Advanced Analog and RF IC Design I - Exam Summer Semester 2017/2018

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Instructions: Attempt all problems. Total allocated time is 3 hours. The points for each problem are indicated in square brackets.

Multiple Choice Questions [6]

Answer to the following questions (note that there is only one exact answer to each question):

1. Into a homodyne receiver baseband, to relax the channel selection filter noise performance, where it is more convenient to place the amplifier? [1]
   (a) After the channel selection filter
   (b) It does not matter, the amplifier has just to perform sufficiently high gain
   (c) Before the channel selection filter
   (d) After the ADC, in digital domain
2. Into a common gate LNA with input impedance matching, what is the correct expression for the noise factor? [1]
   (a) $F = 1 + \delta_{nD}$
   (b) $F = 1$
   (c) $F = \delta_{nD}$
   (d) None of the previous answers
3. What is the relation between the bias current and the oscillation amplitude in a LC cross-coupled NMOS pair oscillator, provided that $A \gg nU_T$ and the cross-coupled pair is biased in WI? [1]
   (a) $I_b = G_m \frac{\pi}{2} A$
   (b) $I_b = G_{m_{crit}} \frac{\pi}{4} A$
   (c) No analytical relation exists
   (d) $I_b = G_{m_{crit}} \frac{\pi}{2} A$
4. The two inputs of a mixer are respectively $x_1(t) = A_1 \cos(\omega_1 t)$ and $x_2(t) = A_2 \cos(\omega_2 t)$. The input port sensing $x_2(t)$ suffers from third-order non-linearity. Determine the output frequency components. [1]
   (a) The output contains only the sum frequencies.
   (b) The output contains only the difference frequencies.
   (c) The output contains only the sum and the difference frequencies.
   (d) The output contains the sum, the difference frequencies, and two spurious components at $\omega_1 + 3\omega_2$ and $\omega_1 - 3\omega_2$
5. Determine the noise factor of a common source stage with an input series resistance R_S and an ideal current source as the drain output load using the Friis formula. [1]
   (a) $F = 1$
   (b) $F = 1 + \gamma_{nD} / (G_m R_S)$
   (c) $F = 4kT (\gamma_{nD} / G_m + R_S)$
   (d) $F = 4kT \gamma_{nD} G_m R_o^2$
6. When the operating points move from strong to moderate or even weak inversion, the advantages for RF circuits are: [1]
   (a) Higher current efficiency, lower electrical fields but higher voltage of operation
   (b) Lower current efficiency, lower electrical fields and higher voltage of operation
   (c) Lower non-linearity
   (d) Higher current efficiency, lower electrical fields, low voltage operation but higher non-linearity

Problem 1: Wideband noise-cancelling LNA [10]

The circuit shown in Fig. 1 is a wideband low-noise amplifier that performs single-ended to differential conversion and noise cancelling. Transistor M_1 is in a separate well (source and bulk connected), and both transistors are biased in saturation, $G_{m1} = G_{m2} = G_m$ and $R_1 = R_2 = R$. In the analysis you can neglect the output conductances of transistors.

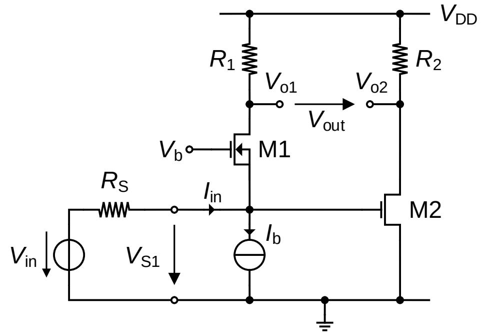

Figure 1: Wideband noise-cancelling LNA

1.1 Small-signal analysis [4]

1. Draw the equivalent small-signal schematic of the circuit. [1]
2. Calculate the input impedance of the LNA and find the value of G_m for impedance matching [1]
3. Calculate the small-signal voltage gain $A_v = \Delta V_{out}/\Delta V_{in}$ and then find its value assuming impedance matching. [2]

1.2 Noise analysis [6]

1. Draw the equivalent small-signal schematic including all the noise sources. [1]
2. Calculate the input-referred thermal noise resistance R_{neq}. [3]
3. Find the value of G_m that cancels the noise contribution of M_1. [1]
4. Find the noise figure F assuming the condition for achieving noise cancelling of M_1 is fulfilled. [1]

Problem 2: Complementary cross-coupled oscillator [8]

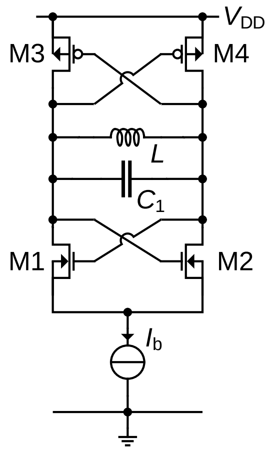

Figure 2: Complementary cross-coupled oscillator

2.1 Oscillator analysis [5]

Fig. 2 shows a complementary cross-coupled oscillator. In the first part of the problem we will derive expressions for quantities that will be used in the second part. All transistors are biased in weak inversion and have transconductances equal to G_m. Quality factor of the inductor is Q_L.

1. Draw the small signal equivalent circuit. [1]
2. Derive the expression for the impedance seen from the inductor Z_c, and find $R_c = -\mathrm{Re}\{Z_c\}$ and $X_c = -\mathrm{Im}\{Z_c\}$. [2]
3. Derive the expression for the oscillation frequency ω_0. [1]
4. Derive the expression for the G_{mcrit}. [1]

2.2 Oscillator design [3]

The derived expressions will now be used to design the oscillator with the following specifications:

$$f_0 = 2.4\,GHz, \quad C = 0.5\,pF, \quad Q_L = 10, \quad V_{out} = 325\,mV, \quad U_T = 25\,mV, \quad n = 1.3$$

Again, assume that all transistors are biased in weak inversion.

1. Find the inductance value for the given oscillation frequency. [1]
2. Find the value of G_{mcrit}. [1]
3. Calculate the bias current needed to achieve the desired amplitude of the output voltage V_{out}. You can assume here that the condition $V_{out} \gg 2nU_T$ is fulfilled. [1]

Solutions to Exam Summer Semester 2017/2018

Multiple Choice Questions [6]

Answer to the following questions (note that there is only one exact answer to each question):

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Problem 1: Wideband noise-cancelling LNA [10]

The circuit shown in Fig. 1 is a wideband low-noise amplifier that performs single-ended to differential conversion and noise cancelling. Transistor M_1 is in a separate well (source and bulk connected), and both transistors are biased in saturation, $G_{m1} = G_{m2} = G_m$ and $R_1 = R_2 = R$. In the analysis you can neglect the output conductances of transistors.

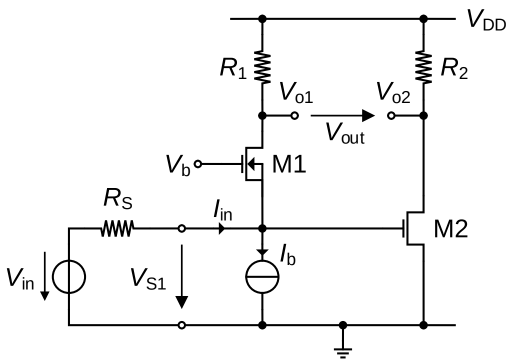

Figure 1: Wideband noise-cancelling LNA

1.1 Small-signal analysis [4]

1. **Draw the equivalent small-signal schematic of the circuit.** **[1]**

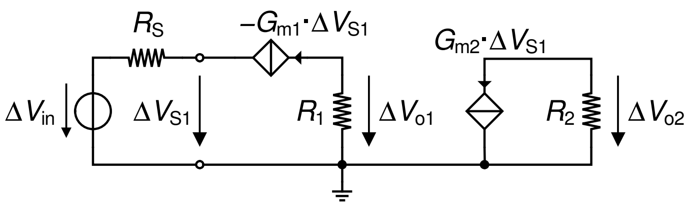

Figure 2: Small signal schematic of the wideband noise-cancelling LNA

2. **Calculate the input impedance of the LNA and find the value of G_m for impedance matching** **[1]**
Since the output conductance of the transistor M_1 can be neglected the input impedance of the LNA is simply

$$Z_{in} = \frac{1}{G_m}. \tag{1}$$

To achieve input matching the transconductance must be

$$G_m = \frac{1}{R_S}. \tag{2}$$

3. **Calculate the small-signal voltage gain $A_v = \Delta V_{out}/\Delta V_{in}$ and then find its value assuming impedance matching.** **[2]**
Looking at the small signal schematic we can write the equations:

$$\Delta V_{o1} = -R(-G_m \Delta V_{S1}) \tag{3}$$

$$\Delta V_{o2} = -R G_m \Delta V_{S1} \tag{4}$$

$$\frac{\Delta V_{in} - \Delta V_{S1}}{R_S} = G_m \Delta V_{S1}. \tag{5}$$

Output differential voltage is

$$\Delta V_{out} = \Delta V_{o1} - \Delta V_{o2} = 2 G_m R \, \Delta V_{S1}. \tag{6}$$

From equation 5 we can express ΔV_{S1} as

$$\Delta V_{S1} = \frac{1}{1 + G_m R_S} \Delta V_{in}, \tag{7}$$

which finally gives

$$\Delta V_{out} = 2 G_m R \frac{1}{1 + G_m R_S} \Delta V_{in} \tag{8}$$

$$A_v = \frac{2 G_m R}{1 + G_m R_S} \tag{9}$$

If the input is matched $G_m = 1/R_S$ and $A_v = R/R_S$.

1.2 Noise analysis [6]

1. **Draw the equivalent small-signal schematic including all the noise sources.** **[1]**

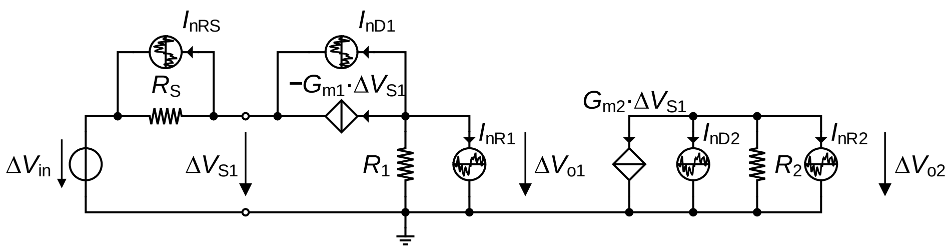

Figure 3: Small-signal schematic of the wideband noise-cancelling LNA with noise sources

2. **Calculate the input-referred thermal noise resistance R_{neq}.** **[3]**
To calculate the input-referred noise resistance we can first calculate output noise voltage V_{nout}. This can be done either by directly solving circuit equations or by calculating the contribution of each noise source separately.

$$V_{nout} = -RI_{\text{nR1}} + RI_{\text{nR2}} + RI_{\text{nD2}} + \frac{2G_{\text{m}}RR_S}{1+G_{\text{m}}R_S} I_{\text{nRs}} + \frac{R(G_{\text{m}}R_S-1)}{1+G_{\text{m}}R_S} I_{\text{nD1}}. \tag{10}$$

The output noise power spectral density is then

$$S_{Vnout} = 4kTR + 4kTR + 4kT\gamma G_{\text{m}}R^2 + \left(\frac{2G_{\text{m}}RR_S}{1+G_{\text{m}}R_S}\right)^2 4kT/R_S + \left(\frac{R(G_{\text{m}}R_S-1)}{1+G_{\text{m}}R_S}\right)^2 4kT\gamma G_{\text{m}}. \tag{11}$$

And the equivalent output noise resistance

$$R_{nout} = 2R + \gamma G_{\text{m}}R^2 + \left(\frac{2G_{\text{m}}R}{1+G_{\text{m}}R_S}\right)^2 R_S + \left(\frac{R(G_{\text{m}}R_S-1)}{1+G_{\text{m}}R_S}\right)^2 \gamma G_{\text{m}}. \tag{12}$$

The input referred noise resistance is calculated as

$$R_{neq} = \frac{R_{nout}}{A_v^2} \tag{13}$$

$$= R_S + 2R\frac{(1+G_{\text{m}}R_S)^2}{4G_{\text{m}}^2R^2} + \gamma G_{\text{m}}R^2\frac{(1+G_{\text{m}}R_S)^2}{4G_{\text{m}}^2R^2} + \gamma G_{\text{m}}\frac{(1+G_{\text{m}}R_S)^2}{4G_{\text{m}}^2R^2}\left(\frac{R(G_{\text{m}}R_S-1)}{1+G_{\text{m}}R_S}\right)^2 \tag{14}$$

$$= R_S + \frac{(1+G_{\text{m}}R_S)^2}{2G_{\text{m}}^2R} + \gamma\frac{(1+G_{\text{m}}R_S)^2}{4G_{\text{m}}} + \gamma\frac{(G_{\text{m}}R_S-1)^2}{4G_{\text{m}}}. \tag{15}$$

3. **Find the value of G_{m} that cancels the noise contribution of M_1.** **[1]**
Output noise voltage that comes from transistor M_1 is

$$V_{nout,M_1} = \frac{R(G_{\text{m}}R_S-1)}{1+G_{\text{m}}R_S} I_{\text{nD1}}. \tag{16}$$

It is easy to see that this factor will disappear if $G_{\text{m}}R_S = 1$ which is the same as the condition to achieve input matching!

4. **Find the noise figure F assuming the condition for achieving noise cancelling of M_1 is fulfilled.** **[1]**
Since $G_{\text{m}} = 1/R_S$ we have:

$$R_{neq} = R_S + 2\frac{R_S^2}{R} + \gamma R_S \tag{17}$$

$$F = \frac{R_{neq}}{R_S} = 1 + 2\frac{R_S}{R} + \gamma. \tag{18}$$

Problem 2: Complementary cross-coupled oscillator [8]

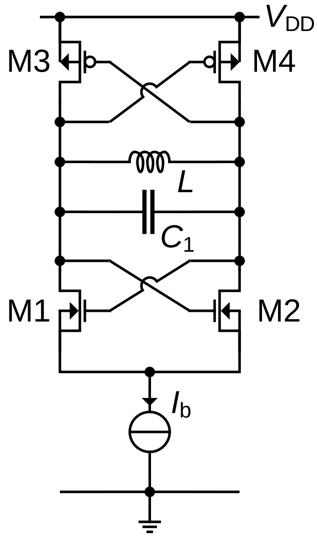

Figure 4: Complementary cross-coupled oscillator

2.1 Oscillator analysis [5]

Fig. 4 shows a complementary cross-coupled oscillator. In the first part of the problem we will derive expressions for quantities that will be used in the second part. All transistors are biased in weak inversion and have transconductances equal to G_m. Quality factor of the inductor is Q_L.

1. **Draw the small signal equivalent circuit.** **[1]**

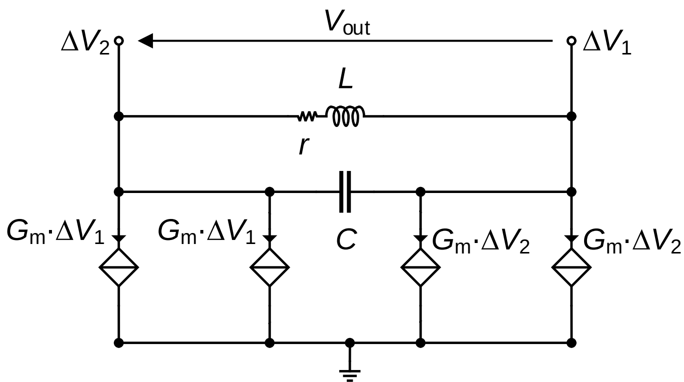

Figure 5: Complementary cross-coupled oscillator

2. **Derive the expression for the impedance seen from the inductor Z_c, and find $R_c = -\mathrm{Re}\{Z_c\}$ and $X_c = -\mathrm{Im}\{Z_c\}$.** **[2]**
As can be seen from the Fig. 5 the complementary oscillator is practically equivalent to the NMOS one. The only difference is the total transconductance that is now equal to the sum of the transconductances of the NMOS and the PMOS transistors. It follows:

$$Z_c = \frac{1}{-G_{\mathrm{m}+} j\omega C}, \tag{19}$$

$$R_c = \frac{G_{\mathrm{m}}}{G_{\mathrm{m}}^2 + \omega^2 C^2}, \tag{20}$$

$$X_c = \frac{j\omega C}{G_{\mathrm{m}}^2 + \omega^2 C^2}. \tag{21}$$

3. **Derive the expression for the G_{mcrit}.** **[1]**

To find the value of critical transconductance we can solve the equation:

$$\frac{X_c(\omega_0, G_{\text{mcrit}})}{R_c(\omega_0, G_{\text{mcrit}})} = Q_L, \tag{22}$$

$$G_{\text{mcrit}} = \frac{\omega_0 C}{Q_L}. \tag{23}$$

4. **Derive the expression for the oscillation frequency ω_0.** **[1]**
   Oscillation frequency can be obtained from the equation:

$$R_c(\omega_0, G_{\text{mcrit}}) = \omega_0 L, \tag{24}$$

$$\omega_0 = \frac{1}{\sqrt{LC\left(1 + \frac{1}{Q_L^2}\right)}} \tag{25}$$

2.2 Oscillator design [3]

The derived expressions will now be used to design the oscillator with the following specifications:

$$f_0 = 2.4\,GHz, \quad C = 0.5\,pF, \quad Q_L = 10, \quad V_{out} = 325\,mV, \quad U_{\text{T}} = 25\,mV, \quad n = 1.3$$

Again, assume that all transistors are biased in weak inversion.

1. **Find the inductance value for the given oscillation frequency.** **[1]**

$$L = \frac{1}{\omega_0^2 C\left(1 + \frac{1}{Q_L^2}\right)} = 8.709\,nH \tag{26}$$

2. **Find the value of G_{mcrit}.** **[1]**

$$G_{\text{mcrit}} = \frac{\omega_0 C}{Q_L} = 754\,\mu S. \tag{27}$$

3. **Calculate the bias current needed to achieve the desired amplitude of the output voltage V_{out}. You can assume here that the condition $V_{out} \gg 2nU_{\text{T}}$ is fulfilled.** **[1]**
   For the given amplitude $V_{out} = 325\,mV$ we can calculate:

$$x = \frac{V_{out}}{2nU_{\text{T}}} = 5, \tag{28}$$

   Due to the high amplitude of the voltage we can write:

$$\frac{G_{\text{m(1)}}}{G_{\text{m}}} = \frac{a_1}{x} = \frac{4}{\pi x} = 0.2547. \tag{29}$$

   For $G_{\text{m(1)}} = G_{\text{mcrit}}$ we have

$$I_b = 2nU_{\text{T}}G_{\text{m}} = 2nU_{\text{T}}\frac{\pi x}{4}G_{\text{mcrit}} = 192\,\mu A \tag{30}$$