

MICRO-461

Low-power Radio Design for the IoT

2. Passive RF Circuits and Impedance Matching

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The logo of the Swiss Federal Institute of Technology, Lausanne (EPFL), consisting of the letters 'EPFL' in a bold, red, sans-serif font.

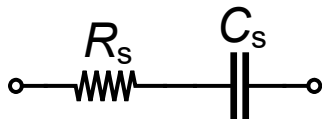
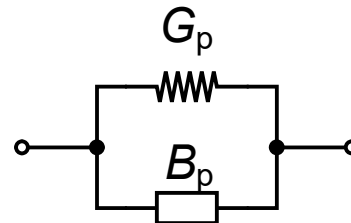
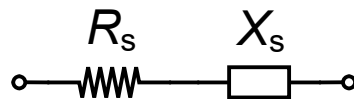
Outline

- Passive RF circuits
- Impedance matching

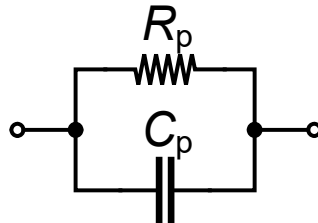
Quality Factor

- The **quality factor** Q is defined as the ratio of the energy stored (in the capacitance or inductance) to the energy dissipated (in the resistance or conductance) per cycle

$$Q \triangleq 2\pi \frac{\text{energy stored}}{\text{energy dissipated per cycle}} = \frac{X_s}{R_s} = \frac{B_p}{G_p} = \frac{R_p}{X_p}$$



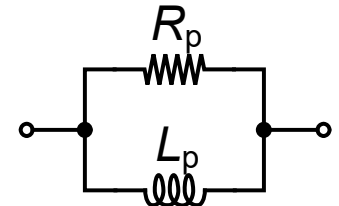
$$Q_s = \frac{1}{\omega R_s C_s}$$



$$Q_p = \omega R_p C_p$$



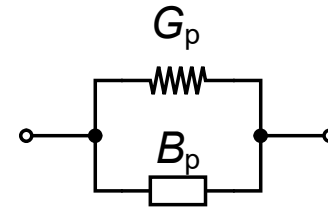
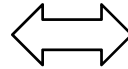
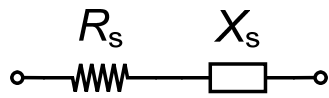
$$Q_s = \frac{\omega L_s}{R_s}$$



$$Q_p = \frac{R_p}{\omega L_p}$$

Series and Parallel Equivalence

- An impedance Z (admittance Y) can always be represented by $Z = R_s + jX_s$ ($Y = G_p + jB_p$) corresponding to the series and parallel circuits shown below



$$Z = R_s + jX_s$$

$$Y = G_p + jB_p$$

$$Z = \frac{1}{Y} = \frac{1}{G_p + jB_p} = \frac{G_p - jB_p}{G_p^2 + B_p^2}$$

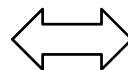
$$Y = \frac{1}{Z} = \frac{1}{R_s + jX_s} = \frac{R_s - jX_s}{R_s^2 + X_s^2}$$

- From which we get

$$R_s = \frac{G_p}{G_p^2 + B_p^2} = \frac{R_p}{Q_p^2 + 1}$$

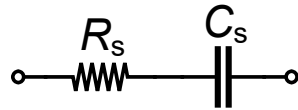
$$G_p = \frac{R_s}{R_s^2 + X_s^2} = \frac{1}{R_s} \cdot \frac{1}{Q_s^2 + 1}$$

$$X_s = -\frac{B_p}{G_p^2 + B_p^2} = \frac{Q_p^2}{Q_p^2 + 1} \cdot X_p$$

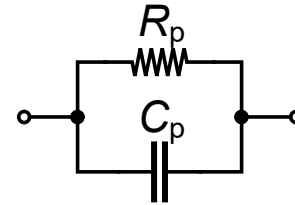


$$B_p = -\frac{X_s}{R_s^2 + X_s^2} = -\frac{1}{X_s} \cdot \frac{Q_s^2}{Q_s^2 + 1}$$

Series and Parallel Equivalence



$$Q_s = Q_p = Q$$

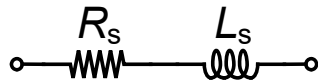


$$R_s = \frac{R_p}{Q^2 + 1} \cong \frac{R_p}{Q^2} \quad \text{for } Q \gg 1$$

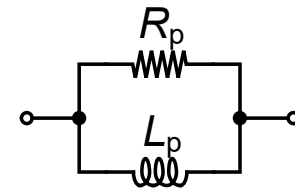
$$R_p = (Q^2 + 1) \cdot R_s \cong Q^2 \cdot R_s \quad \text{for } Q \gg 1$$

$$C_s = \left(1 + \frac{1}{Q^2}\right) \cdot C_p \cong C_p \quad \text{for } Q \gg 1$$

$$C_p = \frac{Q^2}{Q^2 + 1} \cdot C_s \cong C_s \quad \text{for } Q \gg 1$$



$$Q_s = Q_p = Q$$



$$R_s = \frac{R_p}{Q^2 + 1} \cong \frac{R_p}{Q^2} \quad \text{for } Q \gg 1$$

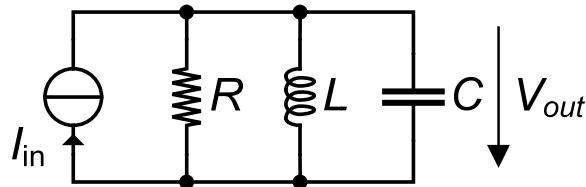
$$R_p = (Q^2 + 1) \cdot R_s \cong Q^2 \cdot R_s \quad \text{for } Q \gg 1$$

$$L_s = \left(\frac{Q^2}{Q^2 + 1}\right) \cdot L_p \cong L_p \quad \text{for } Q \gg 1$$

$$L_p = \left(1 + \frac{1}{Q^2}\right) \cdot L_s \cong L_s \quad \text{for } Q \gg 1$$

- Equivalence **only valid at a given frequency!**

Parallel RLC Network – Admittance



$$Y \triangleq \frac{I_{in}}{V_{out}} = G + \frac{1}{j\omega L} + j\omega C = G + j\omega C \cdot \left[1 - \left(\frac{\omega_0}{\omega} \right)^2 \right]$$

$$\text{with } G \triangleq \frac{1}{R}, \quad \omega_0 \triangleq \frac{1}{\sqrt{LC}}$$

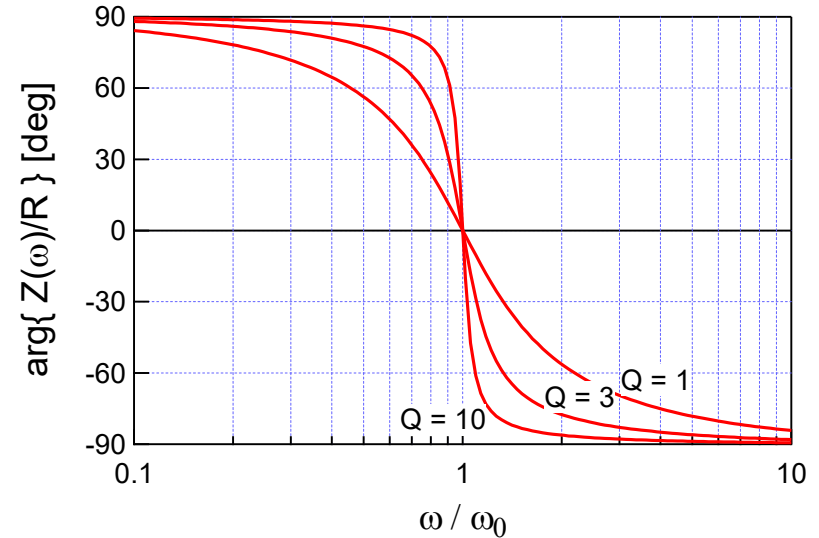
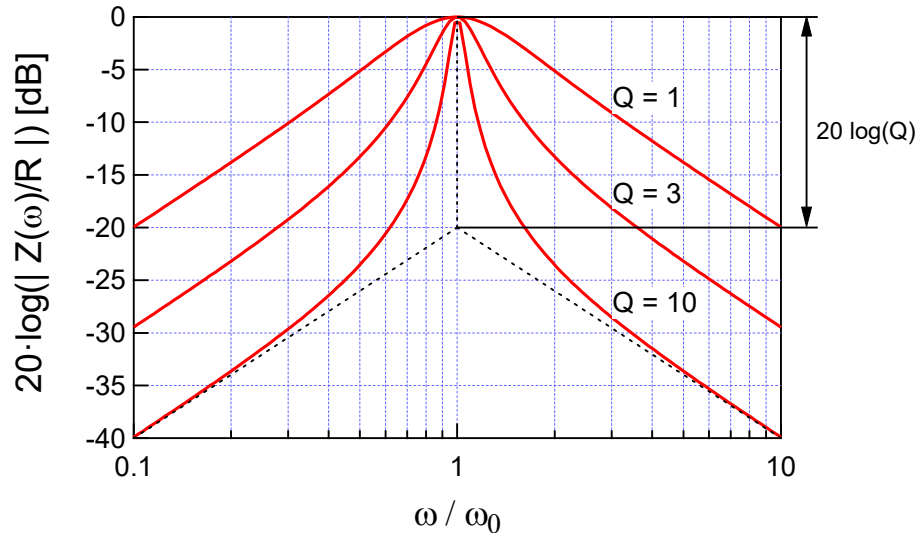
- The **quality factor** Q is defined as the ratio of the energy stored (in the inductance or capacitance) to the energy dissipated (in the conductance) per cycle

$$Q \triangleq 2\pi \frac{\text{energy stored}}{\text{energy dissipated per cycle}} = \omega_0 \frac{C \cdot V_p^2 / 2}{V_p^2 / (2R)} = \omega_0 RC = R \cdot \sqrt{\frac{C}{L}} = \frac{\omega_0 C}{G} = \frac{R}{\omega_0 L} = \frac{1}{\omega_0 GL}$$

- The admittance then writes

$$Y = G \cdot \left[1 + jQ \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \right] \quad \text{with } Q \triangleq \frac{\omega_0 C}{G}$$

Parallel RLC Network – Impedance

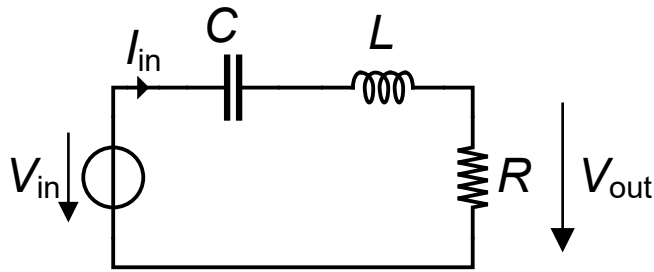


- The impedance of the parallel RLC network is given by

$$Z \triangleq \frac{V_{out}}{I_{in}} = \frac{1}{Y} = \frac{R}{1 + jQ \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)}$$

- At resonance frequency $\omega = \omega_0$, the impedance is maximum and reduces to the parallel resistance $Z = R$
- Frequency components at $\omega \neq \omega_0$ (harmonic components for example) are attenuated proportionally to the Q factor

Series RLC Network – Impedance



$$Z \triangleq \frac{V_{in}}{I_{in}} = R + j\omega L + \frac{1}{j\omega C} = R + j\omega L \cdot \left[1 - \left(\frac{\omega_0}{\omega} \right)^2 \right]$$

with $\omega_0 \triangleq \frac{1}{\sqrt{LC}}$

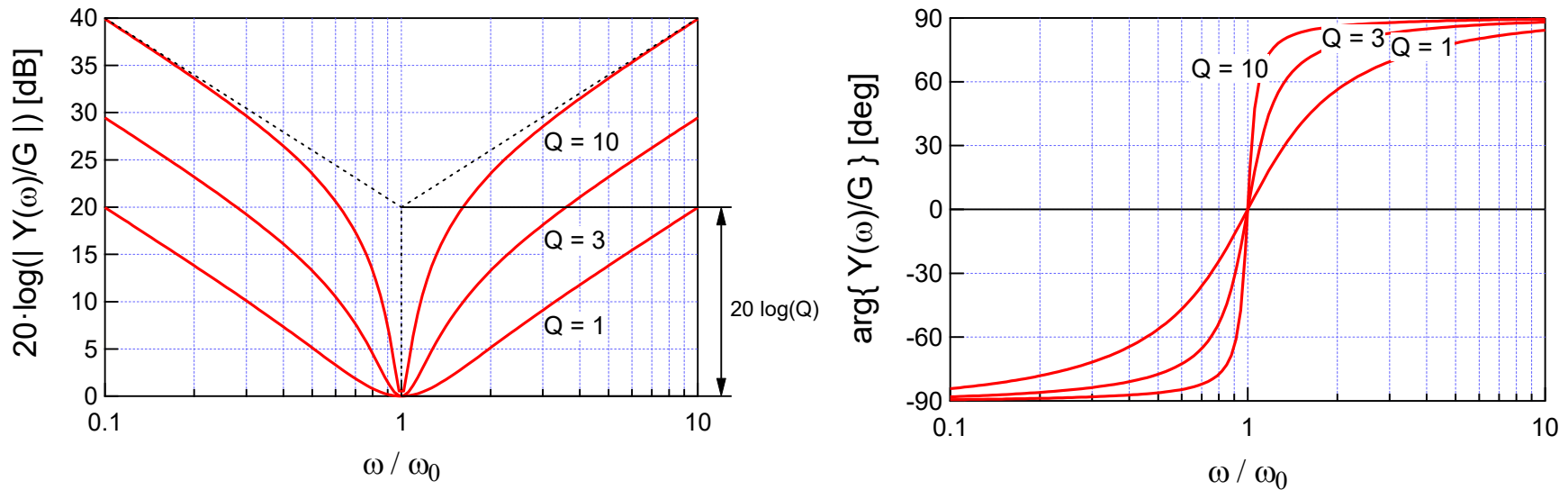
- Similar to the parallel RLC network, the **quality factor** Q is then given by the reciprocal form

$$Q \triangleq 2\pi \frac{\text{energy stored}}{\text{energy dissipated per cycle}} = \omega_0 \frac{L \cdot I_p^2 / 2}{R \cdot I_p^2 / 2} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$$

- The impedance then writes

$$Z = R \cdot \left[1 + jQ \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \right] \quad \text{with} \quad Q \triangleq \frac{\omega_0 L}{R}$$

Parallel (series) RLC Network – Admittance (impedance)

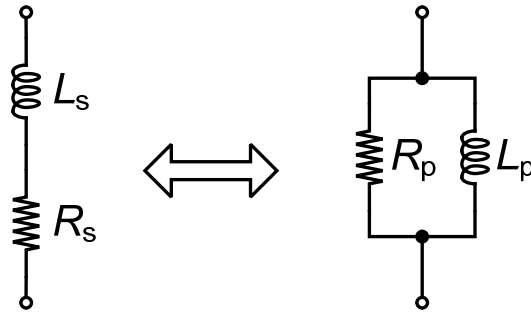


- The normalized admittance (impedance) of the parallel (series) RLC network is given by

$$\frac{Y}{G} = \frac{Z}{R} = 1 + jQ \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)$$

- The admittance (impedance) is minimum at the resonance frequency $\omega = \omega_0$ and reduces to the parallel (series) conductance (resistance) G (R)

Lossy Inductor



$$Z_s = R_s + j\omega L_s$$

$$Y_s = \frac{1}{R_s + j\omega L_s} = \frac{R_s - j\omega L_s}{R_s^2 + (\omega L_s)^2}$$

$$Y_p = G_p + \frac{1}{j\omega L_p} = \frac{1}{R_p} + \frac{1}{j\omega L_p}$$

- Equating the real and imaginary parts and remembering that both circuits should have the same Q (same losses)

$$Q = Q_s = \frac{\omega L_s}{R_s} = Q_p = \frac{1}{\omega G_p L_p}$$

- we get the following relations

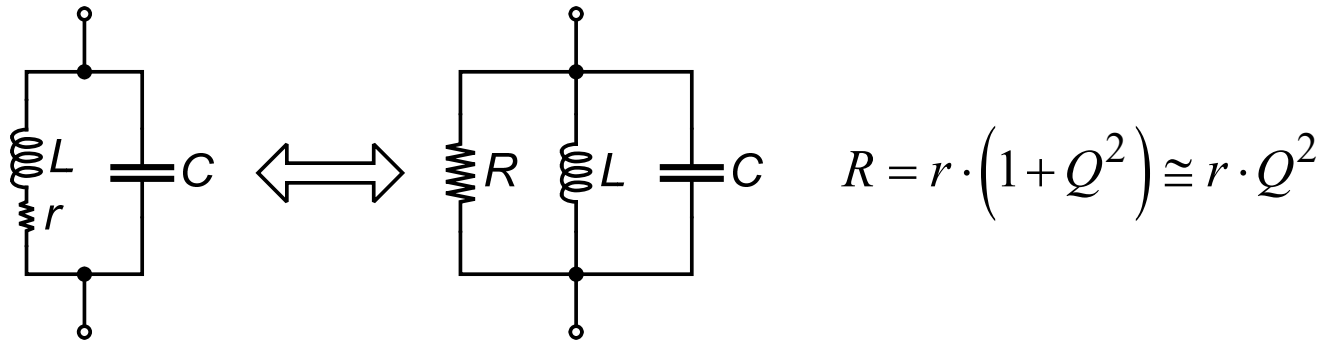
$$R_p = \frac{1}{G_p} = R_s \cdot \left(1 + Q^2\right) \quad L_p = L_s \cdot \left(1 + \frac{1}{Q^2}\right)$$

- If $Q \gg 1$ (typically larger than 10), we get

$$R_p \cong R_s \cdot Q^2 \quad L_p \cong L_s$$

- Note that this equivalence is only **valid at a given frequency**

Lossy LC Tank

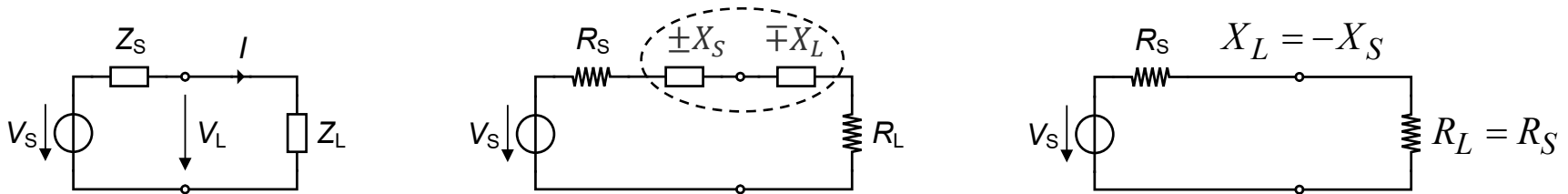


- It is often useful to transform the lossy LC tank into an equivalent parallel RLC circuit
- The series resistance r is transformed into a parallel resistance about Q^2 times larger $R = r \cdot (1 + Q^2) \cong r \cdot Q^2$
- Again, this equivalence is only valid around the resonance frequency ω_0
- The approximation of equal inductance L is valid for $Q \gg 1$ ($Q \cong 3 \dots 4$) leads to about 10% error, whereas $Q > 10$ leads to an error $< 1\%$)

Outline

- Passive RF circuits
- Impedance matching

Maximum Power Transfer and Impedance Matching



- The active power delivered by a voltage source (represented by its Thévenin equivalent circuit with impedance $Z_S = R_S + jX_S$) and dissipated in a load of impedance $Z_L = R_L + jX_L$ is given by

$$P = R_L \cdot I^2 = \frac{R_L}{|Z_S + Z_L|^2} \cdot V_S^2 = \frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} \cdot V_S^2$$

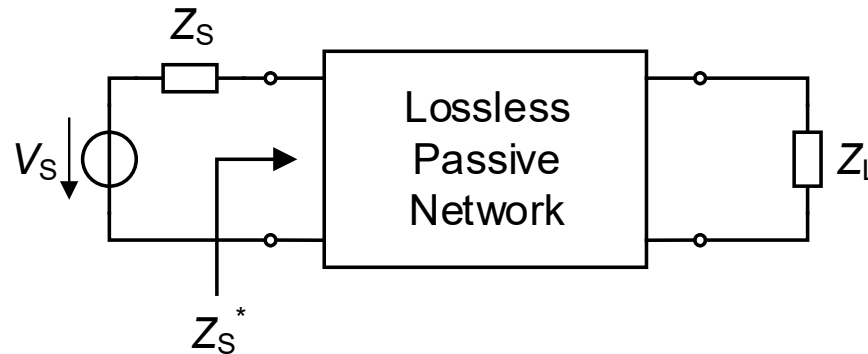
- For a given source impedance, the delivered power is maximum for

$$\frac{\partial P}{\partial X_L} = 0 \Rightarrow X_L = -X_S$$

$$\frac{\partial P}{\partial R_L} = 0 \Rightarrow R_S^2 - R_L^2 + (X_S + X_L)^2 = 0 \Rightarrow R_L = R_S$$

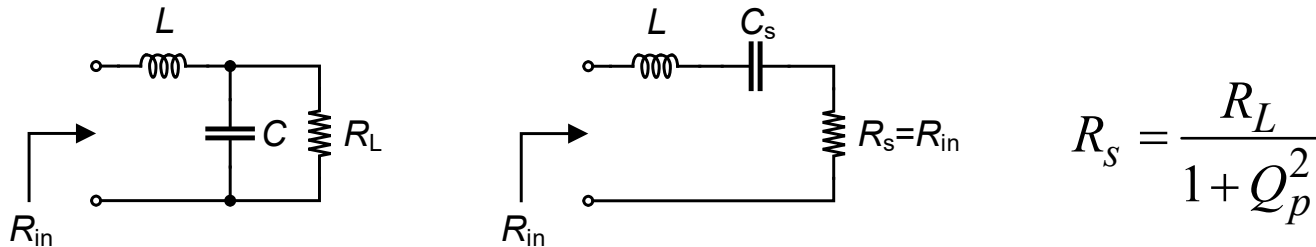
- The power transferred to a load is therefore maximum when $Z_L = Z_S^*$ corresponding to impedance matching which is realized at only a single frequency

Impedance Matching Network



- The load impedance is often different than what is required from the source
- This requires to transform the load impedance to a lower or higher value such that impedance matching occurs
- This can be realized thanks to a lossless passive network (made of capacitors and inductors only)
- The simplest implementation of the impedance matching network is using L circuits (i.e. one capacitor and one inductor)
- The concept is based on the series-parallel equivalent networks discussed above

Inverted L Impedance Matching Network ($R_L > R_S$)



- In the case the source and load impedances are purely resistive and $R_L > R_S$, the load impedance must be transformed into a lower impedance. This can be done by connecting a capacitor C (or an inductor) in parallel with R_L
- The parallel capacitor C and load resistance R_L can be converted to a series circuit as shown above and the series capacitor can be tuned out with a series inductor L . Assuming $R_{in} = 50\Omega$ and $R_L = 3k\Omega$ with $f = 1MHz$, we get

$$Q = \sqrt{\frac{R_L}{R_S} - 1} = \sqrt{\frac{3000}{50} - 1} = 7.7$$

- The reactance and values of the capacitor and inductor are then given by

$$X_C = \frac{R_L}{Q} = \frac{3000}{7.7} = 391 \Omega$$

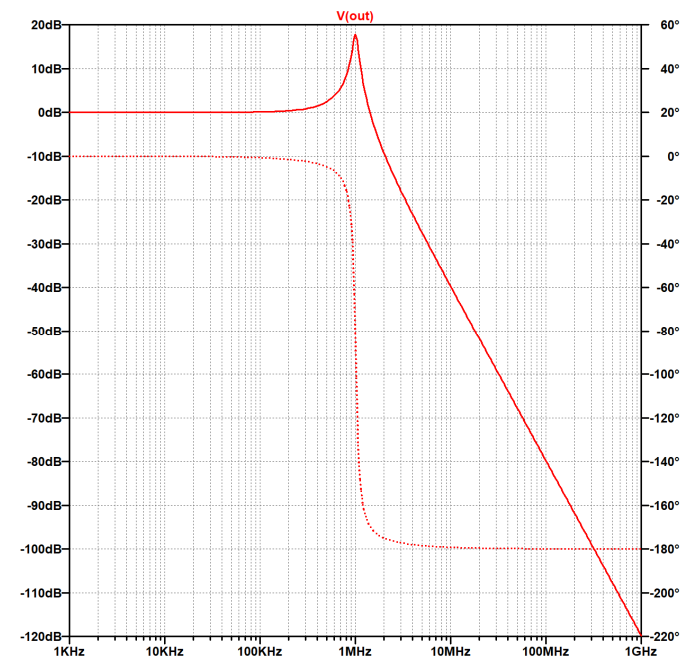
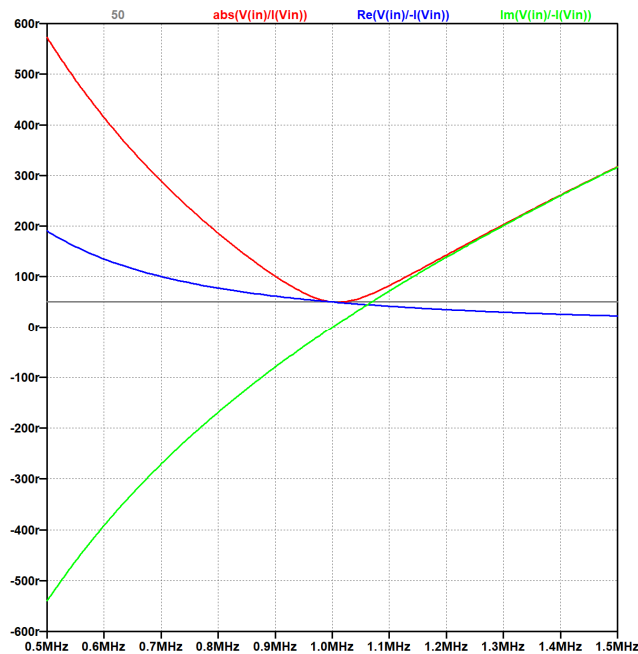
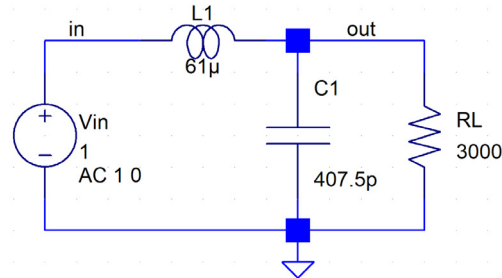
$$X_L = X_{C_s} = Q \cdot R_s = Q \cdot R_{in} = 7.7 \times 50 = 384 \Omega$$

$$C = \frac{1}{2\pi f \cdot X_C} = \frac{1}{2\pi \times 10^6 \times 391} = 407.5 \text{ pF}$$

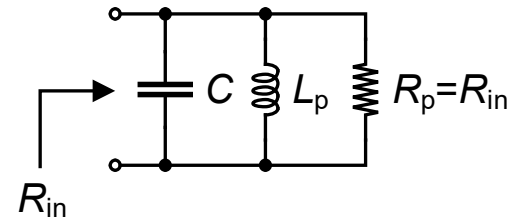
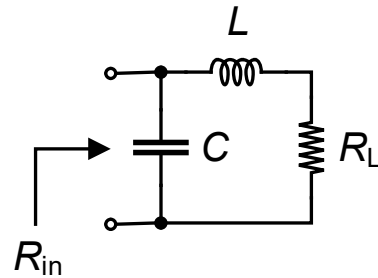
$$L = \frac{X_L}{2\pi f} = \frac{384}{2\pi \times 10^6} = 69 \mu H$$

Example of Inverted L Impedance Matching ($R_L > R_S$)

- Find the inverted L matching network for transforming the load resistance $R_L = 3000\Omega$ into an input resistance $R_L = 50\Omega$ at 1 MHz



L Impedance Matching Network ($R_L < R_S$)



- In case the load impedance is smaller than the source impedance $R_L < R_S$, an inductor (or capacitor) needs to be connected in series with the load to increase its impedance and a capacitor is connected in parallel to tune out the inductor
- Assuming $R_L = 50\Omega$ and $R_{in} = 3k\Omega$ with $f = 1MHz$, we get

$$Q = \sqrt{\frac{R_{in}}{R_L} - 1} = \sqrt{\frac{3000}{50} - 1} = 7.7$$

- The reactance and values of the inductor and capacitor are then given by

$$X_L = Q \cdot R_L = 7.7 \times 50 = 384 \Omega$$

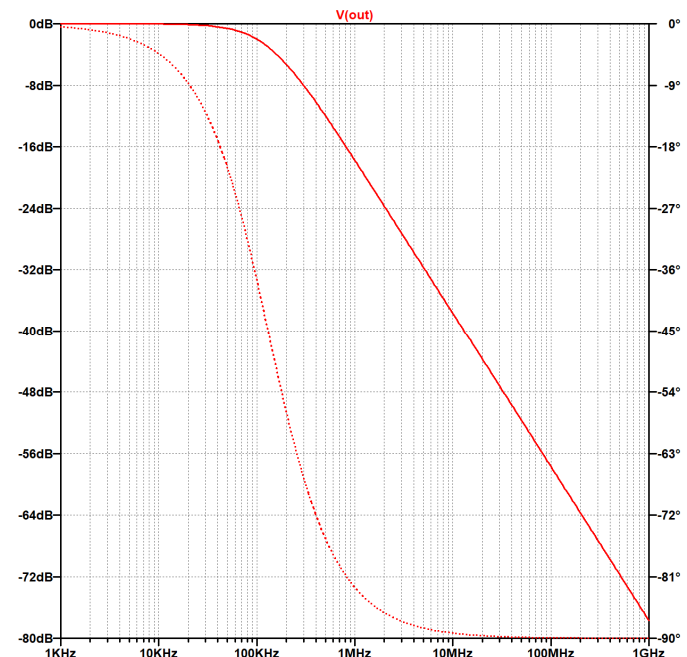
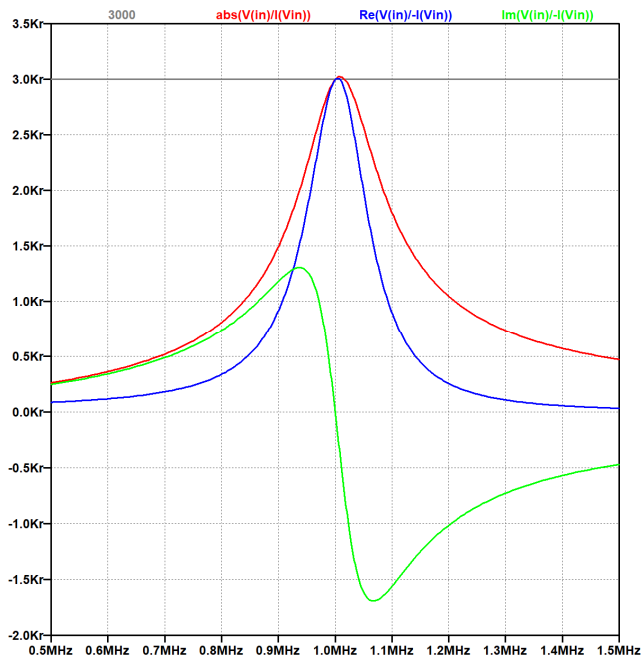
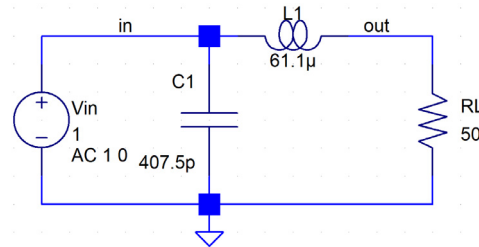
$$L = \frac{X_L}{2\pi f} = \frac{384}{2\pi \times 10^6} = 61.1 \mu H$$

$$X_C = X_{L_p} = \frac{R_p}{Q} = \frac{R_{in}}{Q} = 391 \Omega$$

$$C = \frac{1}{2\pi f \cdot X_C} = \frac{1}{2\pi \times 10^6 \times 391} = 407.5 pF$$

Example of L Impedance Matching Network ($R_L < R_S$)

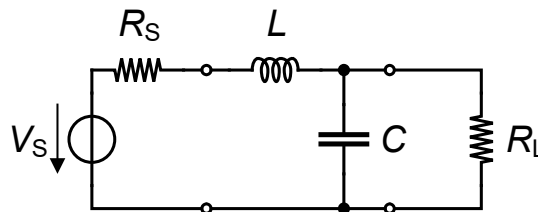
- Find the L matching network for transforming the load resistance $R_L = 50\Omega$ into an input resistance $R_L = 3000\Omega$ at 1 MHz



Basic L Impedance Matching Networks

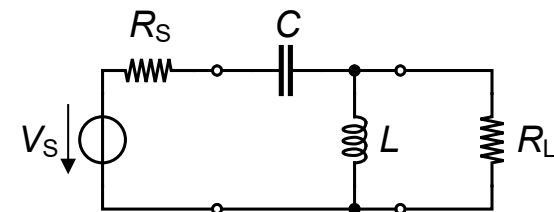
- The previous circuits introduce a low-pass filtering characteristic between the source and the load
- A high-pass characteristic, blocking any dc flow from the source, can be obtained simply by swapping the capacitor and inductor

Low-pass ($R_L > R_S$)

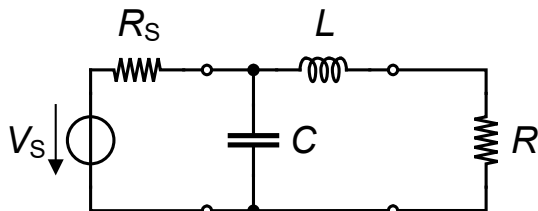


Impedance
reduction

High-pass ($R_L > R_S$)

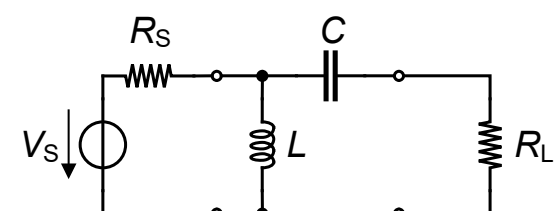


Low-pass ($R_L < R_S$)



Impedance
elevation

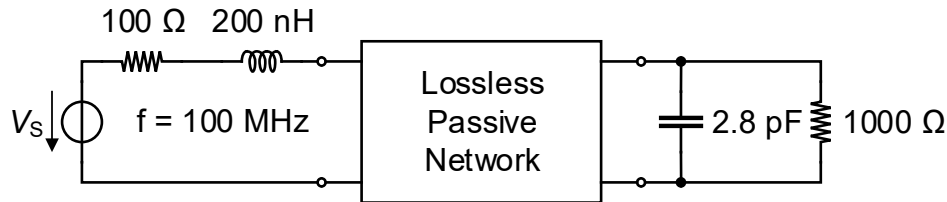
High-pass ($R_L < R_S$)



Complex Source and/or Load Impedances

- In the above examples, the source and load impedances have been assumed to be real, but most of the time they are actually **complex**
- There are basically two ways to account for the source and load reactance:
- **Absorption**: the reactances of the source and load can be included in the matching network such that the parasitic capacitances come in parallel with the functional capacitance and the parasitic inductances in series with the functional inductances. The parasitic reactance should then be subtracted from the desired value. This can only be done if the parasitic components remain smaller than the functional components.
- **Resonance**: eliminate the source or load reactance by resonating it with a parallel or series reactance of opposite sign

Complex Source and Load Impedances Example



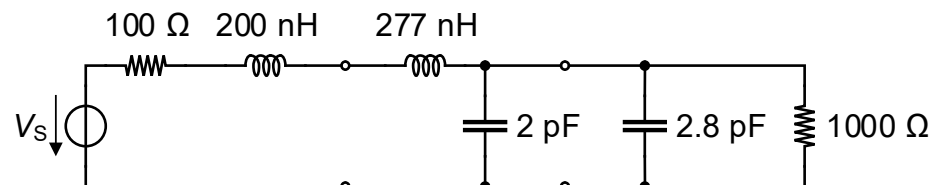
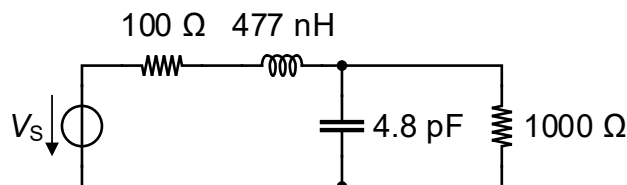
- As an example let's find the matching network for the above circuit. In a first step let's ignore the reactances of the source and load. Since the load resistance is larger than the source resistance we need a L matching network having a quality factor given by

$$Q = \sqrt{\frac{R_L}{R_S} - 1} = \sqrt{\frac{1000}{100} - 1} = 3$$

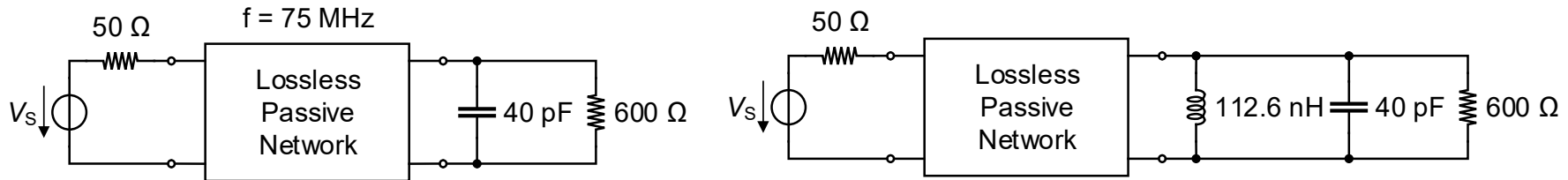
$$X_s = Q \cdot R_S = 300 \Omega \Rightarrow L = \frac{X_s}{2\pi f} = \frac{300}{2\pi \times 10^8} = 477 \text{ nH}$$

$$X_p = \frac{R_p}{Q} = \frac{R_L}{Q} = \frac{1000}{3} = 333 \Omega \Rightarrow C = \frac{1}{2\pi f X_p} = \frac{1}{2\pi \times 10^8 \times 333} = 4.8 \text{ pF}$$

- The resulting inductance and capacitance values are larger than the source inductance and load capacitance so that they can be subtracted resulting in the following impedance matching network



Complex Load Impedance – Resonance Method



- Another example of complex load impedance is given above. In this case the load capacitance is first eliminated by adding a parallel inductance that will resonate at the operating frequency. The value of L is given by

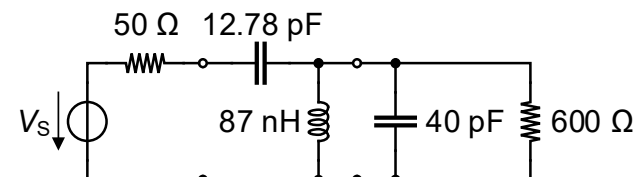
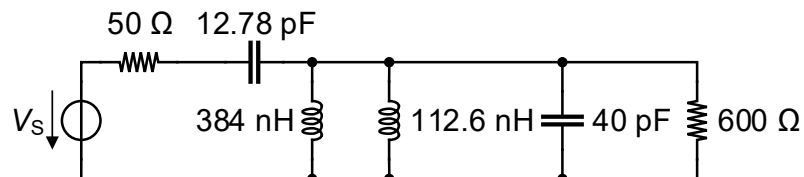
$$L = \frac{1}{(2\pi f)^2 C_L} = \frac{1}{(2\pi \times 75 \times 10^6)^2 \times 40 \times 10^{-12}} = 112.6 \text{ nH}$$

- After adding the parallel inductance we can add an inverted L matching network to reduce the impedance. The inductance of the L circuit can then be combined with that in parallel with the load

$$Q = \sqrt{\frac{R_L}{R_S} - 1} = \sqrt{\frac{600}{50} - 1} = 3.32$$

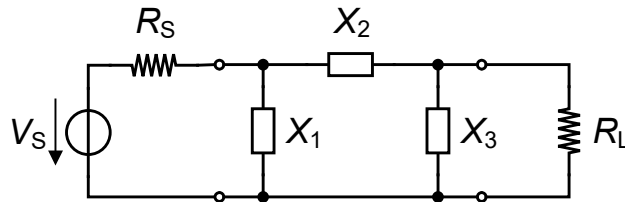
$$X_s = Q \cdot R_S = 3.32 \times 50 = 166 \Omega \Rightarrow C = \frac{1}{2\pi f X_s} = \frac{1}{2\pi \times 75 \times 10^6 \times 166} = 12.78 \text{ pF}$$

$$X_p = \frac{R_p}{Q} = \frac{R_L}{Q} = \frac{600}{3.32} = 181 \Omega \Rightarrow L = \frac{X_p}{2\pi f} = \frac{181}{2\pi \times 75 \times 10^6} = 384 \text{ nH}$$

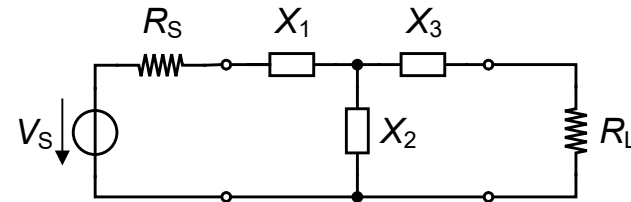


Π or T Impedance Matching Networks

Π Impedance Matching Network

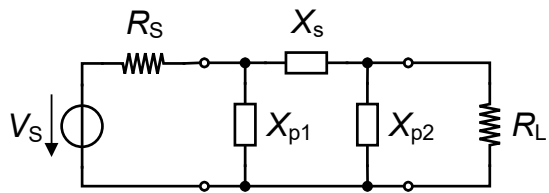


T Impedance Matching Network



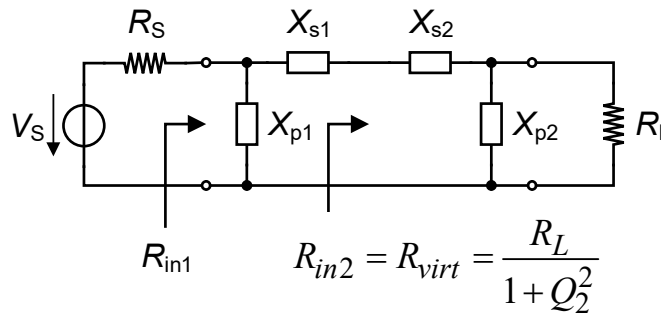
- The main disadvantage of L matching networks is that once the source and load resistances are specified, the Q of the circuit and hence its selectivity is defined
- There are therefore not enough degrees of freedom to choose the Q independently
- This drawback can be alleviated by adding one component leading to the Π or T impedance matching networks
- Note that the Q of the later circuits needs to be higher than that of the L circuit which is therefore the lowest Q matching networks

The Π Impedance Matching Network



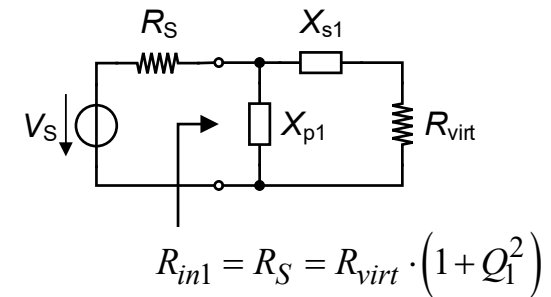
$$Q_{\Pi} = \sqrt{\frac{R_{\max}}{R_{\text{virt}}} - 1}$$

$$R_{\max} \geq \max(R_S, R_L)$$



$$R_{in2} = R_{\text{virt}} = \frac{R_L}{1 + Q_2^2}$$

$$Q_2 = \frac{R_L}{X_{p2}} = \sqrt{\frac{R_L}{R_{\text{virt}}} - 1}$$

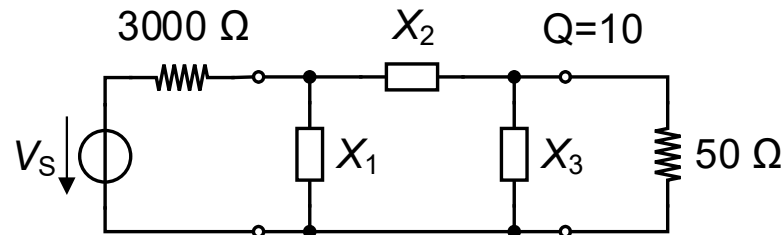


$$R_{in1} = R_S = R_{\text{virt}} \cdot (1 + Q_1^2)$$

$$Q_1 = \frac{X_{s1}}{R_{\text{virt}}} = \sqrt{\frac{R_S}{R_{\text{virt}}} - 1}$$

- The Π network is split into two L sections, first a down transformation circuit towards the load, transforming R_L into R_{virt} , followed to the left by an up transformation circuit transforming R_{virt} into the desired input resistance R_{in}
- The virtual resistance R_{virt} corresponds to the resistance seen when looking into the right L section and constitutes the **additional degree of freedom**
- For the Q factors to be real, the virtual resistance R_{virt} should be higher than the largest resistance between R_S or R_L . The Q of the Π circuit Q_{Π} is therefore associated to the L section having the highest Q and the section having the highest Q is on the side of the highest termination resistance.

II Impedance Matching Network Example



- The virtual resistance is calculated taking the largest termination resistance

$$R_{virt} = \frac{R_{\max}}{1 + Q^2} = \frac{3000}{101} = 29.703 \Omega$$

- After adding the parallel inductance we can add an inverted L matching network to reduce the impedance. The inductance of the L circuit can then be combined with that in parallel with the load

$$X_{p1} = \frac{R_S}{Q} = \frac{3000}{10} = 300 \Omega \quad \text{and} \quad X_{s1} = Q \cdot R_{virt} = 10 \times 29.7 = 297.03 \Omega$$

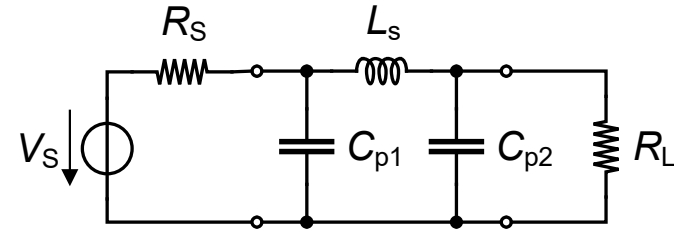
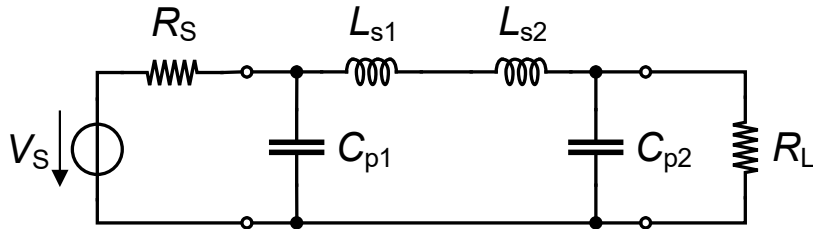
- The Q of the second L section is then obtained from R_{virt} and R_L

$$Q_2 = \sqrt{\frac{R_L}{R_{virt}} - 1} = \sqrt{\frac{50}{29.703} - 1} = 0.8266$$

- The reactances of the second L section are calculated for adapting R_L to R_{virt}

$$X_{s2} = Q_2 \cdot R_{virt} = 0.8266 \times 29.7 = 24.55 \Omega \quad \text{and} \quad X_{p2} = \frac{R_L}{Q_2} = \frac{50}{0.8266} = 60.49 \Omega$$

Π Impedance Matching Network Example



- Choosing inductors for the series branches and capacitors for the parallel branches results in the above left circuit with the following values

$$C_{p1} = \frac{1}{2\pi f X_{p1}} = 530.5 \text{ pF}$$

$$C_{p2} = \frac{1}{2\pi f X_{p2}} = 2631 \text{ pF}$$

$$L_{s1} = \frac{X_{s1}}{2\pi f} = 47.27 \text{ } \mu\text{H}$$

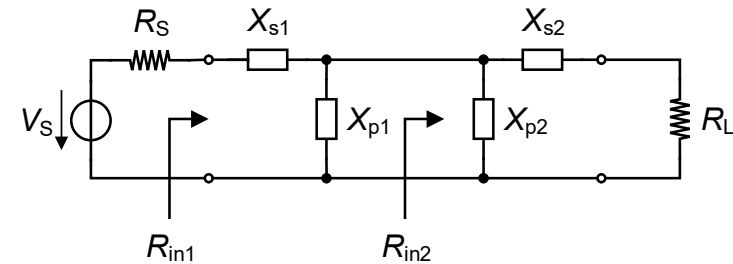
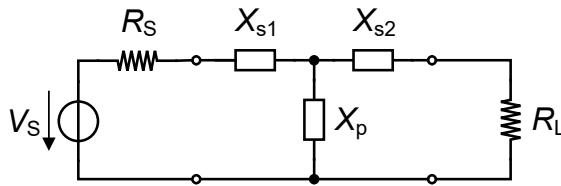
$$L_{s2} = \frac{X_{s2}}{2\pi f} = 3.9 \text{ } \mu\text{H}$$

- The two series inductors can be replaced by a single inductor of value

$$L_s = L_{s1} + L_{s2} = 47.27 + 3.9 = 51.18 \text{ } \mu\text{H}$$

- resulting in the final circuit given above on the right

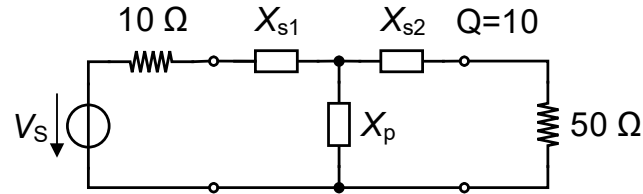
The T Impedance Matching Network



- Similar to the Π network, the T network can also be split into two back-to-back L networks
- The difference with respect to the Π network is that now the virtual resistance is smaller than the smallest termination resistance
- The T network is often used to match low impedances with a high quality factor
- The quality factor of the T network is then set by the L section having the highest Q. By definition the section having the highest Q is on the side of the termination having the lowest resistance R_{min}

$$Q_T = \sqrt{\frac{R_{virt}}{R_{min}} - 1} \quad \text{with} \quad R_{min} \leq \min(R_S, R_L)$$

T Impedance Matching Network Example

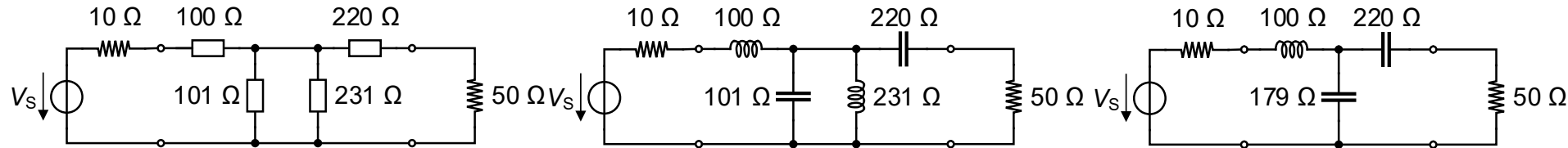


- Design a T matching network for the above circuit with minimum of inductances and with a low-pass characteristic
- The virtual resistance is then given by

$$R_{virt} = R_{\min} \cdot (Q^2 + 1) = R_S \cdot (Q^2 + 1) = 10 \times 101 = 1010 \Omega$$
- The L section having the highest Q is on the side of the source. The reactances of the input L section are then given by

$$X_{s1} = Q \cdot R_S = 10 \times 10 = 100 \Omega \quad \text{and} \quad X_{p1} = \frac{R_{virt}}{Q} = \frac{1010}{10} = 101 \Omega$$

T Impedance Matching Network Example



- The Q of the second L section is then obtained from R_{virt} and R_L

$$Q_2 = \sqrt{\frac{R_{virt}}{R_L} - 1} = \sqrt{\frac{1010}{50} - 1} = 4.4$$

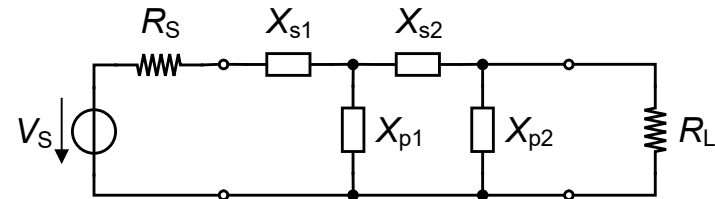
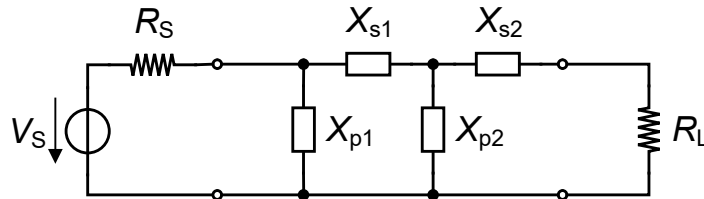
- The reactances of the right side L section are given by

$$X_{p2} = \frac{R_{virt}}{Q_2} = \frac{1010}{4.4} = 230 \Omega \quad \text{and} \quad X_{s2} = Q_2 \cdot R_L = 4.4 \times 50 = 220 \Omega$$

- In order to minimize the number of inductances and to have a low-pass characteristic, the middle circuit is chosen which then reduces to the final circuit shown on the right with a shunt reactance given by

$$X_p = \left(\frac{1}{101} - \frac{1}{231} \right)^{-1} = 179 \Omega$$

Low-Q or Wideband Matching Networks



- The L matching networks have their quality factor defined by the termination resistances
- The Π and T networks allow to select a circuit Q independent of the source or load impedances as long as the chosen Q is larger than that available with the L network
- Π and T networks are therefore appropriate for high-Q matching and hence **narrow band matching**
- For matching impedances over a larger frequency band or equivalently having a quality factor lower than that of the L section, two cascaded L sections can be used as shown above

Low-Q or Wideband Matching Networks

- The minimum quality factor and hence largest frequency band is obtained for

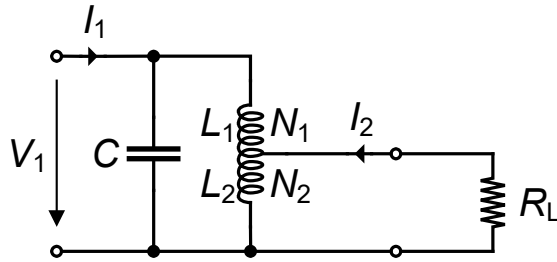
$$R_{virt} = \sqrt{R_S \cdot R_L}$$

- The corresponding quality factor is then given by

$$Q = \sqrt{\frac{R_{virt}}{R_{min}} - 1} = \sqrt{\frac{R_{max}}{R_{virt}} - 1}$$

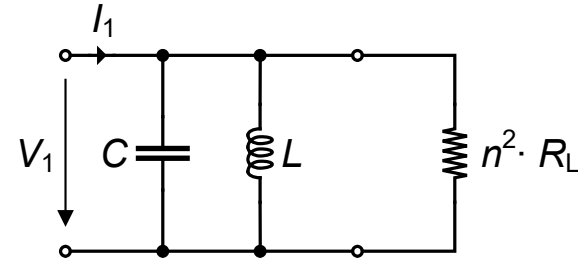
- where R_{virt} is the virtual resistance and R_{min} and R_{max} are the termination resistances with the lowest and highest value, respectively
- The design procedure is then similar to the examples shown above
- To design for a specific Q, solve the second above equation to find R_{virt}
- For a maximum bandwidth choose R_{virt} from the top equation from R_S and R_L and follow the same procedure as above

Tapped Inductor



$$L = L_1 + L_2 + 2M$$

$$k = \frac{M}{\sqrt{L_1 \cdot L_2}}$$



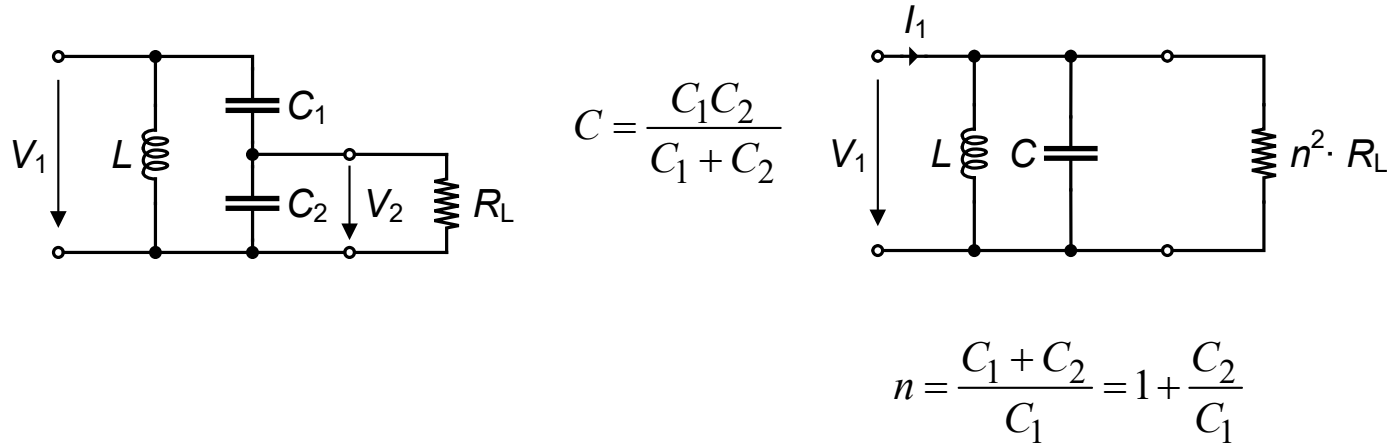
$$n = k \cdot \sqrt{\frac{L}{L_2}} \cong \sqrt{\frac{L}{L_2}} = \frac{N}{N_2}$$

- Impedance matching can also be implemented with a tapped inductor similar to an autotransformer
- The equivalent resistance in parallel to the LC circuit is equal to

$$R'_L = n^2 \cdot R_L \cong \frac{L}{L_2} \cdot R_L = \left(\frac{N}{N_2} \right)^2 \cdot R_L$$

- where N is the total number of turns and N_2 is the number of turns in the secondary

Tapped Capacitor



- For $R_L \gg \omega C_2$, the capacitor tap referred to the primary appears as the circuit shown on the right
- The equivalent resistance in parallel to the LC circuit is then equal to

$$R'_L = n^2 \cdot R_L = \left(1 + \frac{C_2}{C_1}\right)^2 \cdot R_L$$