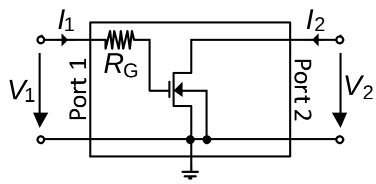

Figure 1: Two-port network corresponding to a common-source amplifier (bias not shown)

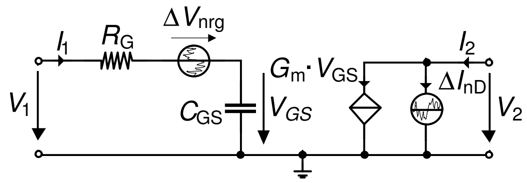

Figure 2: Simplified small-signal schematic of the common-source amplifier of Fig. 1 assuming the transistor is biased in saturation

1.2 Two-Port Noise Parameters

1.2.1 Calculation of two-port noise parameters R_v, G_i, G_c and B_c

From the small-signal schematic of Fig. 2, we can calculate the two-port noise parameters. We first have to calculate the noise sources V_n and I_n from their definitions given by

$$V_n = B \cdot I_2|_{V_1=V_2=0} = -\frac{1}{Y_{21}}\, I_2|_{V_1=V_2=0}\,, \tag{1a}$$

$$I_n = D \cdot I_2|_{I_1=V_2=0} = -\frac{Y_{11}}{Y_{21}}\, I_2|_{I_1=V_2=0}\,, \tag{1b}$$

which require the Y-parameters Y_{11} and Y_{21}, which are calculated as

$$Y_{11} = \frac{sC_{GS}}{1 + sR_G C_{GS}}, \tag{2a}$$

$$Y_{21} = \frac{G_m}{1 + sR_G C_{GS}}. \tag{2b}$$

The output current I_2 when the input and output are short-circuited is given by

$$I_2|_{V_1=V_2=0} = \Delta I_{nD} - \frac{G_m}{1 + sR_G C_{GS}} \cdot \Delta V_{nrg}, \tag{3}$$

from which we obtain V_n

$$V_n = -\frac{1}{Y_{21}} \, I_2|_{V_1=V_2=0} = \Delta V_{nrg} - \frac{1 + sR_G C_{GS}}{G_m} \cdot \Delta I_{nD}. \tag{4}$$

The output current I_2 when the input is open and the output is short-circuited is simply

$$I_2|_{I_1=V_2=0} = \Delta I_{nD}, \tag{5}$$

from which we get I_n

$$I_n = -\frac{Y_{11}}{Y_{21}} \, I_2|_{I_1=V_2=0} = \frac{sC_{GS}}{G_m} \cdot \Delta I_{nD}. \tag{6}$$

Since ΔI_{nD} and ΔV_{nrg} are uncorrelated, the mean square value of V_n is then given by

$$\overline{|V_n|^2} = \frac{1 + (\omega R_G C_{GS})^2}{G_m^2} \cdot \overline{|\Delta I_{nD}|^2} + \overline{|\Delta V_{nrg}|^2}, \tag{7}$$

whereas the mean square value of I_n is given by

$$\overline{|I_n|^2} = \left(\frac{\omega\, C_{GS}}{G_m}\right)^2 \cdot \overline{|\Delta I_{nD}|^2}. \tag{8}$$

The mean-square values $\overline{|\Delta I_{nD}|^2}$ and $\overline{|\Delta V_{nrg}|^2}$ can be expressed in terms of the PSD $S_{\Delta I_{nD}^2}$ and $S_{\Delta V_{nrg}^2}$ as

$$\overline{|\Delta I_{nD}|^2} = S_{\Delta I_{nD}^2} \cdot B = 4kTB \cdot G_{nD}, \tag{9a}$$

$$\overline{|\Delta V_{nrg}|^2} = S_{\Delta V_{nrg}^2} \cdot B = 4kTB \cdot R_G. \tag{9b}$$

The noise parameters R_v and G_i are then given by

$$R_v = \frac{\overline{|V_n|^2}}{4kTB} = R_G + \frac{1 + (\omega R_G C_{GS})^2}{G_m^2} \cdot G_{nD}, \tag{10a}$$

$$G_i = \frac{\overline{|I_n|^2}}{4kTB} = \left(\frac{\omega\, C_{GS}}{G_m}\right)^2 \cdot G_{nD}. \tag{10b}$$

For calculating Y_c according to

$$Y_c = \frac{\overline{I_n \cdot V_n^*}}{\overline{|V_n|^2}}, \tag{11}$$

we need to compute $\overline{I_n \cdot V_n^*}$, which is given by

$$\overline{I_n \cdot V_n^*} = \frac{j\,\omega\, C_{GS}}{G_m} . \frac{(1 - j\,\omega R_G C_{GS})}{Gm} \overline{|\Delta I_{nD}|^2} \tag{12}$$

Substituting $\overline{|\Delta I_{nD}|^2}$ by its expression from Eq. 9a then yields

$$\overline{I_n \cdot V_n^*} = 4kTB \cdot \frac{j\,\omega\, C_{GS} \cdot (1 - j\,\omega R_G C_{GS})}{G_m^2} \cdot G_{nD}, \tag{13}$$

from which we then get

$$Y_c = \frac{j\,\omega\, C_{GS} \cdot (1 - j\,\omega R_G C_{GS})}{1 + G_m^2 R_G / G_{nD} + (\omega R_G C_{GS})^2}. \tag{14}$$

The real and imaginary parts of Y_c are the given by

$$G_c = \frac{\omega^2 R_G C_{GS}^2}{1 + G_m^2 R_G / G_{nD} + (\omega R_G C_{GS})^2}, \tag{15a}$$

$$B_c = \frac{j\,\omega C_{GS}}{1 + G_m^2 R_G / G_{nD} + (\omega R_G C_{GS})^2}. \tag{15b}$$

Introducing the expressions of $G_{nD} = \gamma_{nD} \cdot G_m$ finally results in

$$R_v = R_G + \frac{\gamma_{nD}}{G_m} \cdot \left(1 + (\omega R_G C_{GS})^2\right), \tag{16a}$$

$$G_i = \frac{\gamma_{nD}\omega^2 C_{GS}^2}{G_m}, \tag{16b}$$

$$G_c = \frac{\omega^2 R_G C_{GS}^2}{1 + G_m R_G/\gamma_{nD} + (\omega R_G C_{GS})^2}, \tag{16c}$$

$$B_c = \frac{j\,\omega C_{GS}}{1 + G_m^2 R_G/\gamma_{nD} + (\omega R_G C_{GS})^2}. \tag{16d}$$

The uncorrelated and correlated parts of G_i are then obtained using

$$G_{iu} = G_i - |Y_c|^2 \cdot R_v = (1 - |c|^2) \cdot G_i, \tag{17a}$$

$$G_{ic} = |Y_c|^2 \cdot R_v = |c|^2 \cdot G_i, \tag{17b}$$

$$G_i = G_{iu} + G_{ic}, \tag{17c}$$

and resulting in

$$G_{iu} = \frac{\gamma_{nD}}{G_m} \cdot \frac{\omega^2 C_{GS}^2 (1 + (\omega R_G C_{GS})^2}{1 + G_m R_G/\gamma_{nD} + (\omega R_G C_{GS})^2}, \tag{18a}$$

$$G_{ic} = \frac{\omega^2 R_G C_{GS}^2}{1 + G_m R_G/\gamma_{nD} + (\omega R_G C_{GS})^2}. \tag{18b}$$

1.2.2 Calculation of G_{opt}, B_{opt} and F_{min}

From the R_v, G_i, Y_c parameters we can derive the four noise parameters R_v, G_{opt}, B_{opt} and F_{min} as a function of the circuit parameters using

$$G_{opt} = \sqrt{\frac{G_{iu}}{R_v} + G_c^2} = \sqrt{\frac{G_i}{R_v} - B_c^2}, \tag{19a}$$

$$B_{opt} = -B_c, \tag{19b}$$

$$F_{min} = 1 + 2R_v(G_{opt} + G_c) = 1 + 2R_v \left(\sqrt{\frac{G_{iu}}{R_v} + G_c^2} + G_c \right), \tag{19c}$$

resulting in

$$R_v = R_G + \frac{\gamma_{nD}}{G_m} \cdot \left(1 + (\omega R_G C_{GS})^2\right), \tag{20a}$$

$$G_{opt} = \omega\, C_{GS} \cdot \frac{\sqrt{G_m R_G/\gamma_{nD} + (\omega R_G C_{GS})^2}}{1 + G_m R_G/\gamma_{nD} + (\omega R_G C_{GS})^2}, \tag{20b}$$

$$B_{opt} = -\frac{\omega\, C_{GS}}{1 + G_m R_G/\gamma_{nD} + (\omega R_G C_{GS})^2}, \tag{20c}$$

$$F_{min} = 1 + 2\frac{\gamma_{nD} R_G \omega^2 C_{GS}^2}{G_m} + 2\frac{\omega C_{GS}}{G_m}\sqrt{\gamma_{nD} R_G (G_m + \gamma_{nD} R_G \omega^2 C_{GS}^2)}, \tag{20d}$$

Equations (20) can also be expressed by approximating the G_m/C_{GS} ratio by the transit frequency $\omega_t \cong G_m/C_{GS}$ resulting in

$$R_v = \frac{\gamma_{nD}}{G_m} \cdot \left[1 + \alpha_G + \alpha_G^2 \cdot \gamma_{nD}^2 \cdot (\omega/\omega_t)^2\right], \tag{21a}$$

$$G_{opt} = G_m \cdot \frac{\omega}{\omega_t} \cdot \frac{\sqrt{\alpha_G + \alpha_G^2 \cdot \gamma_{nD}^2 \cdot (\omega/\omega_t)^2}}{1 + \alpha_G + \alpha_G^2 \cdot \gamma_{nD}^2 \cdot (\omega/\omega_t)^2}, \tag{21b}$$

$$B_{opt} = -\frac{G_m \cdot \omega/\omega_t}{1 + \alpha_G + \alpha_G^2 \cdot \gamma_{nD}^2 \cdot (\omega/\omega_t)^2}, \tag{21c}$$

$$F_{min} = 1 + 2\alpha_G \cdot \gamma_{nD}^2 \cdot \left(\frac{\omega}{\omega_t}\right)^2 + 2\frac{\omega}{\omega_t} \cdot \sqrt{\alpha_G \cdot \gamma_{nD}^2 \cdot \left[1 + \alpha_G \cdot \gamma_{nD}^2 \cdot \left(\frac{\omega}{\omega_t}\right)^2\right]}, \tag{21d}$$

where $\alpha_G \triangleq G_m \cdot R_G / \gamma_{nD}$ which represents the ratio of the gate resistance noise to the input-referred channel noise and which is usually much smaller than unity.

Most of the time we can consider $\alpha_G \cdot \gamma_{nD}^2 \cdot (\omega/\omega_t)^2 = \gamma_{nD} \cdot G_m \cdot R_G \cdot (\omega/\omega_t)^2 \ll 1$ and the above equations simplify to

$$R_v \cong \frac{\gamma_{nD}}{G_m} \cdot (1 + \alpha_G), \tag{22a}$$

$$G_{opt} \cong G_m \cdot \frac{\sqrt{\alpha_G}}{1 + \alpha_G} \cdot \frac{\omega}{\omega_t} \cong G_m \cdot \sqrt{\alpha_G} \cdot \frac{\omega}{\omega_t}, \tag{22b}$$

$$B_{opt} \cong -\frac{G_m}{1 + \alpha_G} \cdot \frac{\omega}{\omega_t} \cong -G_m \cdot \frac{\omega}{\omega_t}, \tag{22c}$$

$$F_{min} \cong 1 + 2\gamma_{nD} \cdot \sqrt{\alpha_G} \cdot \frac{\omega}{\omega_t}, \tag{22d}$$

where it has been assumed that $\alpha_G \ll 1$. Note that in the case the gate resistance is ignored ($R_G = 0$ or $\alpha_G = 0$), we find

$$R_v = \frac{\gamma_{nD}}{G_m}, \tag{23a}$$

$$G_{opt} = 0, \tag{23b}$$

$$B_{opt} = -G_m \cdot \frac{\omega}{\omega_t}, \tag{23c}$$

$$F_{min} = 1, \tag{23d}$$

Equations (20) reveal that, due to the noise coming from the gate resistance, the noise matching condition is slightly different than the gain matching condition which would require $B_s = -\omega\, C_{GS}$. Also, the minimum noise factor is strongly depending on the gate resistance. As shown in (23), if gate resistance noise was ignored, the minimum noise factor would be equal to unity! This surprising result can be explained as follows: if the induced gate noise is ignored there is only the drain noise left and the optimum source conductance is null whereas the optimum source susceptance is $-\omega\, C_{GS}$. This noise matching situation corresponds to having an inductor with a susceptance value being $-\omega\, C_{GS}$ and no internal conductance. The input circuit is then an inductance in series with the transistor gate-to-source capacitance. This source inductance will then resonate with the input transistor capacitance at the operating frequency providing an infinite voltage gain at the input. The input referred noise is then nulled, resulting in a unity noise factor.

Equation (22d) indicates that the minimum noise factor increases linearly with frequency for a given bias. For a given operating frequency it can be decreased by increasing the transistor transit frequency. This can be achieved by increasing the transistor bias or by reducing the transistor length (or both). Technology scaling leads to an improved noise factor at a given frequency and bias. It can also be reduced by decreasing the gate resistance.

1.3 Effective noise factor

1.3.1 From Two-Port Noise Parameters

The effective noise factor can be calculated from the four noise parameters using

$$F = F_{min} + \frac{R_v}{G_s} \cdot \left[(G_s - G_{opt})^2 + (B_s - B_{opt})^2 \right]. \tag{24}$$

Replacing F_{min}, R_v, G_{opt} and B_{opt} by the expressions given in (23) results in

$$F = 1 + \left(\gamma_{nD} \frac{G_s}{G_m} + R_G G_s \right) \cdot \left(1 + \left(\frac{B_s}{G_s} \right)^2 \right) + 2\gamma_{nD} \frac{B_s}{G_s} \frac{\omega}{\omega_t} + \gamma_{nD} \frac{G_m}{G_s} \cdot \left[(1 + R_G G_s)^2 + (R_G B_s)^2 \right] \cdot \left(\frac{\omega}{\omega_t} \right)^2. \tag{25}$$

Assuming that $B_s = 0$, (25) reduces to

$$F = 1 + \frac{\gamma_{nD}}{G_m R_S} + \frac{R_G}{R_S} + \gamma_{nD} G_m R_S \left(1 + \frac{R_G}{R_S} \right)^2 \cdot \left(\frac{\omega}{\omega_t} \right)^2, \tag{26}$$

where $R_S \triangleq 1/G_s$.

1.3.2 From direct calculation

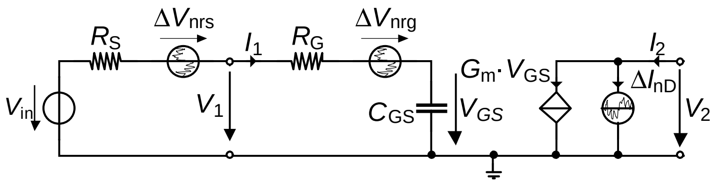

Figure 3: Simplified small-signal schematic of the common-source amplifier for the direct calculation of the noise factor

The effective noise factor can also be calculated directly from the small-signal schematic shown in Fig. 3. We first calculate the effective transconductance as

$$G_{meq} \triangleq \frac{I_2}{V_{in}} = \frac{G_m}{1 + (R_G + R_S) \cdot C_{GS} \cdot s}. \tag{27}$$

The output noise current is given by

$$\Delta I_{nout} = \Delta I_{nD} - G_{meq} \cdot (\Delta V_{nrs} + \Delta V_{nrg}). \tag{28}$$

The corresponding input-referred noise voltage is then given by

$$\Delta V_{neq} = \frac{\Delta I_{nD}}{G_{meq}} - (\Delta V_{nrs} + \Delta V_{nrg}), \tag{29}$$

which leads to the equivalent input-referred noise resistance

$$R_{neq} = R_S + R_G + \frac{G_{nD}}{G_{meq}^2} = R_S + R_G + G_{nD} \cdot \frac{1 + (R_G + R_S)^2 C_{GS}^2 \omega^2}{G_m^2}. \tag{30}$$

Replacing G_m/C_{GS} by the transit frequency ω_t and $G_{nD} = \gamma_{nD} \cdot G_m$, we get

$$R_{neq} = R_S + R_G + \frac{\gamma_{nD}}{G_m} + \gamma_{nD} G_m (R_S + R_G)^2 \cdot \left(\frac{\omega}{\omega_t}\right)^2. \tag{31}$$

The noise factor is then given by

$$F = \frac{R_{neq}}{R_S} = 1 + \frac{R_G}{R_S} + \frac{\gamma_{nD}}{G_m R_S} + \gamma_{nD} G_m R_S \left(1 + \frac{R_G}{R_S}\right)^2 \cdot \left(\frac{\omega}{\omega_t}\right)^2, \tag{32}$$

which is identical to (26).

This shows that, if we are only interested in knowing the effective noise factor, then the direct calculation done in Section 1.3.2 is much easier than deriving the complete noise factors and then the effective noise factor. On the other hand, if we want to know all the noise parameters including the minimum noise factor, we need to derive them following Section 1.2.2.

1.4 F versus IC

Let us analyze the expression of the effective noise factor, F, found previously and reported below.

$$F = 1 + \frac{R_G}{R_S} + \frac{\gamma_{nD}}{G_m R_S} + \gamma_{nD} G_m R_S \left(1 + \frac{R_G}{R_S}\right)^2 \cdot \left(\frac{\omega}{\omega_t}\right)^2. \tag{33}$$

Once R_G and R_S are fixed, the only terms that can be function of IC in this expression are γ_{nD}, G_m and ω_t. The first two terms are constant, the third one is proportional to γ_{nD}/G_m and the last goes like $(\gamma_{nD} \cdot G_m)/\omega_t^2$. Since $\omega_t = \omega_{t,spec} \cdot g_{m,s}$, the entire term is finally proportional to γ_{nD}/G_m. The entire expression for F is then proportional to γ_{nD}/G_m.

1.4.1 Long channel

For a long channel transistor, we have γ_{nD} constant and equal to $\frac{1}{2}n$ in WI and $\frac{2}{3}n$ in SI. On the other hand, G_m follows IC in WI and $\sqrt{IC}$ in SI. The sketch of the asymptotes in WI and SI for the effective noise factor as a function of IC in a long channel transistor is then given in the figure below.

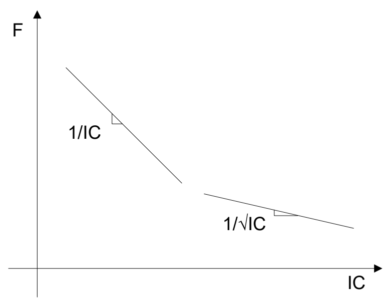

Figure 4: Sketch of the asymptotes in WI and SI for F as a function of IC in a long channel transistor.

1.4.2 Short channel

For a short channel transistor, we have to express γ_{nD} as $1 + \alpha \cdot IC$. Again, in WI G_m follows IC, while in SI the value saturates to $1/\lambda_c$. The sketch of the asymptotes in WI and SI for the effective noise factor as a function of in a short channel transistor is then given in the figure below.

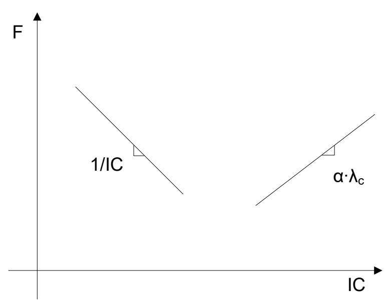

Figure 5: Sketch of the asymptotes in WI and SI for F as a function of IC in a short channel transistor.

1.4.3 Comments

For a long channel transistor, the γ_{nD} is constant, hence the effective noise factor versus IC follows the same trend of $1/G_m$ with IC. In a short channel transistor, the dependency of γ_{nD} with IC implies a decrease of F witch IC in WI and an increasing trend in SI. Hence, in a short channel transistor the effective noise factor, F, will have a minimum value, which will correspond to an optimal value for IC in terms of minimum noise contribution.