MICRO-461
Low-power Radio Design for the IoT

6. Noise Modeling at RF

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Outline

- Noise in Two-port Networks
- Noise in the MOS Transistor at RF
The output noise current $I_{nout}$ depends on the two-port internal noise sources and on the source admittance $Y_S$.

A model of the noisy two-port that is independent of the source admittance $Y_S$ requires at least two noise sources (either two current sources or two voltage sources or a combination).

Using the Y-parameter representation, the noisy two-port can then be modeled by

$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2 + I_{n1}$$
$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2 + I_{n2}$$

$$I_{n1} \triangleq I_1 \bigg|_{V_1=V_2=0}$$
$$I_{n2} \triangleq I_2 \bigg|_{V_1=V_2=0}$$

where $I_{n1}$ and $I_{n2}$ represent all the noise sources within the two-port and are defined as the input and output currents when the input and output are short-circuited.

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Noisy Two-port – ABCD Matrix Representation

- The noisy two-port can also be modelled by referring all the noise sources at the input using the ABCD-parameters as shown below.

\[
\begin{align*}
V_1 &= A \cdot V_2 - B \cdot I_2 + V_n \\
I_1 &= C \cdot V_2 - D \cdot I_2 + I_n
\end{align*}
\]

where \(V_n\) is a noise voltage source that represents all the noise of the device referred to the input when the source impedance is zero (input short-circuited) and \(I_n\) is a noise current source that represents all noise of the device referred to the input when the source admittance is zero (input open circuited).

\[V_n \triangleq B \cdot I_2 | V_1 = V_2 = 0 = - \frac{1}{Y_{21}} \cdot I_2 | V_1 = V_2 = 0\]

\[I_n \triangleq D \cdot I_2 | I_1 = V_2 = 0 = - \frac{Y_{11}}{Y_{21}} \cdot I_2 | I_1 = V_2 = 0\]

Noise in Two-port Networks

Relation Between Noise Sources

- Noise sources of both representations \( I_{n1}, I_{n2} \) and \( V_n, I_n \) are related to each other by

\[
V_n = -\frac{I_{n2}}{Y_{21}} \quad I_{n1} = I_n - Y_{11} \cdot V_n
\]

\[
I_n = I_{n1} - \frac{Y_{11}}{Y_{21}} \cdot I_{n2} \quad I_{n2} = -Y_{21} \cdot V_n
\]

- Since both of these sources (\( V_n \) and \( I_n \) or \( I_{n1} \) and \( I_{n2} \)) are due to the same physical noise sources within the device, they are usually correlated. The mean-square values of sources \( V_n \) and \( I_n \) can be written in terms of the mean square values of sources \( I_{n1} \) and \( I_{n2} \) according to

\[
|V_n|^2 = \frac{|I_{n2}|^2}{|Y_{21}|^2} \quad \text{and} \quad |I_n|^2 = |I_{n1}|^2 + \left|\frac{Y_{11}}{Y_{21}} \right|^2 \cdot |I_{n2}|^2 - 2 \text{Re} \left\{ \frac{Y_{11}^*}{Y_{21}} \cdot I_{n1} \cdot I_{n2}^* \right\}
\]

where the last term accounts for the correlation existing between source \( I_{n1} \) and \( I_{n2} \). The latter has to be evaluated from the internal noise sources.

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Correlation Admittance

- To account for the correlation usually existing between noise sources $V_n$ and $I_n$, noise source $I_n$ can be written as

$$I_n = I_{nu} + I_{nc} = I_{nu} + Y_c \cdot V_n$$

where $I_{nu}$ stands for the part of $I_n$ uncorrelated to $V_n$ and $I_{nc}$ represents the part of $I_n$ that is fully correlated to $V_n$.

- The correlation admittance $Y_c$ is then defined as

$$Y_c \triangleq \frac{I_n \cdot V_n^*}{V_n^2}$$

and the mean-square value of source $I_n$ is then given by

$$|I_n|^2 = |I_{nu}|^2 + |Y_c|^2 \cdot |V_n|^2 + Y_c^* \cdot \overline{I_{nu} \cdot V_n^*} + Y_c \cdot \overline{V_n \cdot I_{nu}^*} = |I_{nu}|^2 + |Y_c|^2 \cdot |V_n|^2$$

$$= |I_{nc}|^2$$

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Correlation Coefficient

- The mean square values of the correlated and uncorrelated sources can also be written as

\[
|I_{nc}|^2 = |c| \cdot |I_n|^2
\]

\[
|I_{nu}|^2 = |I_n|^2 - |c|^2 \cdot |V_n|^2 = \left(1 - |c|^2\right) \cdot |I_n|^2
\]

where \(c\) is the normalized correlation factor defined as

\[
c \triangleq \frac{I_n \cdot V_n^*}{\sqrt{|I_n|^2 \cdot |V_n|^2}} = Y_c \cdot \frac{|V_n|^2}{|I_n|^2}
\]

where the following relation has been used to derive the last equation

\[
I_n \cdot V_n^* = I_{nu} \cdot V_n^* + Y_c \cdot |V_n|^2 = Y_c \cdot |V_n|^2
\]

= 0
The noisy two-port can be modelled by a noiseless two-port in cascade with a noise two-port that includes noise sources $V_n$ and $I_n$ and hence models all the noise of the noisy two-port referred to the input.

The noise two-port can be modelled accounting for the correlation between the current and voltage noise source in the following equivalent ways:

1. $V_1 \rightarrow I_1 \rightarrow V_n \rightarrow I'_1$
2. $I_1 \rightarrow V_1 \rightarrow V_n \rightarrow I'_1$
3. $I_1 \rightarrow I_n \rightarrow Y_c \cdot V_n \rightarrow I'_1$
4. $V_1 \rightarrow Y_c \rightarrow I_{nu} \rightarrow -Y_c \cdot V'_1$

References:
Power Spectral Densities

- Noise sources $V_n$ and $I_n$ are described by their noise power (or rms values) considering a common noise bandwidth.

- For narrow-band systems, sources $V_n$ and $I_n$ can also be described by their power spectral densities $S_v$ and $S_i$.

- The noise current PSD can be split into a fully correlated term $S_{ic}$ and an uncorrelated term $S_{uc}$ according to

$$S_{ic} = |Y_c|^2 \cdot S_v = |c|^2 \cdot S_i \quad S_{iu} = S_i - |Y_c|^2 \cdot S_v = \left(1 - |c|^2\right) \cdot S_i$$

- where the correlation factor $c$ is given by

$$c = Y_c \cdot \sqrt{\frac{S_v}{S_i}}$$
Equivalent Noise Resistance and Conductance

- Considering that the noise sources $V_n$, $I_{nu}$ and $I_{nc}$ are treated as thermal noise (even though they usually are not) produced by an equivalent resistance or conductance, the PSD can be rewritten as

$$S_v \triangleq 4kT \cdot R_v \quad S_i \triangleq 4kT \cdot G_i \quad S_{iu} \triangleq 4kT \cdot G_{iu} \quad S_{ic} \triangleq 4kT \cdot G_{ic}$$

resulting in

$$G_{iu} = G_i - |Y_c|^2 \cdot R_v = \left(1 - |c|^2\right) \cdot G_i \quad G_{ic} = |Y_c|^2 \cdot R_v = |c|^2 \cdot G_i$$

and the correlation factor is then given by

$$c = Y_c \cdot \sqrt{R_v/G_i} \quad \text{and} \quad |c|^2 = \left(G_c^2 + B_c^2\right) \cdot R_v / G_i$$

- Note that $R_v$, $G_i$, $G_{iu}$ and $G_{ic}$ are usually frequency dependent


Noise in Two-port Networks

Noise Factor Definition

- In many circuits and systems we are actually interested in the signal-to-noise ratio $SNR$ defined as the ratio of the signal power to the noise power.
- As the signal is amplified along the signal path, it also accumulates more noise.
- The noise factor evaluates how the $SNR$ is degraded along the path.

$$F \triangleq \frac{SNR_{in}}{SNR_{out}} > 1$$

- For an amplifier having a power gain $G$ we can write

$$F = \frac{S_i / N_i}{G \cdot S_i / \left(G \cdot (N_i + N_a)\right)} = \frac{N_i + N_a}{N_i} = 1 + \frac{N_a}{N_i}$$

where $N_i + N_a$ is the total noise power referred to the amplifier input, $N_a$ is the input-referred noise power added by the amplifier to the noise power already present at the input of the amplifier $N_i$. 
The noise coming from the source $I_{nrs}$ adds to the input-referred noise sources $V_n$ and $I_n$ of the two-port network to generate the total noise at the input of the noiseless two-port.

The total noise can hence be modelled by the equivalent Norton noise current source $I_{ntot}$ which corresponds to the short-circuit current of the above left schematic.

$$I_{ntot} = I_{nrs} + I_{nu} + (Y_S + Y_c) \cdot V_n$$
Noise Factor of the Noisy Two-port Network

- The current delivered at the two-port input is given by
  \[ I_{in} = -\frac{Y_{11}}{Y_s + Y_{11}} \cdot I_{ntot} \]

- The total noise power delivered at the two-port input is then given by
  \[ N_{in|tot} = \Re \{Z_{in}\} \cdot |I_{in}|^2 = \Re \{Z_{in}\} \cdot \frac{|Y_{11}|^2}{|Y_s + Y_{11}|^2} \cdot |I_{ntot}|^2 \]

- Whereas the rms noise power delivered at the input coming from the source is
  \[ N_{in|V_{n}=0,I_{n}=0} = \Re \{Z_{in}\} \cdot \frac{|Y_{11}|^2}{|Y_s + Y_{11}|^2} \cdot |I_{nrs}|^2 \]

- The noise factor is then simply given by
  \[ F = \frac{N_{in|tot}}{N_{in|V_{n}=0,I_{n}=0}} = \frac{|I_{ntot}|^2}{|I_{nrs}|^2} \]
Noise Factor of the Noisy Two-port Network

\[ Y_s = G_s + jB_s \quad \forall s \]

\[ I_{ntot} = I_{nrs} + I_{nu} + (Y_s + Y_c) \cdot V_n \]

- Accounting for the fact that the noise source \( I_{nrs} \) is uncorrelated to the other noise sources \( V_n \) and \( I_{nu} \) and that the noise current source \( I_{nu} \) is by definition uncorrelated with the voltage noise source \( V_n \), the mean-square value of the total noise current is then simply given by

\[
\sqrt{|I_{ntot}|^2} = \sqrt{|I_{nrs}|^2} + \sqrt{|I_{nu}|^2} + \sqrt{|Y_c + Y_s|^2 \cdot |V_n|^2}
\]

- From the above definition, the noise factor is then given by

\[
F = \frac{|I_{ntot}|^2}{|I_{nrs}|^2} = 1 + \frac{|I_{nu}|^2 + |Y_c + Y_s|^2 \cdot |V_n|^2}{|I_{nrs}|^2}
\]

\[
|I_{nrs}|^2 = 4kTB \cdot G_s
\]

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Spot Noise Factor

- The **spot noise factor** is then obtained by replacing the mean square values by the PSD

\[
F = 1 + \frac{S_{iu} + \left| Y_c + Y_s \right|^2 \cdot S_v}{4kT \cdot G_s} = 1 + \frac{G_{iu} + \left| Y_c + Y_s \right|^2 \cdot R_v}{G_s} = \\
= 1 + \frac{G_{iu}}{G_s} + \left[ (G_s + G_c)^2 + (B_s + B_c)^2 \right] \cdot \frac{R_v}{G_s}
\]

where \( G_s, B_s, G_c \) and \( B_c \) are defined as

\[
Y_s = G_s + jB_s \quad \text{and} \quad Y_c = G_c + jB_c
\]

- Note that the spot noise factor is usually frequency dependent, but for narrow-band systems, it is about equal to the noise factor

- \( F \) increases with \( B_s \) but for a given \( B_s \) it has a **minimum** wrt \( G_s \)
Minimum Noise Figure

- The noise factor $F$ reaches a minimum $F_{\text{min}}$ for a particular value of the source admittance $Y_{\text{opt}} = G_{\text{opt}} + j \cdot B_{\text{opt}}$

- The optimum source conductance $G_{\text{opt}}$ and susceptance $B_{\text{opt}}$ can be expressed in terms of the four circuit noise parameters $R_v$, $G_{iu}$, $G_c$ and $B_c$ according to

$$G_{\text{opt}} = \sqrt{G_{iu} + G_c^2} = \sqrt{\frac{G_i}{R_v} - B_c^2} \quad \text{and} \quad B_{\text{opt}} = -B_c$$

- The minimum noise factor $F_{\text{min}}$ is then given by

$$F_{\text{min}} = 1 + 2R_v \cdot \left( G_{\text{opt}} + G_c \right) = 1 + 2R_v \cdot \left( \sqrt{\frac{G_{iu}}{R_v} + G_c^2} + G_c \right)$$

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Noise in Two-port Networks

Noise Factor and Noise Parameters

- The actual noise factor $F$ may also be written in terms of the four noise parameters $F_{min}$, $R_v$, $G_{opt}$ and $B_{opt}$ and the source admittance as

$$F = F_{min} + \frac{R_v}{G_s} \cdot \left[ \left( G_s - G_{opt} \right)^2 + \left( B_s - B_{opt} \right)^2 \right]$$

- In the same way power gain can be maximized by impedance matching, the noise can be minimized by setting the source admittance to $Y_{opt}$

- The situation $F = F_{min}$ obtained for $G_s = G_{opt}$ AND $B_s = B_{opt}$ corresponds to noise matching

- Noise matching usually does not coincide with gain matching

- The $R_v/G_s$ tells us something about the relative sensitivity of the noise factor to departures from the optimum conditions

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**$F_{\text{min}}$ versus Source Conductance $G_s$**

- Assuming that $B_s = B_{\text{opt}}$, the noise increase due to deviation from the noise matching condition is given by

$$F - F_{\text{min}} \bigg|_{B_s = B_{\text{opt}}} = R_v \cdot G_{\text{opt}} \cdot \frac{G_{\text{opt}}}{G_s} \cdot \left( \frac{G_s}{G_{\text{opt}}} - 1 \right)^2$$

![Graph showing $B_s = B_{\text{opt}}$ and $R_v \cdot G_{\text{opt}} = 0.4$]
Noisy Two-port Parameters

- The four noise parameters $F_{\text{min}}$, $R_v$, $G_{\text{opt}}$ and $B_{\text{opt}}$ are usually obtained from measurements.

- They can then be used to derive the circuit noise parameters $R_v$, $G_i$, $G_c$ and $B_c$ of the noisy two-port circuit according to

  \[ G_i = |Y_{\text{opt}}|^2 \cdot R_v = \left( G_{\text{opt}}^2 + B_{\text{opt}}^2 \right) \cdot R_v \]

  \[ G_c = \frac{F_{\text{min}} - 2R_v G_{\text{opt}} - 1}{2R_v} \]

  \[ B_c = -B_{\text{opt}} \]

- $G_{iu}$, $G_{ic}$ can then be calculated as

  \[ G_{iu} = \left( 1 - |c|^2 \right) \cdot G_i \quad G_{ic} = |c|^2 \cdot G_i \]

  with the correlation factor $c$ given by

  \[ c = Y_c \cdot \sqrt{\frac{R_v}{G_i}} = \frac{G_c + jB_c}{\sqrt{G_{\text{opt}}^2 + B_{\text{opt}}^2}} \quad \text{and} \quad |c|^2 = \frac{G_c^2 + B_c^2}{G_{\text{opt}}^2 + B_{\text{opt}}^2} \]
If we are only interested in the actual noise factor $F$, the latter can be derived without calculating $F_{\text{min}}$, $R_v$, $G_{\text{opt}}$ and $B_{\text{opt}}$

It can be calculated from the above circuits where $Z_S = R_S + jX_S$ and where $V_{\text{nrs}}$ corresponds to the noise coming from the real part of the source impedance with a PSD given by $S_{V_{\text{nrs}}} = 4kT R_S$

First, calculate the output noise voltage $V_{\text{nout}}$ and related PSD $S_{V_{\text{nout}}}$ accounting for all the noise sources inside the two-port and the noise source $V_{\text{nrs}}$ with PSD
Simplified Calculation of the Actual Noise Factor

- Calculate the voltage gain \( A_v = \frac{V_{out}}{V_{in}} \) which is in general complex, depends on frequency and on the source impedance \( Z_S \)

- Calculate the input-referred noise PSD

\[
S_{V_{neq}} = \frac{S_{V_{nout}}}{|A_v|^2}
\]

- The noise factor is then given by

\[
F = \frac{S_{V_{neq}}}{S_{V_{nrs}}} = \frac{S_{V_{neq}}}{\frac{4kT}{R_S}} = \frac{R_{neq}}{R_S} = 1 + \frac{R_{namp}}{R_S}
\]

- where \( R_{neq} = \frac{S_{V_{neq}}}{(4kT)} \) and \( R_{namp} = R_{neq}\bigg|_{R_S=0} \) is the input-referred noise resistance due only to the two-port noise sources
Simplified Calculation of the Actual Noise Factor

- This method is usually simpler than deriving $F_{\text{min}}$, $R_v$, $G_{\text{opt}}$ and $B_{\text{opt}}$ particularly when the source impedance is reduced to a resistance which is usually the case for 50Ω systems, but it does not provide any information on the minimum achievable noise factor.

- However $F_{\text{min}}$ could be obtained from the above calculation but for this, the source impedance has to remain complex $Z_S = R_S + jX_S$.

- $F_{\text{min}}$ can then be obtained by differentiating $F$ with respect to $R_S$ and $X_S$.

- $R_{\text{opt}}$ and $X_{\text{opt}}$ are obtained by solving

$$\left. \frac{\partial F}{\partial R_S} \right|_{X_S=\text{const}} = 0 \quad \text{and} \quad \left. \frac{\partial F}{\partial X_S} \right|_{R_S=\text{const}} = 0$$
The Friis Formula

- The noise factor $F_{tot}$ of a cascade of two amplifiers each characterized by their noise factors $F_1$ and $F_2$ and their available power gains $A_{P1}$ and $A_{P2}$ is given by

$$F_{tot} = F_1 + \frac{F_2 - 1}{A_{P1}}$$

- The available power gain $A_{P1}$ is defined as the available power at its output (the power that it would deliver to a matched load) divided by the available source power (the power that the source would deliver to a matched load)

$$A_{P1} = \frac{R_{in1}^2}{(R_S + R_{in1})^2 \cdot A_{v1}^2 \cdot \frac{R_S}{R_{out1}}}$$

- Can be generalized to the cascade of $m$ stages

$$F_{tot} = 1 + (F_1 - 1) + \frac{F_2 - 1}{A_{P1}} + \ldots + \frac{F_m - 1}{A_{P1} \cdot A_{P2} \cdots A_{Pm-1}}$$
Outline

- Noise in Two-port Networks
- Noise in the MOS Transistor at RF
The thermal noise generated by voltage fluctuations in the channel appears at the drain, source but also at the gate and bulk as terminal current fluctuations.

The channel voltage fluctuations are transferred to the drain and source through the (trans)conductances and to the gate and bulk by capacitive coupling.
Noise in the MOS Transistor at RF

General and Simplified HF Thermal Noise Model

- Normally requires one noise source per terminal
- Under quasi-static assumption, the source and drain noise PSD and the drain-source cross-PSD are approximately equal
- At low-frequency ($\omega \ll \omega_{qs}$), the induced gate and substrate noise can be ignored
- The thermal noise model then reduces to the single noise source between drain and source

Thermal Noise at the Drain (long-channel)

- Channel thermal noise power spectral density (PSD)

\[
S_{\Delta I^2_{nD}} = 4kT \cdot G_{nD} \quad \text{with} \quad G_{nD} = \delta_{nD} \cdot G_{ms} = \gamma_{nD} \cdot G_m
\]

where \( \delta_{nD} \) is the **drain thermal noise parameter** and \( \gamma_{nD} = n \cdot \delta_{nD} \) the **thermal noise excess factor**.

- The drain conductance \( G_{nD} \) is **bias dependent** according to

\[
\frac{G_{nD}}{G_{spec}} \approx \begin{cases} 
\frac{2}{3} \cdot \frac{q_s^2 + \frac{3}{4} q_s q_d + \frac{3}{4} q_d + q_d^2}{q_s + q_d + 1} & \text{SI} \\
\frac{1}{2} \cdot (q_s + q_d) & \text{WI}
\end{cases}
\]

- In saturation

\[
\delta_{nD} = \frac{2}{3} \cdot \frac{q_s + 3/4}{q_s + 1} = \begin{cases} 
\frac{1}{2} & \text{WI} \\
\frac{2}{3} & \text{SI}
\end{cases}
\]

\[
\gamma_{nD} = n \cdot \delta_{nD} = \begin{cases} 
\frac{n}{2} \approx 0.8 & \text{WI} \\
n \cdot \frac{2}{3} \approx 1 & \text{SI}
\end{cases}
\]
Effect of Velocity Saturation

- For short-channel devices in SI and saturation → lateral electrical field larger than critical field → carrier velocity saturation

- Carrier velocity limited → additional charge builds up close to the drain → additional thermal noise without increase of $G_m$ → increase of $\delta_{n_{DSat}}$ compared to the long-channel value 2/3
Short-channel Effects on $\gamma_{nD}$ (in saturation)

- The noise excess factor $\gamma_{nD}$ can be modelled versus $IC$ as
  \[ \gamma_{nD} \approx \gamma_{wi} + \alpha_{\gamma_{nD}} \cdot IC \]
- Where $\gamma_{wi}$ and $\alpha_{\gamma_{nD}}$ are empirical factors
- $\alpha_{\gamma_{nD}}$ scales approximatively as $\alpha_{\gamma_{nD}} \approx 2.85/L$ where $L$ is in nm

Example 1: Channel Thermal Noise Only

- The input referred noise sources and the correlation admittance are calculated as
  \[ R_v = \gamma_{nD} / G_m \quad G_i = \gamma_{nD} / G_m \cdot (\omega C_{GS})^2 \quad G_c = 0 \quad B_c = \omega C_{GS} \]

- \( V_n \) and \( I_n \) are fully correlated since there is only one noise source \( I_{nD} \)
  \[ \rho_{GD} = Y_c \cdot \sqrt{R_v / G_i} = j \]

- The optimum source admittance and the minimum noise figure are given by
  \[ G_{opt} = \sqrt{\frac{G_i}{R_v} - B_c^2} = 0 \quad B_{opt} = -B_c = -\omega C_{GS} \quad F_{\text{min}} = 1 + 2R_v \cdot (G_{opt} + G_c) = 1 \]

- The optimum source admittance is simply equal to the conjugate match for maximum gain
Example 1: Channel Thermal Noise Only

- The reason why $F_{min} = 1$ can be explained as follows

\[ Z_{in} \triangleq \frac{V_1}{I_{in}} = \frac{1}{Y_{in}} = \frac{1}{G_s + \frac{1}{j\omega L_s} + j\omega C_{GS}} \]

- For $G_s = G_{opt} = 0$ and $L_s$ is made to resonate with $C_{GS}$ at the operating frequency, the quality factor of the input circuit becomes infinity and the gain at the input becomes infinity, leading to a minimum noise factor of 1!
Example 2: Effect of the Gate Resistance Noise

For \( \omega R_G C_{GS} \ll 1 \), the noise parameters \( R_v, G_i \) and \( Y_c \) of the equivalent noisy two-port are given by

\[
R_v = \frac{\gamma_{nD}}{G_m} + R_G = \frac{\gamma_{nD}}{G_m} \cdot (1 + \alpha_G) \quad G_i = \frac{\gamma_{nD}}{G_m} \cdot (\omega C_{GS})^2 \quad G_c = \frac{(\omega C_{GS})^2 R_G}{1 + \alpha_G} \quad B_c = \frac{\omega C_{GS}}{1 + \alpha_G}
\]

- The gate resistance \( R_G \) directly adds to the input-referred resistance \( R_v \)
- \( \alpha_G \) represents the ratio of the noise PSD of the gate resistance to the input referred channel noise

\[
\alpha_G \triangleq \frac{4kT R_G}{4kT \gamma_{nD}/G_m} = \frac{G_m \cdot R_G}{\gamma_{nD}} \ll 1
\]

- \( V_n \) and \( I_n \) are now partially correlated according to

\[
\rho_{GD} = Y_c \cdot \sqrt{\frac{R_v}{G_i}} = \frac{\omega R_G C_{GS} + j}{\sqrt{1 + \alpha_G}} \approx \frac{j}{\sqrt{1 + \alpha_G}} \quad \text{for} \quad \omega R_G C_{GS} \ll 1
\]
Example 2: Effect of the Gate Resistance Noise

- The optimum source admittance and the minimum noise factor are then given by

\[
G_{opt} = \sqrt{\frac{G_i}{R_v} - B_c^2} = \omega C_{GS} \cdot \frac{\sqrt{\alpha_G}}{1 + \alpha_G} \cong \omega C_{GS} \cdot \sqrt{\alpha_G} \quad \text{for} \quad \alpha_G \ll 1
\]

\[
B_{opt} = -B_c = -\frac{\omega C_{GS}}{1 + \alpha_G} \cong -\omega C_{GS} \quad \text{for} \quad \alpha_G \ll 1
\]

\[
F_{\min} = 1 + 2R_v \cdot (G_{opt} + G_c) \cong 1 + 2 \frac{\gamma_n D}{G_m} \cdot \omega C_{GS} \cdot \left(\sqrt{\alpha_G} + \omega R_G C_{GS}\right)
\]

\[
\cong 1 + 2\gamma_n D \cdot \sqrt{\alpha_G} \cdot \frac{\omega}{\omega_t} \quad \text{with} \quad \alpha_G = \frac{G_m \cdot R_G}{\gamma_n D}
\]

where \(\omega_t = \frac{G_m}{C_{GS}}\)
Noise in the MOS Transistor at RF

Comparison to Measurements on 40 nm Device

- **$V_G-V_{T0} = 0.55V$ (IC = 29)**
- Analytical Model
- BSIM6 Model

- $F_t \approx 190 \text{ GHz}$
- $\gamma_{nD} \approx 3$
- $R_G \approx 4.5 \Omega$
- $G_m \approx 100 \text{ mA/V}$
- $\alpha_G \approx 0.15$

\[ F_{\text{min}} \approx 1 + 2\gamma_{nD} \cdot \sqrt{\alpha G} \cdot \frac{\omega}{\omega_t} \quad \text{with} \quad \alpha_G \triangleq \frac{G_m \cdot R_G}{\gamma_{nD}} \]

- This simple model works very well for $\omega \ll \omega_t$
Measured and Simulated Noise Parameters

- $N_f = 10$
- $W_t = 12 \mu m$
- $L_t = 0.36 \mu m$

- $G_m = 10.7 \text{ mA/V}$
- $C_{GS} = 129 \text{ fF}$
- $f_t = 13.2 \text{ GHz}$
- $R_G = 6.59 \Omega$
- $R_B = 119 \Omega$
- $Z_0 = 50 \Omega$

- $V_G = 0.743 \text{ V}$
- $V_D = 1.5 \text{ V}$
- $I_D = 1.116 \text{ mA}$

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The previous values of the noise factors $\gamma_{nD}$, $\delta_{nG}$ and $c_G$ are given for long-channel transistor ignoring the short-channel effects (SCE) such as velocity saturation (VS).

SCE and particularly VS tend to degrade the noise performance resulting in an increase of $\gamma_{nD}$ and $\delta_{nG}$ particularly in SI where VS is predominant.

This is confirmed by the values shown below extracted from noise measurements made on a 40 nm bulk CMOS device.

- **M. Chalkiadaki and C. Enz, TMTT, 2015.**
- **M. Chalkiadaki, PhD Thesis No. 7030, 2016**
**$N_{F \text{min}}$ and $R_n$ versus $IC$ for 40nm Bulk CMOS Process**

- The minimum noise figure $N_{F \text{min}}$ and input-referred noise resistance $R_n$ show a minimum in MI due to the sharp increase of $\gamma_{nD}$ at high $IC$.

\[ F_{\text{min}} \approx 1 + 2 \omega C_{GS} \cdot \frac{\gamma_{nD}}{G_m} \cdot \sqrt{\frac{\beta nG}{\gamma_{nD}}} \cdot \left(1 - c_g^2\right) \]

\[ R_n \approx \frac{\gamma_{nD}}{G_m} + R_G \]

The actual noise figure also shows a minimum in MI

\[ F - 1 = \frac{1}{50\Omega} \cdot \left( \frac{\gamma_{nD}}{G_m} + R_G \right) \]
Noise Parameters for a 40nm Bulk CMOS Process

N-channel, $M=6$, $N_f = 10$, $W_f = 2 \mu m$, $L_f = 40$ nm, $V_S = 0$ V, $V_D = 1.1$ V

- $V_G - V_{T0} = 0.05$ V
- $V_G - V_{T0} = 0.15$ V
- $V_G - V_{T0} = 0.55$ V

Analytical Model

BSIM6 Model

Shot Noise of the Gate Leakage Current

- In addition to the IGN (thermal noise), there is also an additional component coming from the gate leakage current which shows shot noise and has a PSD given by

\[ S_{\Delta I_G} = 2q \cdot I_G \]

\[ W=10\mu m, \ L=10\mu m \]
\[ V_{GS}=2 \ V; \ V_{DS}=1 \ V \]
\[ I_G=66.2 \ \mu A \]
Effect of the Gate Tunneling Current Shot Noise

- The effect of the shot noise coming from the tunneling current can be added by redefining $G_{nG}$ as

$$G_{nG}(\omega) = \beta_{nG} \cdot \left(\frac{\omega C_{GS}}{G_m}\right)^2 + \frac{2qI_G}{4kT} = \beta_{nG} \cdot \left(\frac{\omega C_{GS}}{G_m}\right)^2 + \frac{I_G}{2U_T}$$

- The minimum noise factor is then given by

$$F_{\text{min}} = 1 + \sqrt{\frac{2\gamma_{nD} \cdot I_G}{G_m \cdot U_T}} \cdot \left(1 - c_g^2\right) \cdot \left(1 + \left(\frac{\omega}{\omega_c}\right)^2\right)$$

with

$$\omega_c \triangleq \omega_t \cdot \sqrt{\frac{I_G}{2\beta_{nG} \cdot G_m \cdot U_T}}$$

- The gate tunneling current sets a non-zero minimum value of $F_{\text{min}}$ at frequencies below $\omega_c$

$$F_{\text{min}} = \begin{cases} 1 + \sqrt{\frac{2\gamma_{nD} \cdot I_G}{G_m \cdot U_T}} \cdot \left(1 - c_g^2\right) & \text{for } \omega \ll \omega_c \\ 1 + 2 \frac{\omega}{\omega_t} \cdot \sqrt{\gamma_{nD} \cdot \beta_{nG} \cdot \left(1 - c_g^2\right)} & \text{for } \omega \gg \omega_c \end{cases}$$
Shot Noise of the Gate Leakage Current

- The gate leakage shot noise is independent of frequency whereas the IGN and the noise coming from the gate resistance are proportional to $\omega^2$.

- The plot (simulations) below shows that the gate noise for $L=100\,\text{nm}$ is dominated by shot noise for $f<1\,\text{GHz}$ and by IGN and gate resistance noise for $f>1\,\text{GHz}$.

![Graph showing noise comparison](image-url)
References

Most of this Chapter is based on Chapter 13 of Reference [1]


