# MICRO-461
# Low-power Radio Design for the IoT

## Summary of the EKV MOS Transistor Model

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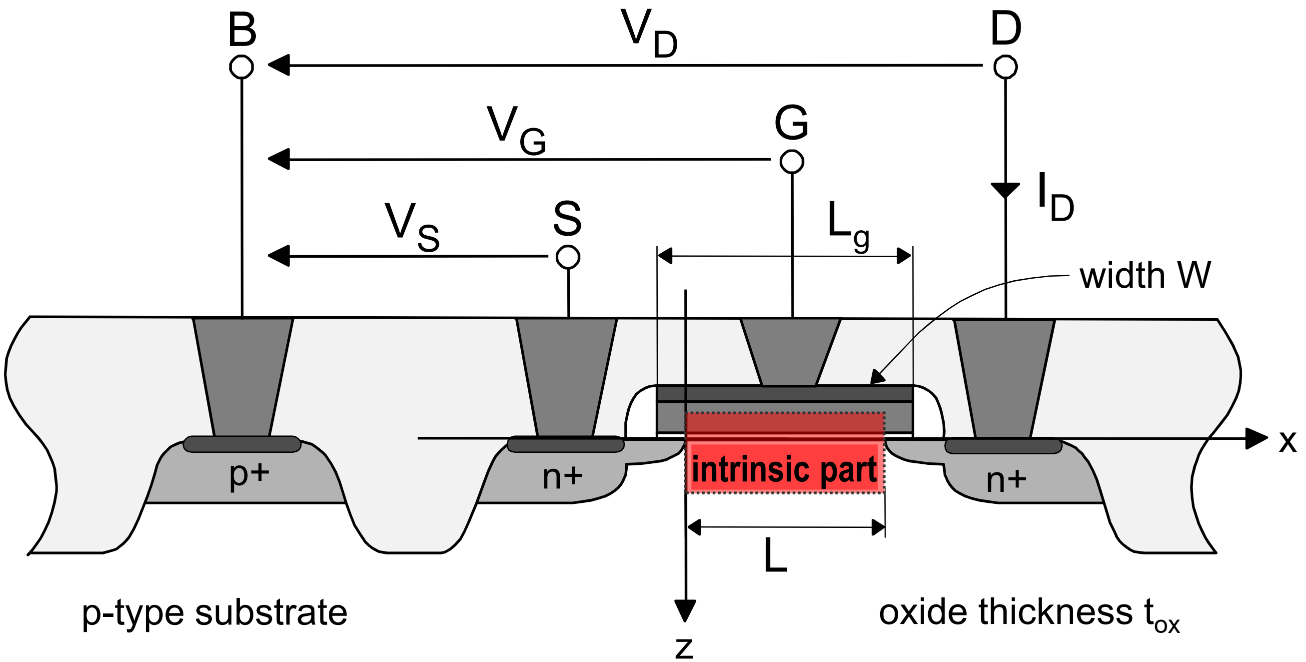
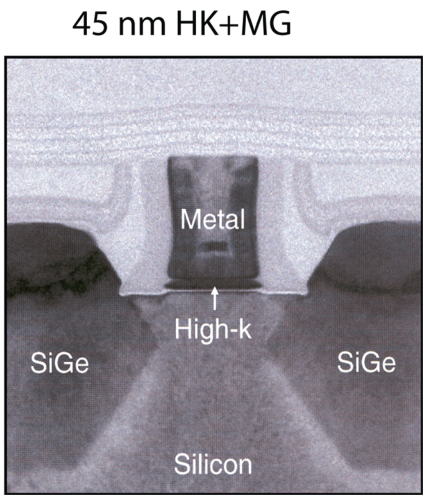
# Outline

- **The long-channel static model**
- The long-channel small-signal model
- The long-channel noise model
- The extended model
- The simplified EKV model

# Device Symmetry



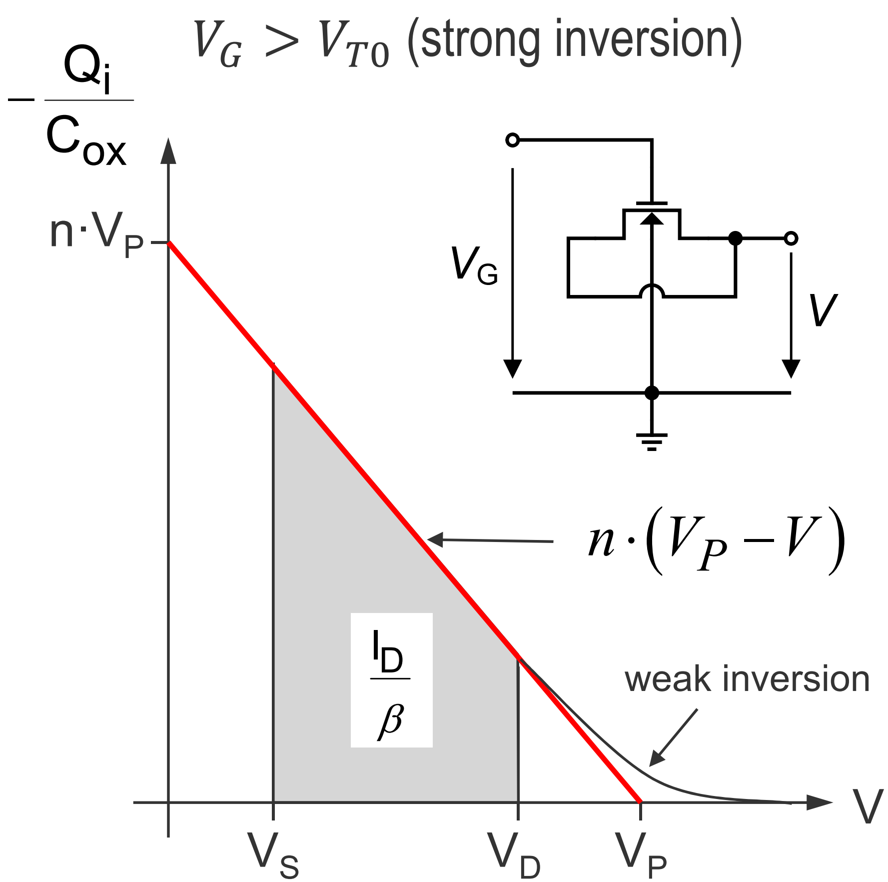




- **Symmetrical** with respect to source (S) and drain (D)
- Terminal voltages are **referred to the local substrate**
- Leads to symmetrical model

C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Drain Current

$$I_D = \mu \cdot W \cdot (-Q_i) \cdot \frac{dV}{dx} \Longrightarrow I_D = \beta \cdot \int_{V_S}^{V_D} \frac{-Q_i}{C_{ox}} \cdot dV \text{ with } \beta \triangleq \mu \cdot C_{ox} \cdot \frac{W}{L}$$




- $Q_i$ is the **inversion mobile charge density** (electrons for n-channel)
- $V$ is the **channel voltage** (electrons quasi-Fermi potential), equal to $V_S$ at the source and $V_D$ at the drain
- $V_P$ is the **pinch-off voltage** given by

$$V_P \cong \frac{V_G - V_{T0}}{n}$$

- $V_{T0}$ is the **threshold voltage** (at $V = 0$)
- $n$ is the **slope factor**
- $\mu$ is the electron mobility (at low field)

C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Pinch-off Voltage Definition

- The value of $V$ for which $Q_i$ becomes zero in a non-equilibrium situation is defined as the **pinch-off voltage** $V_P$

$$V_P = V_G - V_{T0} - \Gamma_b \cdot \left[\sqrt{V_G - V_{T0} + \left(\sqrt{\Psi_0} + \frac{\Gamma_b}{2}\right)^2} - \left(\sqrt{\Psi_0} + \frac{\Gamma_b}{2}\right)\right]$$

- $\Gamma_b$ is the **body effect factor** defined as

$$\Gamma_b \triangleq \frac{\sqrt{2q\varepsilon_{si}N_b}}{C_{ox}} \text{ with } C_{ox} \triangleq \frac{\varepsilon_{ox}}{t_{ox}}$$

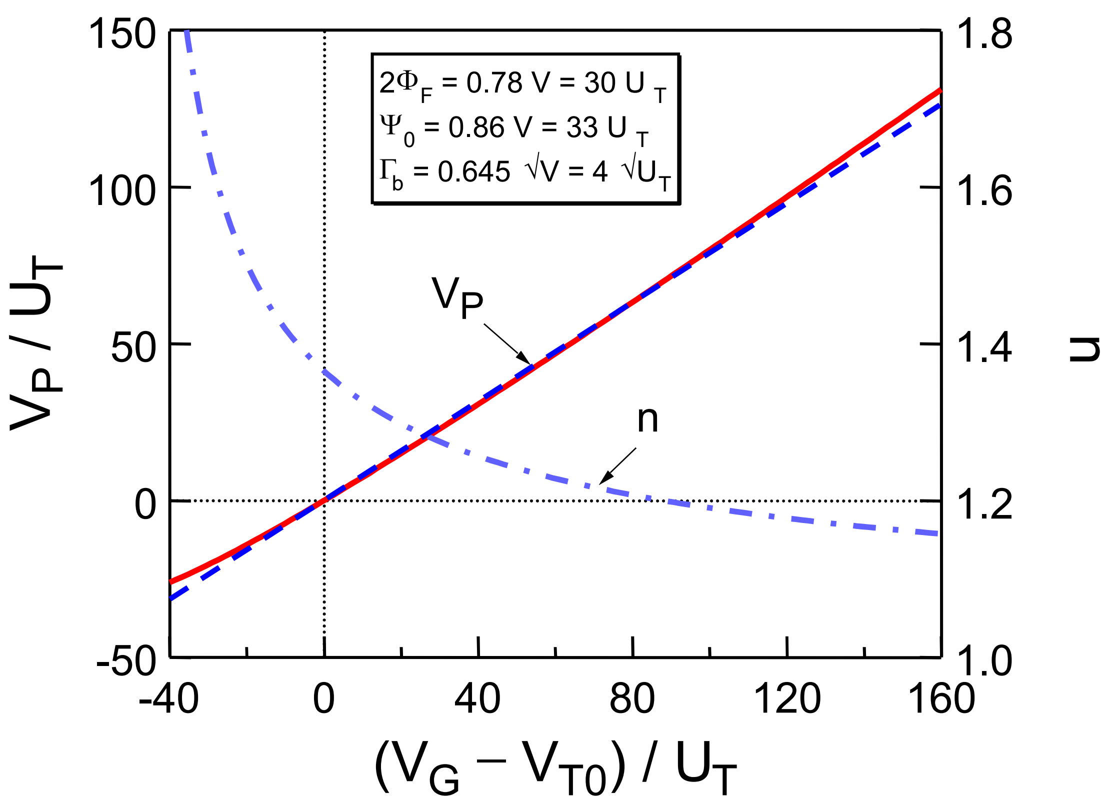
- $V_{T0}$ is the **threshold voltage** defined as $V_G$ such as $Q_i = 0$ when the channel is at equilibrium ($V = 0$)

$$V_{T0} \triangleq V_{FB} + \Psi_0 + \Gamma_b \cdot \sqrt{\Psi_0}$$

- $V_{FB}$ is the **flat-band voltage** and $\Psi_0 = 2\Phi_F + m \cdot U_T$ is the approximation of the surface potential in strong inversion at equilibrium with $m \cong 3$ to $5U_T$

# Pinch-off Voltage

$$V_P = V_G - V_{T0} - \Gamma_b \cdot \left[ \sqrt{V_G - V_{T0} + \left( \sqrt{\Psi_0} + \frac{\Gamma_b}{2} \right)^2} - \left( \sqrt{\Psi_0} + \frac{\Gamma_b}{2} \right) \right]$$




- Note that $V_P = 0$ for $V_G = V_{T0}$
- The pinch-off voltage can be approximated by

$$V_P \cong \frac{V_G - V_{T0}}{n_0}$$

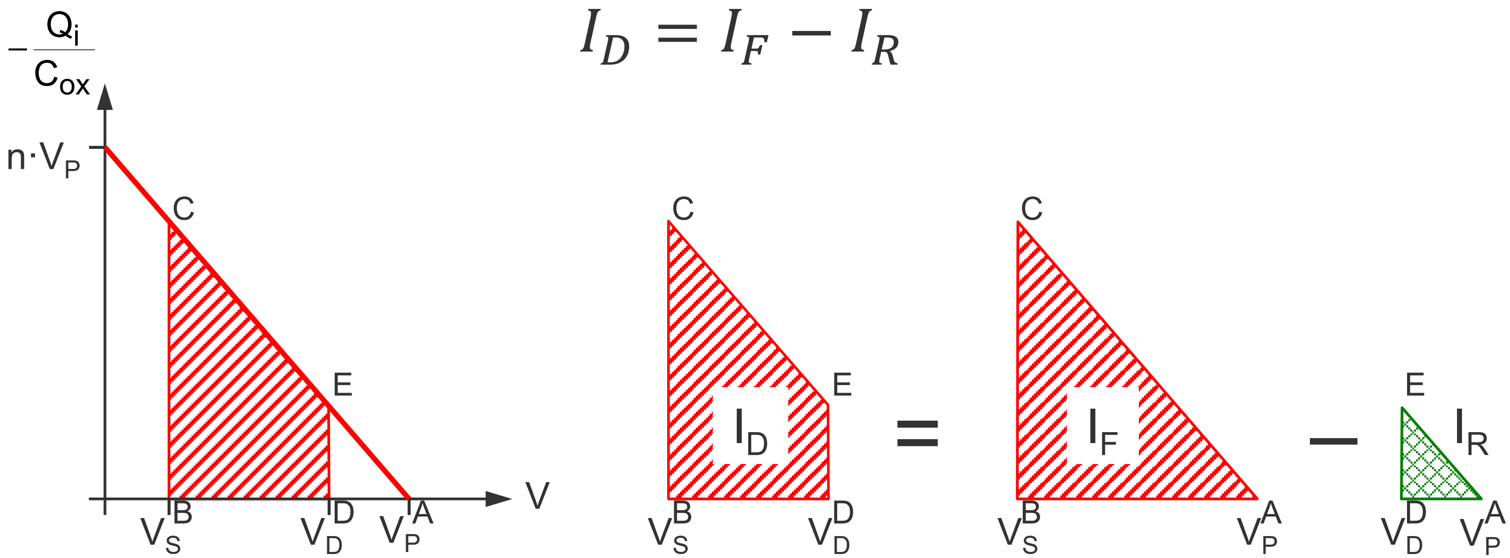
- where

$$n_0 \cong 1 + \frac{\Gamma_b}{2\sqrt{\Psi_0}} \cong 1 + \frac{\Gamma_b}{2\sqrt{2\Phi_F}}$$

- with $\Psi_0 = 2\Phi_F + m \cdot U_T$

# Forward and Reverse Currents

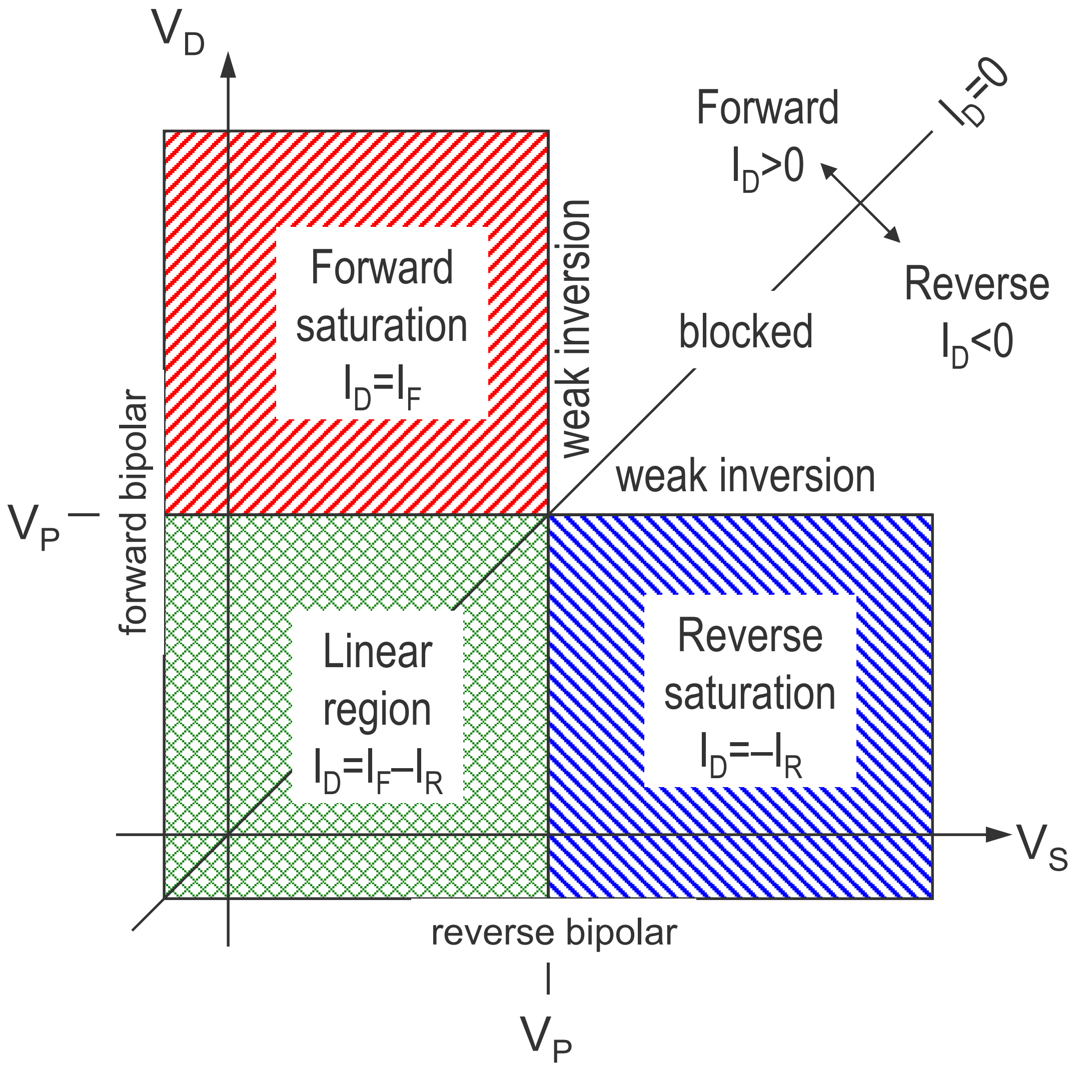
$$I_D = I_F - I_R$$



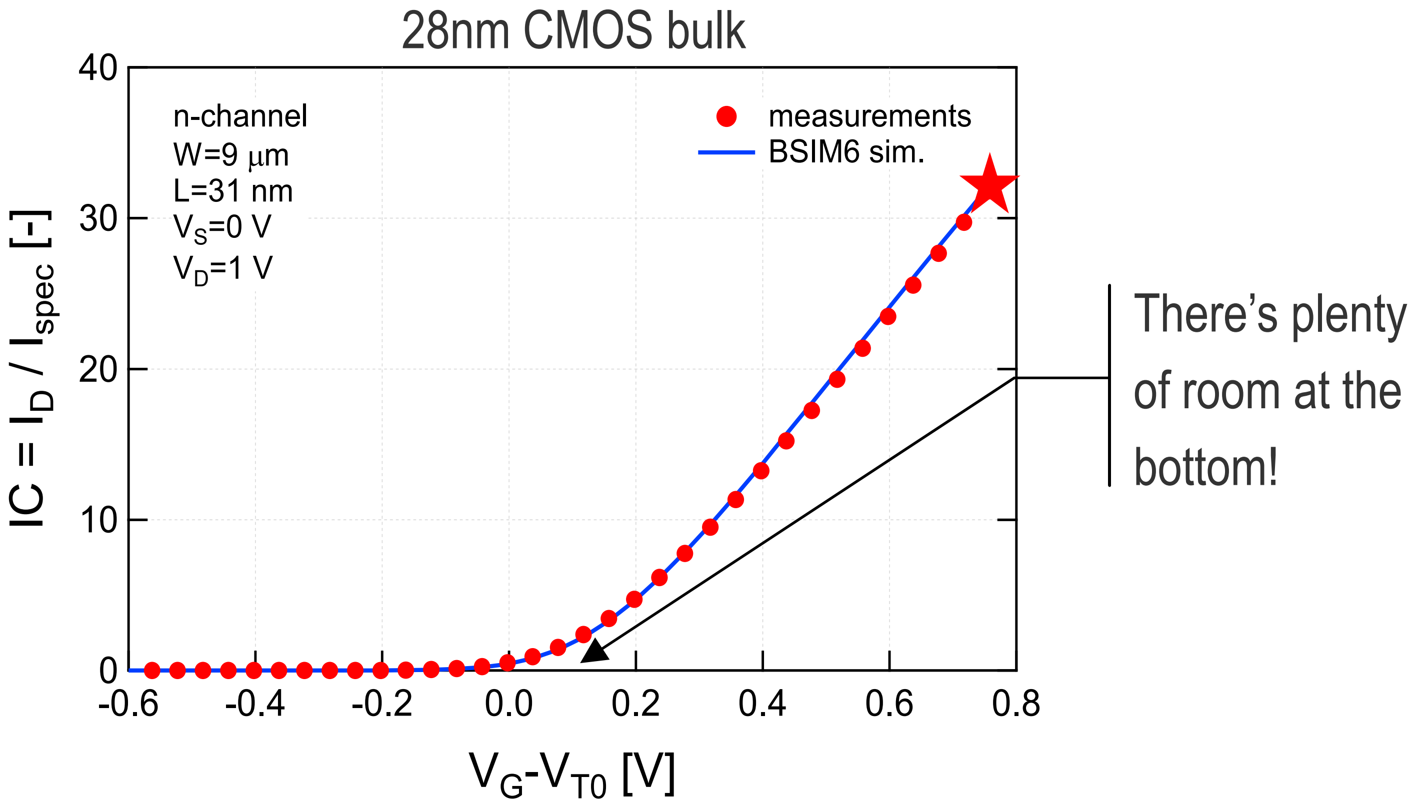

$$I_D = \beta \cdot \int_{V_S}^{V_D} \frac{-Q_i}{C_{ox}} \cdot dV = \underbrace{\beta \cdot \int_{V_S}^{+\infty} \frac{-Q_i}{C_{ox}} \cdot dV}_{\substack{\text{forward current } I_F \\ \text{controlled by } V_P - V_S}} - \underbrace{\beta \cdot \int_{V_D}^{+\infty} \frac{-Q_i}{C_{ox}} \cdot dV}_{\substack{\text{reverse current } I_R \\ \text{controlled by } V_P - V_D}}$$

C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Modes of Operation

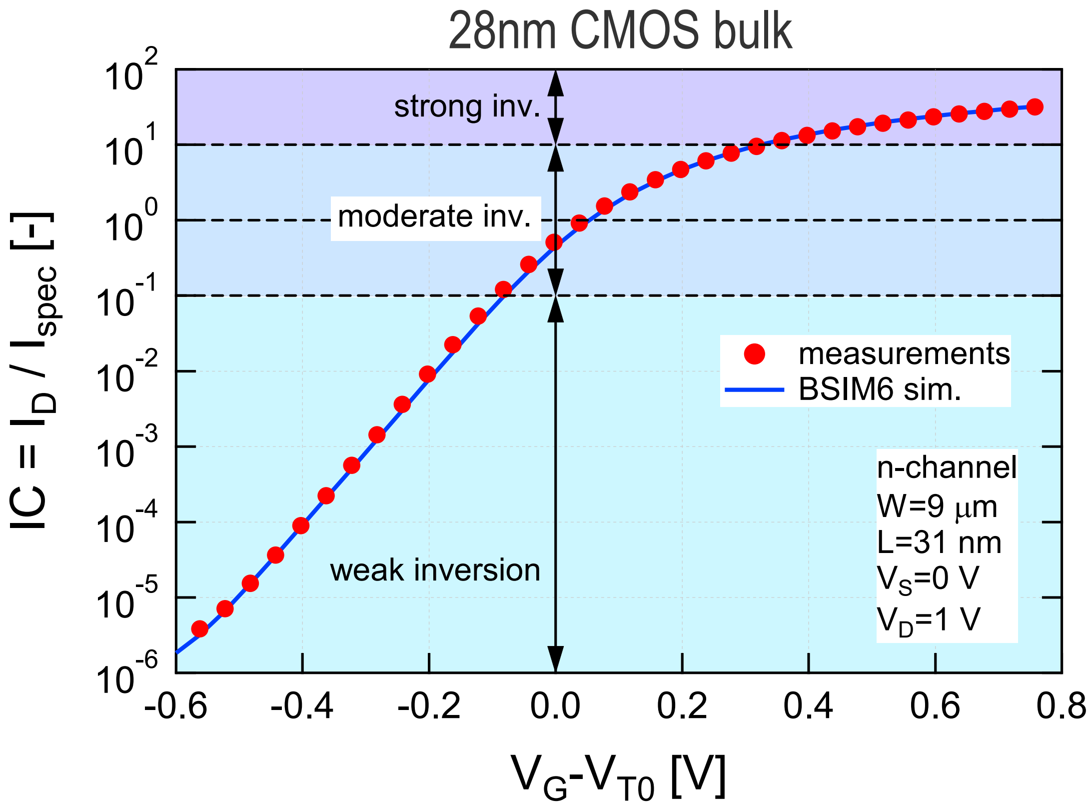


# Moderate and Weak Inversion in 28nm Bulk CMOS



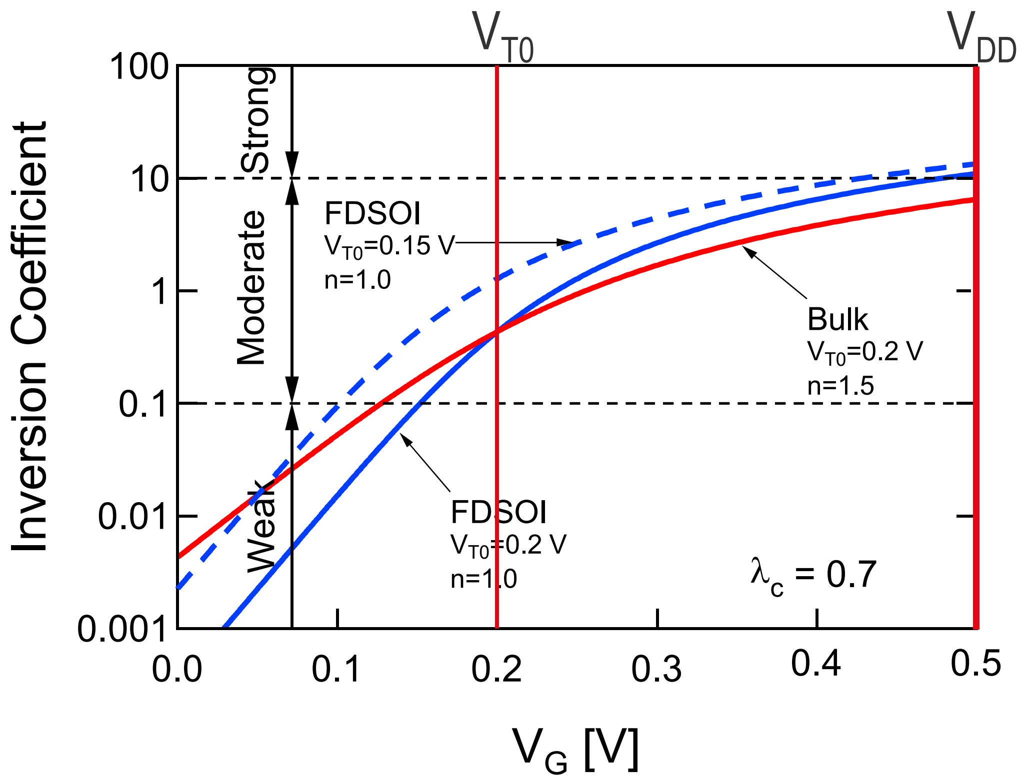

- Strong inversion spans over wide range of voltage, but…

# Moderate and Weak Inversion in 28nm Bulk CMOS




- Strong inversion spans over wide range of voltage, but…
- **Moderate** and **weak inversion** span over **6 decades of current**, whereas strong inversion is limited to less than 1 decade
- How to derive an I-V expression valid in all regions of operations?

# Strong Inversion will Disappear at Low-Voltage!



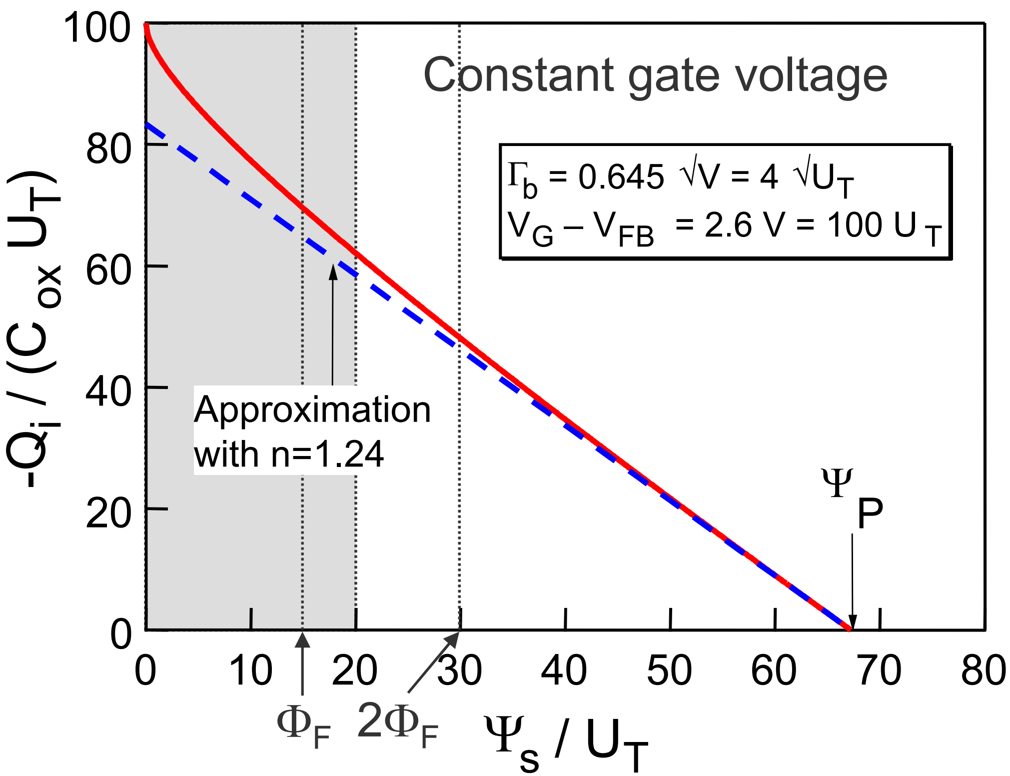

- The above plot clearly illustrates that the strong inversion region is reducing dramatically because of voltage scaling and ultimately is disappearing

# Inversion Charge Linearization

- The inversion (mobile) charge is given by

$$-Q_i = C_{ox}\left(V_G - V_{FB} - \Psi_s - \Gamma_b\sqrt{\Psi_s}\right)$$

- where $\Psi_s$ is the surface potential and $V_{FB}$ the flat-band voltage




- $Q_i$ can be approximated by

$$-Q_i \cong -nC_{ox}(\Psi_s - \Psi_P)$$

- where $n$ is the slope factor

$$n \cong 1 + \frac{\Gamma_b}{2\sqrt{\Psi_P}}$$

- where $\Psi_P$ is the pinch-off surface potential which depends on the gate voltage according to

$$\Psi_P \triangleq \Psi_s(Q_i = 0)$$

$$= V_G - V_{FB} - \Gamma_b^2\left(\sqrt{\frac{V_G - V_{FB}}{\Gamma_b^2} + \frac{1}{4}} - \frac{1}{2}\right)$$

C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Charge-based Drain Current Expression

- The drain current can also be written in terms of drift and diffusion components as

$$I_D = \mu \cdot W \cdot \left( \underbrace{-Q_i \cdot \frac{d\Psi_s}{dx}}_{\text{drift}} + \underbrace{U_T \cdot \frac{dQ_i}{dx}}_{\text{diffusion}} \right)$$

- From the mobile charge linearization we get

$$\Psi_s \cong \frac{Q_i}{nC_{ox}} + \Psi_P$$

- which can be used to express the gradient of the surface potential in terms of the inversion charge according to

$$\frac{d\Psi_s}{dx} = \frac{1}{nC_{ox}} \cdot \frac{dQ_i}{dx}$$

- Replacing in the above equation leads to charge-based expression of the drain current valid from weak to strong inversion

$$I_D = \mu \cdot W \cdot \left( \frac{-Q_i}{nC_{ox}} + U_T \right) \cdot \frac{dQ_i}{dx}$$

📖 C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Normalization

- It is convenient to normalize the drain current expression according to

$$i_d \triangleq \frac{I_D}{I_{spec}} = -(2q_i + 1) \cdot \frac{dq_i}{d\xi}$$

- where the drain current is normalized as

$$i_d \triangleq \frac{I_D}{I_{spec}}$$

- where $I_{spec} \triangleq 2n\beta U_T^2$ is the **specific current** with $\beta \triangleq \mu C_{ox} W/L$
- the inversion charge is normalized as

$$q_i \triangleq \frac{Q_i}{Q_{spec}}$$

- where $Q_{spec} \triangleq -2nC_{ox}U_T$ (note that since both $Q_i$ and $Q_{spec}$ are negative, the normalized inversion charge is positive)
- and finally the distance is normalized to the channel length

$$\xi \triangleq \frac{x}{L}$$

C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Charge-based Forward and Reverse Currents

- Drain current can be integrated in **charge domain** from source to drain, leading to

$$i_d = -\int_{q_s}^{q_d} (2q_i + 1)\, dq_i = \left| q_i^2 + q_i \right|_{q_d}^{q_s} = \underbrace{\left(q_s^2 + q_s\right)}_{=i_f} - \underbrace{\left(q_d^2 + q_d\right)}_{=i_r} = i_f - i_r$$

- Drain current depends only on **charge densities** at the **source $q_s$** and at the **drain $q_d$** defined as

$$q_s \triangleq q_i(\xi = 0) = \frac{Q_i(x=0)}{Q_{spec}} \text{ and } q_d \triangleq q_i(\xi = 1) = \frac{Q_i(x=L)}{Q_{spec}}$$

- The normalized **forward current $i_f$** and **reverse current $i_r$** can be written as

$$i_f \triangleq \frac{I_F}{I_{spec}} = q_s^2 + q_s \text{ and } i_r \triangleq \frac{I_R}{I_{spec}} = q_d^2 + q_d$$

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# Charges versus Currents

- The source and drain charges $q_s$ and $q_d$ can be expressed as a function of the forward and reverse currents $i_f$ and $i_r$ by solving

$$i_f = q_s^2 + q_s \text{ and } i_r = q_d^2 + q_d$$

- for $q_s$ and $q_d$ resulting in

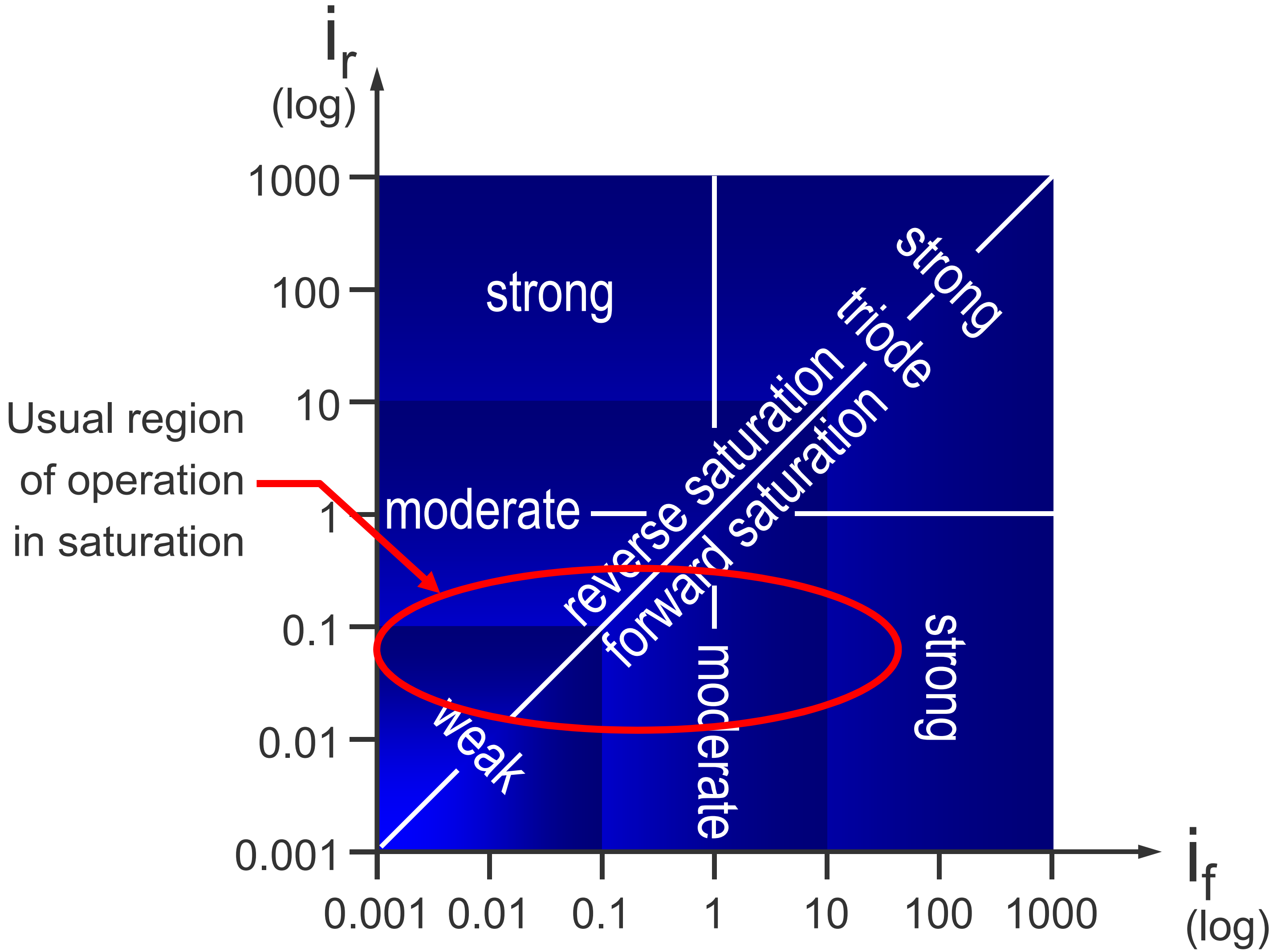
$$q_s = \frac{2 i_f}{\sqrt{4 i_f + 1} + 1} = \tfrac{1}{2}\left(\sqrt{4 i_f + 1} - 1\right)$$

$$q_d = \frac{2 i_r}{\sqrt{4 i_r + 1} + 1} = \tfrac{1}{2}\left(\sqrt{4 i_r + 1} - 1\right)$$

- which can be approximated in weak inversion (WI) and strong inversion (SI) by

$$i_{f(r)} = \begin{cases} q_{s(d)} \text{ for } q_{s(d)} \ll 1 \text{ (WI)} \\ q_{s(d)}^2 \text{ for } q_{s(d)} \gg 1 \text{ (SI)} \end{cases} \text{ and } q_{s(d)} = \begin{cases} i_{f(r)} \text{ for } i_{f(r)} \ll 1 \text{ (WI)} \\ \sqrt{i_{f(r)}} \text{ for } i_{f(r)} \gg 1 \text{ (SI)} \end{cases}$$

# Modes of Operation (versus $i_f$ and $i_r$)

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# Voltages versus Charges and Currents

- The inversion charge $Q_i$ is linked to the normalized pinch-off voltage $v_p \triangleq V_P/U_T$ and the normalized channel voltage $v \triangleq V/U_T$ by the following relation which is **valid all along the channel**

$$v_p - v(\xi) = \ln\big(q_i(\xi)\big) + 2q_i(\xi) \text{ for } 0 \leq \xi \leq 1$$

- in particular at the **source** ($\xi = 0$) where $q_i(\xi = 0) = q_s$ and **drain** ($\xi = 1$) where $q_i(\xi = 1) = q_d$ leading to

$$v_p - v_s = \ln(q_s) + 2q_s \text{ and } v_p - v_d = \ln(q_d) + 2q_d$$

- Replacing the charges with the expression in terms of currents leads to

$$v_p - v_{s(d)} \triangleq \frac{V_P - V_{S(D)}}{U_T} = \ln\left(\sqrt{4i_{f(r)} + 1} - 1\right) + \sqrt{4i_{f(r)} + 1} - 1 - \ln 2$$

- Relation **cannot be inverted analytically** to give an explicit expression of the drain versus voltages valid in all regions of operation
- But can **easily be inverted numerically**

C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Drain Current in Strong Inversion (1/3)

- In SI, $q_s \gg 1$ and $q_d \gg 1$ and similarly $i_f \gg 1$ and $i_r \gg 1$ and the voltage-charge relation simplifies to

$$v_p - v_{s(d)} \cong 2q_{s(d)} \cong 2\sqrt{i_{f(r)}}$$

- Which can now be inverted to express the current in terms of voltages

$$i_{f(r)} \triangleq \frac{I_{F(R)}}{I_{spec}} \cong \left(\frac{v_p - v_{s(d)}}{2}\right)^2 = \left(\frac{V_P - V_{S(D)}}{2U_T}\right)^2$$

- Or in denormalized form

$$I_{F(R)} = \begin{cases} \frac{n\beta}{2}\left(V_P - V_{S(D)}\right)^2 & \text{for} \quad V_{S(D)} \leq V_P \\ 0 & \text{for} \quad V_{S(D)} > V_P \end{cases}$$

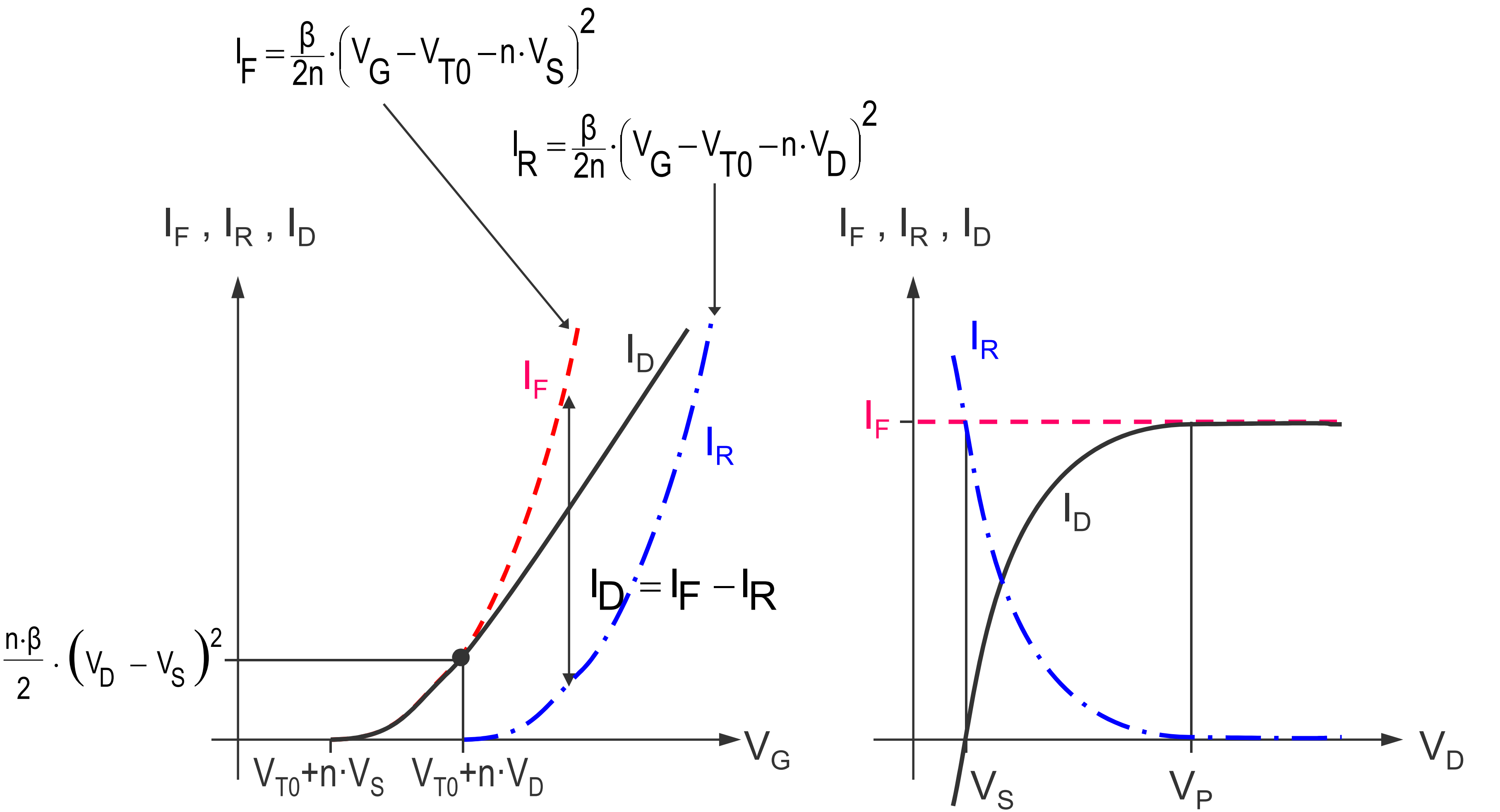
- Using the **approximation** of the pinch-off voltage $V_P \cong (V_G - V_{T0})/n$ leads to

$$I_{F(R)} = \begin{cases} \frac{n\beta}{2}\left(V_G - V_{T0} - nV_{S(D)}\right)^2 & \text{for} \quad V_{S(D)} \leq (V_G - V_{T0})/n \\ 0 & \text{for} \quad V_{S(D)} > (V_G - V_{T0})/n \end{cases}$$

# Drain Current in Strong Inversion (2/3)

| MODE | $I_D = I_F - I_R$ | CONDITION |
|---|---|---|
| Linear Region | $n \cdot \beta \cdot \left(V_P - \frac{V_D + V_S}{2}\right) \cdot (V_D - V_S)$ $\cong \beta \cdot \left(V_G - V_{T0} - \frac{n}{2} \cdot (V_D + V_S)\right) \cdot (V_D - V_S)$ | $V_S < V_P$ $V_D < V_P$ |
| Forward Saturation | $\frac{n \cdot \beta}{2} \cdot (V_P - V_S)^2 \cong \frac{\beta}{2n} \cdot (V_G - V_{T0} - n \cdot V_S)^2$ | $V_S < V_P$ $V_D \geq V_P$ |
| Reverse Saturation | $-\frac{n \cdot \beta}{2} \cdot (V_P - V_D)^2 \cong -\frac{\beta}{2n} \cdot (V_G - V_{T0} - n \cdot V_D)^2$ | $V_S \geq V_P$ $V_D < V_P$ |
| Blocked | $I_F = I_R = I_D = 0$ | $V_S \geq V_P$ $V_D \geq V_P$ |

# Drain Current in Strong Inversion (3/3)



# Drain Current in Weak Inversion (1/2)

- In WI, $q_s \ll 1$ and $q_d \ll 1$ and similarly $i_f \ll 1$ and $i_r \ll 1$ and the voltage versus charge relation can be approximated by

$$v_p - v_{s(d)} \cong \ln\left(\sqrt{i_{f(r)} + \frac{1}{4}} - \frac{1}{2}\right) \cong \ln\left(i_{f(r)}\right)$$

- Which can now be inverted to express the current in terms of voltages

$$i_{f(r)} \triangleq \frac{I_{F(R)}}{I_{spec}} \cong e^{\left(v_p - v_{s(d)}\right)} = e^{\frac{V_P - V_{S(D)}}{U_T}}$$

- Or in denormalized form

$$I_{F(R)} = I_{spec}\, e^{\frac{V_P - V_{S(D)}}{U_T}} = I_{spec}\, e^{\frac{V_G - V_{T0} - nV_{S(D)}}{nU_T}} = I_{D0} \cdot e^{\frac{V_G - n \cdot V_S}{n \cdot U_T}}$$

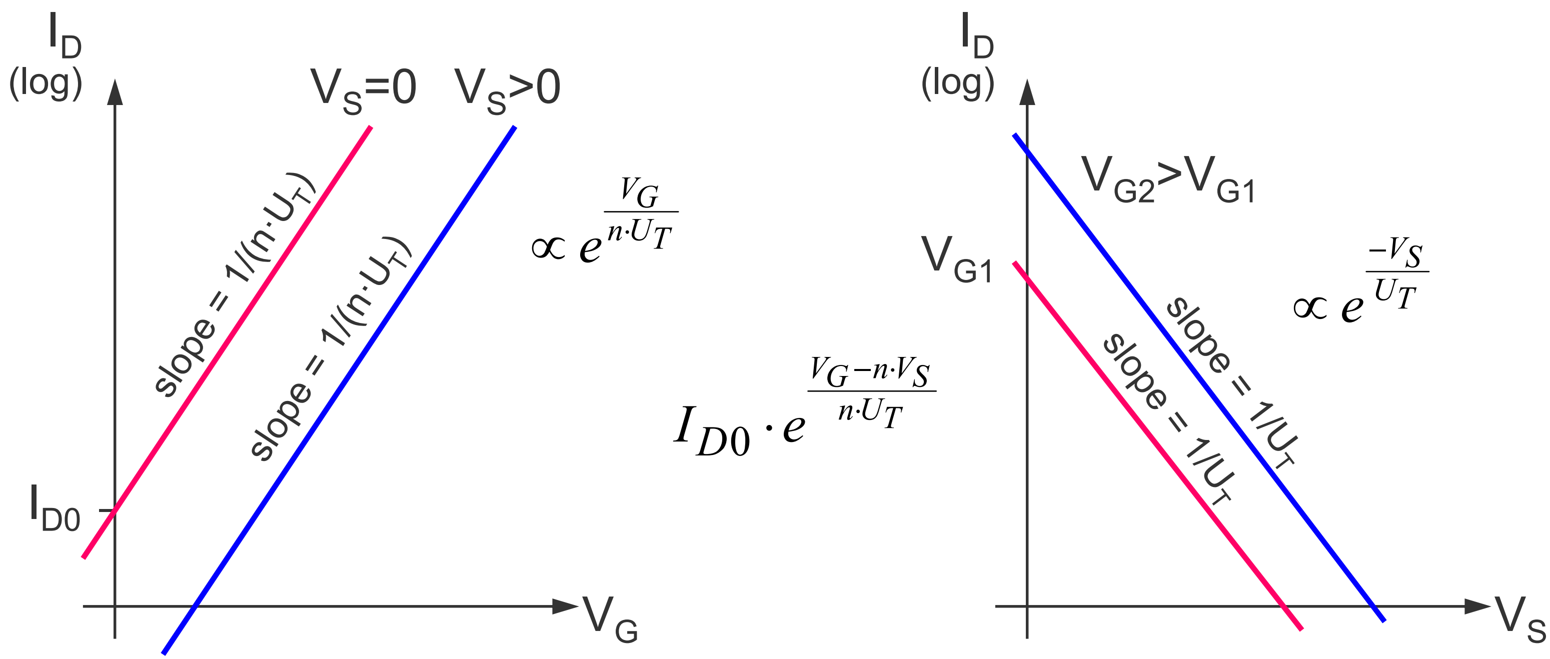
- Where $I_{D0}$ is the **leakage current** (drain current that flows in saturation when gate voltage is set to zero)

$$I_{D0} \triangleq I_D\big|_{V_G=0} = I_{spec}\, e^{\frac{-V_{T0}}{nU_T}}$$

# Drain Current in Weak Inversion (2/2)

| MODE | $\boldsymbol{I_D = I_F - I_R}$ | CONDITION |
|---|---|---|
| Linear Region | $I_{spec} \cdot e^{\frac{V_P}{U_T}} \cdot \left[e^{-\frac{V_S}{U_T}} - e^{-\frac{V_D}{U_T}}\right]$ $\cong I_{D0} \cdot e^{\frac{V_G}{n \cdot U_T}} \cdot \left[e^{-\frac{V_S}{U_T}} - e^{-\frac{V_D}{U_T}}\right]$ | $V_S > V_P$ $V_D > V_P$ |
| Forward Saturation | $I_{spec} \cdot e^{\frac{V_P - V_S}{U_T}} \cong I_{D0} \cdot e^{\frac{V_G - n \cdot V_S}{n \cdot U_T}}$ | $V_D - V_S \gg U_T$ |
| Reverse Saturation | $-I_{spec} \cdot e^{\frac{V_P - V_D}{U_T}} \cong -I_{D0} \cdot e^{\frac{V_G - n \cdot V_D}{n \cdot U_T}}$ | $V_S - V_D \gg U_T$ |
| Blocked | $I_F = I_R \Rightarrow I_D = 0$ | $V_S \gg U_T$ *and* $V_D \gg U_T$ *or* $V_D = V_S$ |

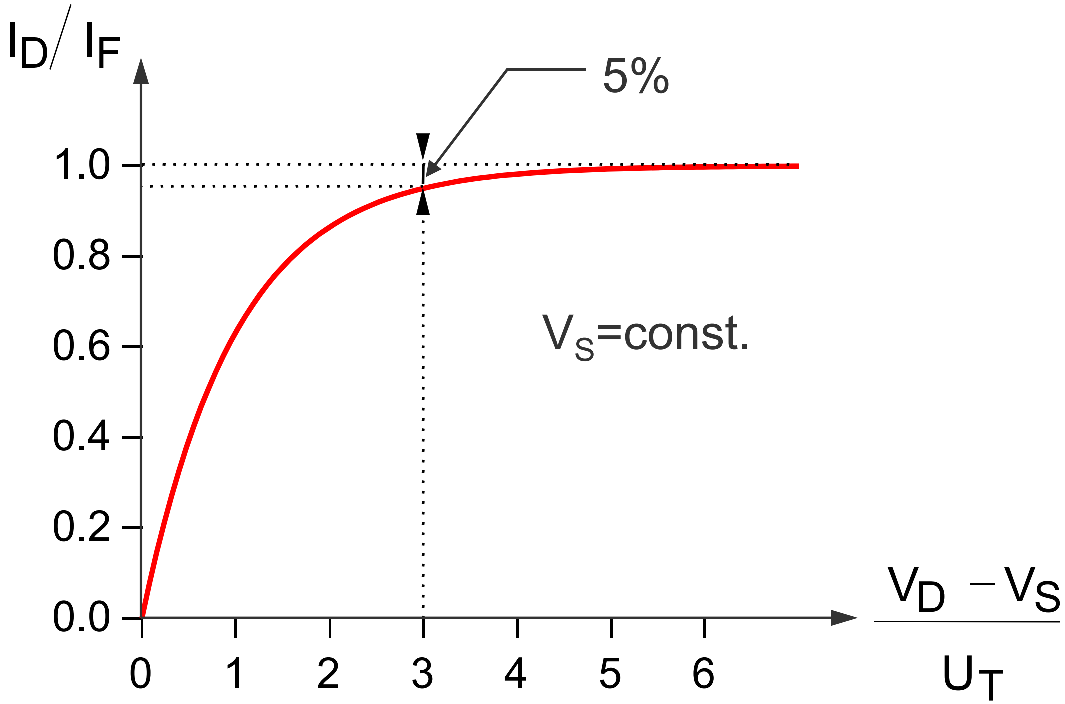
# Transfer Characteristic in WI and Saturation




- **Leakage current $I_{D0}$** depends **exponentially** on the threshold voltage $V_{T0}$ and is therefore not well controlled

$$I_{D0} = I_{spec}\, e^{\frac{-V_{T0}}{nU_T}}$$

# Output Characteristics in Weak Inversion




$$I_D = I_F - I_R = I_F\left(1 - \frac{I_R}{I_F}\right) = I_F\left(1 - e^{-\frac{V_D - V_S}{U_T}}\right)$$

- Saturation reached for only a few $U_T$ (typically $3\ to\ 5U_T \cong 78\ to\ 120\ mV$)

# The Subthreshold Slope

- The **subthreshold slope** or **gate swing** is defined as the increase of gate voltage (in mV) required for the drain current to increase by one order of magnitude (x10)

$$\Delta V_G = nU_T \ \ln 10 = 2.3\, nU_T = 2.3\, n\, \frac{kT}{q} \quad \left[\frac{\mathrm{mV}}{\mathrm{decade}}\right]$$

- with $U_T$ expressed in mV
- At room temperature it is typically equal to **90 mV/dec**
- It can be compared to the 60 mV/dec obtained for a bipolar transistor or a fully depleted SOI MOS device for which $n \cong 1$
- Scales with temperature

# Summary

- Take advantage of the **symmetry** of the device by referring the terminal voltages to the local substrate resulting in symmetrical model
- Drain current can be split into a **forward** $I_F$ and a **reverse** component $I_R$ as $I_D = I_F - I_R$
- Normalized forward (reverse) current $i_f \triangleq I_F/I_{spec}$ ($i_r \triangleq I_R/I_{spec}$) can be expressed in terms of the normalized source (drain) charge $q_s$ ($q_d$) as $i_f = q_s^2 + q_s$ ($i_r = q_d^2 + q_d$)
- The **specific current** is defined as $I_{spec} = I_{spec\square} \cdot W/L$ where $I_{spec\square} \triangleq 2n\mu C_{ox}U_T^2$ is the most fundamental parameter for a given type of transistor in a given process
- The normalized charge are related to the saturation voltage by $v_p - v_s = \ln(q_s) + 2q_s$ where $v_p \triangleq V_P/U_T$, $v_s \triangleq V_S/U_T$ where the **pinch-off voltage** is given by $V_P = (V_G - V_{T0})/n$

# Outline

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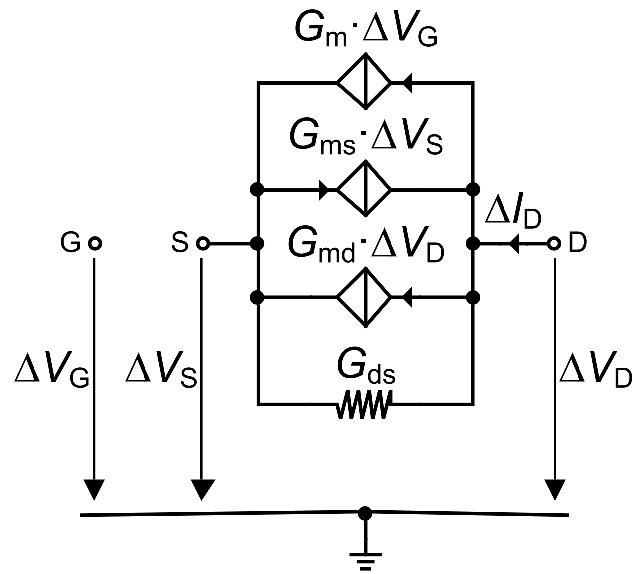
- **The long-channel small-signal model**
- The long-channel noise model
- The extended model
- The simplified EKV model

# Transconductances

- Since there are **3 control voltages**, there are also **3 transconductances**
- For the bulk-referenced model they are defined as

$$\Delta I_D = \frac{\partial I_D}{\partial V_G}\Delta V_G - \frac{\partial I_D}{\partial V_S}\Delta V_S + \frac{\partial I_D}{\partial V_D}\Delta V_D$$
$$= G_m \Delta V_G - (G_{ms} + G_{ds})\Delta V_S + (G_{md} + G_{ds})\Delta V_D$$

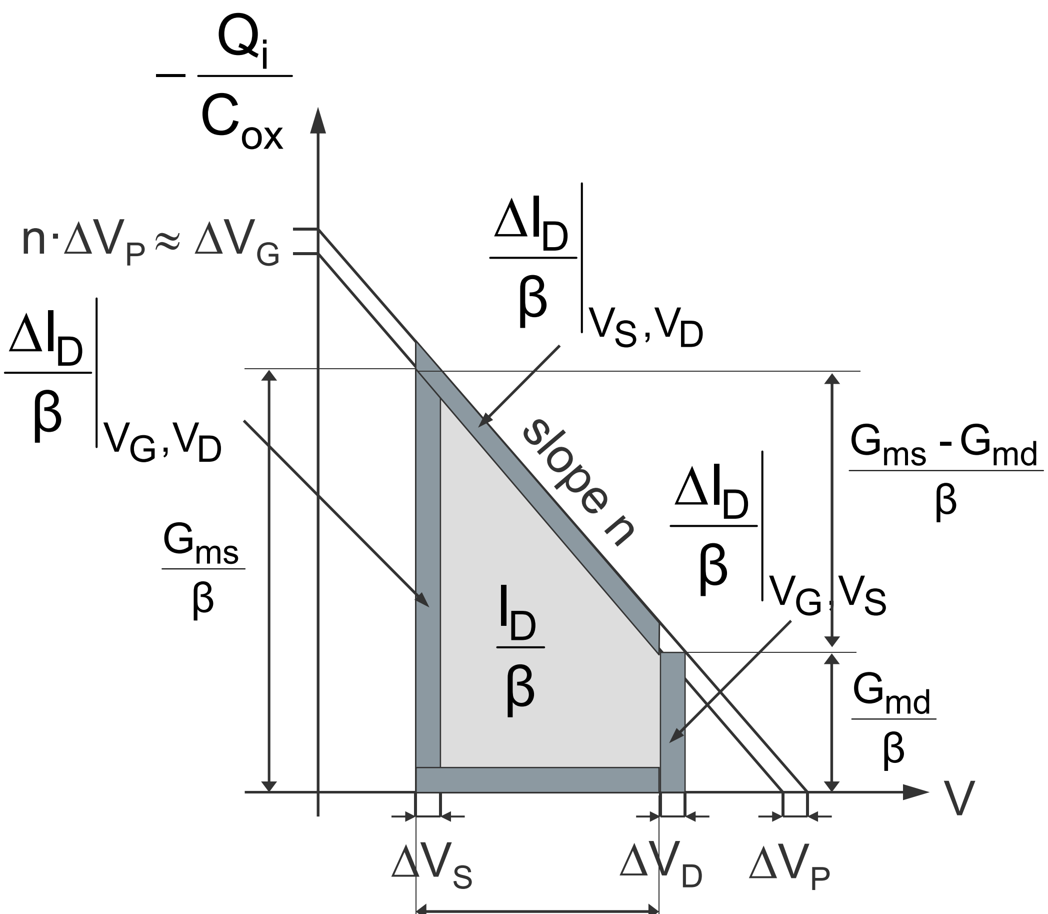
- In **forward saturation**, $G_{md} = 0$ and hence
$\Delta I_D = G_m \Delta V_G - G_{ms}\Delta V_S + G_{ds}\Delta V_{DS}$
- Note that $G_{md}$ is **not** the output conductance $G_{ds}$




- In forward saturation, the **source-referenced** transconductances are related to the **bulk-referenced** transconductances by

$$G_m\Big|_{source} = G_m\Big|_{bulk} = G_m$$
$$G_{mb}\Big|_{source} = G_{ms}\Big|_{bulk} - G_m\Big|_{bulk} = (n-1)G_m$$
$$G_{ds}\Big|_{source} = G_{ds}\Big|_{bulk} = G_{ds}$$

# Transconductances versus Charges




$$G_{ms} = \beta \left.\frac{-Q_i}{C_{ox}}\right|_{V=V_S} = G_{spec} \cdot q_s$$

$$G_{md} = \beta \left.\frac{-Q_i}{C_{ox}}\right|_{V=V_D} = G_{spec} \cdot q_d$$

$$G_{spec} \triangleq \frac{I_{spec}}{U_T}$$

$$G_m = \frac{G_{ms} - G_{md}}{n} = G_{spec} \frac{q_s - q_d}{n}$$

$$G_{ms} \triangleq \left.\frac{\Delta I_D}{\Delta V_S}\right|_{V_G, V_D} \qquad \frac{G_m}{\beta} = \frac{G_{ms} - G_{md}}{n \cdot \beta} \qquad G_{md} \triangleq \left.\frac{\Delta I_D}{\Delta V_D}\right|_{V_G, V_S}$$

# Transconductances versus Charges

- The **source** and **drain transconductances** $G_{ms}$ and $G_{md}$ are directly related to the charges at source and drain according to

$$I_D = \beta \int_{V_S}^{V_D} \frac{-Q_i}{C_{ox}} dV \Rightarrow \begin{cases} G_{ms} \triangleq -\frac{\partial I_D}{\partial V_S} = \beta \left.\frac{-Q_i}{C_{ox}}\right|_{V=V_S} = \beta \frac{-Q_i(x=0)}{C_{ox}} = G_{spec} \cdot q_s \\ G_{md} \triangleq \frac{\partial I_D}{\partial V_D} = \beta \left.\frac{-Q_i}{C_{ox}}\right|_{V=V_D} = \beta \frac{-Q_i(x=L)}{C_{ox}} = G_{spec} \cdot q_d \end{cases}$$

- with $G_{spec} \triangleq I_{spec}/U_T = 2n\beta U_T$
- The **gate transconductance** $G_m$ depends on $G_{ms}$ and $G_{md}$ according to

$$G_m \triangleq \frac{\partial I_D}{\partial V_G} = \frac{G_{ms} - G_{md}}{n} = G_{spec} \frac{q_s - q_d}{n}$$

- In **forward saturation** $G_{md} = 0$ and hence

$$G_m = \frac{G_{ms}}{n}$$

C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Charges and Transconductances versus Currents

- The **source** and **drain charges** $q_s$ and $q_d$ can be expressed as a function of the forward and reverse currents $i_f$ and $i_r$ by solving $i_f = q_s^2 + q_s$ and $i_r = q_d^2 + q_d$ for $i_f$ and $i_r$ resulting in

$$q_s = \frac{2 i_f}{\sqrt{4 i_f + 1} + 1} = \frac{1}{2}\left(\sqrt{4 i_f + 1} - 1\right)$$

$$q_d = \frac{2 i_r}{\sqrt{4 i_r + 1} + 1} = \frac{1}{2}\left(\sqrt{4 i_r + 1} - 1\right)$$

- The **source** and **drain transconductances** can then also be expressed in terms of the forward and reverse currents $i_f$ and $i_r$ according to

$$G_{ms} = G_{spec} \cdot q_s = G_{spec} \frac{2 i_f}{\sqrt{4 i_f + 1} + 1} = \frac{G_{spec}}{2}\left(\sqrt{4 i_f + 1} - 1\right)$$

$$G_{md} = G_{spec} \cdot q_d = G_{spec} \frac{2 i_r}{\sqrt{4 i_r + 1} + 1} = \frac{G_{spec}}{2}\left(\sqrt{4 i_r + 1} - 1\right)$$

- Note that these expressions are **valid in all regions of inversion**

C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Transconductances in Strong inversion

- In **strong inversion**, the transconductances are related to the forward and reverse currents according to

$$G_{ms} = n\beta(V_P - V_S) = \sqrt{2n\beta I_F} = \frac{2I_F}{V_P - V_S}$$

$$G_{md} = n\beta(V_P - V_D) = \sqrt{2n\beta I_R} = \frac{2I_R}{V_P - V_D}$$

$$G_m = \frac{G_{ms} - G_{md}}{n} = \beta V_{DS} = \sqrt{2\beta/n}\left(\sqrt{I_F} - \sqrt{I_R}\right)$$

- In **saturation** $I_R \cong 0$, $I_D \cong I_F$ and therefore

$$G_{ms} = n\beta(V_P - V_S) = \sqrt{2n\beta I_D} = \frac{2I_D}{V_P - V_S}$$

$$G_{md} \cong 0$$

$$G_m = \beta(V_P - V_S) = \sqrt{2\beta I_D/n} = \frac{2I_D}{n(V_P - V_S)}$$

- In strong inversion, $G_m$ depends on two design parameters (either $\beta$ and $V_P - V_S$ or $\beta$ and $I_D$ or $I_D$ and $V_P - V_S$

# Transconductances in Weak Inversion

- In **weak inversion**, the transconductances are related to the forward and reverse currents according to

$$G_{ms} = \frac{I_F}{U_T}$$

$$G_{md} = \frac{I_R}{U_T}$$

$$G_m = \frac{G_{ms} - G_{md}}{n} = \frac{I_F - I_R}{nU_T} = \frac{I_D}{nU_T}$$

- In **saturation** $I_R \cong 0$, $I_D \cong I_F$ and therefore

$$G_{ms} = \frac{I_D}{U_T}$$

$$G_{md} \cong 0$$

$$G_m = \frac{G_{ms}}{n} = \frac{I_D}{nU_T}$$

- In weak inversion, $G_m$ depends only on the drain current $I_D$

# Inversion Coefficient Definition

- Overdrive voltage $V_G - V_{T0}$ or $V_{GS} - V_T$ **not convenient for weak inversion**
- Replaced by the **inversion coefficient $IC$** characterizing the global level of inversion of the transistor and formerly defined as $IC \triangleq max(i_f, i_r)$
- In (forward) **saturation** $i_f \gg i_r$ and therefore

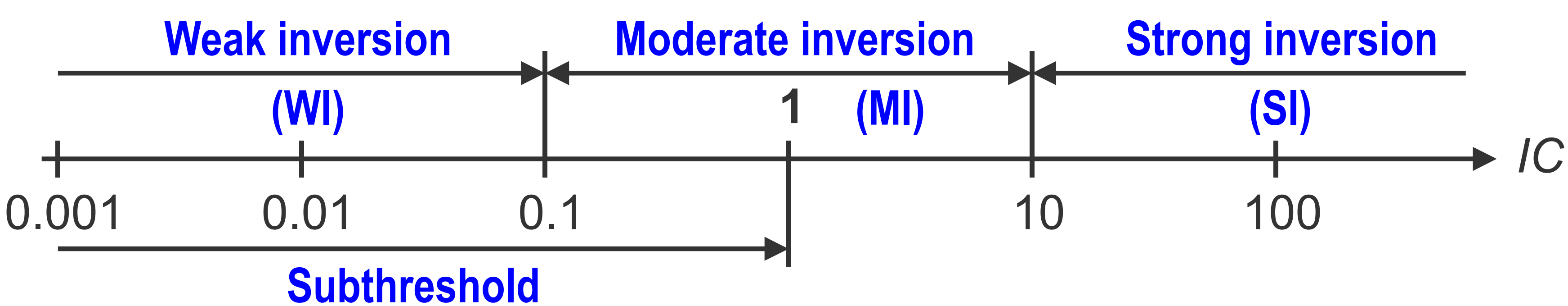
$$IC \triangleq \frac{I_D|_{saturation}}{I_{spec}}$$

Typical values of $I_{spec\square}$ for 28-nm:
750 nA for NMOS
200 nA for PMOS

- Where the specific current $I_{spec}$ is defined as

$$I_{spec} \triangleq I_{spec\square} \cdot \frac{W}{L} \text{ with } I_{spec\square} \triangleq 2n\mu C_{ox} U_T^2 \text{ and } U_T \triangleq \frac{kT}{q}$$

- The different regions of operation in **saturation** can then be defined as



# Current Efficiency or $G_m/I_D$ Ratio (long-channel)

- The **current efficiency** or **transconductance efficiency** is a figure-of-merit that evaluates how much transconductance you get for a given current
- The transconductance in saturation is related to the inversion coefficient (or normalized current) according to

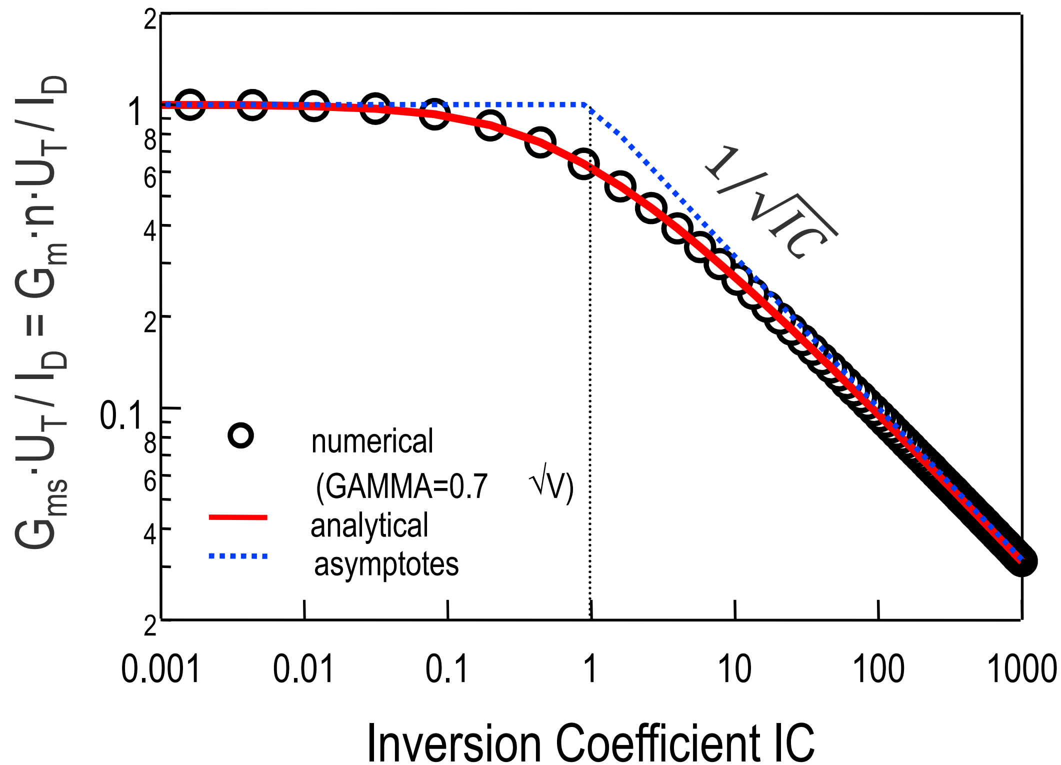
$$G_{ms} = nG_m = G_{spec} \cdot g_{ms}(IC) \text{ with } g_{ms}(IC) \triangleq \frac{G_{ms}}{G_{spec}} = \frac{2IC}{\sqrt{4IC+1}+1}$$

- The normalized current efficiency is then given by dividing the transconductance by the transconductance in weak inversion $G_{ms} = I_D/U_T$ resulting in

$$\frac{G_{ms} \cdot U_T}{I_D} = \frac{G_m \cdot nU_T}{I_D} = \frac{g_{ms}(IC)}{IC} = \frac{2}{\sqrt{4IC+1}+1} = \begin{cases} 1 & \text{in WI and saturation} \\ \dfrac{1}{\sqrt{IC}} & \text{in SI and saturation} \end{cases}$$

- The current efficiency is therefore **maximum in WI** which means that for a given current budget it is better to bias the transistor in WI to get the maximum transconductance (careful, it does not mean that the maximum transconductance is reached in WI)
- Or alternatively, to achieve a given transconductance biasing the transistor in WI saves current

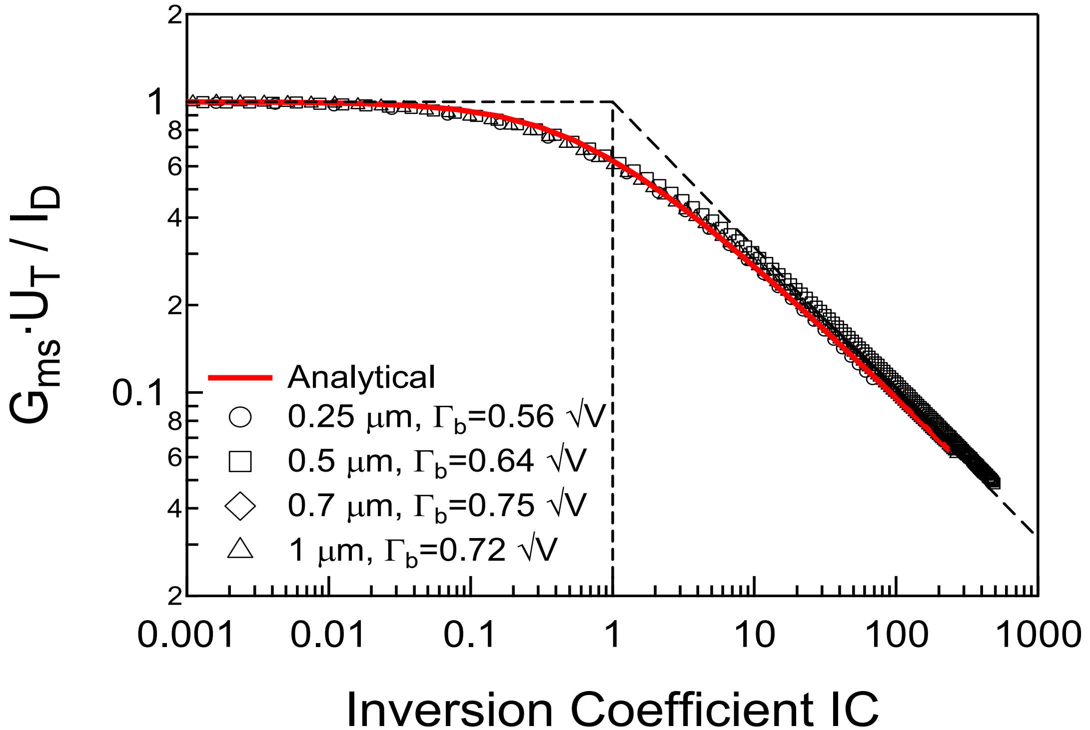
# Current Efficiency or $G_m/I_D$ Ratio (long-channel)




- The **current efficiency** (in saturation) is maximum in weak inversion

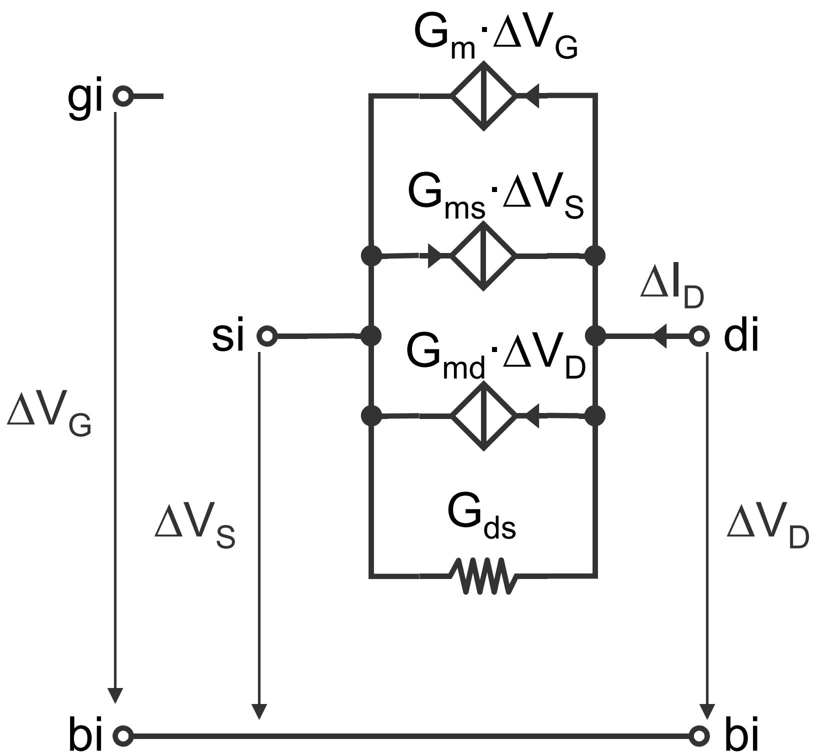
$$\frac{G_{ms} \cdot U_T}{I_D} = \frac{G_m \cdot nU_T}{I_D} = \frac{2}{\sqrt{4IC+1}+1} = \begin{cases} 1 & \text{in WI and saturation} \\ \dfrac{1}{\sqrt{IC}} & \text{in SI and saturation} \end{cases}$$

# $G_m/I_D$ Characteristics – Invariant to CMOS Scaling




- The normalized $G_m/I_D$ characteristic is almost **invariant** to CMOS technology scaling

# Low-frequency (dc) Small-signal Model




$$G_{ms} = \begin{cases} \dfrac{I_F}{U_T} & WI \\ n \cdot \beta \cdot (V_P - V_S) = \sqrt{2n \cdot \beta \cdot I_F} = \dfrac{2 I_F}{V_P - V_S} & SI \end{cases}$$

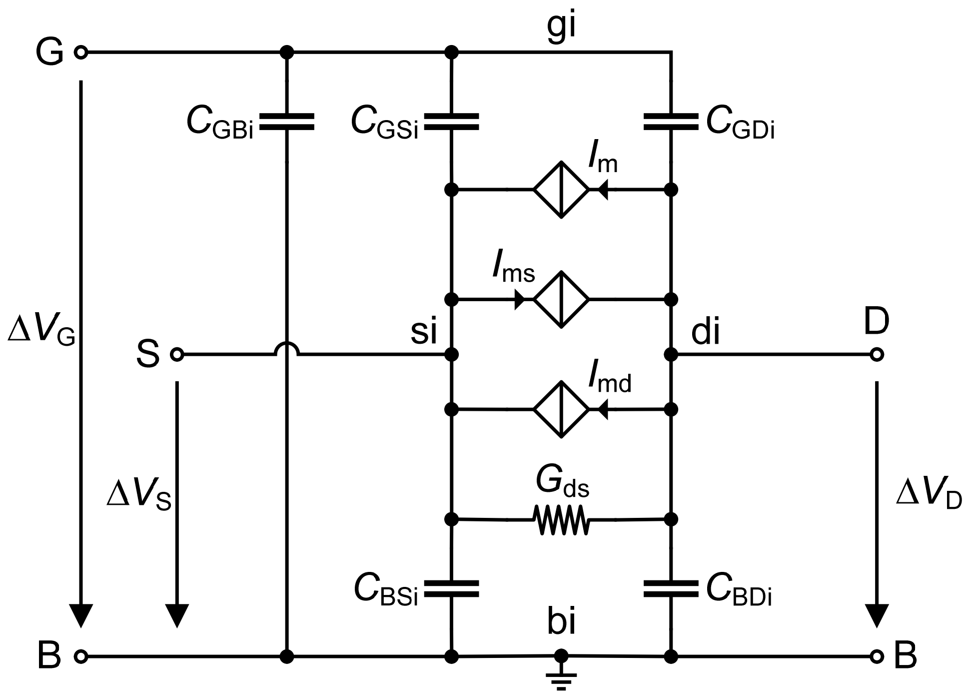
$$G_{md} = \begin{cases} \dfrac{I_R}{U_T} & WI \\ n \cdot \beta \cdot (V_P - V_D) = \sqrt{2n \cdot \beta \cdot I_R} = \dfrac{2 I_R}{V_P - V_D} & SI \end{cases}$$

$$G_m = \frac{G_{ms} - G_{md}}{n}$$

$$G_{ms} = G_{spec} \cdot q_s = G_{spec} \cdot \frac{2 i_f}{\sqrt{4 i_f + 1} + 1} = \frac{G_{spec}}{2} \cdot \sqrt{4 i_f + 1} - 1$$

$$G_{md} = G_{spec} \cdot q_d = G_{spec} \cdot \frac{2 i_r}{\sqrt{4 i_r + 1} + 1} = \frac{G_{spec}}{2} \cdot \sqrt{4 i_r + 1} - 1$$

# Intrinsic Quasi-static Small-signal Model




$$I_m = Y_m \cdot \Delta V_G$$

$$I_{ms} = Y_{ms} \cdot \Delta V_S$$

$$I_{md} = Y_{md} \cdot \Delta V_D$$

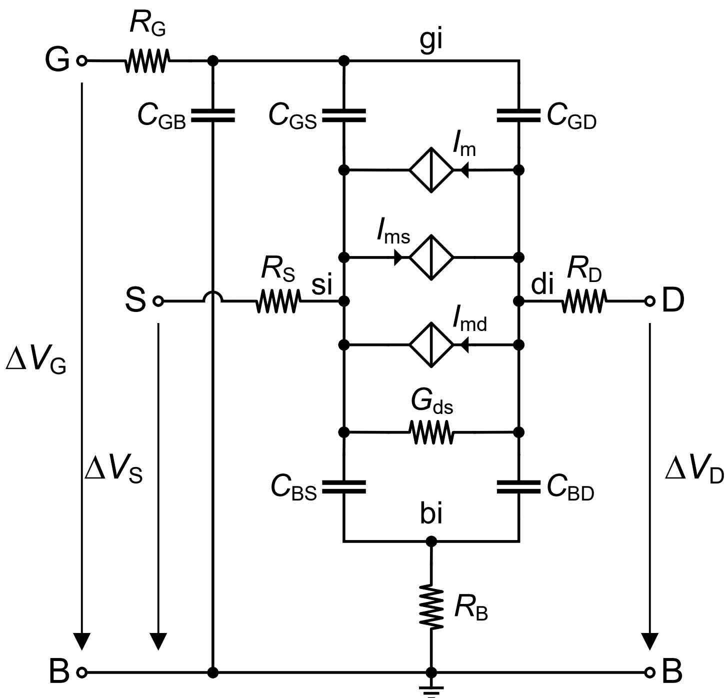
$$Y_m = G_m \cdot \left(1 - j\omega \cdot \tau_{qs}\right) = G_m - j\omega \cdot C_m$$

$$Y_{ms} = G_{ms} \cdot \left(1 - j\omega \cdot \tau_{qs}\right) = G_{ms} - j\omega \cdot C_{ms}$$

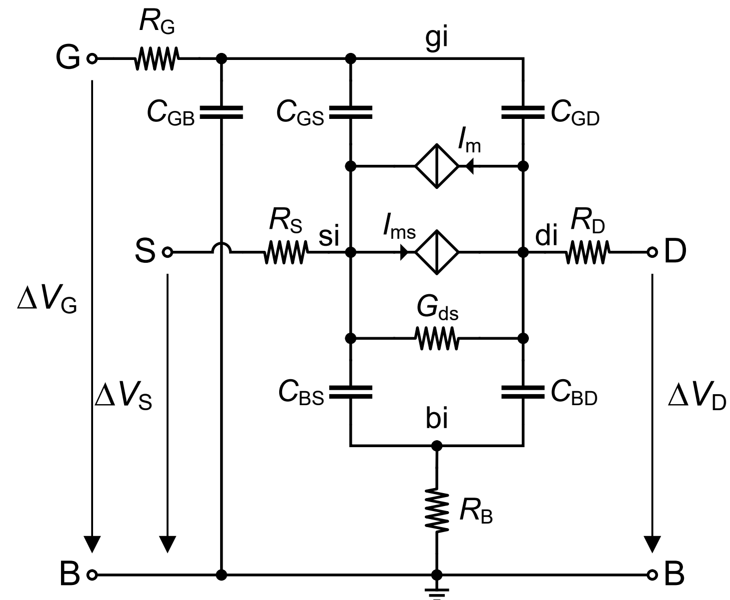
$$Y_{md} = G_{md} \cdot \left(1 - j\omega \cdot \tau_{qs}\right) = G_{md} - j\omega \cdot C_{md}$$

# Complete Quasi-static Small-signal Model

Linear region




Saturation




$$C_{GS} = C_{GSi} + C_{GSe}$$
$$C_{GD} = C_{GDi} + C_{GDe}$$
$$C_{GB} = C_{GBi} + C_{GBe}$$
$$C_{BS} = C_{BSi} + C_{BSj}$$
$$C_{BD} = C_{BDi} + C_{BDj}$$

$$I_m = Y_m \cdot \left(V(gi) - V(bi)\right)$$
$$I_{ms} = Y_{ms} \cdot \left(V(si) - V(bi)\right)$$
$$I_{md} = Y_{md} \cdot \left(V(di) - V(bi)\right)$$

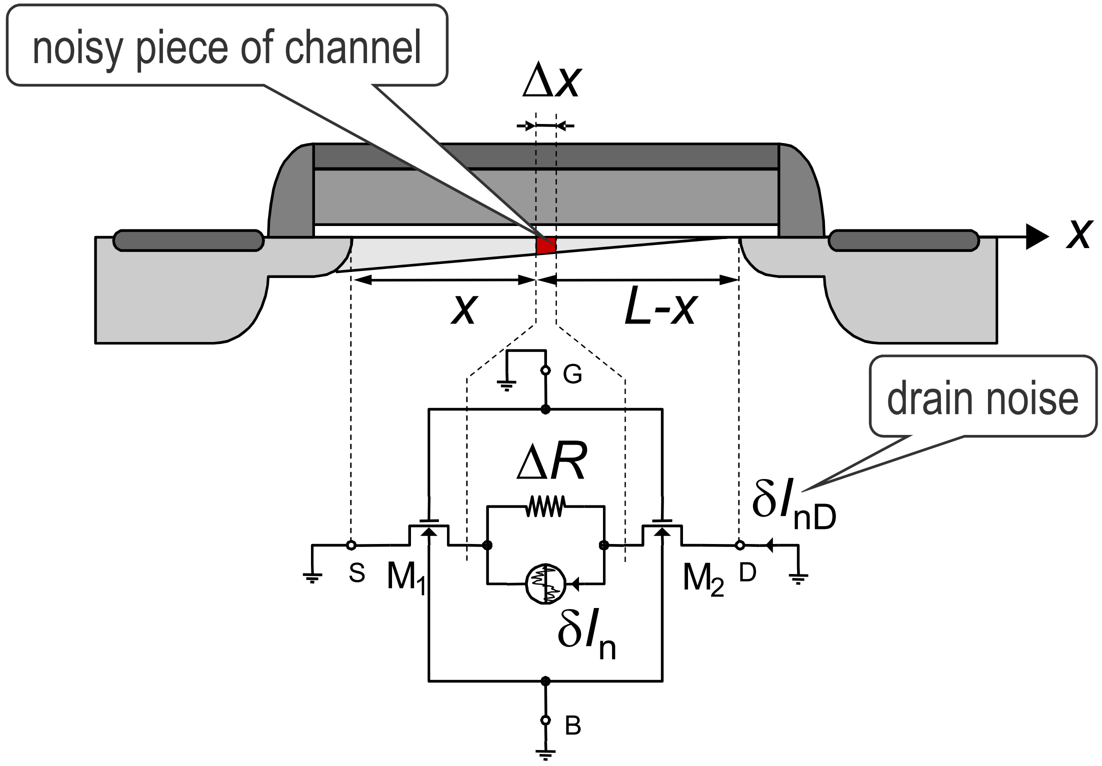
$$Y_m = G_m \cdot \left(1 - j\omega\tau_{qs}\right) = G_m - j\omega C_m$$
$$Y_{ms} = G_{ms} \cdot \left(1 - j\omega\tau_{qs}\right) = G_{ms} - j\omega C_{ms}$$
$$Y_{md} = G_{md} \cdot \left(1 - j\omega\tau_{qs}\right) = G_{md} - j\omega C_{md}$$

$$Y_m = \frac{Y_{ms} - Y_{md}}{n}$$
$$G_m = \frac{G_{ms} - G_{md}}{n}$$
$$C_m = \frac{C_{ms} - C_{md}}{n}$$

# Outline

- The long-channel static model
- The long-channel small-signal model
- **The long-channel noise model**
- The extended model
- The simplified EKV model

# General MOST Noise Calculation

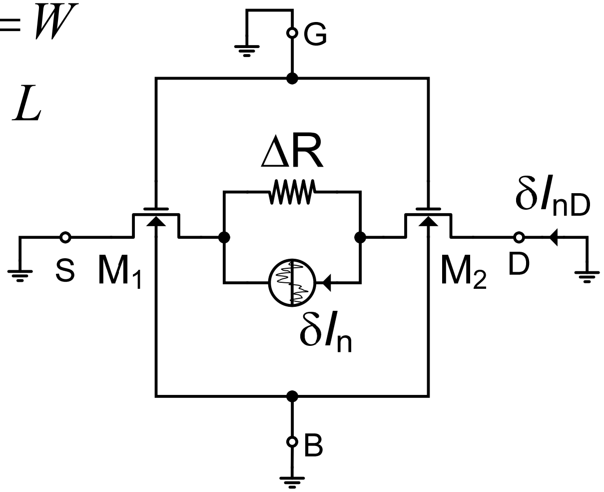


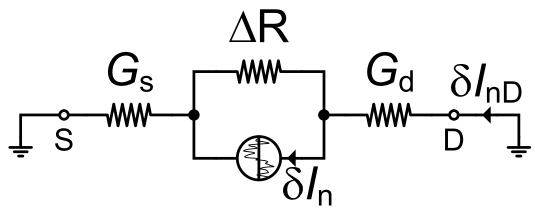

- Noiseless channel except for a slice of channel comprised between $x$ and $x + \Delta x$ and having a resistance $\Delta R$
- Local noise (including both thermal and flicker noise) modeled by current source $\delta I_n$ which induces a fluctuation of the drain current $\delta I_{nD}$ through the (trans)conductance

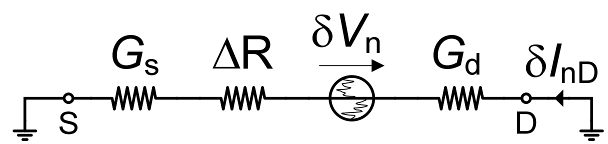
# Two-Transistors Approach

$$W_1 = W_2 = W$$

$$L_1 + L_2 = L$$




- Drain current fluctuation due to local noise source (assuming $1/\Delta R \gg G_{ch}$)

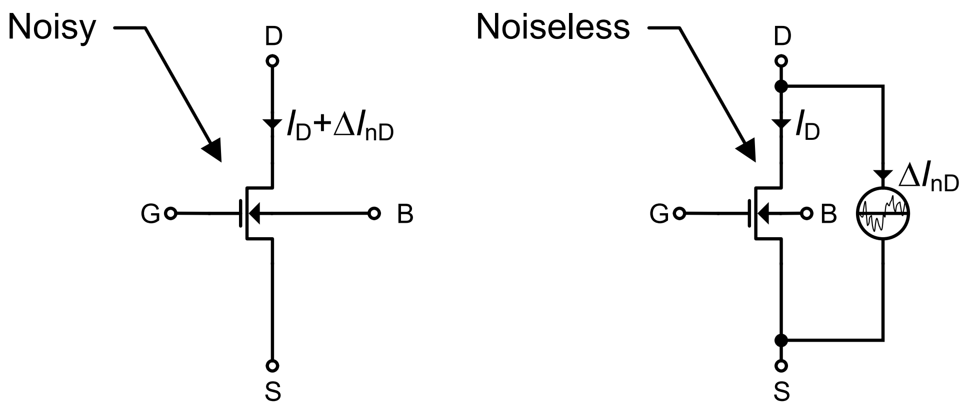
$$\delta I_{nD} = G_{ch} \cdot \Delta R \cdot \delta I_n = G_{ch} \cdot \delta V_n$$

where $G_{ch}$ is the channel conductance seen from point $x$

$$\frac{1}{G_{ch}} = \frac{1}{G_s} + \frac{1}{G_d} \quad \text{with:} \quad G_s = G_{md1} \quad \text{and} \quad G_d = G_{ms2}$$

- Note that local fluctuation of quantity $x$ is noted $\delta x$ whereas global fluctuation due to the whole channel is noted $\Delta x$

# Long-Channel Thermal Noise




- The **thermal noise** at low-frequency can be modeled as a current source between source and drain having a PSD given by

$$S_{\Delta I_{nD}^2} = 4kT \cdot G_{nD} \text{ where } G_{nD} = \frac{\mu}{L^2} \cdot |Q_I| = G_{spec} \cdot q_I$$

- with $G_{spec} \triangleq \frac{I_{spec}}{U_T} = 2n\beta U_T$

- $|Q_I|$ is the total mobile charge in the channel and $q_I$ its normalized form given by

$$q_I \triangleq \frac{|Q_I|}{Q_{spec}} = \frac{1}{6} \frac{4q_s^2 + 3q_s + 4q_s q_d + 3q_d + 4q_d^2}{q_s + q_d + 1} = \begin{cases} q_s & \text{for} \quad q_s = q_d \text{ (linear)} \\ q_s \dfrac{2}{3} \dfrac{q_s + \frac{3}{4}}{q_s + 1} & \text{for} \quad q_s \gg q_d \text{ (saturation)} \end{cases}$$

# Channel Thermal Noise in Weak Inversion

- The total normalized inversion charge in weak inversion is given by

$$q_I \cong \frac{q_s + q_d}{2} = \frac{i_f + i_r}{2}$$

- The noise PSD can be rewritten as

$$G_{nD} = G_{spec}\, q_I = \frac{I_{spec}}{U_T} \frac{i_f + i_r}{2} = \frac{I_F + I_R}{2U_T}$$

- and therefore

$$S_{\Delta I_D^2} = 4kTG_{nD} = 4kT \frac{I_F + I_R}{2U_T} = 2q(I_F + I_R)$$

- which corresponds to **full shot noise** of **both forward and reverse** components
- This result is consistent with the fact that the current in weak inversion is dominated by the **diffusion current**

# Thermal Noise Excess Factor

- The **thermal noise excess factor** $\gamma_{nD}$ is defined as

$$\gamma_{nD} \triangleq \frac{G_{nD}}{G_m}$$

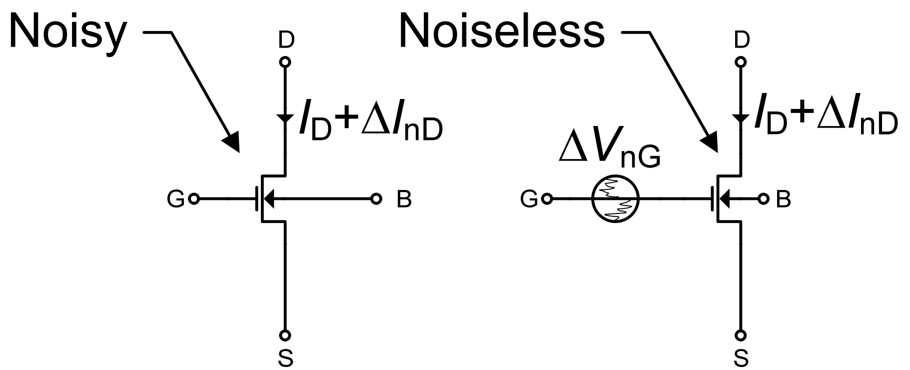
- where $G_m$ is the gate transconductance
- $\gamma_{nD}$ is actually a figure-of-merit (FoM) showing how much noise is generated at the drain for a given $G_m$

$$\gamma_{nD} \triangleq n \frac{q_I|_{saturation}}{q_s} = n \frac{2}{3} \frac{q_s + \frac{3}{4}}{q_s + 1} = \begin{cases} \frac{n}{2} & \text{WI and sat.} \\ \frac{2n}{3} \cong 1 & \text{SI and sat.} \end{cases}$$

- Since $G_m \rightarrow 0$ for $V_{DS} \rightarrow 0$, $\gamma_{nD}$ is becoming large for small $V_{DS}$
- The thermal noise conductance (in saturation) is then given by

$$G_{nD} = \gamma_{nD} \cdot G_m = \begin{cases} \frac{n}{2} G_m & \text{WI and sat.} \\ \frac{2n}{3} G_m \cong G_m & \text{SI and sat.} \end{cases}$$

# Gate-referred Thermal Noise




- For $G_m \neq 0$ and in particular in saturation, the thermal noise can also be referred to the gate as a voltage source having a PSD given by

$$S_{\Delta V_{nG}^2} = \frac{S_{\Delta I_{nD}^2}}{G_m^2} = 4kT\ R_{nG}$$

- where the input (or gate) referred thermal noise resistance $R_{nG}$ is given by

$$R_{nG} = \frac{G_{nD}}{G_m^2} = \frac{\gamma_{nD}}{G_m} = \begin{cases} \frac{1}{2}\frac{n}{G_m} & \text{WI and sat.} \\ \frac{2}{3}\frac{n}{G_m} \cong \frac{1}{G_m} & \text{SI and sat.} \end{cases}$$

# Flicker Noise – Origin and Gate-referred PSD

- Basically two main causes to this 1/f noise:
  - **Carrier number fluctuation** ΔN (Mc Worther model): trapping of mobile charge in traps located in the oxide close to the Si-SiO2 interface resulting in fluctuations of the inversion charge
  - **Carrier mobility fluctuation** Δμ (Hooge model)
- The PSD of the **input referred gate voltage fluctuations** is given by

$$S_{\Delta V_{nG}^2}(f) = S_{\Delta V_{nG}^2}(f)\Big|_{\Delta N} + S_{\Delta V_{nG}^2}(f)\Big|_{\Delta \mu}$$

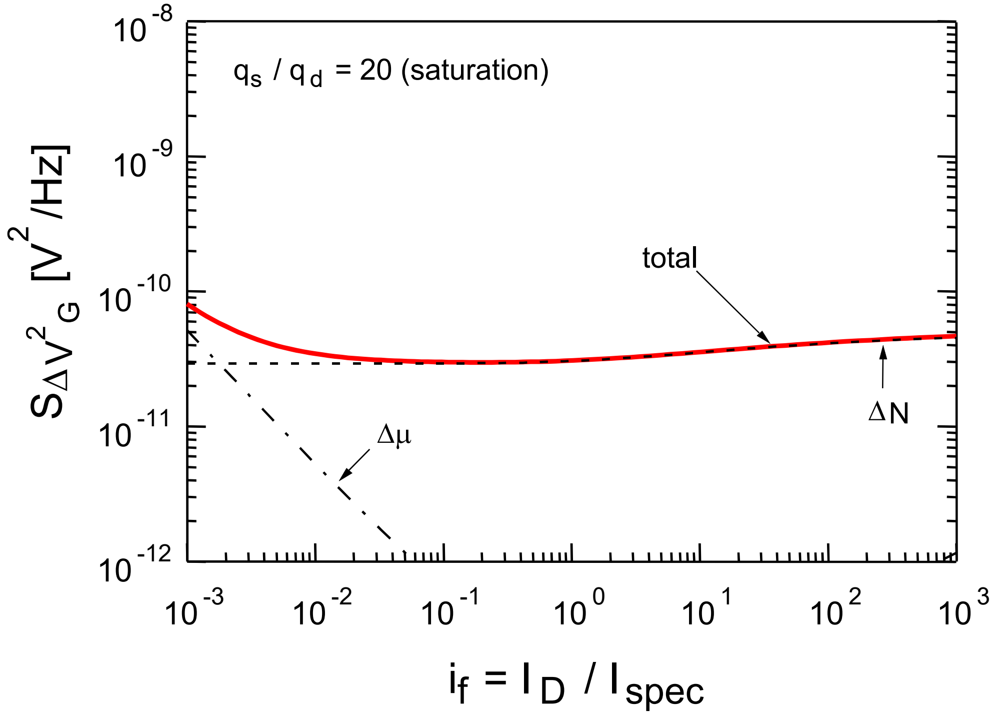
- where

$$S_{\Delta V_{nG}^2}(f)\Big|_{\Delta N} = \frac{K_{\Delta N}}{W \cdot L \cdot C_{ox}^2 \cdot f} \text{ and } S_{\Delta V_{nG}^2}(f)\Big|_{\Delta \mu} = \frac{K_{\Delta \mu}}{W \cdot L \cdot C_{ox} \cdot f}$$

- Inversely proportional to **frequency** and to **gate area**
- Note that $K_{\Delta N}$ and $K_{\Delta \mu}$ are slightly bias dependent

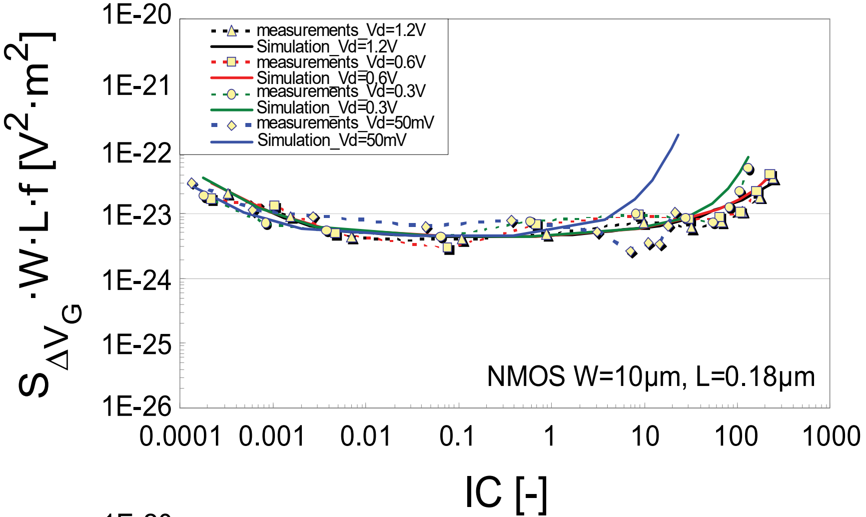
# Flicker Noise – Bias Dependence

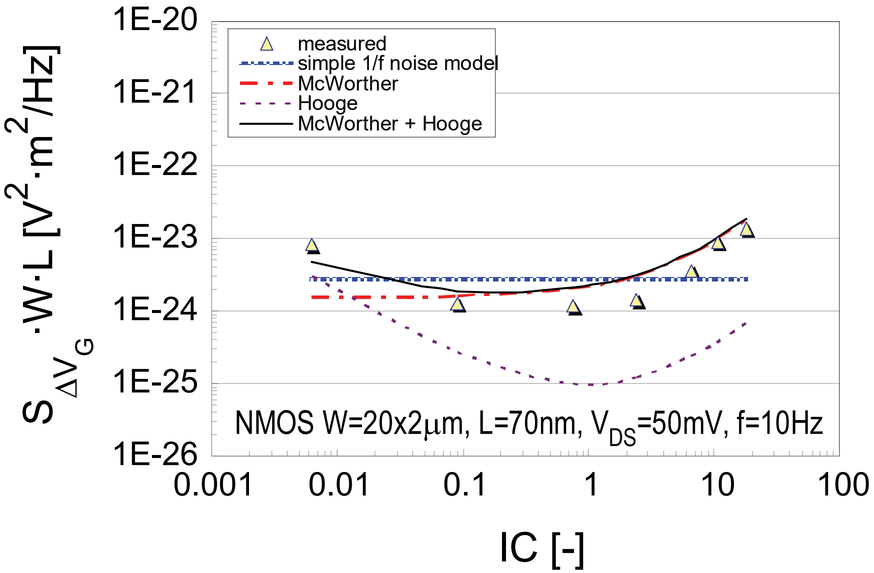
$$S_{\Delta V_{nG}^2} = S_{\Delta V_{nG}^2}\Big|_{\Delta N} + S_{\Delta V_{nG}^2}\Big|_{\Delta \mu}$$

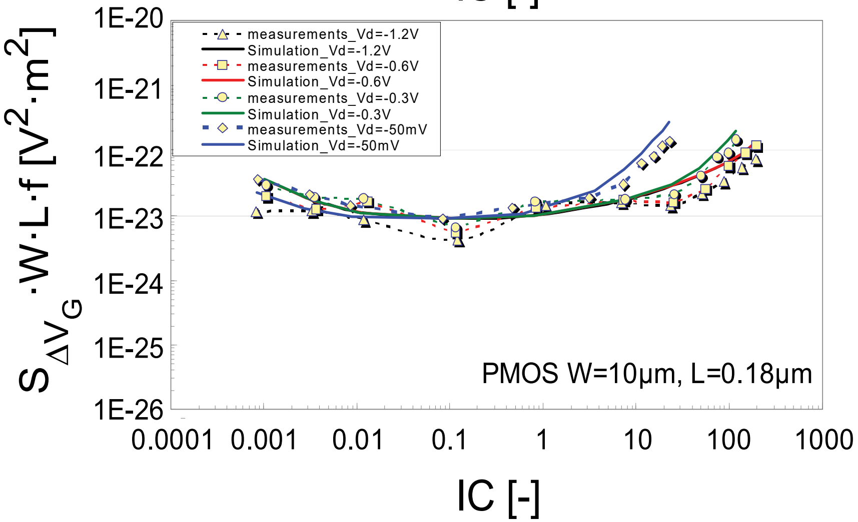


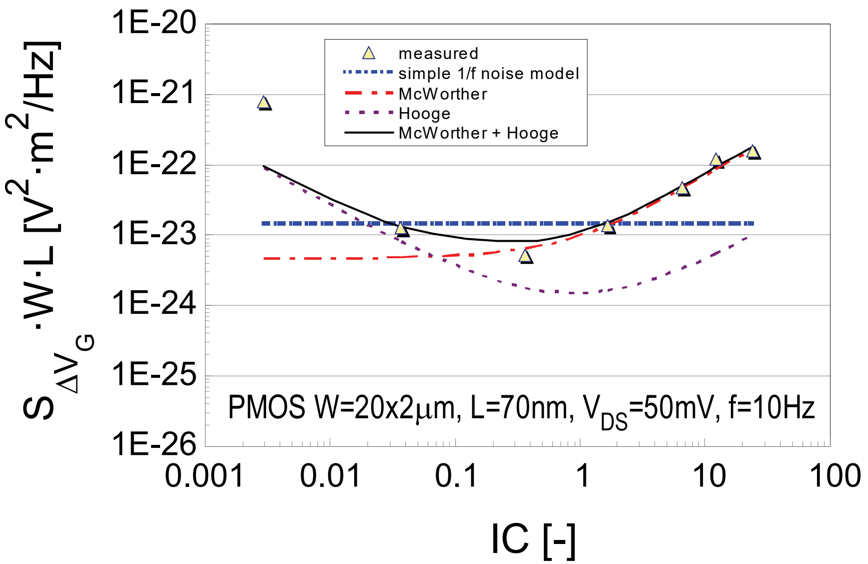

- Usually number fluctuation dominates over mobility fluctuation
- For design purpose, the gate referred noise PSD can be considered at first order as **bias independent**

# Flicker Noise – Measurements



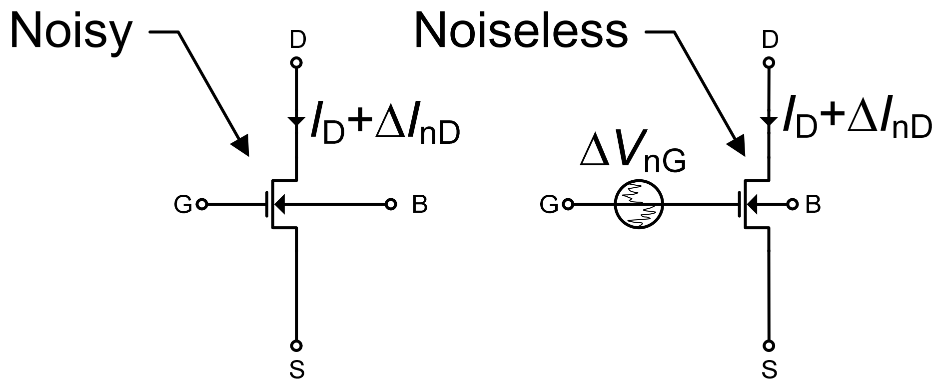





N. Mavredakis, A. Antonopoulos and M. Bucher, Nanotech-WCM, 2010.
N. Mavredakis, A. Antonopoulos and M. Bucher, ECCSC 2010.

# Flicker Noise – Gate-referred Flicker Noise Resistance



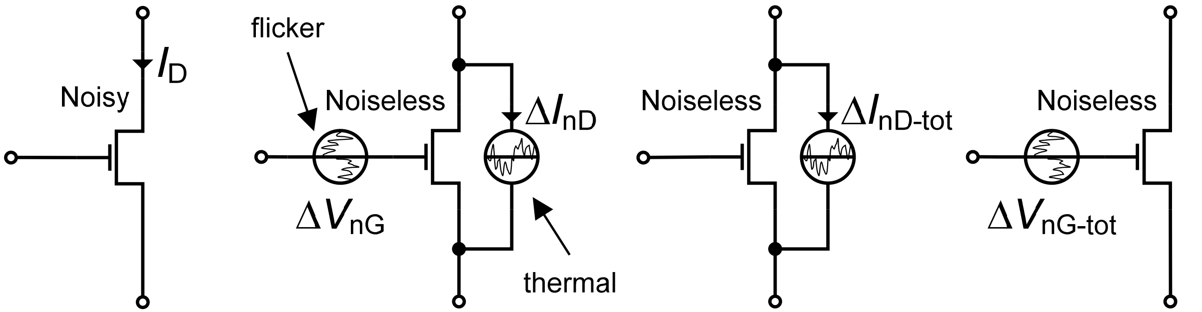

- Similarly to the thermal noise, the gate-referred flicker noise can be expressed in terms of a noise resistance (but frequency dependent)

$$S_{\Delta V_{nG}^2}(f) = 4kT \cdot R_{nG}(f) \text{ with } R_{nG}(f) = \frac{\rho}{W \cdot L \cdot f}$$

- If number fluctuation dominates mobility fluctuation, $\rho$ is related to the previous flicker noise parameters by

$$\rho = \frac{K_{\Delta N}}{4kT \cdot C_{ox}^2}$$

# Total Noise in Saturation




- The total output noise is given by

$$S_{\Delta I^2_{nD,tot}} = S_{\Delta I^2_{nD}} + G_m^2 \cdot S_{\Delta V^2_{nG}}(f) = 4kT \cdot \gamma_{nD} G_m + G_m^2 \cdot S_{\Delta V^2_{nG}}(f)$$

- which for a given current and a given gate area is **minimum in SI**
- The total gate-referred noise is given by

$$S_{\Delta V^2_{nG,tot}}(f) = \frac{S_{\Delta I^2_{nD,tot}}}{G_m^2} = 4kT \cdot \frac{\gamma_{nD}}{G_m} + S_{\Delta V^2_{nG}}(f) = 4kT \cdot R_{nG,tot}(f)$$

- Where the total gate-referred noise resistance $R_{nG,tot}(f)$ is given by

$$R_{nG,tot}(f) = \frac{\gamma_{nD}}{G_m} + \frac{\rho}{W \cdot L \cdot f}$$

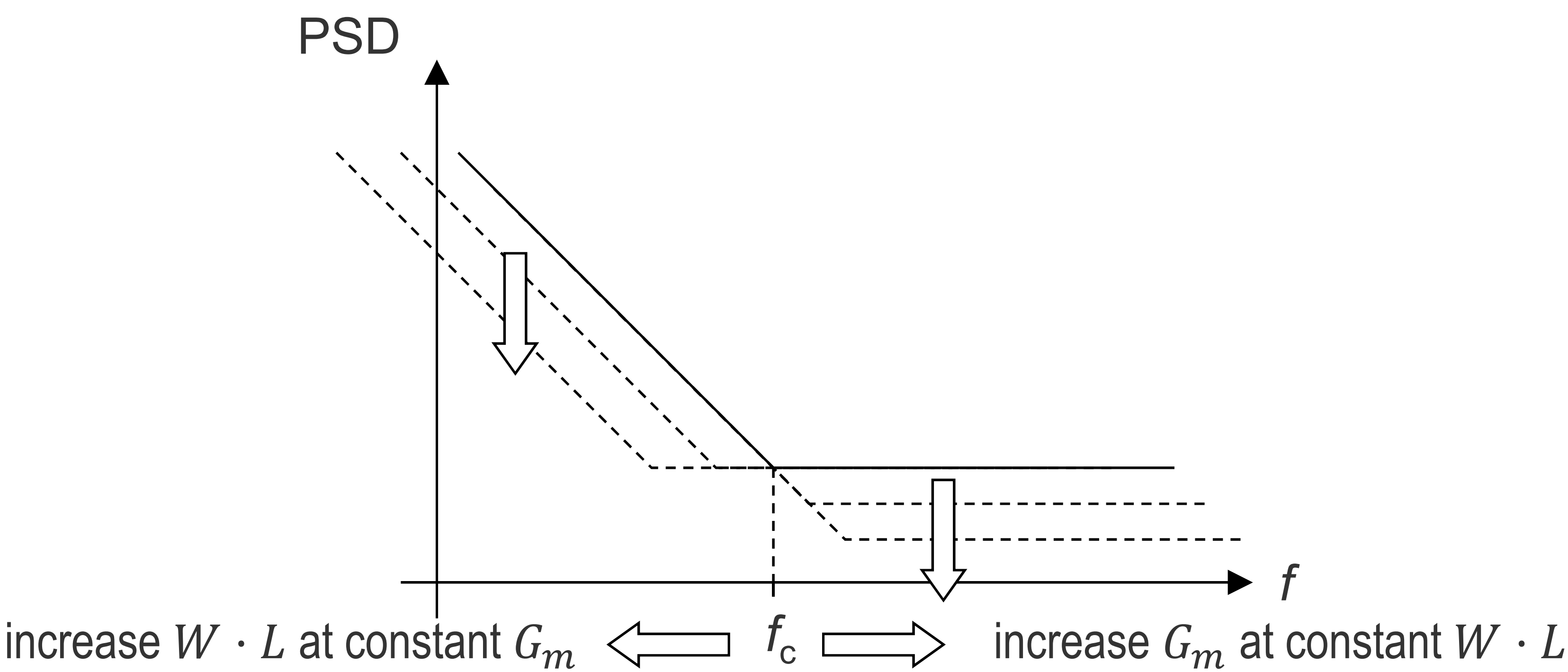
- For a given current and gate area, the total gate-referred noise is **minimum in WI**

# Corner Frequency

- The **corner frequency** is defined as the frequency for which the 1/f noise PSD is equal to the thermal noise PSD

$$\frac{\rho}{W \cdot L \cdot f_c} = \frac{\gamma_{nD}}{G_m} \;\Rightarrow\; f_c = \frac{G_m \rho}{\gamma_{nD} W L} = \frac{K_{\Delta N}\mu}{2q\gamma_{nD} C_{ox}} \frac{g_{ms}(IC)}{L^2}$$

- The corner frequency scales approximately as $1/(C_{ox}L^2)$



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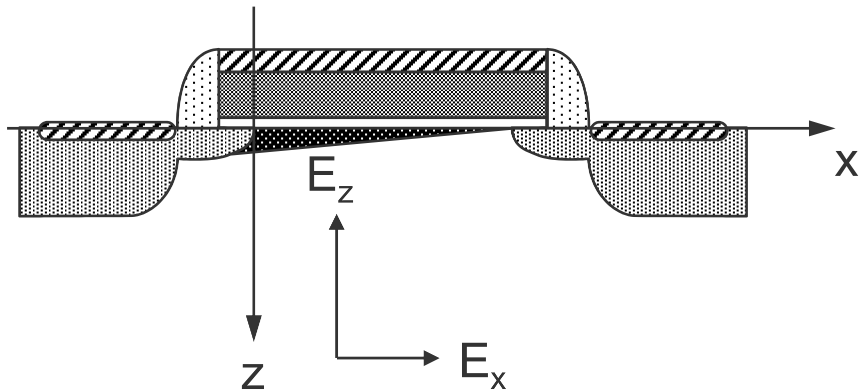
- The simplified EKV model

# Mobility Reduction Mechanisms



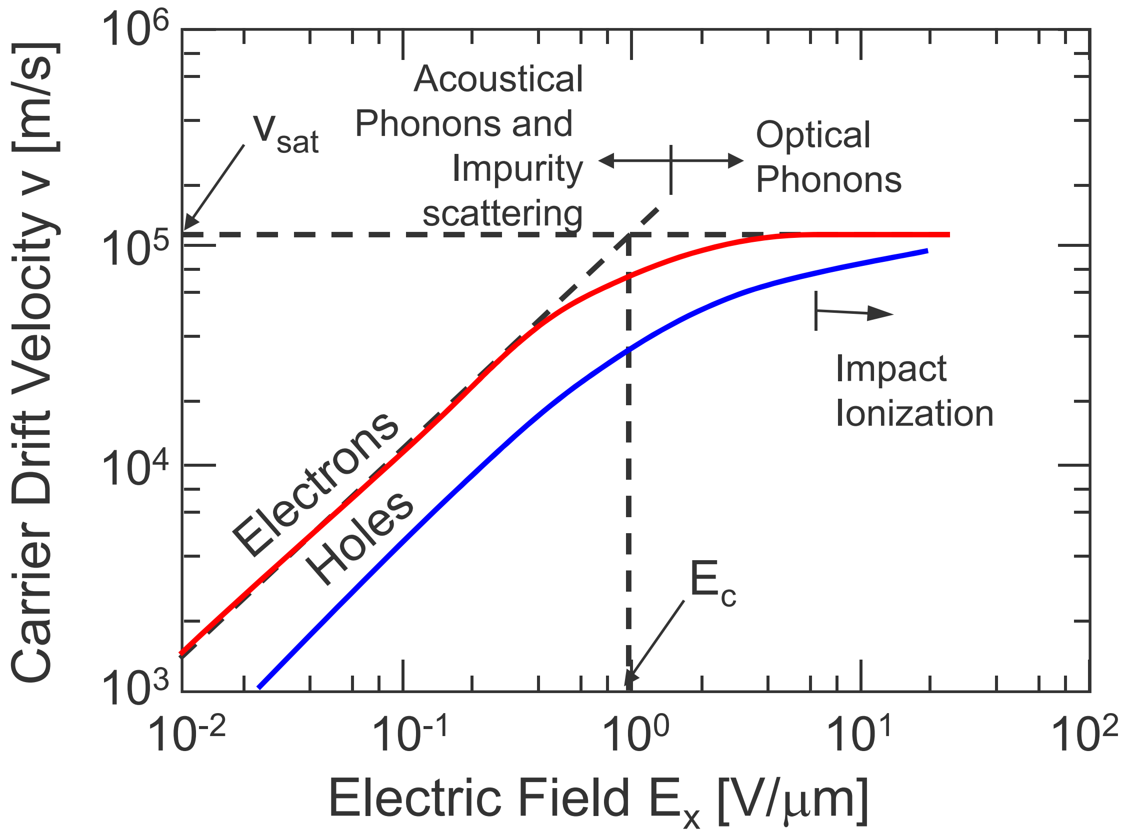

- Mainly two different mobility reduction mechanisms:
  - due to **vertical field** $E_z$
  - due to **longitudinal field** $E_x$
- Mobility reduction due to the vertical field $E_z$ is caused by several **scattering mechanisms**, namely:
  - **Coulomb scattering**: interaction with ionized impurity atoms (at low field)
  - **Phonon scattering**: interaction with lattice vibrations (at medium field)
  - **Surface roughness**: roughness of the Si-$SiO_2$ interface (at high field)
- Mobility reduction due to longitudinal field $E_x$ is caused by the **saturation of carrier velocity**

# Small Geometry Effects

- Short channel length
  - **Velocity saturation (VS)**
  - **Channel length modulation (CLM)**
  - Charge sharing
  - **Drain induced barrier lowering (DIBL)**
  - Reverse short-channel effect
- Narrow gate width
- Thin gate oxide
  - Polysilicon depletion
  - Gate leakage current
- **Effects on thermal noise**

# Velocity Saturation

- A high longitudinal electric field $E_x$ causes the carrier velocity $v_{drift}$ to saturate to $v_{sat}$




For Si

Electrons:

$v_{sat} \cong 10^5$ m/s

$E_c \cong 1$ V/µm

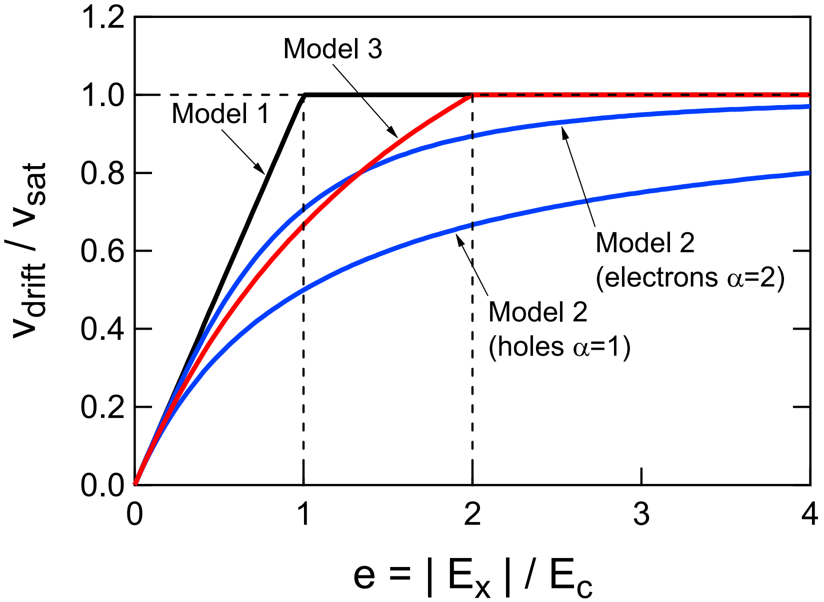
Holes:

$v_{sat} \cong 8\text{x}10^4$ m/s

$E_c \cong 3$ V/µm

C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Velocity-Field Models




- Model 1

$$\frac{v_{drift}}{v_{sat}} = \begin{cases} e & \text{for} \quad e \leq 1 \\ 1 & \text{for} \quad e > 1 \end{cases}$$

with $e \triangleq |E_x|/E_c$ and $E_c \triangleq 2v_{sat}/\mu$

- Model 2

$$\frac{v_{drift}}{v_{sat}} = \frac{e}{(1+e^{\alpha})^{1/\alpha}}$$

where $\alpha = 2$ for electrons and $\alpha = 1$ for holes

- Model 3 (BSIM Model)

$$\frac{v_{drift}}{v_{sat}} = \begin{cases} \dfrac{e}{1+e/2} & \text{for} \quad e < 2 \\ 1 & \text{for} \quad e \geq 1 \end{cases}$$

# Effect of VS on the Drain Current in SI

- Assuming Model 1 for the velocity-field function, the current in SI and saturation, neglecting the effect of mobility reduction due to the vertical field, is given by

$$i_d = \frac{2q_s^2}{1+\sqrt{1+(\lambda_c q_s)^2}} \text{ where } \lambda_c \triangleq \frac{L_{sat}}{L} \text{ with } L_{sat} \triangleq \frac{2\mu_0 U_T}{v_{sat}} = \frac{2U_T}{E_c}$$

- $L_{sat}$ represents the **portion of the channel that is under full VS**
- For very short channel and/or high overdrive voltage

$$\lambda_c q_s = \frac{\mu_0}{v_{sat}} \cdot \frac{V_P - V_S}{L} \gg 1 \Longrightarrow i_d \cong \frac{2q_s}{\lambda_c}$$

- Remembering that in SI $q_s \cong (V_P - V_S)/(2U_T)$ leads to the denormalized drain current given by

$$I_D \cong WnC_{ox}v_{sat}(V_P - V_S) = WC_{ox}v_{sat}(V_G - V_{T0} - nV_S)$$

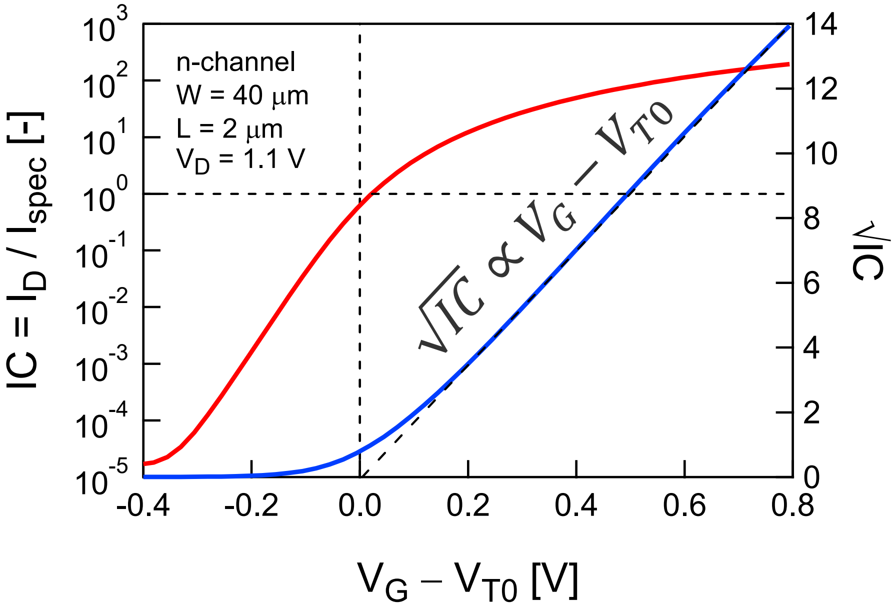
- The current becomes a **linear function** of the charge and therefore of the overdrive voltage and also **independent of the length**

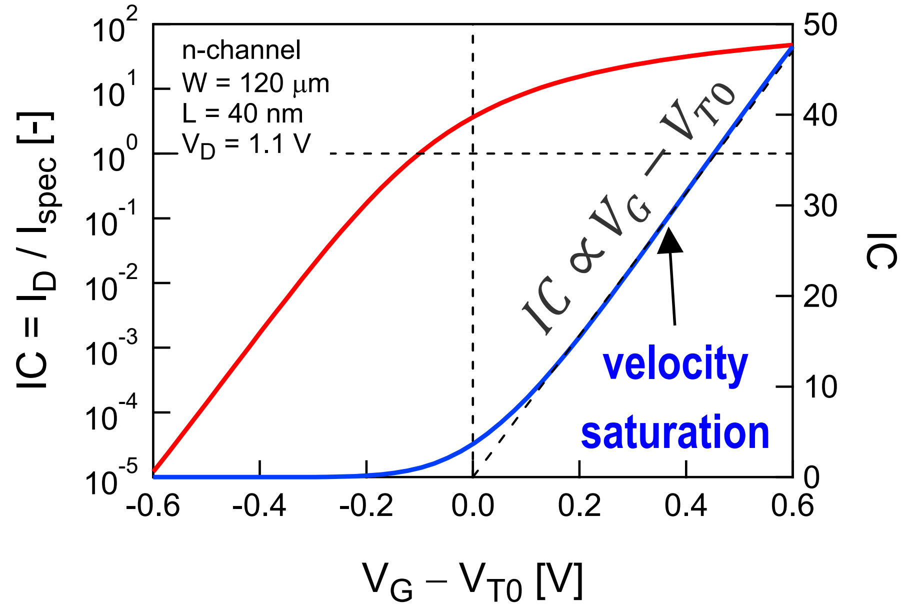
# Effect of VS on the Drain Current (40nm Process)

**Long-channel** | **Short-channel**

$$I_D \cong \frac{\beta}{2n}(V_G - V_{T0})^2$$

$$I_D \cong W C_{ox} v_{sat} (V_G - V_{T0})$$




- Velocity saturation has a **strong impact** on the drain current in **strong inversion**
- The current becomes proportional to $V_G - V_{T0}$
- Hence the gate and source transconductances become **independent of the current** (and **independent of the length**)

# Effect of VS on the Transconductance in SI

- The effect of VS on the **source transconductance** in SI is given by

$$g_{ms} \triangleq \frac{G_{ms}}{G_{spec}} = \frac{q_s}{\sqrt{1 + (\lambda_c q_s)^2}}$$

- For $\lambda_c \cdot q_s \gg 1$, $g_{ms}$ saturates to $1/\lambda_c$

$$g_{ms} \cong \frac{1}{\lambda_c} = \frac{L}{L_{sat}} \text{ in SI and saturation}$$

- or in denormalized form

$$G_{ms} \cong \frac{G_{spec}}{\lambda_c} = nWC_{ox}v_{sat}$$

- $G_{ms}$ becomes **independent of the length and of the current**
- It only depends on $v_{sat}$ and increases with $W$

C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, John Wiley, 2006.

# Effect of VS on the Drain Current in WI

- Velocity saturation also affects the current in weak inversion (in saturation)

$$i_d = \frac{q_s}{1 + \frac{\lambda_c}{2}}$$

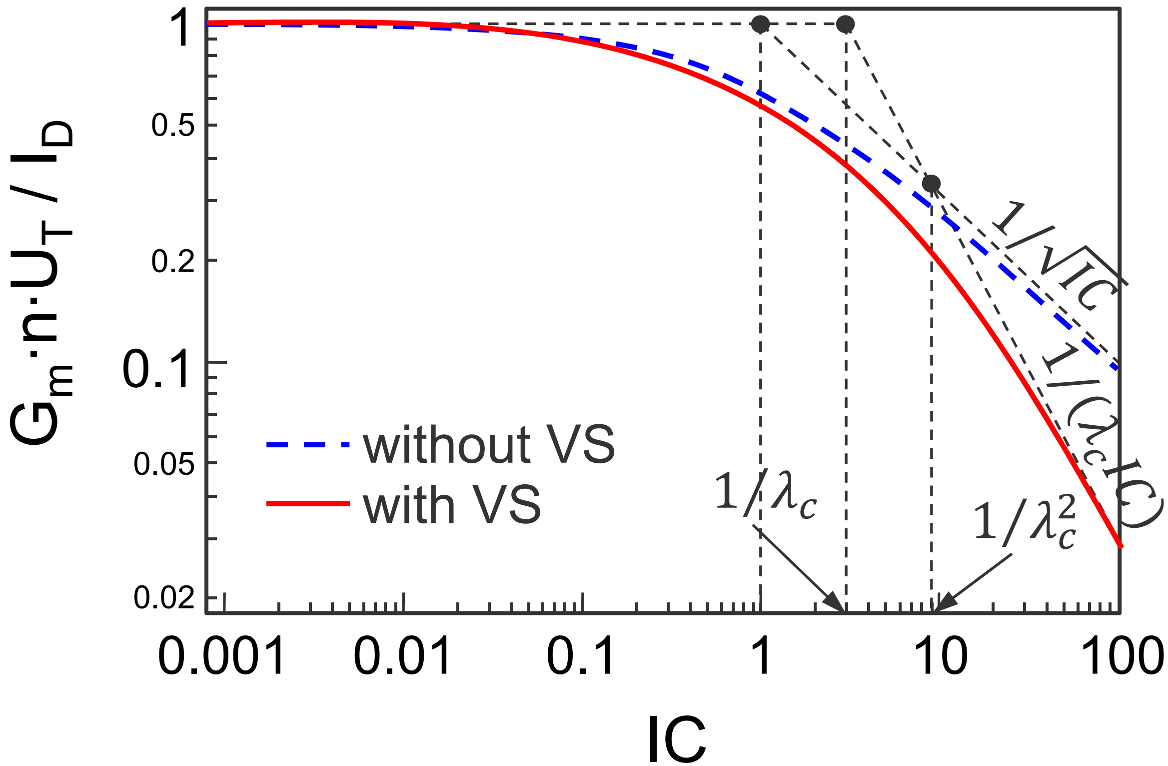
- The source transconductance is then given by

$$g_{ms} = \frac{q_s}{1 + \frac{\lambda_c}{2}} = i_d$$

- The source (gate) transconductances **remain proportional to the current**
- The $G_{ms}/I_D$ ratio **remains equal to unity** as for the long channel case

A. Mangla, J.-M. Sallese and C. Enz, MIXDES 2011

# Effect of Velocity Saturation (VS) on $G_m/I_D$



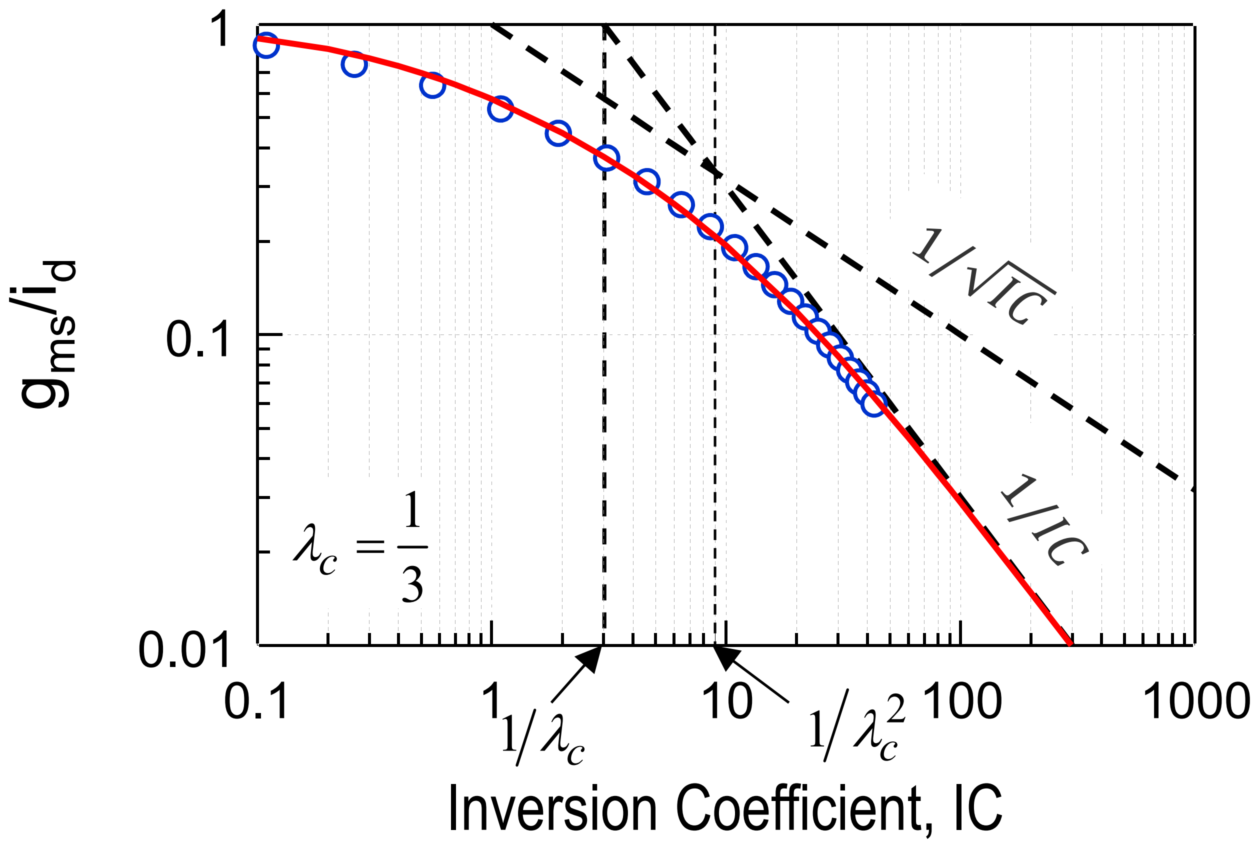

$$\lambda_c \triangleq \frac{L_{sat}}{L}$$

$$L_{sat} \triangleq \frac{2\mu_0 U_T}{v_{sat}} = \frac{2U_T}{E_c}$$

$$\frac{g_{ms}}{i_d} = \frac{G_{ms}U_T}{I_D} = \frac{G_m n U_T}{I_D} = \frac{\sqrt{(\lambda_c IC + 1)^2 + 4IC} - 1}{IC(\lambda_c(\lambda_c IC + 1) + 2)} = \begin{cases} 1 & \text{WI and sat.} \\ \dfrac{1}{\lambda_c IC} & \text{SI and sat.} \end{cases}$$

A. Mangla, M. A. Chalkiadaki, F. Fadhuile, T. Taris, Y. Deval, and C. C. Enz, Microelectronics Journal, vol. 44, pp. 570-575, July 2013.

# Effect of VS on the Current Efficiency (130 nm Process)




$$\lambda_c \triangleq \frac{L_{sat}}{L}$$

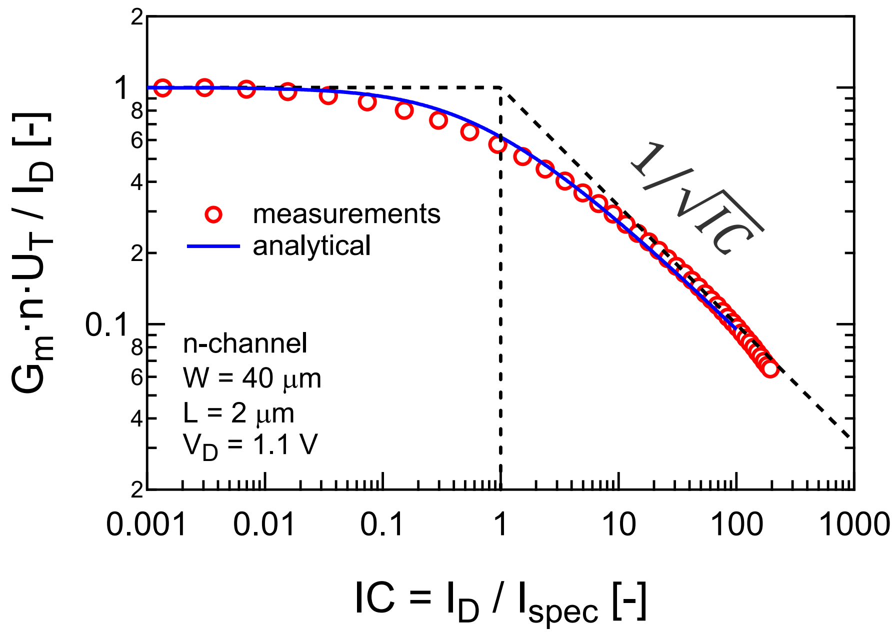
$$L_{sat} \triangleq \frac{2\mu_0 U_T}{v_{sat}} = \frac{2U_T}{E_c}$$

- **Current efficiency is maximum** in weak inversion
- Strongly degrades in strong inversion due to high field effects such as **velocity saturation** and **mobility reduction** due to the vertical field

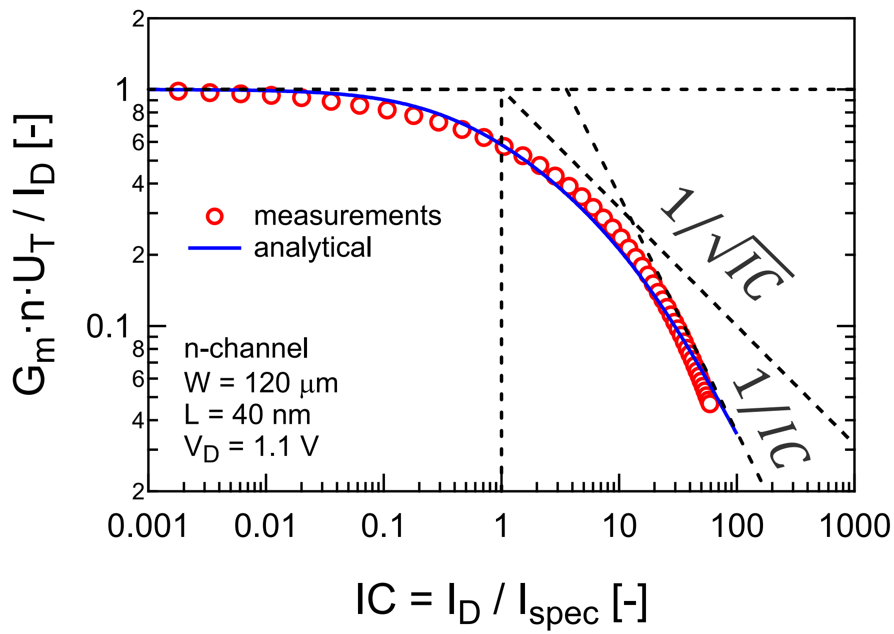
📖 A. Mangla, J.-M. Sallese and C. Enz, MIXDES 2011

# Effect of VS on the Current Efficiency (40 nm Process)

## Long-channel

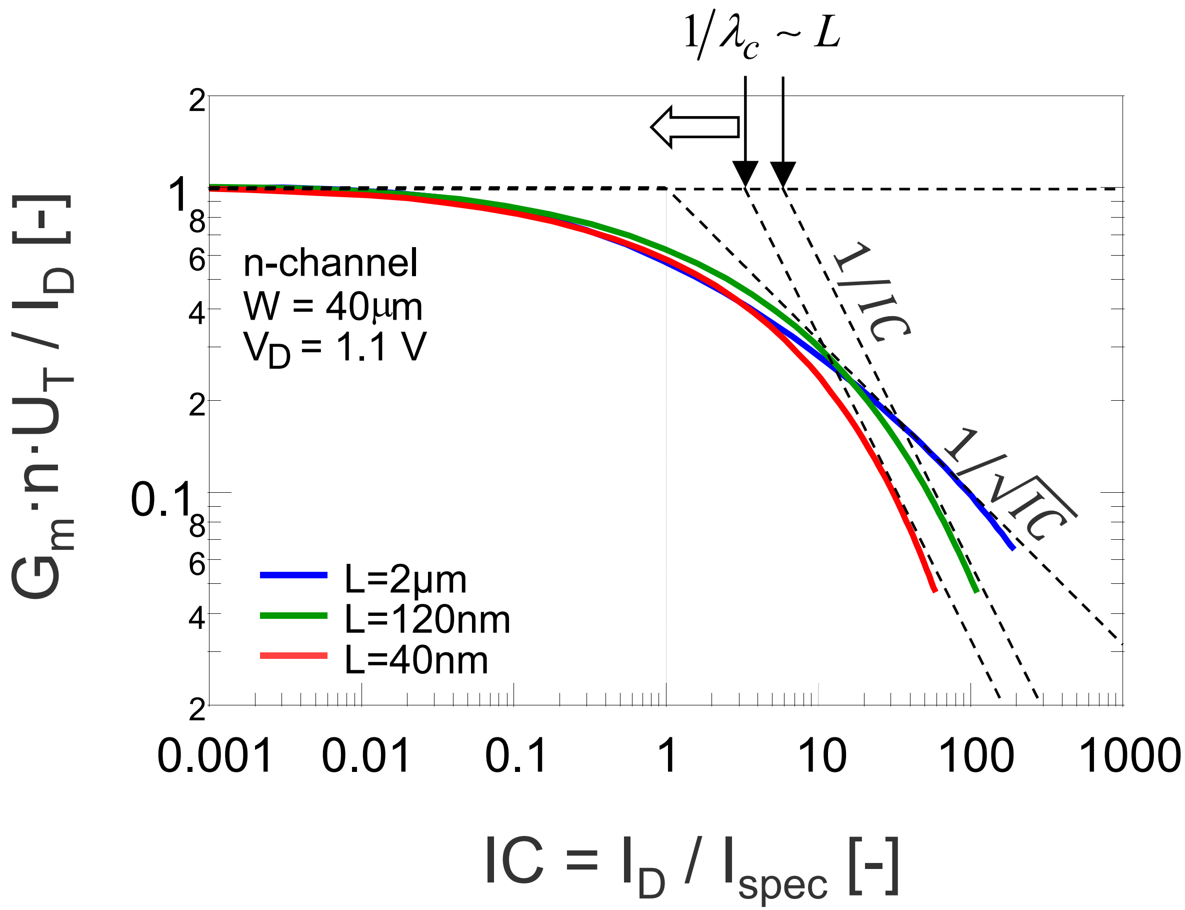



## Short-channel




- Velocity saturation (VS) further degrades the current efficiency in strong inversion
- VS has little impact on the current efficiency in weak and moderate inversion

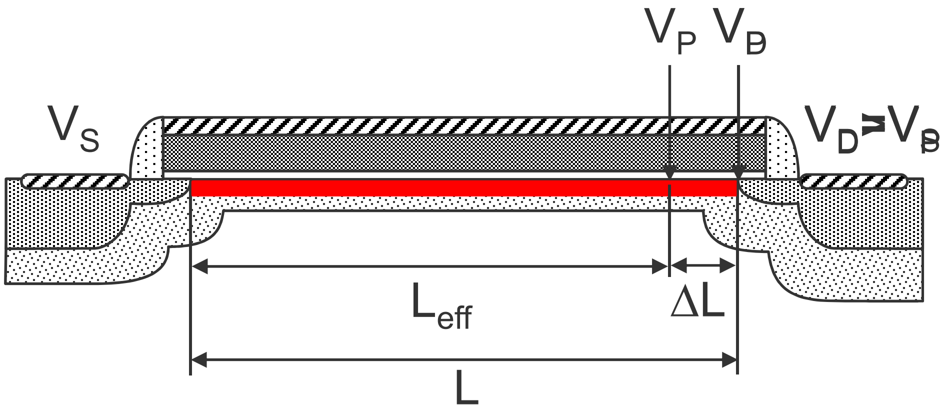
# Effect of VS on the Current Efficiency




$$\lambda_c \triangleq \frac{2\mu_0 U_T}{v_{sat} L} = \frac{2U_T}{E_c L}$$

- The VS intersection point is moving to the left with shorter channel length removing the region where $G_m/I_D$ scales like $1/\sqrt{IC}$

# Channel Length Modulation (CLM)



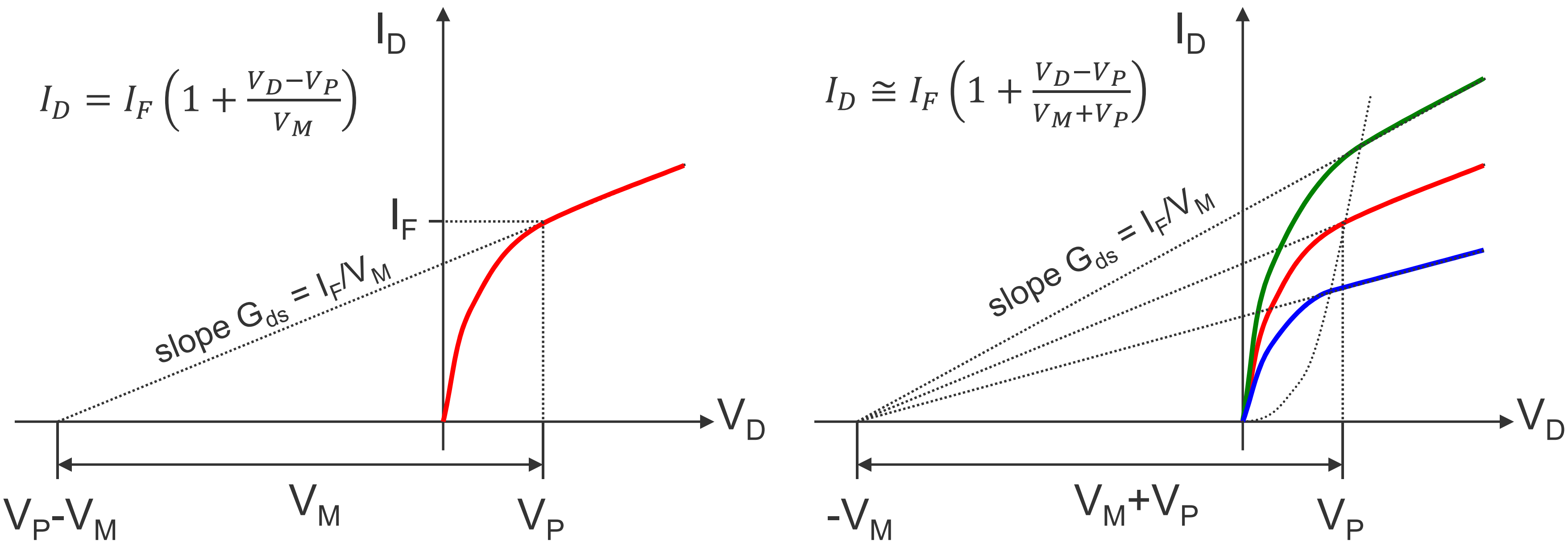

- Increasing the drain voltage above $V_P$ moves the pinch-off point toward the source and creates a depletion region of length $\Delta L$

$$\frac{\Delta L}{L} \cong \frac{\varsigma}{L}\left[\sqrt{\Phi_D + V_D - V_P} - \sqrt{\Phi_D}\right] \cong \frac{\varsigma}{L}\,\frac{V_D - V_P}{\Phi_D} = \frac{V_D - V_P}{V_M} \text{ with } \varsigma \triangleq \sqrt{\frac{2\varepsilon_{si}}{qN_b}}$$

- where $V_M$ is the **channel length modulation voltage** (corresponding to the Early voltage in a bipolar) and defined as

$$V_M \triangleq \lambda\, L \text{ with } \lambda = \frac{2\sqrt{\Phi_D}}{\varsigma}$$

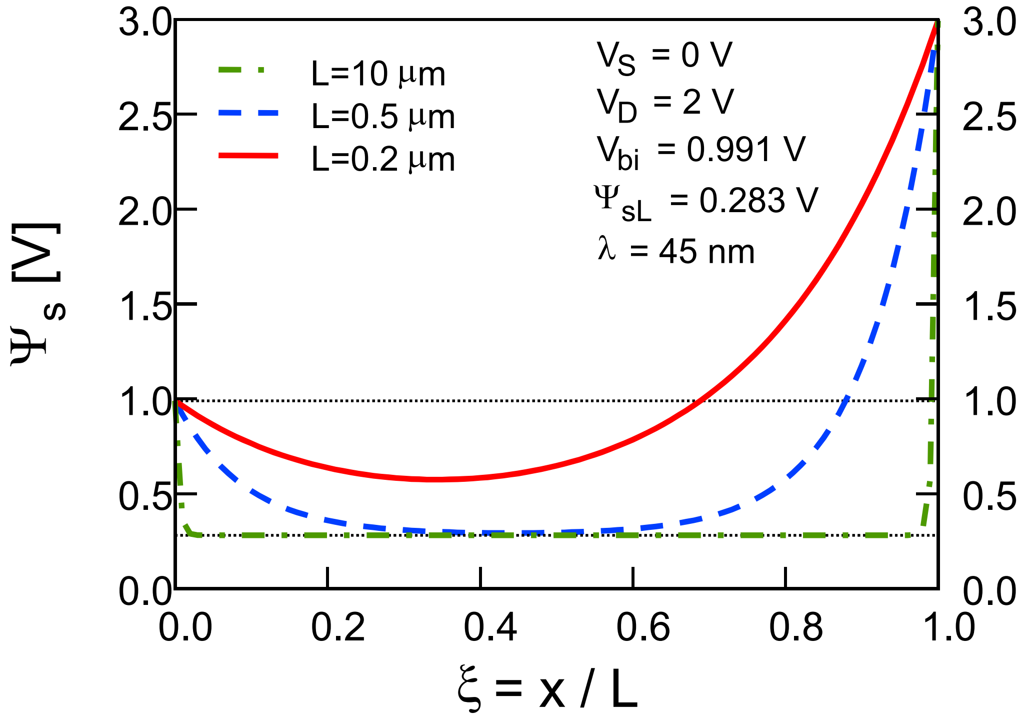
# Output Characteristics




- Assuming $\Delta L\,/L \ll 1$, the drain current in saturation including channel modulation effect can be written as

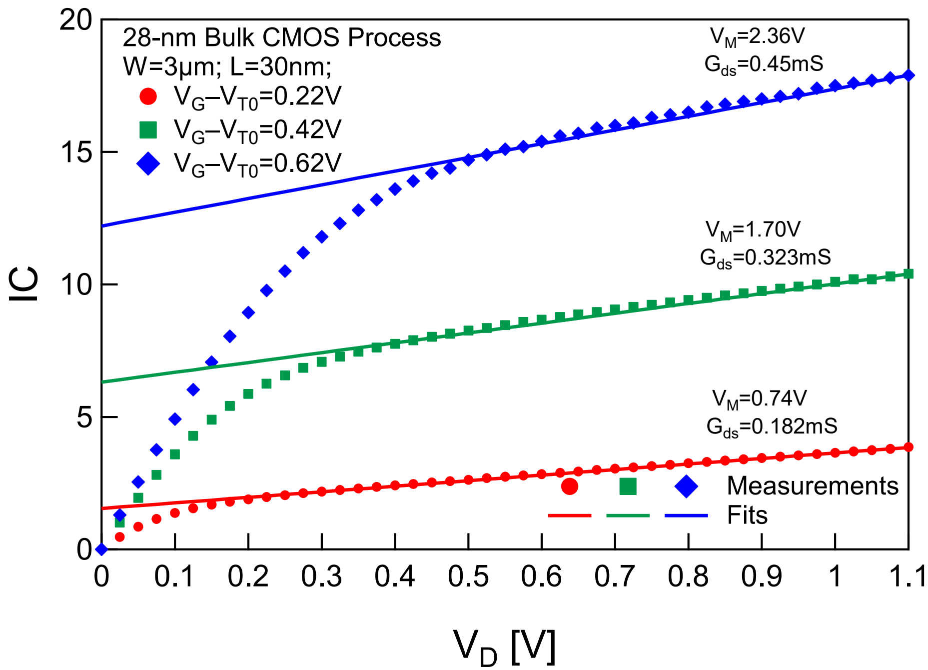
$$I_D = \frac{I_F}{1-\frac{\Delta L}{L}} \cong I_F\left(1+\frac{\Delta L}{L}\right) = I_F\left(1+\frac{V_D-V_P}{V_M}\right) \cong I_F\left(1+\frac{V_D-V_P}{V_M+V_P}\right)$$

# Drain Induced Barrier Lowering (DIBL)




- For short channel devices, the voltages at the source and drain influence the channel surface potential and tend to make it larger than what would be obtained from the long-channel approximation
- This results in an increase of the drain current, particularly in weak and moderate inversion

# Output Characteristic in SI of 28nm Bulk CMOS Process




- The output characteristics in SI is can be modelled by

$$I_D \cong G_{ds}(V_D + V_M)$$

- Where $G_{ds}$ is the output conductance in saturation and $V_M$ is the Early voltage that actually accounts for both CLM and DIBL

C. Enz, F. Chicco, and A. Pezzotta, IEEE Solid-State Circuits Magazine, vol. 9, no. 3, pp. 26-35, Summer 2017.

# Output Conductance due to DIBL

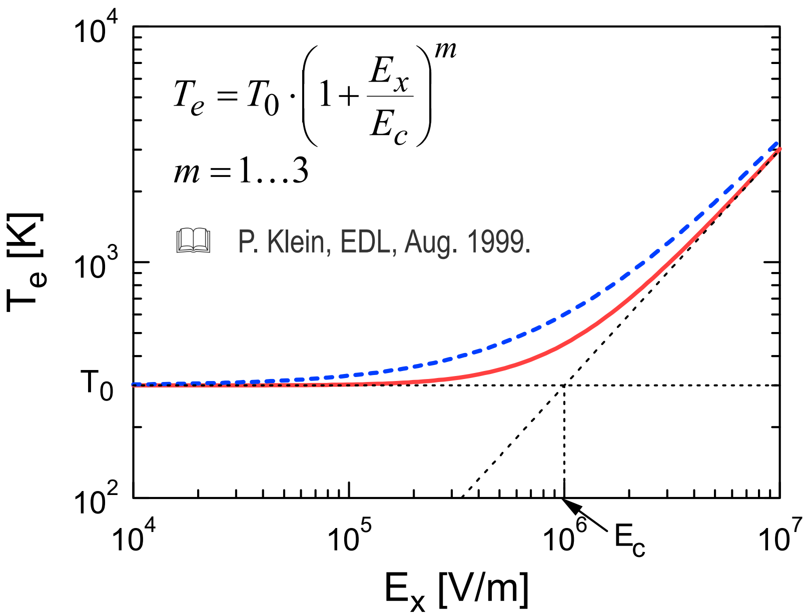
- In advanced short-channel devices biased in MI or WI, the output conductance is dominated by DIBL
- DIBL is defined as the variation of the threshold voltage with respect to the drain-to-source voltage

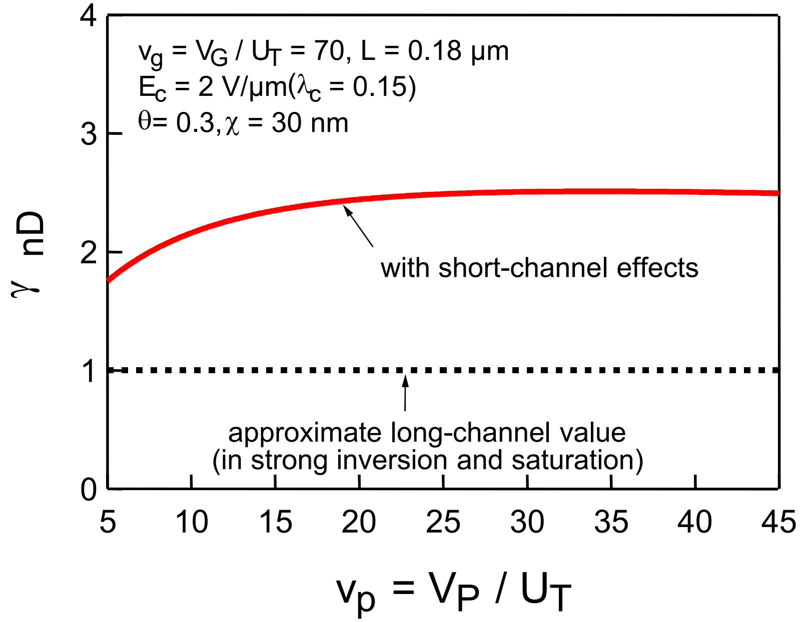
$$V_T \cong V_{T0}(1 - \sigma_d V_{DS}) \text{ where } \sigma_d \triangleq \frac{\partial V_T}{\partial V_{DS}}$$

- The output conductance can then be written as

$$G_{ds} \triangleq \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_G, V_S} \cong \frac{\partial I_D}{\partial V_T} \frac{\partial V_T}{\partial V_{DS}} = \sigma_d \cdot G_m$$

# Short-channel Effects on Thermal Noise

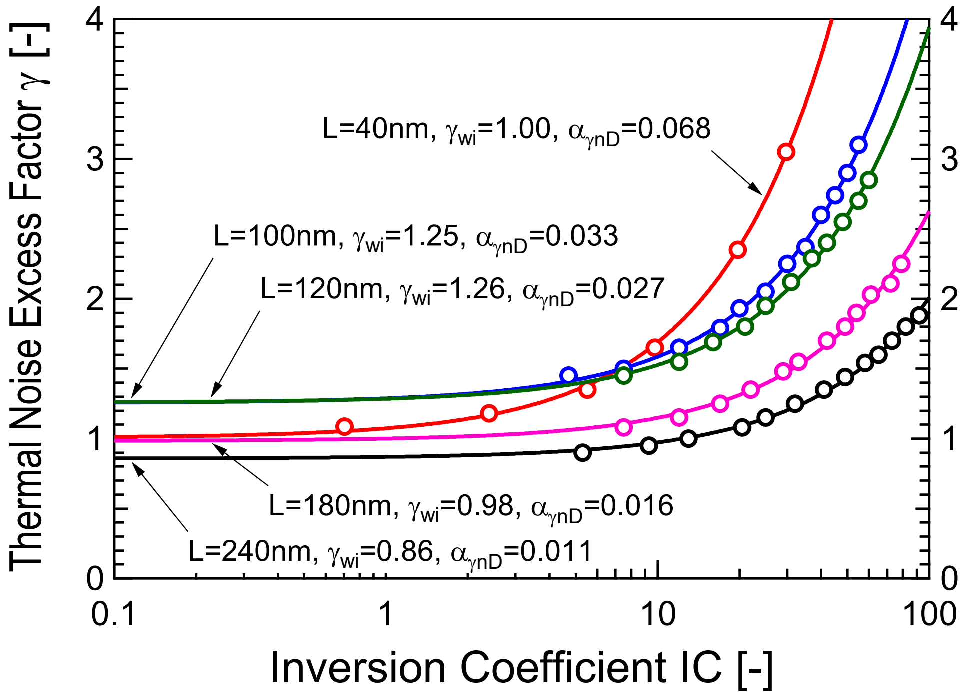


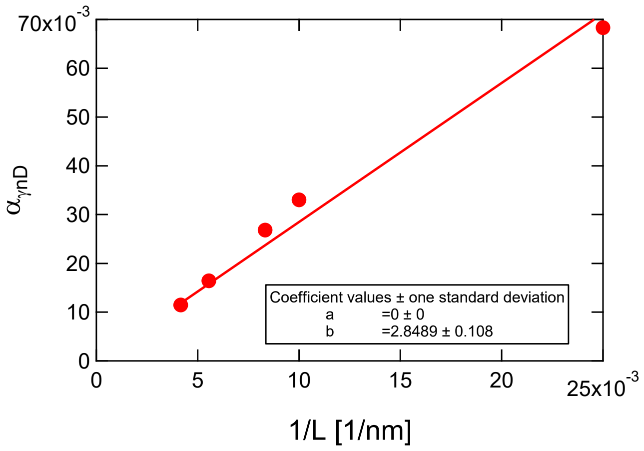




- Thermal noise is affected by following effects:
  - **Velocity saturation** (VS)
  - **Carrier heating** (CH)
  - **Mobility reduction due to the vertical field** (MRV)
  - **Channel length modulation** (CLM)
- Due to the opposite effects of **VS** and **CH** as well as opposite effects of **MRV** and **CLM**, the degradation in $\gamma_{nD}$ is not as dramatic as expected initially

A. S. Roy and C. C. Enz, TED, April 2005.

# Short-channel Effects on $\gamma_{nD}$ (in saturation)




- The noise excess factor $\gamma_{nD}$ can be modelled versus $IC$ as

$$\gamma_{nD} \cong \gamma_{wi} + \alpha_{\gamma_{nD}} \cdot IC$$

- Where $\gamma_{wi}$ and $\alpha_{\gamma_{nD}}$ are empirical factors
- $\alpha_{\gamma_{nD}}$ scales approximatively as $\alpha_{\gamma_{nD}} \cong 2.85/L$ where $L$ is in nm

A. Antonopoulos et al., "CMOS Small-Signal and Thermal Noise Modeling at High Frequencies," TED, vol. 60, No. 11, Nov. 2013.
M. Chalkiadaki, PhD Thesis 2016.

# Outline

- The long-channel static model
- The long-channel small-signal model
- The long-channel noise model
- The extended model
- **The simplified EKV model**

# Simplified EKV Charge-based Model (in saturation)

- The **normalized drain current** in saturation or **inversion coefficient** is given by

$$IC = \frac{I_D|_{saturation}}{I_{spec}} = \frac{4(q_s^2 + q_s)}{2 + \lambda_c + \sqrt{\lambda_c^2(2q_s + 1)^2 + 4(1 + \lambda_c)}}$$

- $q_s \triangleq Q_i(x = 0)/Q_{spec}$ is the **normalized inversion charge** at the source where $Q_{spec} = -2nC_{ox}U_T$
- $\lambda_c$ is the **velocity saturation** (VS) parameter corresponding to the fraction of the channel under full VS

$$\lambda_c = \frac{L_{sat}}{L} \text{ with } L_{sat} = \frac{2\mu_0 U_T}{v_{sat}} = \frac{2U_T}{E_c}$$

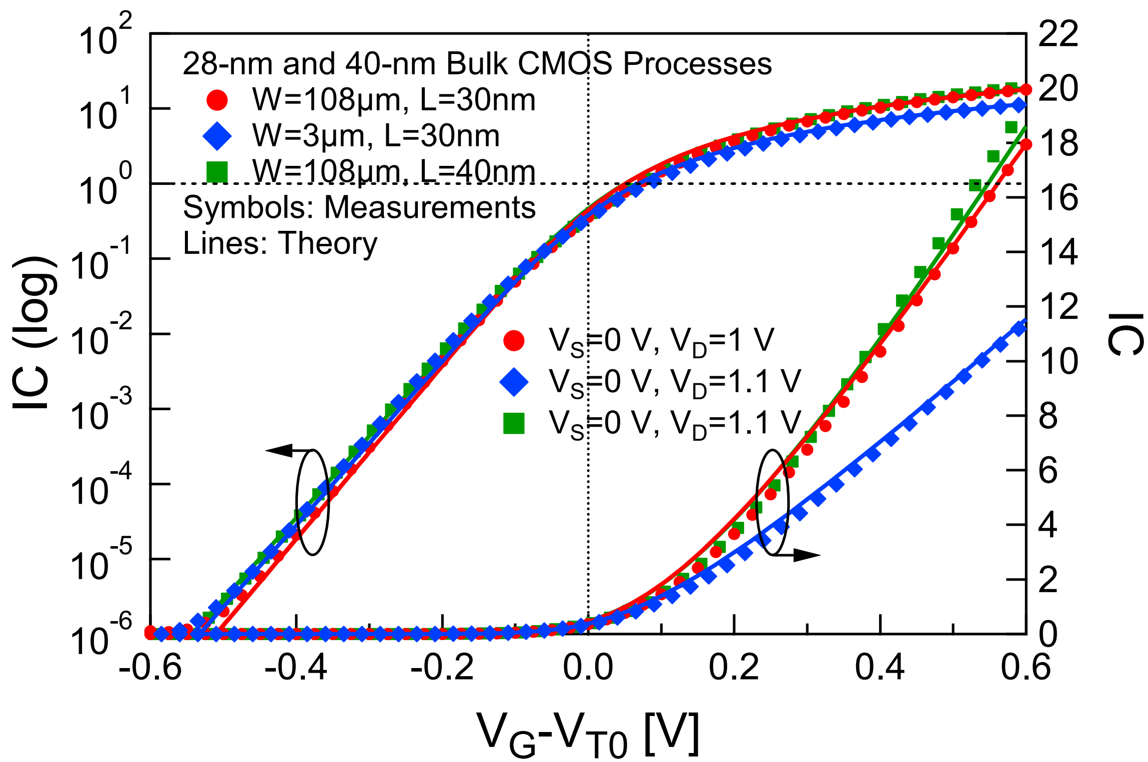
- $q_s$ is related to the gate and source voltage according to

$$v_p - v_s = \ln(q_s) + 2q_s \text{ with } v_p = \frac{V_P}{U_T} = \frac{V_G - V_{T0}}{nU_T},\ v_s = \frac{V_S}{U_T},\ U_T = \frac{kT}{q}$$

- Only **requires the following 4 parameters**: $n, I_{spec\square}, V_{T0}, L_{sat}$

C. Enz, F. Chicco, and A. Pezzotta, IEEE Solid-State Circuits Magazine, vol. 9, no. 3, pp. 26-35, Summer 2017.

# Drain Current for 28 and 40-nm Bulk CMOS Processes

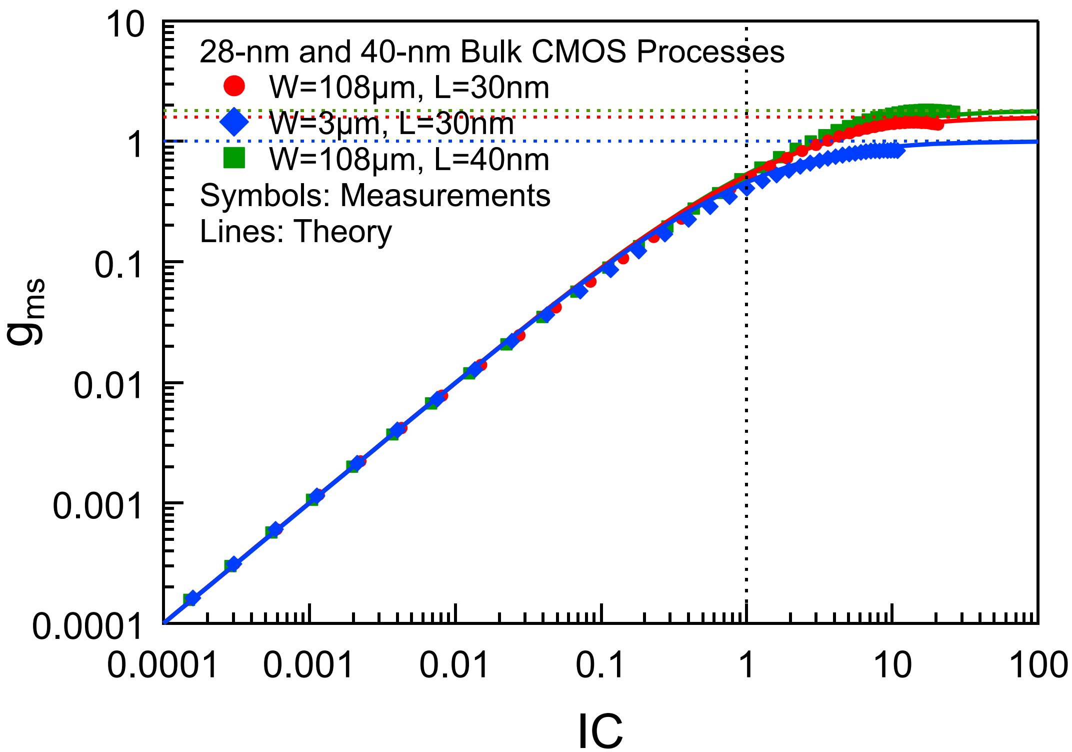



- Simple model validated on **28-nm and 40-nm bulk CMOS processes** over **more than 6 decades of current** despite only requiring **few parameters, namely**:

$$n, I_{spec\square}, V_{T0}, L_{sat}$$

📖 C. Enz, F. Chicco, and A. Pezzotta, IEEE Solid-State Circuits Magazine, vol. 9, no. 3, pp. 26-35, Summer 2017.

# $G_m$ vs. $IC$ for 28 and 40-nm Bulk CMOS Process




$$g_{ms}(IC) \triangleq \frac{G_{ms}}{G_{spec}} = \frac{nG_m}{G_{spec}}$$

$$= \frac{\sqrt{(\lambda_c IC + 1)^2 + 4IC} - 1}{\lambda_c(\lambda_c IC + 1) + 2}$$

$\cong \frac{1}{\lambda_c}$ for $IC \gg 1$

with $\lambda_c = \frac{L_{sat}}{L}$

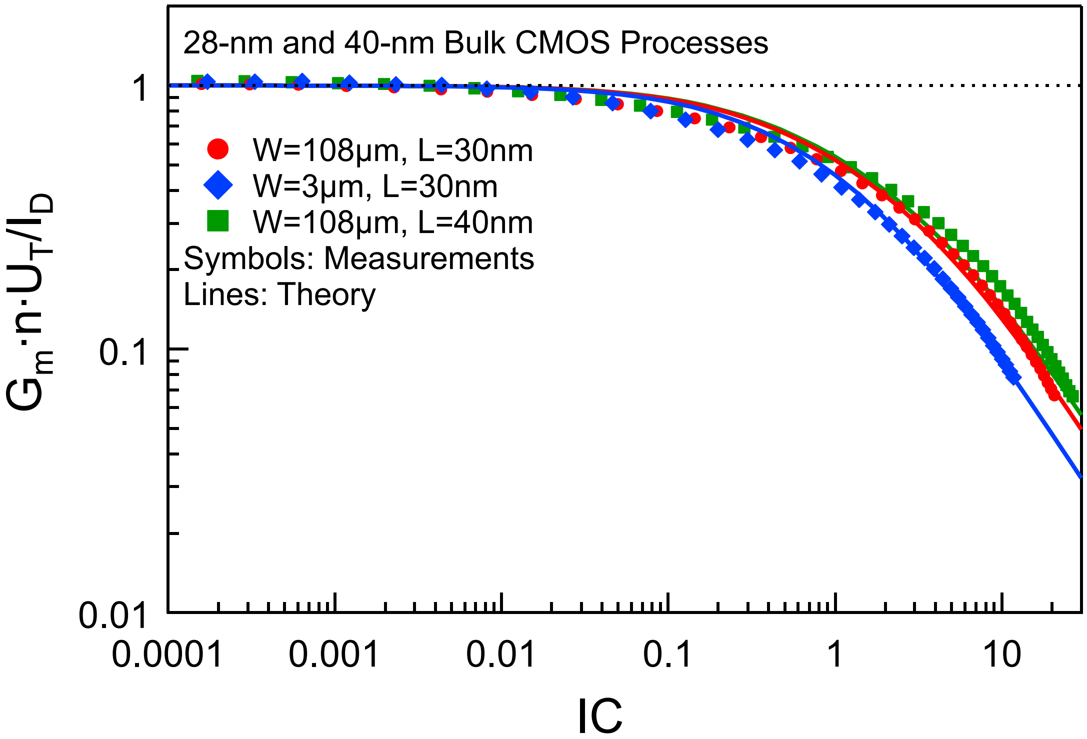
and $L_{sat} = \frac{2\mu_0 U_T}{v_{sat}} = \frac{2U_T}{E_c}$

- Simple model of **transconductance** validated on 28-nm and 40-nm bulk CMOS processes over more 5 decades of current despite only requiring few parameters, namely:

$$n, I_{spec\square}, V_{T0}, L_{sat}$$

C. Enz, F. Chicco, and A. Pezzotta, IEEE Solid-State Circuits Magazine, vol. 9, no. 3, pp. 26-35, Summer 2017.

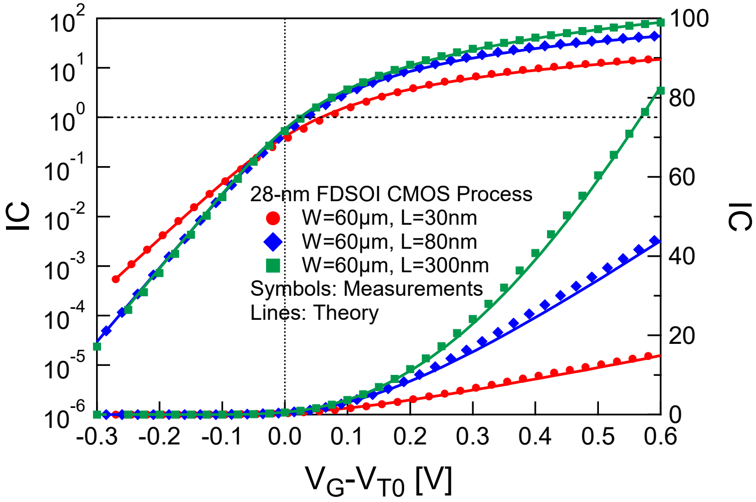
# $G_m/I_D$ vs. $IC$ for 28 and 40-nm Bulk CMOS

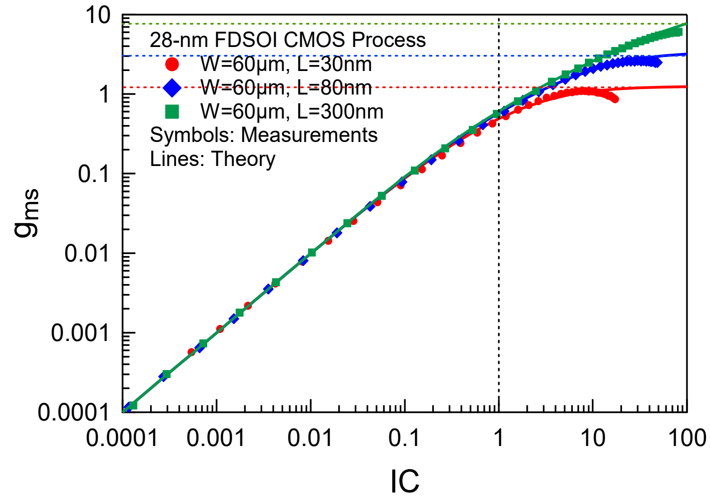


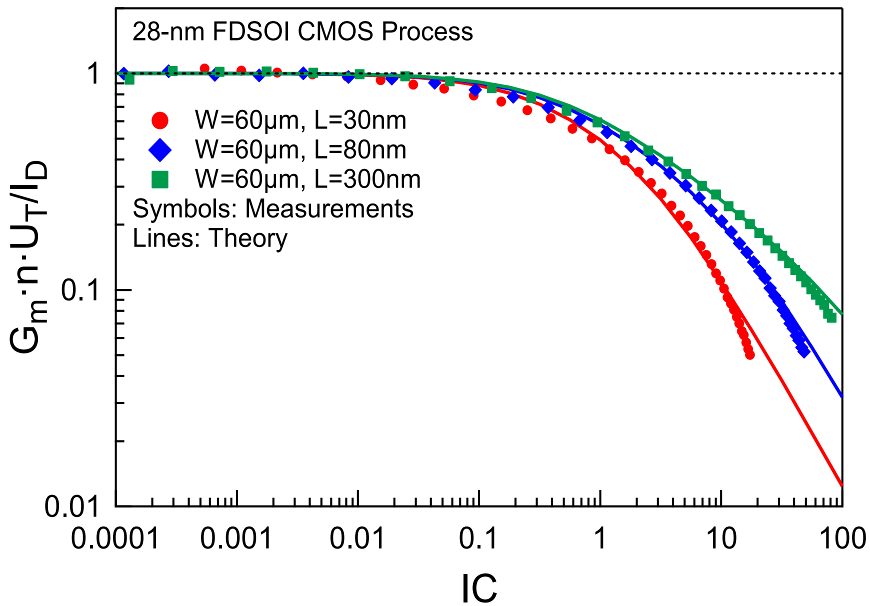

$$\frac{g_{ms}}{i_d} = \frac{G_{ms}U_T}{I_D} = \frac{G_{ms}nU_T}{I_D} = \frac{\sqrt{(\lambda_c IC + 1)^2 + 4IC} - 1}{IC(\lambda_c(\lambda_c IC + 1) + 2)}$$

C. Enz, F. Chicco, and A. Pezzotta, IEEE Solid-State Circuits Magazine, vol. 9, no. 3, pp. 26-35, Summer 2017.

# $IC$, $G_m$ and $G_m/I_D$ for 28-nm FDSOI Process

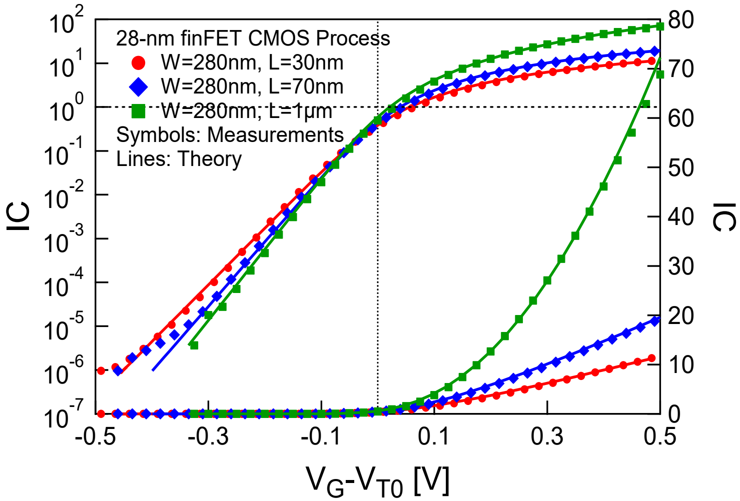


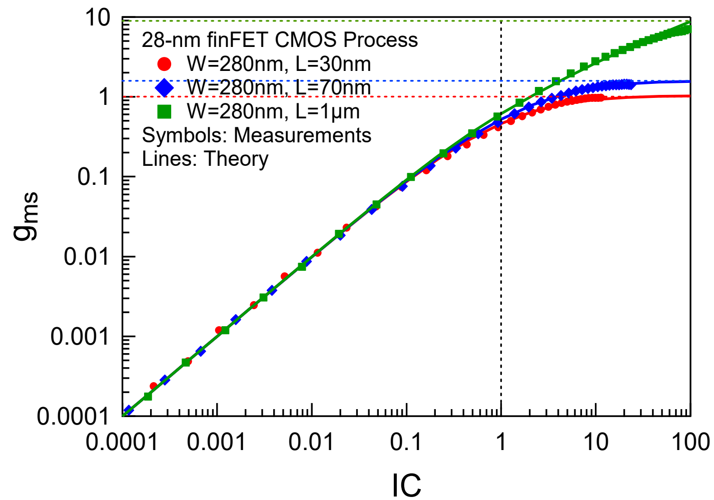



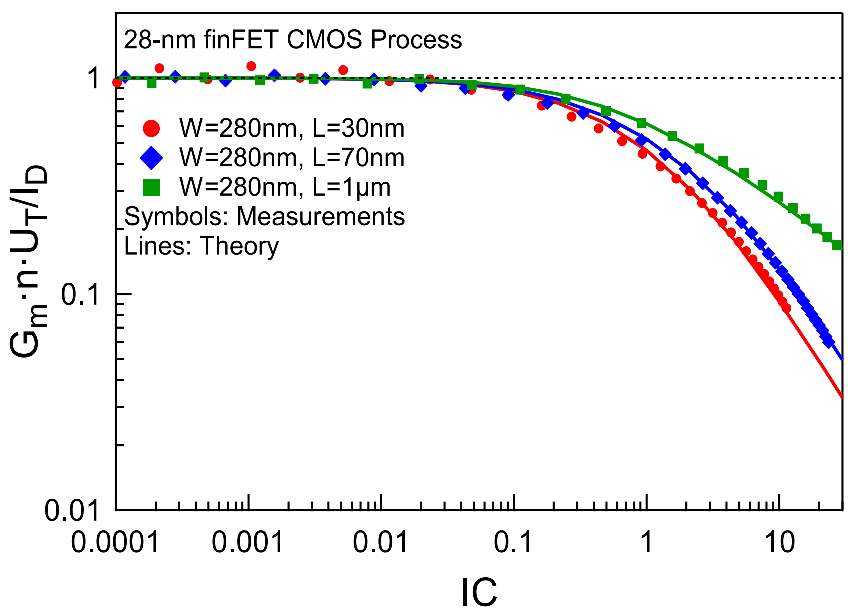




C. Enz, F. Chicco, and A. Pezzotta, IEEE Solid-State Circuits Magazine, vol. 9, no. 3, pp. 26-35, Summer 2017.

# $IC$, $G_m$ and $G_m/I_D$ for 28-nm FinFET Process



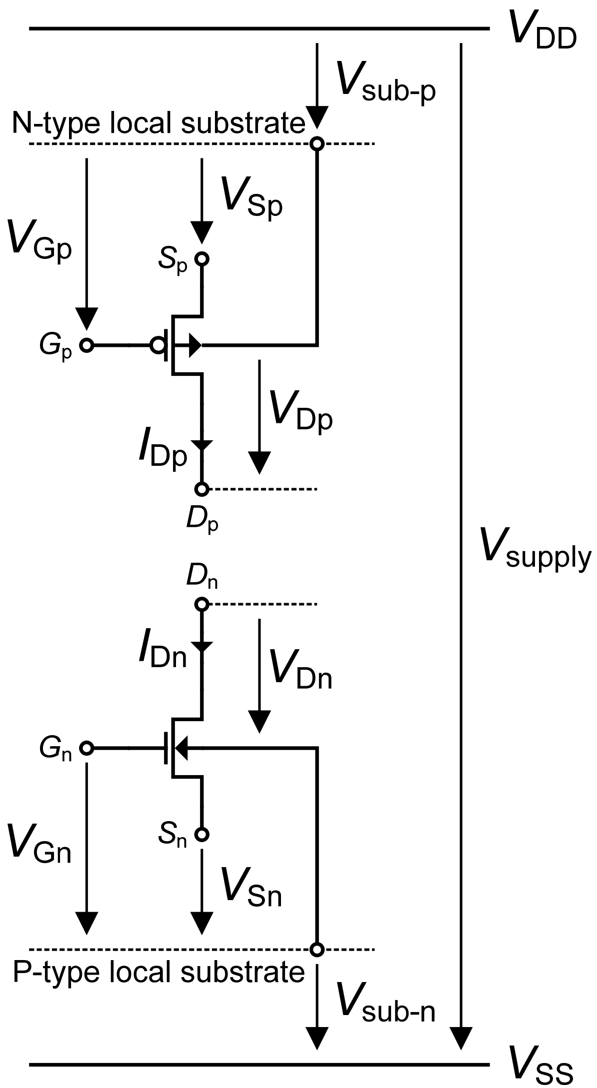




C. Enz, F. Chicco, and A. Pezzotta, IEEE Solid-State Circuits Magazine, vol. 9, no. 3, pp. 26-35, Summer 2017.

# Summary

- The drain current in saturation of the BSIM6-EKV compact model can be simplified in the region of interest for designers to a model requiring only **4 parameters** $n$, $I_{spec\square}$, $V_{T0}$, $L_{sat}$
- The transconductance $G_m$ and transconductance efficiency or **current efficiency** $G_m/I_D$ can easily be expressed in terms of $IC$ using very simple expressions accounting for velocity saturation
- The simplified model holds for advanced bulk CMOS processes and has been successfully validated down to 28-nm
- It can also be used for FD-SOI and FinFET processes!

# Symbol, Voltages and Currents Definition




- Definitions of currents and voltages result in identical equations for N- and P-channel transistors
- Local substrates are defined as follows:
  - For n-well processes, the substrate is P-type and hence $V_{sub\text{-}n}=0$
  - For p-well processes, the substrate is P-type and hence $V_{sub\text{-}p}=0$
  - Some processes offer twin-wells. Then $V_{sub\text{-}n}$ and $V_{sub\text{-}p}$ can be both non-zero.
- When the substrate connection is not shown it means that the substrate is connected to $V_{SS}$ ($V_{sub\text{-}n}=0$) for N-channel transistors and to $V_{DD}$ ($V_{sub\text{-}p}=0$) for P-channel transistors

# Normalization Factors

| Quantity | Normalization factor | Unit |
|---|---|---|
| Voltage | $U_T \triangleq kT/q$ | V |
| Current | $I_{spec} \triangleq 2n \cdot \beta \cdot U_T^2$ | A |
| Charge density | $Q_{spec} \triangleq 2n \cdot C_{ox} \cdot U_T$ | A·s / m² |
| Capacitance | $C_{OX} \triangleq W \cdot L \cdot C_{ox}$ | F |
| Position | $L$ | m |
| Frequency | $\omega_{spec} \triangleq \mu \cdot U_T/L^2$ | Hz |
| Admittance | $G_{spec} \triangleq I_{spec}/U_T = 2n \cdot \beta \cdot U_T$ | A / V |

# Main Process Parameters

- Numerical values corresponding to a typical 0.18 µm CMOS process

| Parameter | | NMOS | | | PMOS | | Units |
|---|---|---|---|---|---|---|---|
| Definition | Symbol | 1.8V | 3.3V | Native | 1.8V | 3.3V | |
| Oxyde thickness | $t_{ox}$ | 4.08E-09 | 6.8E-09 | 4.08E-09 | 4.08E-09 | 6.77E-09 | m |
| Oxyde capacitances per unit area | $C_{ox}$ | 8.46E-03 | 5.08E-03 | 8.46E-03 | 8.46E-03 | 5.10E-03 | F/m² |
| Threshold voltage | $V_{T0}$ | 0.455 | 0.62 | -0.0177 | 0.4572 | 0.66 | V |
| Body effect factor | $\Gamma_b$ | 0.54 | 0.54 | 0.54 | 0.609 | 0.609 | √V |
| Approximation of surface potential in SI | $\Psi_0$ | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | V |
| Transconductance parameter | $\beta_{\square}$ | 4.20E-04 | 1.85E-04 | 4.60E-04 | 9.90E-05 | 6.18E-05 | A/V² |
| Slope factor (for $V_P$=0) | $n_0$ | 1.27136 | 1.27136 | 1.27136 | 1.306034 | 1.306034 | - |
| Specific current for W/L=1 | $I_{spec\square}$ | 0.7139 | 0.314463 | 0.782577 | 0.172866 | 0.107827 | µA |
| Output conductance parameter | $\lambda$ | 10 | 10 | 10 | 10 | 10 | V/µm |
| Threshold voltage mismatch parameter | $A_{VT}$ | 5 | 8.97 | 5 | 5.49 | 6.39 | mV·µm |
| Beta factor mismatch parameter | $A_{\beta}$ | 1 | 0.72 | 0.18 | 1.13 | 0.64 | %·µm |
| Flicker noise frequency exponent | AF | 0.8265 | 1 | 1 | 1.3358 | 1 | - |
| Flicker noise parameter | KF | 8.10E-24 | 3.56E-24 | 2.66E-24 | 6.75E-23 | 2.93E-23 | J |
| Flicker noise parameter | $\rho$ | 0.057766 | 0.042314 | 0.01897 | 0.481385 | 0.346725 | V·m²/(A·s) |

# Large-signal Model

Pinch-off voltage: $V_P = V_G - V_{T0} - \Gamma_b \cdot \left[ \sqrt{V_G - V_{T0} + \left( \sqrt{\Psi_0} + \frac{\Gamma_b}{2} \right)^2} - \left( \sqrt{\Psi_0} + \frac{\Gamma_b}{2} \right) \right] \cong \frac{V_G - V_{T0}}{n}$ Slope factor: $n = 1 + \frac{\Gamma_b}{2\sqrt{\Psi_0 + V_P}}$

Transconductance parameter: $\beta = \mu \cdot C_{ox} \cdot \frac{W}{L}$ Specific current: $I_{spec} = 2n \cdot \beta \cdot U_T^2$ Drain current: $I_D = I_F - I_R = I_{spec} \cdot (i_f - i_r)$

Normalized forward and reverse current: $i_{f(r)} = \left( \frac{V_P - V_{S(D)}}{2U_T} \right)^2 \cong \left( \frac{V_G - V_{T0} - nV_{S(D)}}{2nU_T} \right)$ in SI $i_{f(r)} = \exp\left[ \frac{V_P - V_{S(D)}}{U_T} \right]$ in WI

Current in weak inversion: $I_D = I_{spec} \cdot e^{\frac{V_P}{U_T}} \cdot \left[ e^{-\frac{V_S}{U_T}} - e^{-\frac{V_D}{U_T}} \right] = I_{spec} \cdot e^{\frac{V_G - V_{T0}}{nU_T}} \cdot \left[ e^{-\frac{V_S}{U_T}} - e^{-\frac{V_D}{U_T}} \right] = I_{D0} \cdot e^{\frac{V_G}{nU_T}} \cdot \left[ e^{-\frac{V_S}{U_T}} - e^{-\frac{V_D}{U_T}} \right]$

Current in strong inversion: $I_{F(R)} = \begin{cases} \frac{n \cdot \beta}{2} \cdot (V_P - V_{S(D)})^2 & for: V_{S(D)} \leq V_P \\ 0 & for: V_{S(D)} > V_P \end{cases}$

Saturation ($V_D > V_P$): $I_D = I_F = \frac{n \cdot \beta}{2} \cdot (V_P - V_S)^2 = \frac{\beta}{2n} \cdot (V_G - V_{T0} - n \cdot V_S)^2$

Linear ($V_D \leq V_P$): $I_D = I_F - I_R = n \cdot \beta \cdot \left( V_P - \frac{V_D + V_S}{2} \right) \cdot (V_D - V_S) = \beta \cdot \left( V_G - V_{T0} - \frac{n}{2} \cdot (V_D + V_S) \right) \cdot (V_D - V_S)$

Continuous expressions valid from weak to strong inversion

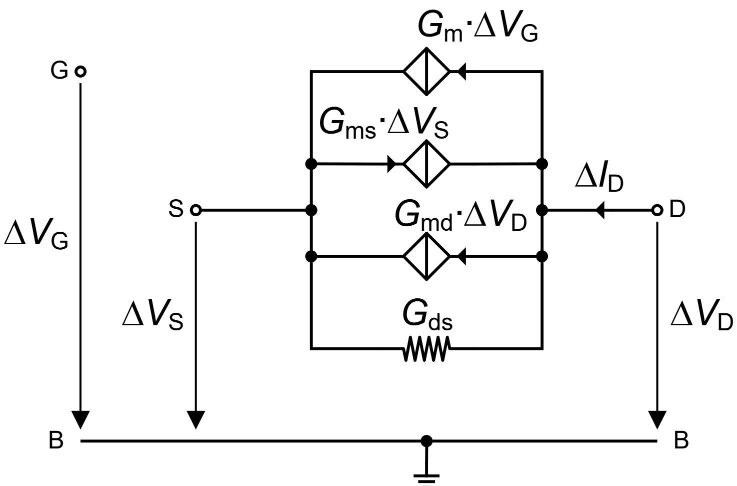
Normalized currents: $i_f = F\left( \frac{V_P - V_S}{U_T} \right)$ $i_r = F\left( \frac{V_P - V_D}{U_T} \right)$

Inverse relations: $\frac{V_P - V_S}{U_T} = F^{-1}(i_f) = 2g(i_f) + \ln[g(i_f)]$ $\frac{V_P - V_D}{U_T} = F^{-1}(i_r) = 2g(i_r) + \ln[g(i_r)]$ $g(i) = \frac{\sqrt{4i+1} - 1}{2} = \frac{2i}{\sqrt{4i+1} + 1}$

Approximate invertible function: $i_f = F\left( \frac{V_P - V_S}{U_T} \right) = \ln^2\left[ 1 + \exp\left( \frac{V_P - V_S}{2U_T} \right) \right]$ $i_r = F\left( \frac{V_P - V_D}{U_T} \right) = \ln^2\left[ 1 + \exp\left( \frac{V_P - V_D}{2U_T} \right) \right]$

Approximate inverse function: $\frac{V_P - V_S}{U_T} = F^{-1}(i_f) = 2\ln\left[ \exp\left( \sqrt{i_f} \right) - 1 \right]$ $\frac{V_P - V_D}{U_T} = F^{-1}(i_r) = 2\ln\left[ \exp\left( \sqrt{i_r} \right) - 1 \right]$

# DC Small-signal Model




In saturation $G_{md}$ can be neglected

General relation between transconductances:

$$G_m = \frac{G_{ms} - G_{md}}{n} \quad \text{in saturation:} \quad G_{md} = 0 \quad \text{and} \quad G_m = \frac{G_{ms}}{n}$$

Transconductances in weak inversion:

$$G_{ms} = \frac{I_D}{U_T} \qquad G_m = \frac{I_D}{n \cdot U_T}$$

Transconductances in strong inversion:

$$G_{ms} = n \cdot \beta \cdot (V_P - V_S) = \sqrt{2n \cdot \beta \cdot I_D} = \frac{2I_D}{V_P - V_S} \cong \frac{2nI_D}{V_G - V_{T0} - nV_S}$$

$$G_m = \beta \cdot (V_P - V_S) = \sqrt{\frac{2\beta \cdot I_D}{n}} = \frac{2I_D}{n \cdot (V_P - V_S)} \cong \frac{2I_D}{V_G - V_{T0} - nV_S}$$

Output conductance (very approximative!):

$$G_{ds} \cong \frac{I_D}{V_M} \quad \text{where} \quad V_M \cong \lambda \cdot L \quad \text{with } \lambda \text{ in } \frac{V}{\mu m}$$

Continuous expressions valid from weak to strong inversion

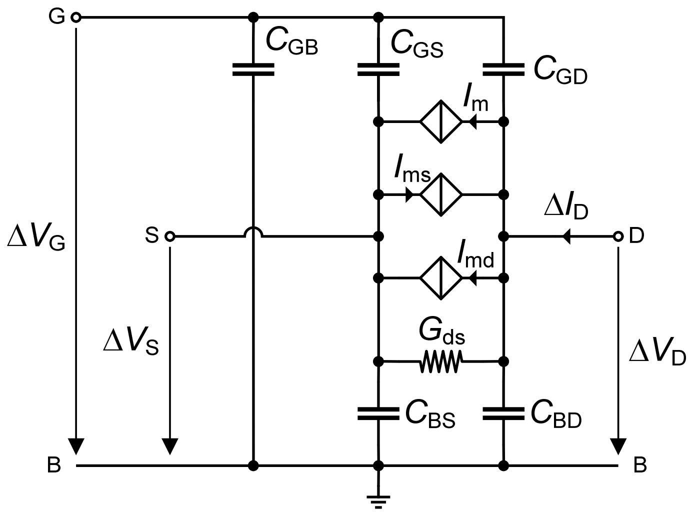
Transconductance-to-current ratio (in saturation $I_D = I_F$):

$$\frac{G_{ms} \cdot U_T}{I_F} = \frac{G_m \cdot nU_T}{I_F} = \frac{2}{\sqrt{4i_f + 1} + 1} = \begin{cases} 1 & WI \\ 1/\sqrt{i_f} & SI \end{cases} \quad \text{or inverse function:} \quad i_f = \frac{I_F}{G_{ms} \cdot U_T} \cdot \left( \frac{I_F}{G_{ms} \cdot U_T} - 1 \right)$$

Transconductances:

$$g_{ms} = \frac{G_{ms}}{G_{spec}} = g(i_f) = \frac{\sqrt{4i_f + 1} - 1}{2} = \frac{2i_f}{\sqrt{4i_f + 1} + 1} \quad g_{md} = \frac{G_{md}}{G_{spec}} = g(i_r) = \frac{\sqrt{4i_r + 1} - 1}{2} = \frac{2i_r}{\sqrt{4i_r + 1} + 1} \quad G_{spec} = \frac{I_{spec}}{U_T} = 2n\beta U_T$$

# Quasi-static Small-signal Equivalent Circuit




$$I_m = Y_m \cdot \Delta V_G$$
$$I_{ms} = Y_{ms} \cdot \Delta V_S$$
$$I_{md} = Y_{md} \cdot \Delta V_D \quad (Y_{md} = 0 \quad \text{in saturation})$$

$$Y_m = G_m \cdot (1 - j\omega \cdot \tau_{qs}) = G_m - j\omega \cdot C_m$$
$$Y_{ms} = G_{ms} \cdot (1 - j\omega \cdot \tau_{qs}) = G_{ms} - j\omega \cdot C_{ms}$$
$$Y_{md} = G_{md} \cdot (1 - j\omega \cdot \tau_{qs}) = G_{md} - j\omega \cdot C_{md}$$

$$Y_m = \frac{Y_{ms} - Y_{md}}{n}$$
$$G_m = \frac{G_{ms} - G_{md}}{n}$$
$$C_m = \frac{C_{ms} - C_{md}}{n}$$

Total capacitances:

$$C_{GS} = C_{GSi} + C_{GSo}$$
$$C_{GD} = C_{GDi} + C_{GDo}$$
$$C_{GB} = C_{GBi} + C_{GBo}$$
$$C_{BS} = C_{BSi} + C_{BSj}$$
$$C_{BD} = C_{BDi} + C_{BDj}$$

Overlap capacitances:

$$C_{GSo} = W \cdot L_{ov} \cdot C_{ox}$$
$$C_{GDo} = W \cdot L_{ov} \cdot C_{ox}$$
$$C_{GBo} = CGBO \cdot L$$

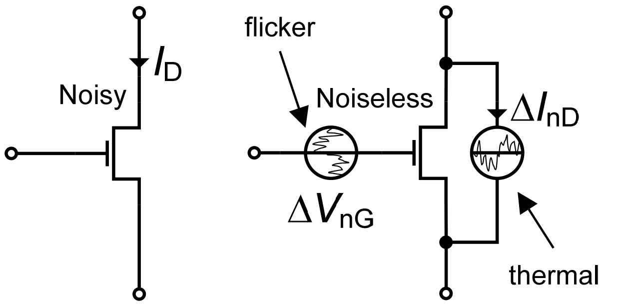
Junction capacitances:

$$C_{BSj} = A_S \cdot C_{jbw} + (P_S - W) \cdot C_{jsw} + W \cdot C_{jswg}$$
$$C_{BDj} = A_D \cdot C_{jbw} + (P_D - W) \cdot C_{jsw} + W \cdot C_{jswg}$$
$$C_j = \frac{C_{j0}}{\sqrt{1 + V_{S(D)B} / \Phi_B}}$$

Intrinsic capacitances normalized to $C_{OX} = W\,L\,C_{ox}$

| | WI (IC<<1) | SI | |
|---|---|---|---|
| | | $V_D = V_S$ | $V_D > V_P$ |
| $C_{GSi}$ | <<1 | 1/2 | 2/3 |
| $C_{GDi}$ | <<1 | 1/2 | <<1 |
| $C_{GBi}$ | 1−1/n | 0 | (n−1)/(3n) |
| $C_{BSi}$ | $(n-1)C_{GSi}$ | | |
| $C_{BDi}$ | $(n-1)C_{GDi}$ | | |
| $C_m$ | <<1 | 0 | 4/15 |
| $C_{ms}$ | <<1 | n/6 | 4n/15 |
| $C_{md}$ | <<1 | n/6 | <<1 |

# Noise Model

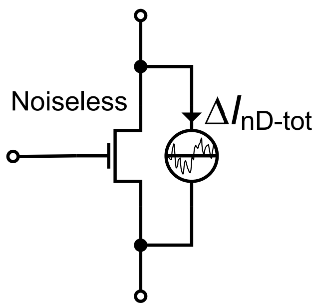



PSD of the thermal noise at the drain: $S_{\Delta I_{nD}^2} = 4kT \cdot G_{nD}$ $\quad G_{nD} = \delta_{nD} \cdot G_{ms} = \gamma_{nD} \cdot G_m$

Noise parameter and noise excess factor: $\delta_{nD} = \begin{cases} \frac{1}{2} & \text{WI} \\ \frac{2}{3} & \text{SI} \end{cases}$ $\quad \gamma_{nD} = n \cdot \delta_{nD}$

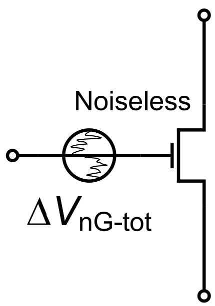
PSD of the flicker noise at the gate: $S_{\Delta V_{nG}^2} = \dfrac{KF}{C_{ox}^2 \cdot W \cdot L \cdot f} = 4kT \cdot R_{nG}(f)$

$$R_{nG}(f) = \frac{\rho}{W \cdot L \cdot f} \qquad \rho = \frac{KF}{4kT \cdot C_{ox}^2}$$




PSD of the total noise referred at the drain:

$$S_{\Delta I_{nD-tot}^2} = 4kT \cdot G_{nD-tot} \qquad G_{nD-tot} = \gamma_{nD} G_m + G_m^2 \frac{\rho}{W \cdot L \cdot f}$$



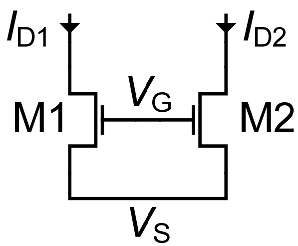

PSD of the total noise referred at the gate:

$$S_{\Delta V_{nG-tot}^2} = 4kT \cdot R_{nG-tot} \qquad R_{nG-tot} = \frac{\gamma_{nD}}{G_m} + \frac{\rho}{W \cdot L \cdot f}$$

# Transistor Mismatch

## Drain current relative mismatch (same gate and source voltages)

$$\Delta I_D = I_{D2} - I_{D1}$$




Standard deviation of relative current difference:

$$\sigma_{\frac{\Delta I_D}{I_D}} = \sqrt{\sigma^2_{\frac{\Delta\beta}{\beta}} + \left(\frac{G_m}{I_D}\right)^2 \sigma^2_{\Delta V_T}} = \sqrt{\sigma^2_{\frac{\Delta\beta}{\beta}} + \left(\frac{2}{\sqrt{4IC+1}+1}\frac{\sigma_{\Delta V_T}}{nU_T}\right)^2} \quad \text{with} \quad \sigma^2_{\frac{\Delta\beta}{\beta}} = \frac{A^2_\beta}{W \cdot L} \quad \text{and} \quad \sigma^2_{\Delta V_T} = \frac{A^2_{VT}}{W \cdot L}$$

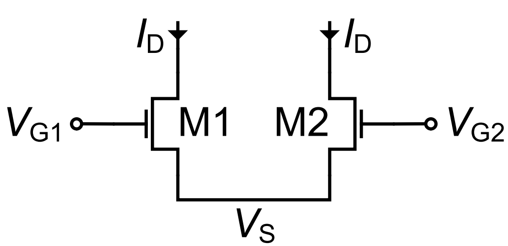
$A_\beta$ in %·µm (typically 0.2% to 20%) and $A_{VT}$ in mV·µm (typically 1 to 10 mV)

Minimum in strong inversion:

$$\sigma_{\frac{\Delta I_D}{I_D}} \cong \sqrt{\sigma^2_{\frac{\Delta\beta}{\beta}} + \left(\frac{2\sigma_{\Delta V_T}}{V_G - V_{T0} - nV_S}\right)^2} \cong \sigma_{\frac{\Delta\beta}{\beta}}$$

## Gate voltage absolute mismatch (same drain current and source voltage)

$$\Delta V_G = V_{G2} - V_{G1}$$




Standard deviation of gate voltage difference:

$$\sigma_{\Delta V_G} = \sqrt{\sigma^2_{\Delta V_T} + \left(\frac{I_D}{G_m}\right)^2 \sigma^2_{\frac{\Delta\beta}{\beta}}} = \sqrt{\sigma^2_{\Delta V_T} + \left(\frac{nU_T\left(\sqrt{4IC+1}+1\right)}{2}\sigma_{\frac{\Delta\beta}{\beta}}\right)^2}$$

Minimum in weak inversion:

$$\sigma_{\Delta V_G} \cong nU_T\sqrt{\left(\frac{\sigma_{\Delta V_T}}{nU_T}\right)^2 + \sigma^2_{\frac{\Delta\beta}{\beta}}} \cong \sigma_{\Delta V_T}$$