Low-power radio design for the IoT
Exercise 9 (20.05.2021)

Christian Enz

Swiss Federal Institute of Technology (EPFL), Lausanne, Switzerland

Problem 1 Dual Gate Mixer

Consider the dual-gate mixer shown in Fig. 1. Assume when M_1 is on, it has an on-resistance of R_{on1}. Also, assume abrupt edges and a 50% duty cycle for the LO, also $G_{ds1} = G_{ds2} = 0$ and the connection between bulk and source.

- Compute the voltage conversion gain of the circuit. Assume M_2 does not enter the linear region and denote its transconductance by g_{m2}.
- If R_{on1} is very small, determine the IP_2 of the circuit. Assume M_2 has an overdrive of $V_{GS0} - V_{TH}$ in the absence of signals (when it is on).

Problem 2 Active mixer with load mismatch

Consider the active mixer shown in Fig. 2, where LO has abrupt edges and a 50% duty cycle. Also, assume $G_{ds1} = G_{ds2} = 0$ and the connection between bulk and source. The load resistors exhibit mismatch, but the circuit is otherwise symmetric. Assume M_1 carries a bias current of I_{ss}.

- Determine the output offset voltage.
- Determine the IP_2 of the circuit in terms of the overdrive and bias current of M_1.

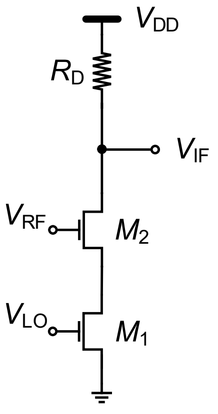

Figure 1: Dual-gate mixer

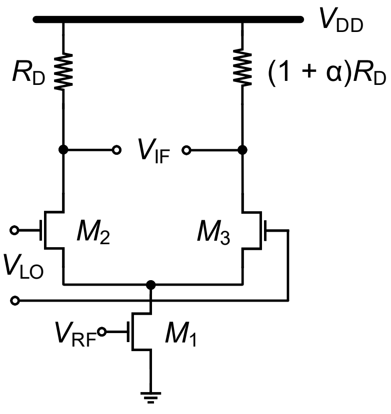

Figure 2: Active mixer with load mismatch

Solutions to Exercise 9 (20.05.2021)

Problem 1 Dual-Gate mixer

Consider the dual-gate mixer shown in Fig. 1. Assume when M_1 is on, it has an on-resistance of R_{on1}. Also, assume abrupt edges and a 50% duty cycle for the LO, also $G_{ds1} = G_{ds2} = 0$ and the connection between bulk and source.

- Compute the voltage conversion gain of the circuit. Assume M_2 does not enter the linear region and denote its transconductance by g_{m2}.

The output voltage of the mixer can be written as

$$V_{IF}(t) = I_{RF}(t) \cdot R_D \cdot S_{LO}(t) \tag{1}$$

where S_{LO} is the square wave toggling between 1 and 0 applied to the gate of M_1. This square wave can be approximated by its first component and give raise to the following approximation:

$$V_{IF}(t) = I_{RF}(t) \cdot R_D \cdot \frac{2}{\pi} \cos(\omega_{LO} t) \tag{2}$$

The I_{RF} current can be derived by solving the following small signal equivalent circuit from which

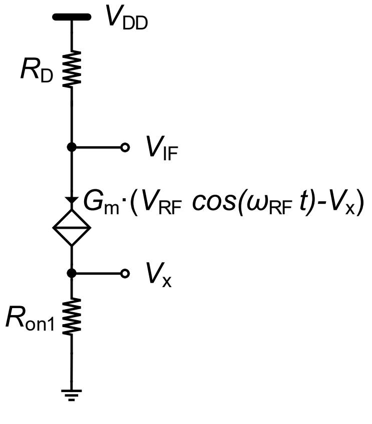

Figure 1: Small signal equivalent of the dual-gate mixer

$$I_{RF}(t) = \frac{G_{m2}}{1 + G_{m2} R_{on1}} V_{RF} \cos(\omega_{RF} t) \tag{3}$$

The output V_{IF} can be then re-written as

$$V_{IF}(t) = \frac{G_{m2}}{1 + G_{m2} R_{on1}} V_{RF} \cos(\omega_{RF} t) \cdot R_D \cdot \frac{2}{\pi} \cos(\omega_{LO} t) \tag{4}$$

which for the $\cos((\omega_{RF} - \omega_{IF})t)$ components results into

$$V_{IF}(t) = \frac{G_{m2}}{1 + G_{m2} R_{on1}} V_{RF} \cdot R_D \cdot \frac{1}{\pi} \cos((\omega_{RF} - \omega_{LO})t) \tag{5}$$

Finally, the conversion gain can be expressed as

$$\frac{V_{IF_p}}{V_{RF_p}} = \frac{1}{\pi} \cdot \frac{G_{m2}}{1 + G_{m2} R_{on1}} \cdot R_D \tag{6}$$

- If R_{on1} is very small, determine the IP_2 of the circuit. Assume M_2 has an overdrive of $V_{GS0} - V_{TH}$ in the absence of signals (when it is on).

The total voltage applied to the gate of M_2 to evaluate the IP_2 is equal to

$$V_{RF} = V_m \cos(\omega_1 t) + V_m \cos(\omega_2 t) + V_{GS0} \tag{7}$$

This determines a second order intermodulation component that can be written as

$$I_{IM2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_m^2 \cos((\omega_1 - \omega_2)t) \tag{8}$$

To derive the IP2, V_{IIP2}, we equate the two expressions

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{IIP2}^2 R_D = \frac{1}{\pi}\cdot\frac{G_{m2}}{1 + G_{m2}R_{on1}}\cdot R_D V_{IIP2} \tag{9}$$

and we obtain, for $G_{m1} >> 1/R_{on}$, the following expression

$$V_{IIP2} = \frac{2}{\pi}(V_{GS0} - V_{th})_2 \tag{10}$$

Problem 2 Active mixer with load mismatch

Consider the active mixer shown in Fig. 2, where LO has abrupt edges and a 50% duty cycle. Also, assume $G_{ds1} = G_{ds2} = 0$ and the connection between bulk and source. The load resistors exhibit mismatch, but the circuit is otherwise symmetric. Assume M_1 carries a bias current of I_{ss}.

- Determine the output offset voltage.

In the circuit in Fig. 2, transistor M_1 produces a small-signal drain current equal to $g_{m1}V_{\text{RF}}$. With abrupt LO switching, M_2 multiplies I_{ss} by a square wave toggling between 0 and 1, $S(t)$, and M_3 by $S\big(t - \frac{T_{\text{LO}}}{2}\big)$ because the input pair is differentially driven. It follows that:

$$I_2 = I_{\text{ss}} \cdot S(t) \tag{11a}$$

$$I_3 = I_{\text{ss}} \cdot S\left(t - \frac{T_{\text{LO}}}{2}\right) \tag{11b}$$

It follows:

$$\begin{aligned} V_{\text{out}}(t) = I_3 R_{\text{D}}(1+\alpha) - I_2 R_{\text{D}} &= \underbrace{I_{\text{ss}}\alpha R_{\text{D}} S\left(t - \frac{T_{\text{LO}}}{2}\right)}_{\text{Offset term}} \qquad + \underbrace{I_{\text{ss}} R_{\text{D}}\left(S\left(t - \frac{T_{\text{LO}}}{2}\right) - S(t)\right)}_{\text{Original term}} \approx \\ &\approx I_{\text{ss}}\alpha R_{\text{D}}\frac{2}{\pi}\cos(\omega_{\text{LO}}t) \qquad + I_{\text{ss}} R_{\text{D}}\frac{4}{\pi}\cos(\omega_{\text{LO}}t); \end{aligned} \tag{12}$$

where the approximation takes into consideration the fundamental amplitude of the square wave. Therefore the output offset is $I_{\text{ss}}\alpha R_{\text{D}}\frac{2}{\pi}$.

- Determine the IP_2 of the circuit in terms of the overdrive and bias current of M_1.

In order to calculate the IP_2 we assume M_1 gate voltage to be:

$$V_{\mathrm{RF}} = V_{\mathrm{ov}} \cos(\omega_1 t) + V_{\mathrm{ov}} \cos(\omega_2 t) + V_{\mathrm{GS0}}; \tag{13}$$

where V_{GS0} is the bias gate-source voltage of M_1 and V_{ov} is the overdrive voltage. With a square-law device, the second order intermodulation IM_2 product emerges in the current of M_1 as

$$\mathrm{IM}_2 = \frac{1}{2} \mu C_{\mathrm{ox}} \frac{W}{L} V_{\mathrm{m}}^2 \cos\left((\omega_1 - \omega_2)t\right) \tag{14}$$

Multiplying this quantity by $\alpha R_{\mathrm{D}} \frac{2}{\pi}$ yields the direct feed-through to the output:

$$V_{\mathrm{IM2,out}} = \left(\frac{1}{2} \mu C_{\mathrm{ox}} \frac{W}{L} V_{\mathrm{m}}^2 \cos\left((\omega_1 - \omega_2)t\right) \right) \alpha R_{\mathrm{D}} \frac{2}{\pi} \tag{15}$$

To calculate the IP_2, the value of V_{ov} must be raised until the amplitude of $V_{\mathrm{IM2,out}}$ becomes equal to the amplitude of the main downconverted components. This amplitude is simply given by $(2/\pi) g_{m1} R_{\mathrm{D}} V_{\mathrm{ov}}$. Thus

$$\begin{aligned} \frac{1}{2} \mu C_{\mathrm{ox}} \frac{W}{L} V_{\mathrm{m}}^2 \alpha R_{\mathrm{D}} \frac{2}{\pi} &= \frac{2}{\pi} g_{m1} R_{\mathrm{D}} V_{\mathrm{IIP2}} \\ &= \frac{2}{\pi} \mu C_{\mathrm{ox}} \frac{W}{L} V_{\mathrm{ov1}} R_{\mathrm{D}} V_{\mathrm{IIP2}} \end{aligned} \tag{16}$$

where $g_{m1} = \mu C_{\mathrm{ox}} (W/L) V_{\mathrm{ov1}}$. Finally,

$$V_{\mathrm{IIP2}} = \frac{\alpha V_{\mathrm{ov1}}}{2} \tag{17}$$