

Partial derivatives in ConvNets

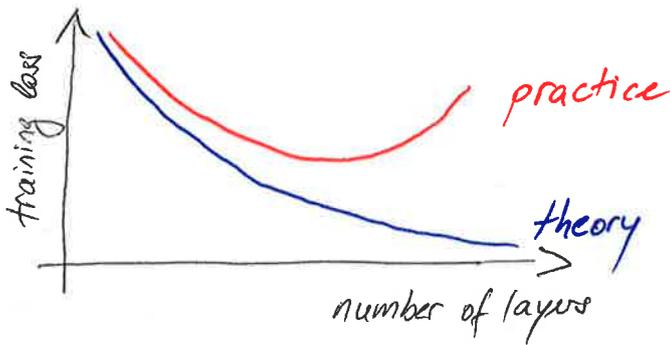
$$\begin{aligned} a) \quad \frac{\partial L}{\partial w_{1111}^{(1)}} &= - \sum_o (t_o - \hat{y}_o) \frac{\partial \hat{y}_o}{\partial w_{1111}} \\ &= - \sum_o (t_o - \hat{y}_o) \sum_{ijk} w_{ijko}^{(2)} \sigma'(a_{ijk}) \frac{\partial a_{ijk}}{\partial w_{1111}} \\ &= - \sum_o (t_o - \hat{y}_o) \sum_{ijk} w_{ijko}^{(2)} \sigma'(a_{ijk}) I_{ij1} \quad (\star) \end{aligned}$$

$$b) \quad \frac{\partial L}{\partial w_{1111}^{(1)}} = - \sum_o (t_o - \hat{y}_o) \sum_k w_{ok}^{(2)} \sigma'(a_k) I_{111}$$

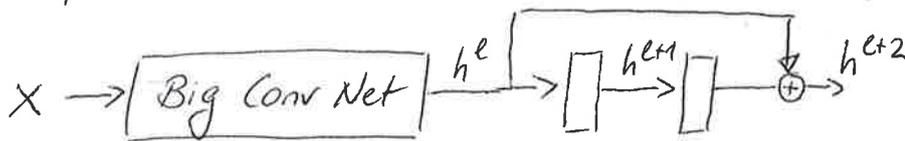

c) Sum in (\star) runs only over i and j that where the $\arg \max$ for some max operation.

ResNet

without skip connections



with skip connections



$$h^{e+2} = \sigma(a^{e+2} + h^e)$$

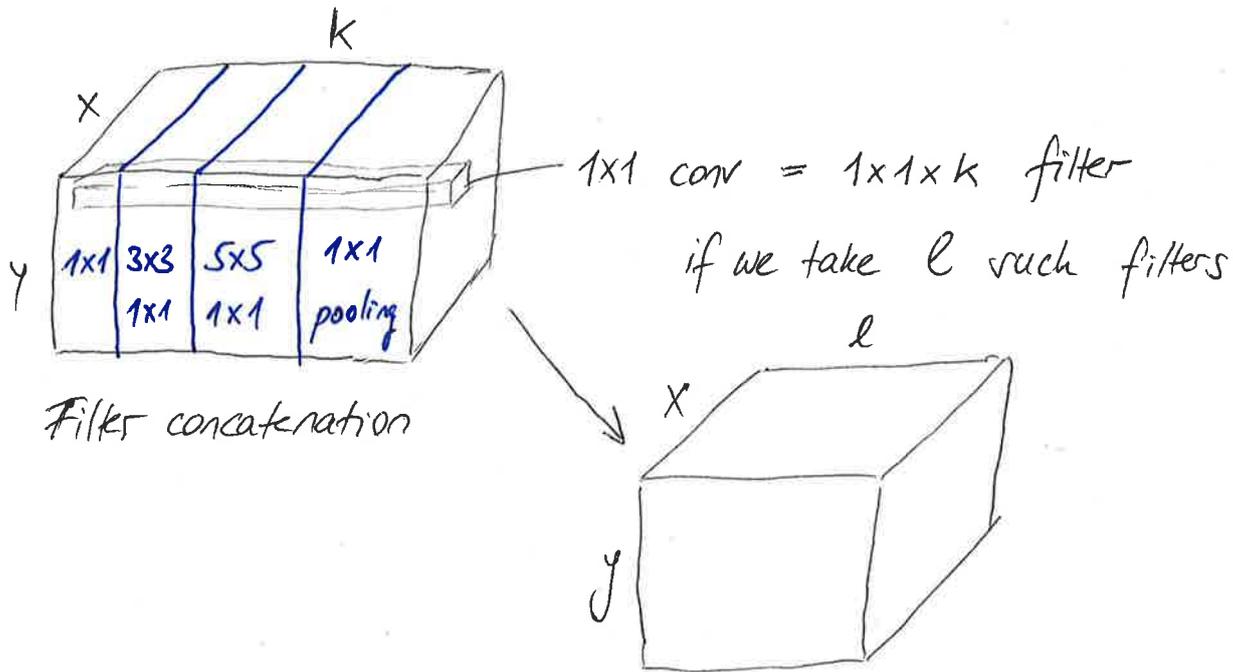
$$= \sigma(\underbrace{w^{e+2} \cdot h^{e+1} + b^{e+2}}_{\neq 0} + h^e)$$

$$\begin{array}{l|l} \swarrow = 0 & \searrow \neq 0 \\ = \sigma(h^e) = h^e & h^{e+2} \neq h^e \\ \uparrow & \\ \sigma = \text{relu} & \end{array}$$

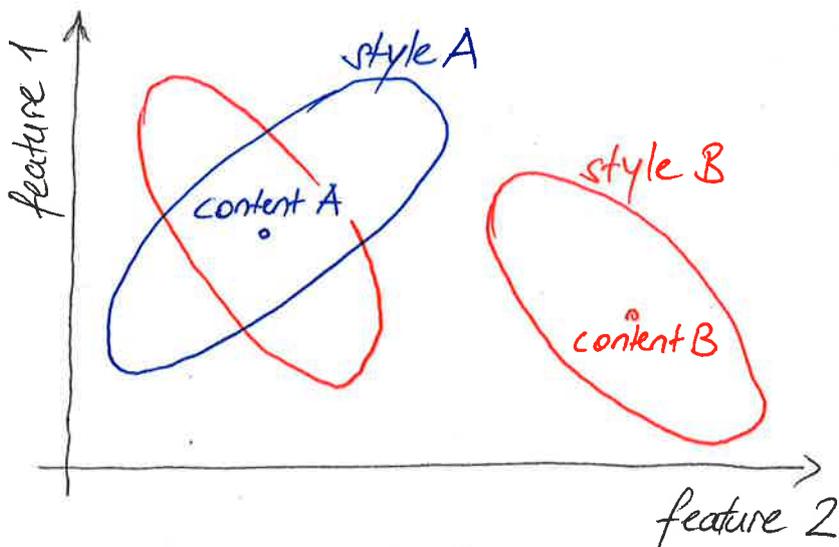
network with
 l -layers

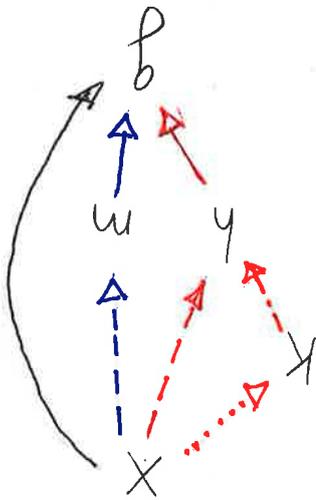
network with
 $l+2$ -layers

Inception module



Neural style





$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial m} \frac{\partial m}{\partial x} + \left(\frac{\partial f}{\partial y} + \frac{\partial f}{\partial k} \frac{\partial k}{\partial y} \right) \frac{\partial y}{\partial x} =$$

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial m} \frac{\partial m}{\partial x} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial x} = \frac{df}{dx}$$

$$f(x) = g(h(k(x), x), m(x), x)$$

Example:

Automatic reverse mode differentiation