MICRO-461
Low-power Radio Design for the IoT

9. Mixers

Christian Enz

Integrated Circuits Lab (ICLAB), Institute of Microengineering (IMT), School of Engineering (STI)

Swiss Federal Institute of Technology, Lausanne (EPFL), Switzerland

Outline

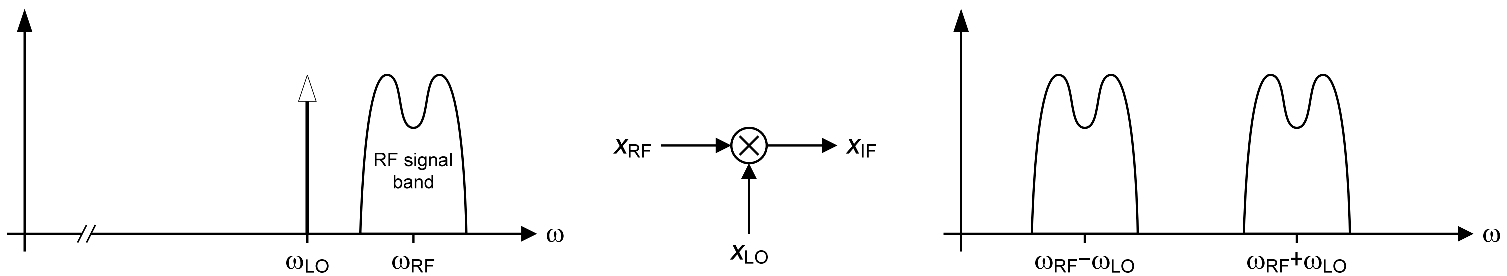
- **Mixer fundamentals**
- Non-linearity based mixers
- Multiplier-based mixers
- Detailed analysis of the single balanced mixer (SBM)

Mixers – General Considerations

- Component used for **frequency translation** by multiplication of two signals (RF signal with a periodic signal called the LO signal for down conversion mixers)
- Mainly two types of mixers
  - **Passive** mixers
  - **Active** mixers
- Passive mixers do the frequency translation but without any gain (they actually introduce some loss), whereas active mixers can achieve some gain

Mixers Fundamentals

$$x_{IF}(t) = x_{RF}(t) \cdot x_{LO}(t) \quad \leftrightarrow \quad X_{IF}(f) = X_{RF}(f) * X_{LO}(f)$$



$$x_{LO}(t) = A_{LO} \cdot \cos\left(\omega_{LO} \cdot t\right) \quad \leftrightarrow \quad X_{LO}(f) = \frac{A_{LO}}{2} \cdot \left[\delta(f - f_0) + \delta(f + f_0)\right]$$

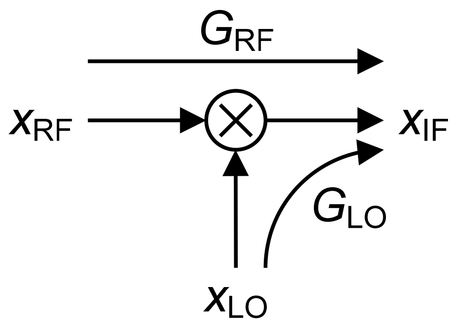
$$x_{IF}(t) = x_{RF}(t) \cdot A_{LO} \cdot \cos\left(\omega_{LO} \cdot t\right)$$

$$X_{IF}(f) = \frac{A_{LO}}{2} \cdot X_{RF}(f) * \left[\delta(f - f_0) + \delta(f + f_0)\right] = \frac{A_{LO}}{2} \cdot \left[X_{RF}(f - f_0) + X_{RF}(f + f_0)\right]$$

- Mixer generates frequency components not present at the input, it is therefore not a LTI system
- However, with respect to the RF and IF inputs only, the mixing of the sum of two RF signals by the same LO is a linear process

$$mix\left(a \cdot x_{RF1} + b \cdot x_{RF2}\right) = a \cdot mix\left(x_{RF1}\right) + b \cdot mix\left(x_{RF2}\right)$$

Mixer Characteristics – Conversion and LO Gains



- Different gains can be defined
- The **conversion gain** is defined as the ratio of the rms voltage of the IF (output or downconverted) signal to the rms voltage of the RF signal (input)

$$G_{RF} = \frac{X_{IF}}{X_{RF}}$$

- The **LO gain** is defined as

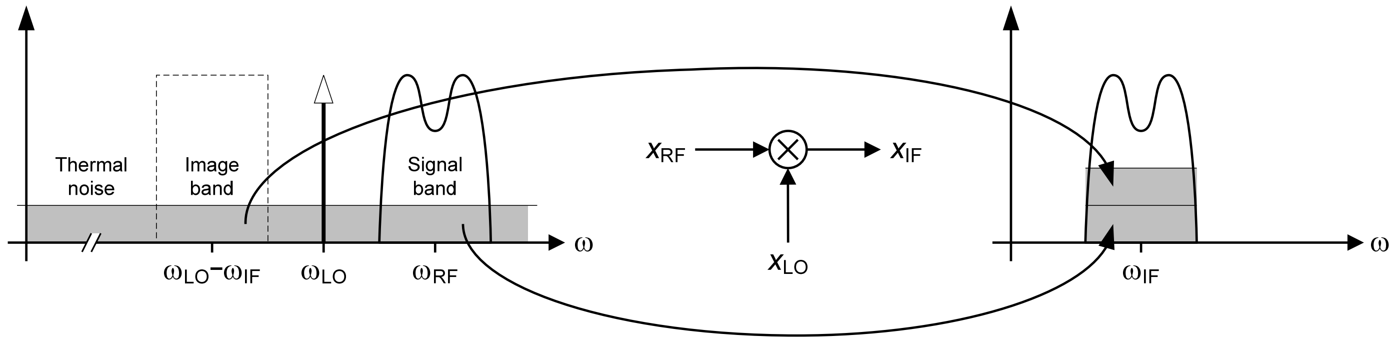
$$G_{LO} = \frac{X_{IF}}{X_{LO}}$$

Mixer Characteristics – Linearity

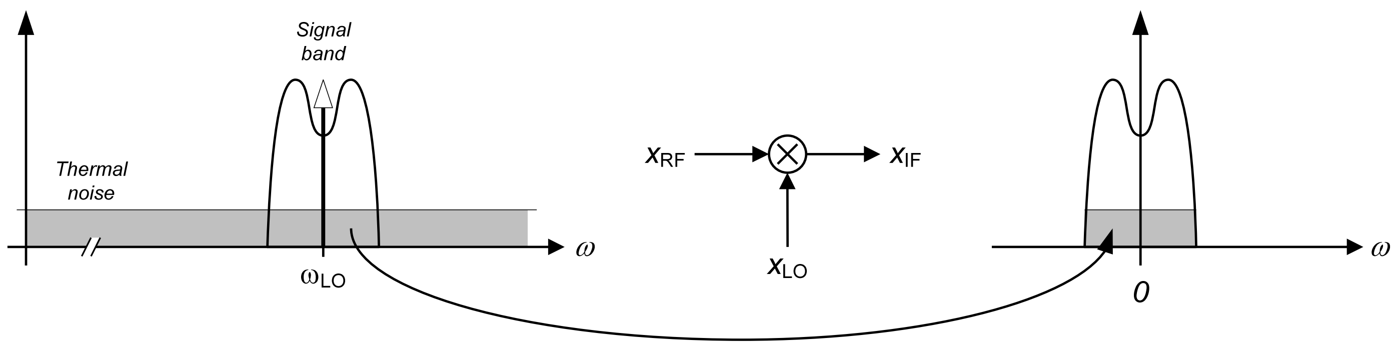
- Similar to amplifiers except that IF signal is at a different frequency than RF signal
- Nonlinearities in the RF section of the mixer can cause in-band intermodulation products
- Two-tone third order intercepts, IIP3 and OIP3, can be defined for input signal and desired output signal respectively
- 1dB compression point is also used in conjunction with mixer gain to allow fair comparison among different mixers topologies
- Typical mixers exhibit an IIP3 of 10dBm or less

Mixer Characteristics – Mixer Noise Figures

- **Single side-band** (SSB) noise factor is the ratio of SNR at the desired output frequency (IF) to the SNR at input frequency (RF) measured in a single side band.



- **Double-sideband** (DSB) noise factor is the ratio of SNR at the output (IF) to the SNR at the input measured in both signal and image side bands (input signal spectrum resides on both sides of LO frequency, a common case in homodyne or zero-IF systems)



Mixer Characteristics – Mixer Noise Figures

- SSB NF is 3 dB higher than DSB NF in the ideal noise-less mixer with a sinusoid LO signal
- Typically mixers are noisier than amplifiers due to the noise folding nature of mixers
- DSB NF ranges from 10-15 dB

Mixer Characteristics – Port-to-port isolation

- Isolation is defined as the amount of feedthrough of input RF and LO signal to the desired output band
- Large LO component at IF may desensitize subsequent stages
- RF component in the IF rises the issue with respect to even-order distortion problem in homodyne receiver
- Reverse isolation of IF and LO signal back to the input (RF) port is also of great importance especially in minimizing interference to other receivers
- Isolation of around 30-50 dB is considered adequate for most communication systems

Mixer Characteristics – Spurious response

- Mixers can generate numerous cross-products of the LO and RF signals and their harmonics
- Need to ensure none of the spurious components fall in the desired (IF) band

$$\omega_{spur} = m \cdot \omega_{RF} + n \cdot \omega_{LO}$$

where n and m can be any positive or negative integers

- Analysis is tedious, need CAD tools
- Adequate input filtering as well as the choice (or quality) of LO and IF can limit the amount of spurs in the desired band to acceptable levels

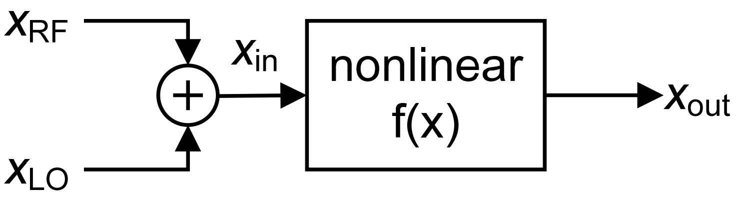
Outline

- Mixer fundamentals
- **Non-linearity based mixers**
- Multiplier-based mixers
- Detailed analysis of the single balanced mixer (SBM)

Nonlinearity-based Mixers – Principle

- Instead of implementing the multiplication operator, frequency translation can also be obtained by using a nonlinear device

$$x_{in}(t) = A_{RF} \cdot \cos(\omega_{RF} t) + A_{LO} \cdot \cos(\omega_{LO} t)$$



- If the mixer can be considered as a memoryless nonlinear system, the output is then given by

$$x_{out}(t) = f\left(x_{in}(t)\right) \cong \alpha_1\, x_{in}(t) + \alpha_2\, x_{in}^2(t) + \alpha_3\, x_{in}^3(t) + \ldots$$

- Situation is then similar to a nonlinear amplifier with two-tone input
- In addition to the harmonics, the useful component for the mixing operation correspond actually to one of the 2nd-order IM products

$$x_{IF}(t) = \alpha_2 \cdot A_{RF} \cdot A_{LO} \cdot \cos\left((\omega_{RF} - \omega_{LO})t\right)$$

- Other components need to be filtered out by an appropriate bandpass filter

Nonlinearity-based Mixers – Principle

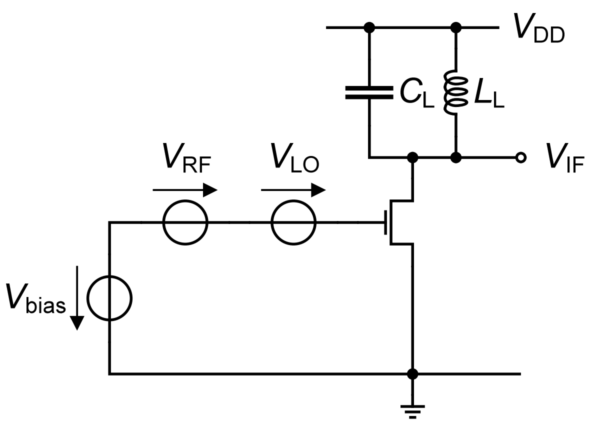
- For a fixed LO amplitude, the IF output amplitude is linearly proportional to the RF input amplitude. The nonlinear-based mixer hence implements a linear mixer since the output is proportional to the input
- The conversion gain for this nonlinearity is then given by

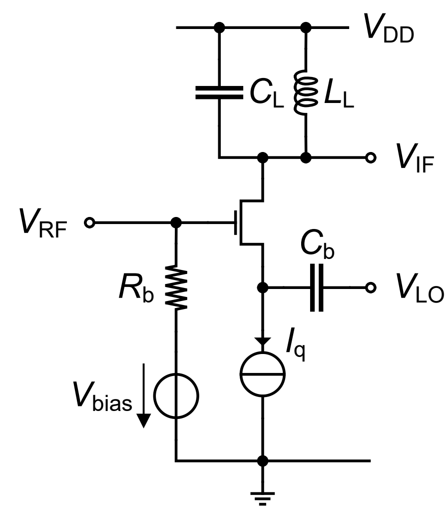
$$G_{RF} = \alpha_2 \cdot A_{LO}$$

- Note that a square-law nonlinear mixer has the advantage of producing only frequency components at DC, 2nd harmonics $2\omega_{RF}$, $2\omega_{LO}$, $\omega_{RF} + \omega_{LO}$ and $\omega_{RF} - \omega_{LO}$, hence the filter can be very simple since the components to be filtered are at very different frequencies than the desired one

Square-law MOSFET Mixers

- The square-law mixer can be implemented by using a single MOS biased in strong inversion





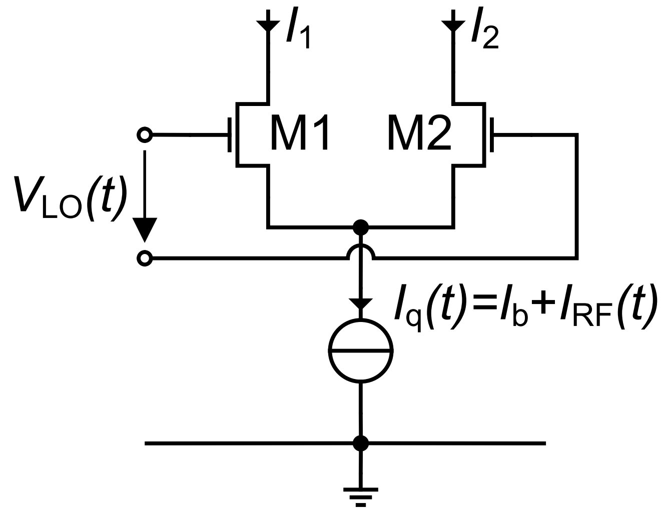
$$G_{RF} = \frac{\beta}{2n} \cdot A_{LO} = \frac{\mu C_{ox} \cdot W}{2n \cdot L} \cdot A_{LO} \qquad G_{RF} = \frac{\beta}{2n} \cdot nA_{LO} = \frac{\mu C_{ox} \cdot W}{2 \cdot L} \cdot A_{LO}$$

- For long-channel devices, the conversion gain (actually transconductance if we look at the drain current) is independent of the bias (assuming in SI and constant mobility)
- **Poor isolation** between LO and RF (a bit better for the right one)

Outline

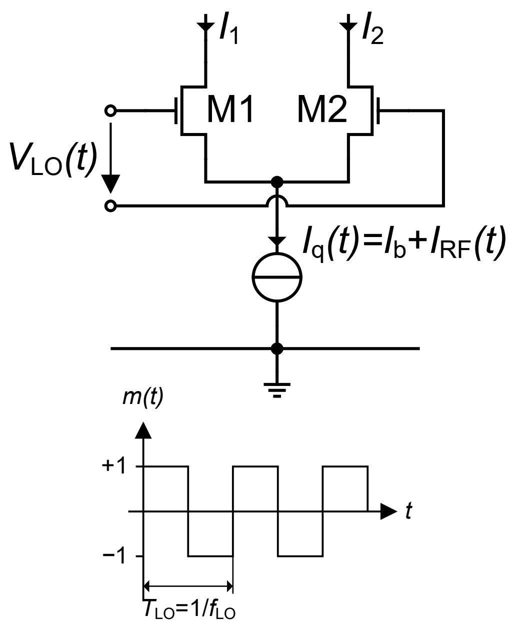
- Mixer fundamentals
- Non-linearity based mixers
- **Multiplier-based mixers**
- Detailed analysis of the single balanced mixer (SBM)

Multiplier-based Mixers



- Nonlinearity-based mixers produce many unwanted frequency components and usually have a poor LO-to-RF isolation
- Multiplier-based mixers don't have these drawbacks
- They are usually based on the differential pair, where the RF signal is added to the bias current provided by the queue current source and the LO signal is applied to the differential input

The Single Balanced Mixer (SBM)



$$I_{out} = I_1 - I_2$$

$$V_{LO}(t) = A_{LO} \cdot \cos(\omega_{LO} t)$$

$$I_{RF}(t) = A_{RF} \cdot \cos(\omega_{RF} t)$$

- The RF signal is added to the bias current I_b and switched to either output by the differential pair
- The differential output current is equal to the queue current modulated by a periodic function $m(t)$
- If the differential pair is hard-switched, then $m(t)$ corresponds to a square wave with no dc component
- The output spectrum contains only odd harmonics of the LO and the components resulting from mixing with the RF signal

$$I_{out}(t) = I_q(t) \cdot m(t) = \left(I_b + I_{RF}(t)\right) \cdot m(t) = I_b \cdot m(t) + I_{RF}(t) \cdot m(t)$$

T. Melly et al., JSSC, Jan. 2001.

The Single Balanced Mixer – LO Feedthrough

- The differential output current is given by

$$I_{out}(t) = I_q(t) \cdot m(t) = \left(I_b + I_{RF}(t)\right) \cdot m(t) = I_b \cdot m(t) + I_{RF}(t) \cdot m(t)$$

- The first term corresponds to the **LO feedthrough** to the harmonics of the LO by the modulation signal $m(t)$
- If the differential pair is hard-switched, the modulation signal can be assumed to be a simple square wave
- The fundamental component is the strongest and is given by

$$I_{out\text{-}LO}(t) = \frac{4}{\pi} \cdot I_b \cdot \sin\left(2\pi f_{LO} t\right)$$

- Since it is proportional to the bias current, if the later is too large, this component might desensitize the next stage
- A detailed analysis valid for any LO amplitude and from weak to strong inversion is given next

The Single Balanced Mixer – Conversion Gain

- The output spectrum is given by

$$I_{out}(f) = \underbrace{I_b \cdot \sum_{\substack{n=-\infty \\ n\ odd}}^{+\infty} M_n \cdot \delta(f - n \cdot f_{LO})}_{\text{up-converted dc to odd harmonics of } f_{LO}} + \underbrace{\sum_{\substack{n=-\infty \\ n\ odd}}^{+\infty} M_n \cdot I_{RF}(f - n \cdot f_{LO})}_{\text{includes down-converted RF signal } (n=\pm 1)} \quad \text{with} \quad M_n = \frac{2}{j\pi n}$$

- Assuming the RF signal is a cosine $I_{RF}(t) = A_{RF} \cdot cos(\omega_{RF} t)$ and that the mixer is hard-switched, the signal at IF $f_{IF} = f_{RF} - f_{LO}$ is given by[1]

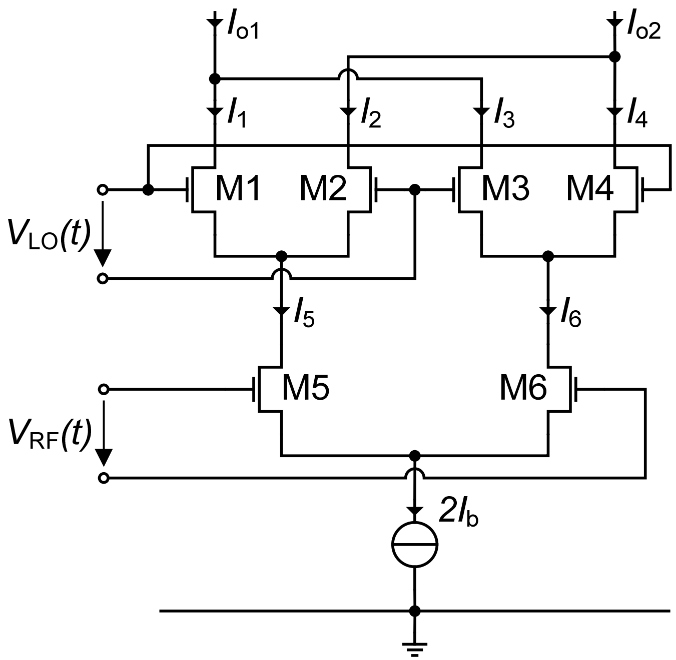
$$I_{out}(t) = -\frac{2}{\pi} \cdot A_{RF} \cdot \sin\left(2\pi f_{IF} t\right)$$

- The conversion gain under hard-switching conditions is then given by

$$G_{RF} = \frac{I_{out-rms}\big|_{\omega=\omega_{IF}}}{\frac{A_{RF}}{\sqrt{2}}} = \frac{\frac{2}{\pi} \cdot \frac{A_{RF}}{\sqrt{2}}}{\frac{A_{RF}}{\sqrt{2}}} = \frac{2}{\pi}$$

1) see Appendix 1 for the detailed derivation

Multiplier-based Mixers – The Double Balanced Mixer



- The LO feedthrough of the single balanced mixer can be avoided by using the double balanced mixer
- All the LO feedthrough components are canceled in the differential output current

$$I_{out} = I_{o1} - I_{o2} = I_1 - I_2 - (I_4 - I_3)$$

$$I_1 - I_2 = I_5 \cdot m(t), \quad I_4 - I_3 = I_6 \cdot m(t)$$

$$I_{out} = (I_5 - I_6) \cdot m(t)$$

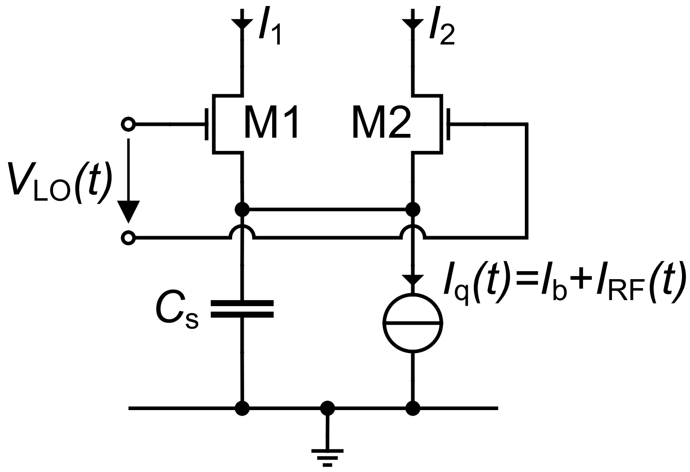
$$I_5 = I_b + \frac{\Delta I_{RF}}{2}, \quad I_6 = I_b - \frac{\Delta I_{RF}}{2}$$

$$I_{out} = \Delta I_{RF} \cdot m(t)$$

Outline

- Mixer fundamentals
- Non-linearity based mixers
- Multiplier-based mixers
- **Detailed analysis of the single balanced mixer (SBM)**

Detailed Analysis of the SBM – Definitions



$$I_{out} = I_1 - I_2$$
$$V_{LO}(t) = A_{LO} \cdot \cos(\omega_{LO} t)$$
$$I_{RF}(t) = A_{RF} \cdot \cos(\omega_{RF} t)$$

- Normalizing the currents to twice the specific current of a single transistor

$$i_{out}(t) \triangleq \frac{I_{out}(t)}{2I_{spec}} \quad i(t) \triangleq \frac{I_q(t)}{2I_{spec}} = IC + i_{RF}(t) \quad IC \triangleq \frac{I_b/2}{I_{spec}} \quad i_{RF}(t) \triangleq \frac{I_{RF}(t)}{2I_{spec}}$$

- Normalizing the differential LO voltage to $2nU_T$

$$u(t) = \frac{V_{LO}(t)}{2nU_T} = v_{LO} \cdot \cos(\omega_{LO} t) \quad \text{with} \quad v_{LO} \triangleq \frac{A_{LO}}{2nU_T}$$

T. Melly et al., JSSC, Jan. 2001.

Definitions

- The normalized differential output current can then be written in general form

$$i_{out} = i \cdot f(u,i)$$

where the nonlinear function $f(u,i)$ in **WI** is independent of i and simply given by

$$f(u,i) = \tanh(u)$$

whereas in **SI**, the nonlinear function $f(u,i)$ depends on both u and i and is given by

$$f(u,i) = \begin{cases} \dfrac{u}{\sqrt{i}} \cdot \sqrt{1 - \dfrac{u^2}{4i}} & \text{for } u^2 \le 2i \\ \operatorname{sgn}(u) & \text{for } u^2 > 2i \end{cases}$$

T. Melly et al., JSSC, Jan. 2001.

Output Signal

- The mixer output current is simply the product of the RF signal times a periodic modulation signal $m(t)$ of period T_{LO}

$$i_{out}(t) = i(t) \cdot \underbrace{f\left(u(t), i(t)\right)}_{m(t)} = i(t) \cdot m(t) = \left(IC + i_{RF}(t)\right) \cdot m(t) = IC \cdot m(t) + i_{RF}(t) \cdot m(t)$$

- The shape of the modulation signal $m(t)$ depends on the LO amplitude A_{LO} (v_{LO}), the bias current I_b (IC) and the DP nonlinearity $f(u, i)$
- The spectrum of $i_{out}(t)$ is given by (double-sided FT)

$$I_{out}(f) \triangleq F\{i_{out}(t)\} = I(f) * M(f) = I(f) * \sum_{n=-\infty}^{+\infty} M_n \cdot \delta(f - nf_{LO}) = \sum_{n=-\infty}^{+\infty} M_n \cdot I(f - nf_{LO})$$

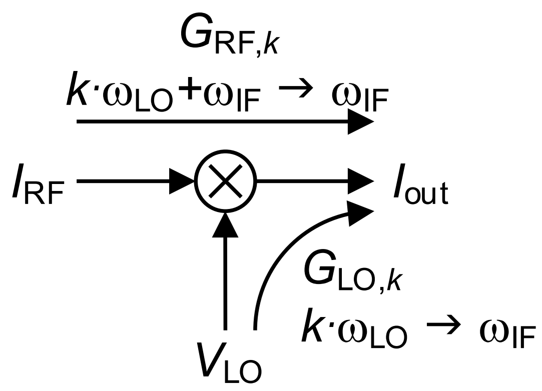
with $\quad I(f) \triangleq F\{i(t)\} \quad$ and $\quad M(f) \triangleq F\{m(t)\}$

- The Fourier coefficient M_n are defined as

$$M_n = \frac{1}{T_{LO}} \cdot \int_0^{T_{LO}} m(t) \cdot e^{-jn2\pi f_{LO}t} \cdot dt \quad \text{with} \quad m(t) \triangleq f\left(u(t), i(t)\right)$$

- Note that it is not always possible to calculate these coefficients M_n analytically

Gains Definition



- Different conversion gains $G_{RF,k}$ can be defined as the current gains from the RF signal in the band $k \cdot \omega_{LO} + \omega_{IF}$ and the IF band at ω_{IF}
- The **main conversion gain** of interest for the mixer is the gain for $k = 1$, $G_{RF,1}$
- Similarly, different LO gains $G_{LO,k}$ can be defined from the LO input in the band $k \cdot \omega_{LO}$ and the IF band at ω_{IF}
- The LO gain of most interest is the gain for $k = 0$ (DC to ω_{IF} up-conversion), $G_{LO,0}$ corresponding to the **up-conversion gain of the flicker noise to the IF band**

Conversion Gains

- Since the RF current amplitude can be assumed much smaller than the bias current, the output current can be approximated by a Taylor series as

$$i_{out} = i \cdot f(u,i) \cong \frac{di_{out}}{di_{RF}} \cdot i_{RF} = m(t) \cdot i_{RF}(t) \quad \text{with} \quad m(t) \cong \frac{di_{out}}{di_{RF}} = \left[f(u,i) + i \cdot \frac{df(u,i)}{di} \right]_{i=IC}$$

- The conversion gains for the different harmonics can be computed as the average dc output current when the input signal is a cosine

$$i_{RF}(t) = a_{RF} \cdot \cos(\omega_{RF} \cdot t) \quad \text{with} \quad a_{RF} = \frac{A_{RF}}{I_{spec}}$$

$$G_{RF,k} = \left| \frac{1}{T_{LO}} \int_0^{T_{LO}} m(t) \cdot \cos(k \cdot \omega_{LO} \cdot t) \cdot dt \right|$$

Note: see Appendix for the demonstration of this formula

- Note that, if the transistors are perfectly matched, the conversion gains for k even are all null

$$G_{RF,k} = 0 \quad \text{for } k \text{ even}$$

- This means that, since $G_{RF,0} = 0$, the **flicker noise** coming from the stages preceding the mixer (such as the LNA) is **suppressed** (actually only limited by the mismatch)

Conversion Gain $G_{RF,1}$

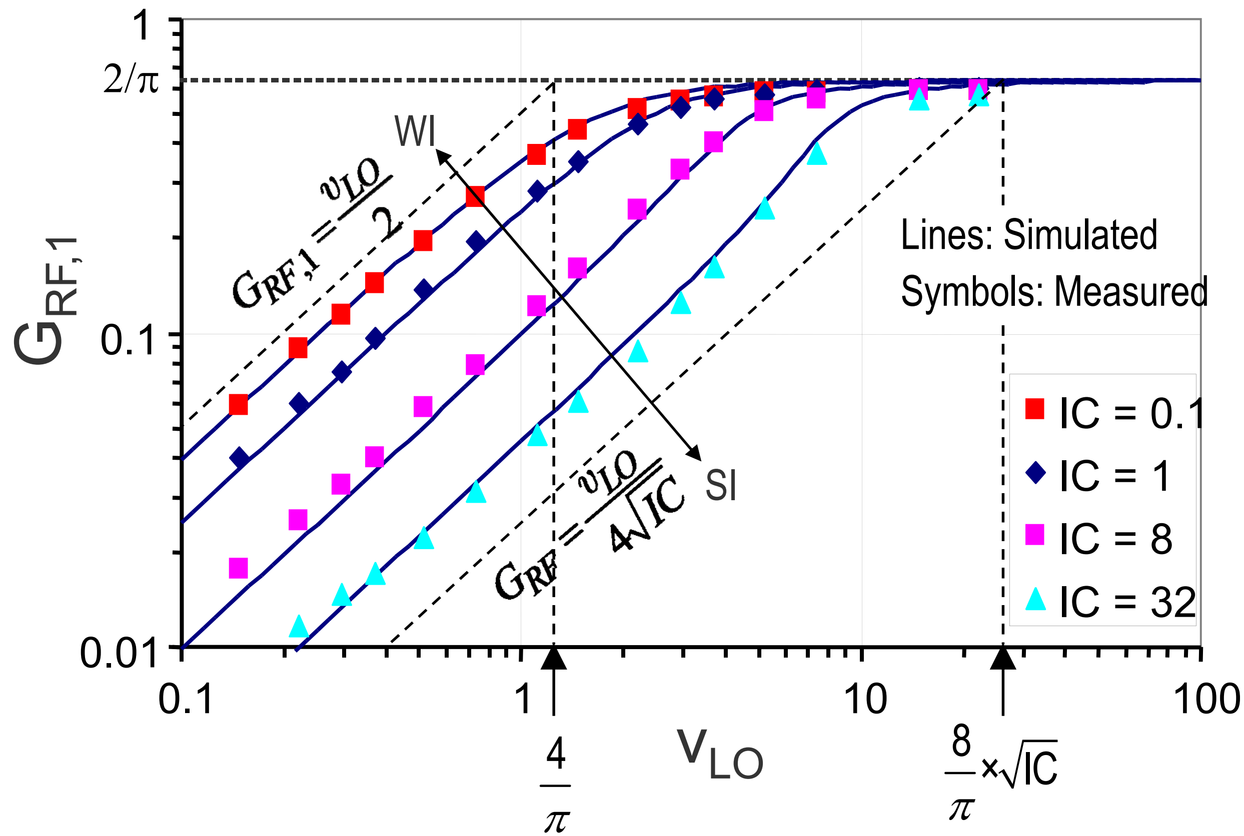
- The conversion gain at the fundamental $G_{RF,1}$ cannot be calculated analytically except for the asymptotic case of small and large LO amplitudes
- For **large** LO amplitudes, the modulation signal can be considered as a square wave and the conversion gain is given by

$$G_{RF,1} \cong \frac{2}{\pi} \quad \text{for} \quad v_{LO} \gg \sqrt{1+2IC}$$

- This is valid for both WI and SI when the differential pair is hard switched
- For **small** LO amplitude we have

$$G_{RF,1} \cong \begin{cases} \dfrac{v_{LO}}{2} & \text{for} \quad v_{LO} \ll 1 \quad \text{(WI)} \\ \dfrac{v_{LO}}{4\sqrt{IC}} & \text{for} \quad v_{LO} \ll \sqrt{2IC} \quad \text{(SI)} \end{cases}$$

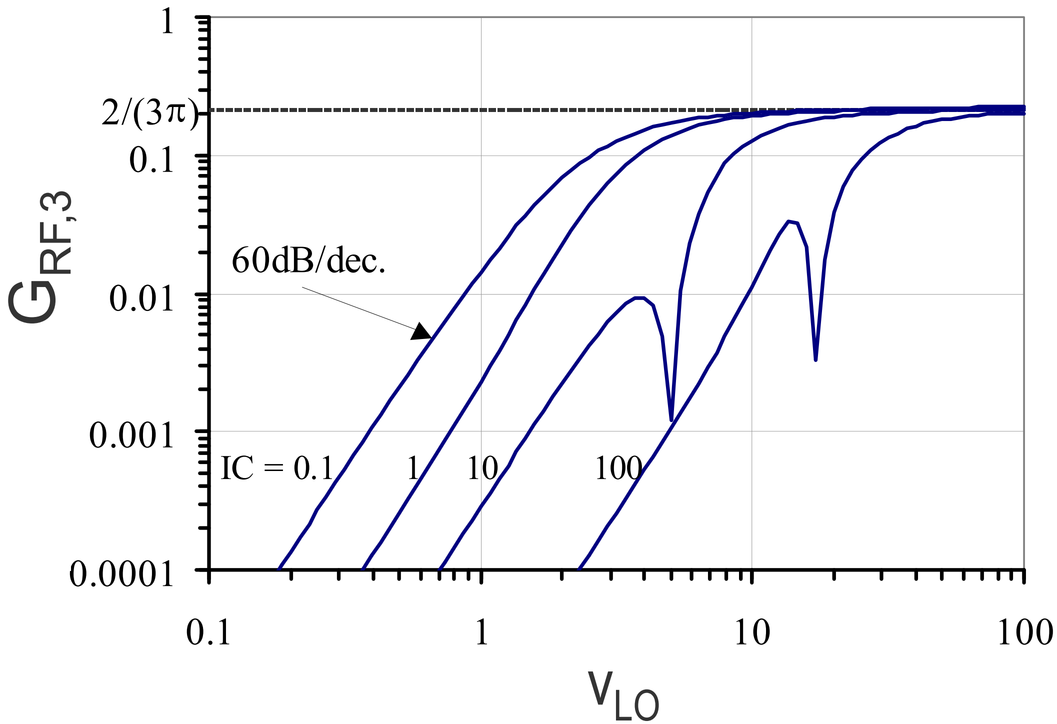
Conversion Gain $G_{RF,1}$



- The conversion gain $G_{RF,1}$ is not easy to calculate between the asymptotes
- It is plotted below versus the normalized LO amplitude v_{LO} for different inversion coefficients IC

T. Melly et al., JSSC, Jan. 2001.

Conversion Gain $G_{RF,3}$



- The gain for small LO amplitudes is proportional to the k^th power of v_{LO}
- The maximum current gain, when the input current is instantaneously switched from one output terminal to the other, is simply given by

$$G_{RF,k} = \frac{2}{k \cdot \pi} \quad \text{for} \quad v_{LO} \gg \sqrt{1+2IC} \quad \text{and} \quad k = 1,3,5,\ldots$$

LO Gain

- The normalized transconductance gain $G_{LO,k}$ between the single ended LO voltage and the differential output current can be calculated as

$$G_{LO,k} = G_{m0} \cdot \frac{1}{T_{LO}} \int_0^{T_{LO}} \frac{df(u,IC)}{du} \cdot \cos\left(k \cdot \omega_{LO} \cdot t\right) \cdot dt \quad \text{where} \quad G_{m0} \triangleq \frac{I_b}{2nU_T}$$

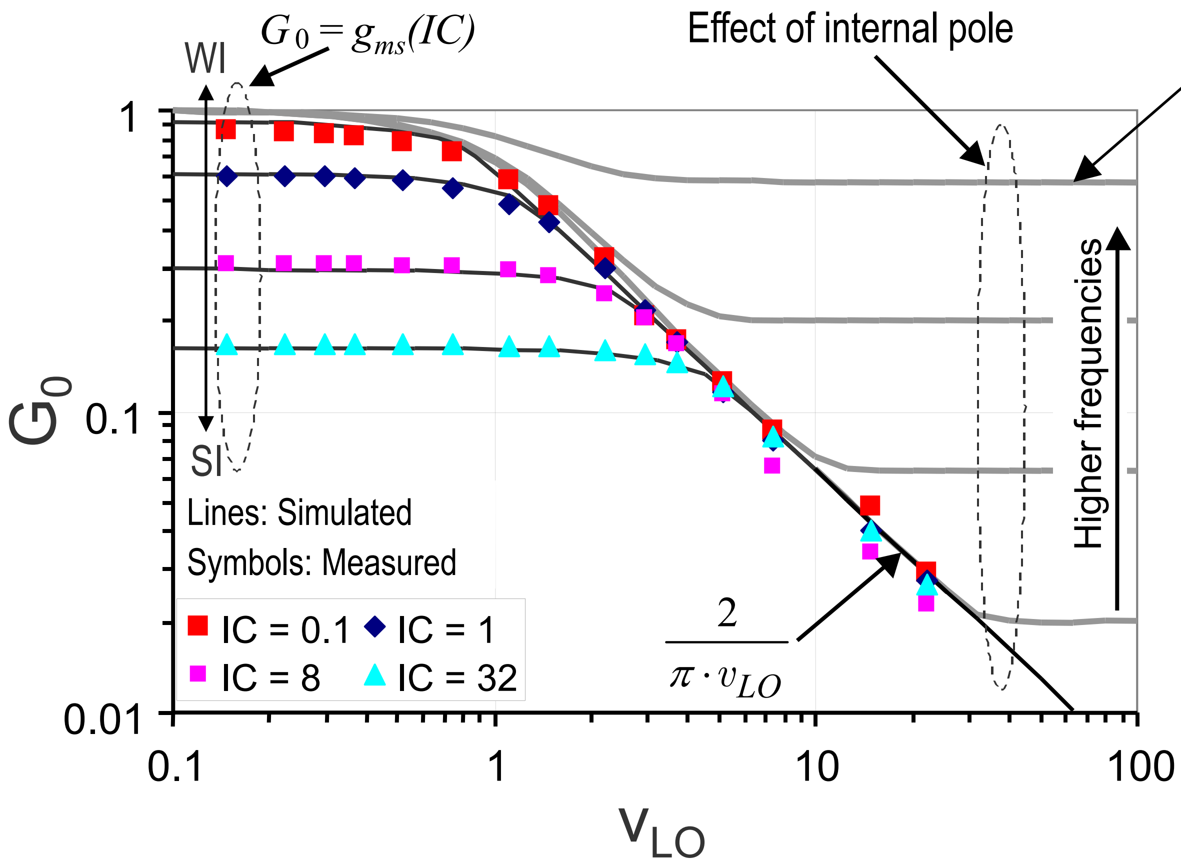
- The LO gain for $k = 0$ is of particular interest to evaluate the flicker noise and offset up-conversion to IF

$$G_{LO,0} = G_{m0} \cdot \underbrace{\frac{1}{T_{LO}} \int_0^{T_{LO}} \frac{df(u,IC)}{du} \cdot dt}_{=G_0} = G_{m0} \cdot G_0$$

- G_0 can unfortunately not be calculated analytically, but a good approximation is given by

$$G_0 \cong \begin{cases} g_{ms}(IC)\sqrt{1-\left(\dfrac{\pi}{4} \cdot g_{ms}(IC) \cdot v_{LO}\right)^2} & \text{for} \quad v_{LO} \leq \dfrac{2}{\pi} \cdot \dfrac{\sqrt{2}}{g(IC)} \\ \dfrac{2}{\pi \cdot v_{LO}} & \text{for} \quad v_{LO} > \dfrac{2}{\pi} \cdot \dfrac{\sqrt{2}}{g(IC)} \end{cases} \quad \text{where} \quad g(IC) \triangleq \frac{G_{ms} \cdot U_T}{I_D} = \frac{\sqrt{1+4IC}-1}{2IC}$$

LO Gain G_0



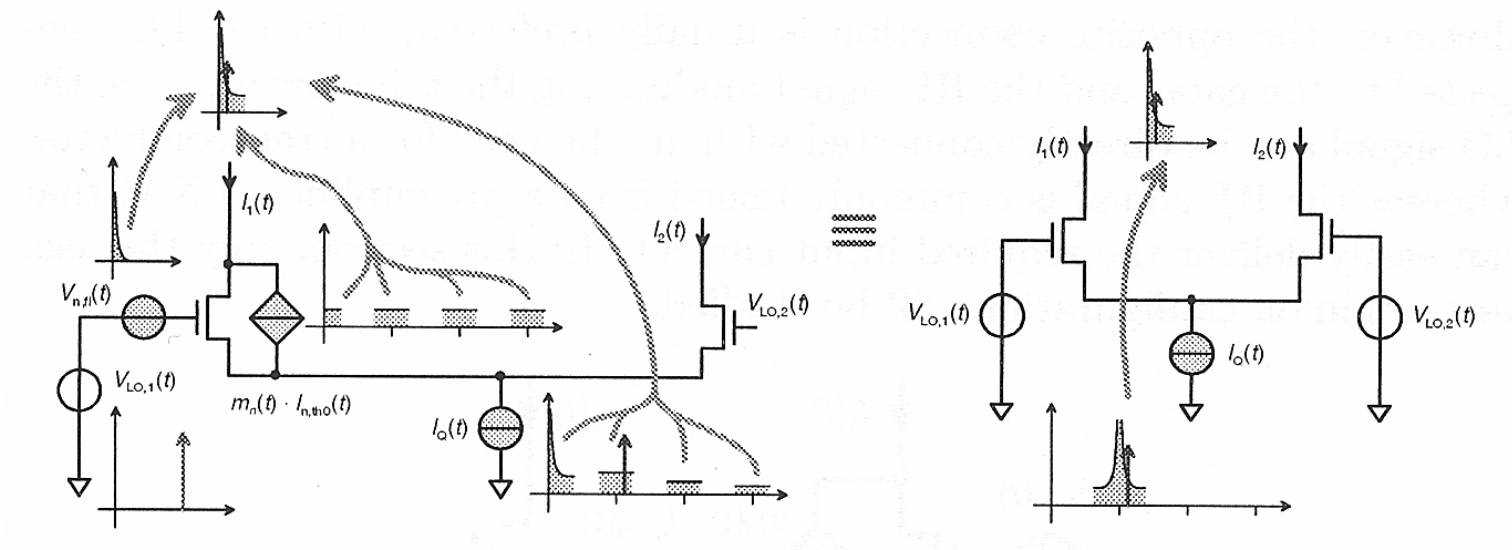
$$G_{0,\min} \cong \frac{1}{\sqrt{1+\left(\frac{\pi}{2}\cdot\frac{\omega_c}{\omega_{LO}}\right)^2}}$$

where

$$\omega_c \triangleq \frac{2G_{ms}}{C_s}$$

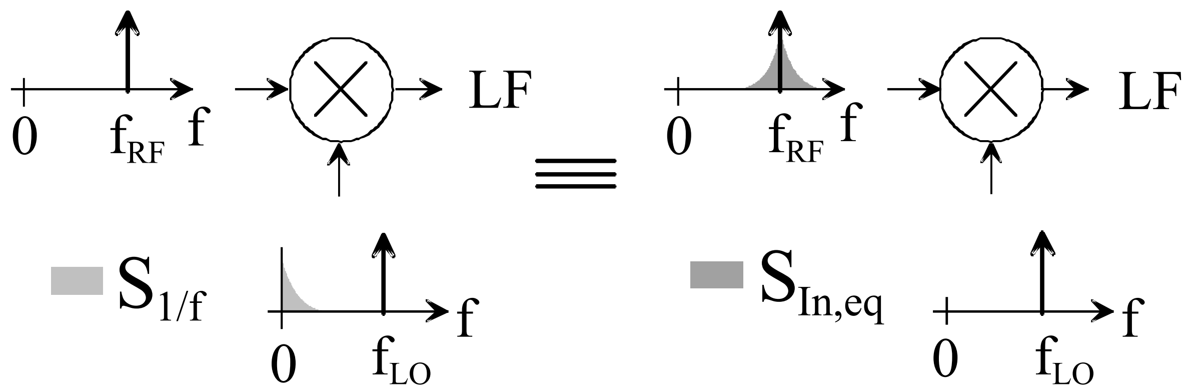
- For large LO amplitude, the LO gain $G_{LO,0}$ decreases with $2/(\pi \cdot v_{LO})$ up to a point limited by the parasitic capacitance C_s at the common source node

Noise Sources and Noise Mixing



- There are several noise sources that should be accounted for
  - The noise from the RF input $I_q(t)$ which is downconverted to the IF
  - The noise from the LO input and particularly the flicker noise $V_{n,fl}(t)$ at the DP gates that is up-converted to the IF
  - The channel noise of the DP which is modulated by the periodic drain current $m_n(t) \cdot I_{n,th0}(t)$
- Each of these sources have to be treated separately deriving the appropriate gains and PSD at the output and referring them to the input

Input Referred DSB Noise PSD



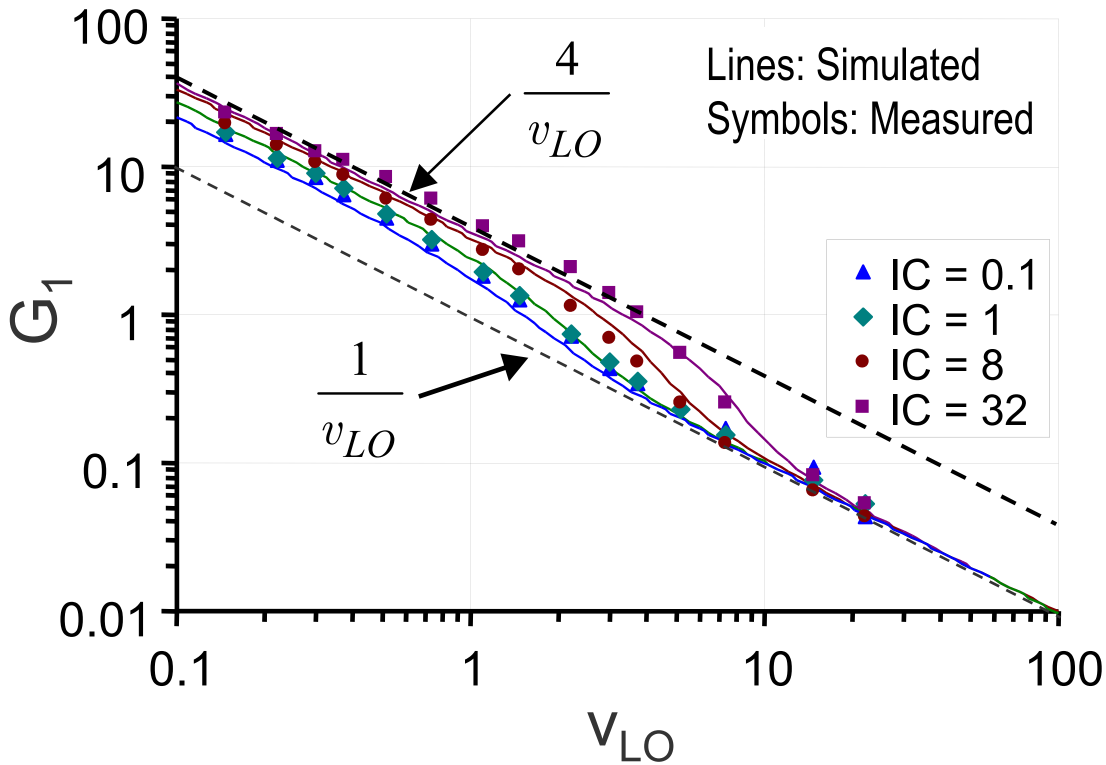
- The noise and particularly the flicker noise coming from the LO input of the DP and found at the mixer output can advantageously be referred to the mixer input

$$\frac{S_{I_{in,eq}}(\omega \pm \omega_{IF})}{S_{1/f}(\omega_{IF})} = \frac{1}{2}\left(\frac{G_{LO,0}}{G_{RF,1}}\right)^2 = \frac{1}{2} \cdot G_{m0}^2 \cdot G_1^2(IC, v_{LO})$$

- With the asymptotes obtained previously for large and small v_{LO}, we get

$$G_1 = \begin{cases} \dfrac{1}{v_{LO}} & \text{for} \quad v_{LO} \gg \sqrt{1+2IC} \\ \dfrac{4}{v_{LO}} & \text{for} \quad v_{LO} \ll 1 \quad \text{(WI)} \end{cases}$$

Input Referred DSB Noise



- Assuming the same flicker noise PSD at the gate of the mixer transistors, the input-referred flicker noise PSD is 4 times (12dB) larger at low LO amplitude and in SI than at large LO amplitude
- At small LO amplitudes, the input referred current flicker noise is 6 dB higher in SI compared to WI

Thermal Noise

- Since the current in the DP transistor is strongly varying, the thermal noise sources of the DP transistors are non-stationary
- They are also modulated by the mixer
- The output noise due to the thermal noise sources in WI is given by

$$\frac{S_{I_{out,th}}}{S_{I_{out,0}}} = \frac{2}{\pi \cdot v_{LO}} \cdot \tanh\left(\frac{\pi \cdot v_{LO}}{2}\right) \cong \frac{2}{\pi \cdot v_{LO}} \quad \text{for } v_{LO} \gg 1$$

Summary

- At small LO amplitudes, the input referred current flicker noise is 6 dB higher in strong inversion mode than in weak inversion
- Without taking into account linearity issues, the ideal mode of operation for the mixing transistors is therefore in weak inversion mode, since the mixing gain is maximum, the equivalent input flicker noise is minimum and gain saturation is reached for smaller LO amplitude
- However, the effect of the input pole degrades both gain and flicker-noise rejection for the small inversion factor, because, if the bias current is kept constant, transistors of larger active area are needed
- As a result, in practice and for a sufficiently advanced technology, the ideal mode of operation is usually situated in the moderate inversion region (i.e., $IC \cong 1$)

Appendix 1 – Output Spectrum for SBM

- The differential output current is given by

$$I_{out}(t) = \left(I_b + I_{RF}(t)\right) \cdot m(t)$$

where $m(t)$ is the modulation signal which under hard-switching (i.e. sufficiently lare LO amplitude) corresponds to a square wave having a Fourier Transform (FT) given by

$$M(f) = \sum_{\substack{n=-\infty \\ n\ odd}}^{+\infty} M_n \cdot \delta(f - n \cdot f_{LO}) \quad \text{with} \quad M_n = \frac{2}{j\pi n}$$

- The FT of $I_{out}(t)$ is given by

$$I_{out}(f) = \left(I_b \cdot \delta(f) + I_{RF}(f)\right) * \sum_{\substack{n=-\infty \\ n\ odd}}^{+\infty} M_n \cdot \delta(f - n \cdot f_{LO})$$

$$= \underbrace{I_b \cdot \sum_{\substack{n=-\infty \\ n\ odd}}^{+\infty} M_n \cdot \delta(f - n \cdot f_{LO})}_{\text{up-converted dc to odd harmonics of } f_{LO}} + \underbrace{\sum_{\substack{n=-\infty \\ n\ odd}}^{+\infty} M_n \cdot I_{RF}(f - n \cdot f_{LO})}_{\text{includes down-converted RF signal } (n=\pm 1)}$$

Appendix 1 – LO Feedtrough for SBM

- The LO feedthrough comes from the up-conversion of the dc bias current I_b
- The one at f_{LO} is the strongest and its FT is given for $n = \pm 1$

$$I_{out-LO}(f) = I_b \cdot \left[-\frac{2}{j\pi} \cdot \delta(f + f_{LO}) + \frac{2}{j\pi} \cdot \delta(f - f_{LO}) \right] = \frac{4}{\pi} \cdot \frac{j}{2} \cdot [\delta(f + f_{LO}) - \delta(f - f_{LO})]$$

which corresponds to a sinewave in the time domain given by

$$I_{out-LO}(t) = \frac{4}{\pi} \cdot I_b \cdot \sin\left(2\pi f_{LO} t\right)$$

Appendix 1 – Output Signal of the SBM

- The FT corresponding to the RF signal downconverted to the IF frequency $f_{IF} = f_{RF} - f_{LO}$ is obtained by setting $n = \pm 1$ in the 2nd term of the output spectrum

$$I_{out}(f) = \frac{2}{j\pi} \cdot \left[I_{RF}(f - f_{LO}) - I_{RF}(f + f_{LO}) \right]$$

- Assuming the RF signal is a cosine given by

$$I_{RF}(t) = A_{RF} \cdot \cos(\omega_{RF} t) \quad \leftrightarrow \quad I_{RF}(f) = \frac{A_{RF}}{2} \cdot \left(\delta(f + f_{RF}) + \delta(f - f_{RF}) \right)$$

- Replacing IRF(f) in the above expression leads to

$$I_{out}(f) = \frac{2}{j\pi} \cdot \frac{A_{RF}}{2} \cdot \left[\delta(f + f_{RF} - f_{LO}) + \delta(f - f_{RF} - f_{LO}) - \delta(f + f_{RF} + f_{LO}) - \delta(f - f_{RF} + f_{LO}) \right]$$

$$= \frac{2}{j\pi} \cdot \frac{A_{RF}}{2} \cdot \left[\delta(f + f_{IF}) + \delta(f - f_{IF} - 2f_{LO}) - \delta(f + f_{IF} + 2f_{LO}) - \delta(f - f_{IF}) \right]$$

- The output FT at IF is then given by

$$I_{out}(f) = \frac{2}{j\pi} \cdot \frac{A_{RF}}{2} \cdot \left[\delta(f + f_{IF}) - \delta(f - f_{IF}) \right] = -\frac{2}{\pi} \cdot A_{RF} \cdot \frac{j}{2} \cdot \left[\delta(f + f_{IF}) - \delta(f - f_{IF}) \right]$$

- which corresponds to a sinewave given by

$$I_{out}(t) = -\frac{2}{\pi} \cdot A_{RF} \cdot \sin(2\pi f_{IF} t)$$

Appendix 2 – Conversion Gain of the SBM

- The mixer output current is simply the product of the RF signal times a periodic modulation signal $m(t)$ of period T_{LO}

$$i_{out}(t) = i(t) \cdot \underbrace{f\left(u(t), i(t)\right)}_{m(t)} = i(t) \cdot m(t)$$

- The shape of the modulation signal $m(t)$ depends on the LO amplitude A_{LO} (v_{LO}), the bias current I_b (IC) and the DP nonlinearity $f(u,i)$
- The spectrum of $i_{out}(t)$ is given by (double-sided FT)

$$I_{out}(f) \triangleq F\left\{i_{out}(t)\right\} = I(f) * M(f) = I(f) * \sum_{n=-\infty}^{+\infty} M_n \cdot \delta(f - nf_{LO}) = \sum_{n=-\infty}^{+\infty} M_n \cdot I(f - nf_{LO})$$

with $I(f) \triangleq F\left\{i(t)\right\}$ and $M(f) \triangleq F\left\{m(t)\right\}$

- The Fourier coefficient M_n are defined as

$$M_n = \frac{1}{T_{LO}} \cdot \int_0^{T_{LO}} m(t) \cdot e^{-jn2\pi f_{LO} t} \cdot dt \quad \text{with} \quad m(t) \triangleq f\left(u(t), i(t)\right)$$

- Note that it is not always possible to calculate these coefficients M_n analytically

Appendix 2 – Conversion Gain of the SBM

- The conversion from the band around $k \cdot \omega_{LO}$ to the band around ω_{IF} is obtained by assuming that the RF signal is a cosine at $k \cdot \omega_{LO} + \omega_{IF}$

$$i_{RF}(t) = a_{RF} \cdot \cos(\omega_{RF} \cdot t) \quad \text{with} \quad a_{RF} = \frac{A_{RF}}{I_{spec}} \quad \text{and} \quad \omega_{RF} = k \cdot \omega_{LO} + \omega_{IF}$$

$$I_{RF}(f) = F\{i_{RF}(t)\} = \frac{a_{RF}}{2} \cdot [\delta(f + f_{RF}) + \delta(f - f_{RF})]$$

- The signal at IF is obtained by first evaluating the are shifted by $k \cdot \omega_{LO}$ or equivalently isolating the terms $n = \pm k$ in the Fourier transform expression of $I_{out}(f)$

$$\begin{aligned} I_{out}(f)\big|_{n=-k,+k} &= M_{-k} \cdot \frac{a_{RF}}{2} \cdot [\delta(f + f_{RF} + k \cdot f_{LO}) + \delta(f - f_{RF} + k \cdot f_{LO})] \\ &+ M_{+k} \cdot \frac{a_{RF}}{2} \cdot [\delta(f + f_{RF} - k \cdot f_{LO}) + \delta(f - f_{RF} - k \cdot f_{LO})] \\ &= M_{-k} \cdot \frac{a_{RF}}{2} \cdot [\delta(f + f_{IF} + 2k \cdot f_{LO}) + \delta(f - f_{IF})] \\ &+ M_{+k} \cdot \frac{a_{RF}}{2} \cdot [\delta(f + f_{IF}) + \delta(f - f_{IF} - 2k \cdot f_{LO})] \end{aligned}$$

Appendix 2 – Conversion Gain of the SBM

- Isolating the frequency components that fall into the IF band around $f_{IF} = f_{RF} - k \cdot f_{LO}$ and $-f_{IF} = k \cdot f_{LO} - f_{RF}$ and dropping the others leads to

$$I_{IF}(f) = \frac{a_{RF}}{2} \cdot \left[M_{-k} \cdot \delta(f - f_{IF}) + M_{+k} \cdot \delta(f + f_{IF}) \right]$$

where the Fourier coefficients M_{+k} and M_{-k} are given by

$$M_{\pm k} = \frac{1}{T_{LO}} \cdot \int_0^{T_{LO}} m(t) \cdot e^{\mp jk2\pi f_{LO} t} \cdot dt$$

- The signal power at IF is then given by

$$P_{IF} = \left(\frac{a_{RF}}{2} \right)^2 \cdot \left(\left| M_{-k} \right|^2 + \left| M_{+k} \right|^2 \right) = \left(\frac{a_{RF}}{2} \right)^2 \cdot 2 \left| M_{+k} \right|^2 = \frac{a_{RF}^2}{2} \cdot \left| M_{+k} \right|^2$$

since for a real signal we have

$$M_{-k} = M_{+k}^* \quad \text{and hence} \quad \left| M_{-k} \right| = \left| M_{+k} \right|$$

- The conversion gain is then given by

$$G_{RF,k} = \frac{\sqrt{P_{IF,k}}}{\sqrt{P_{RF}}} = \frac{\frac{a_{RF}}{\sqrt{2}} \cdot \left| M_{+k} \right|}{\frac{a_{RF}}{\sqrt{2}}} = \left| M_{+k} \right|$$

Appendix 2 – Conversion Gain of the SBM

- The magnitude of the Fourier coefficient M_k can be related to the cosine and sine Fourier coefficients a_n and b_n respectively

$$|M_k| = \frac{1}{2} \cdot \sqrt{a_k^2 + b_k^2}$$

where a_n and b_n are given by

$$a_k = \frac{2}{T_{LO}} \cdot \int_0^{T_{LO}} m(t) \cdot \cos(2\pi k f_{LO} t) \cdot dt \quad \text{and} \quad b_k = \frac{2}{T_{LO}} \cdot \int_0^{T_{LO}} m(t) \cdot \sin(2\pi k f_{LO} t) \cdot dt$$

- Since the nonlinear function f(u,i) is odd, the Fourier coefficients b_n are all null, hence we have

$$|M_k| = \frac{|a_k|}{2}$$

- The conversion gain can then be written in terms of a_n as

$$G_{RF,k} = |M_{+k}| = \frac{|a_k|}{2} = \left| \frac{1}{T_{LO}} \cdot \int_0^{T_{LO}} m(t) \cdot \cos(2\pi k f_{LO} t) \cdot dt \right|$$

Appendix 2 – Conversion Gain of the SBM in WI

- In WI, the nonlinear function $f(u,i)$ is independent of i and is simply given by

$$f(u,i) = \tanh(u)$$

- For $v_{LO} \gg 1$, $m(t)$ can be considered as a square wave (unity amplitude), hence

$$|M_{+k}| = \frac{2}{k\pi}$$

- The conversion gain (which is actually a current gain) in this case is given by

$$G_{RF,k} = \frac{2}{k\pi}$$

Appendix 2 – Conversion Gain of the SBM in WI

- For $v_{LO} \ll 1$, $m(t)$ remains a cosine with amplitude v_{LO}, hence

$$M_{-1} = M_{+1} = \frac{v_{LO}}{2}$$

- The signal at f_{IF} is then given by

$$I_{IF}(f) = \frac{v_{LO}}{2} \cdot \frac{a_{RF}}{2} \cdot \left[\delta(f + f_{IF}) + \delta(f - f_{IF})\right]$$

$$i_{IF}(t) = F^{-1}\left\{I_{IF}(f)\right\} = \frac{v_{LO}}{2} \cdot a_{RF} \cdot \cos\left(\omega_{IF} \cdot t\right)$$

- The conversion gain then becomes proportional to the LO amplitude

$$G_{RF} = \frac{v_{LO}}{2} = \frac{A_{LO}}{4nU_T}$$

- Finally, the conversion gain for the DP in WI is given by

$$G_{RF} = \begin{cases} \dfrac{v_{LO}}{2} = \dfrac{A_{LO}}{4nU_T} & \text{for } v_{LO} \ll 1 \\ \dfrac{2}{\pi} & \text{for } v_{LO} \gg 1 \end{cases}$$

Appendix 2 – Conversion Gain of the SBM in SI

- In SI, the nonlinear function $f(u,i)$ is given by

$$f(u,i) = \begin{cases} \dfrac{u}{\sqrt{i}} \cdot \sqrt{1 - \dfrac{u^2}{4i}} & \text{for } u^2 \leq 2i \\ \operatorname{sgn}(u) & \text{for } u^2 > 2i \end{cases}$$

- The Fourier coefficients cannot be calculated analytically
- On the other hand, the RF current can usually be considered as much smaller than the bias current, which allows for the following approximation

$$i_{out} \cong \frac{di_{out}}{di_{RF}} \cdot i_{RF} = m(t) \cdot i_{RF}(t) \quad \text{with} \quad m(t) \cong \frac{di_{out}}{di_{RF}} = \left[f(u,i) + i \cdot \frac{df(u,i)}{di} \right]_{i=IC}$$

- In SI we have

$$\frac{di_{out}}{di_{RF}} = \begin{cases} \dfrac{u}{\sqrt{4IC - u^2}} & \text{for } u^2 \leq 2IC \\ \operatorname{sgn}(u) & \text{for } u^2 > 2IC \end{cases}$$

Appendix 2 – Conversion Gain of the SBM in SI

- For large LO amplitudes, the modulation signal is again a square wave and we get

$$G_{RF} \cong \frac{2}{\pi} \cong 0.64 \quad \text{for} \quad v_{LO} \gg \sqrt{2IC}$$

- For small LO amplitudes, we have

$$m(t) = \frac{di_{out}}{di_{RF}} \cong \frac{u(t)}{2\sqrt{IC}} = \frac{v_{LO}}{2\sqrt{IC}} \cdot \cos(\omega_{LO} t) \quad \text{for} \quad v_{LO} \ll \sqrt{2IC}$$

- and hence

$$M_{-1} = M_{+1} = \frac{v_{LO}}{4\sqrt{IC}} \quad \text{for} \quad v_{LO} \ll \sqrt{2IC}$$

- The signal at f_{IF} is then given by

$$I_{IF}(f) = \frac{v_{LO}}{4\sqrt{IC}} \cdot \frac{a_{RF}}{2} \cdot \left[\delta(f + f_{IF}) + \delta(f - f_{IF})\right]$$

$$i_{IF}(t) = F^{-1}\{I_{IF}(f)\} = \frac{v_{LO}}{4\sqrt{IC}} \cdot a_{RF} \cdot \cos(\omega_{IF} \cdot t)$$

- Finally, the conversion gain for the DP in SI is given by

$$G_{RF} = \begin{cases} \dfrac{v_{LO}}{4\sqrt{IC}} = \dfrac{A_{LO}}{8nU_T\sqrt{IC}} = \dfrac{A_{LO}}{4 \cdot (V_G - V_{T0})} & \text{for } v_{LO} \ll \sqrt{2IC} = \sqrt{2} \cdot \dfrac{V_G - V_{T0}}{2nU_T} \\ \dfrac{2}{\pi} & \text{for } v_{LO} \gg \sqrt{2IC} = \sqrt{2} \cdot \dfrac{V_G - V_{T0}}{2nU_T} \end{cases}$$