

# Neural Networks and Biological Modeling

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## QUESTION SET 12

### Exercise 1: From adaptive integrate-and-fire to the SRM

Consider a leaky integrate-and-fire neuron with a spike triggered adaptive current  $w$

$$\begin{aligned}\tau \frac{du}{dt} &= -(u - u_{rest}) - \alpha R w(t) + RI(t), \\ \tau_w \frac{dw}{dt} &= -w + \tau_w \beta S(t),\end{aligned}\tag{1}$$

where the membrane potential  $u$  is reset to  $u_{rest}$  at the threshold  $u = \theta$ . Here, we assume the neuron fires at given times  $t^f > 0$ , which gives the spike train  $S(t) = \sum_f \delta(t - t^f)$ .

**1.1** Set  $\alpha = 0$  and assume the neuron is at rest at time  $t = t_0$ . Integrate Eq. 1, explicitly including a reset of the membrane potential at the spike times. Hint: Use an adequate pulse current injection for the reset.

After this you can set  $t_0 \rightarrow -\infty$  since the initial time is arbitrary here. Then write the result in the following closed form

$$u(t) = u_{rest} + \int_0^\infty \epsilon(s) I(t-s) ds + \int_0^\infty \eta(s) S(t-s) ds,\tag{2}$$

with two kernels  $\epsilon(t)$  and  $\eta(t)$ . What are the two kernels?

**1.2** Now set  $\alpha = 1$  and additionally assume that at  $t = 0$  the adaptation variable is  $w = 0$ . Derive a closed form expression similar to the one in the last question, by first integrating  $w$  and then  $u$ . What are the two kernels now?

### Exercise 2: Integrate-and-fire model with linear escape rates

Consider a leaky integrate-and-fire neuron with linear escape rate,

$$\rho_I(t|\hat{t}) = \beta[u(t|\hat{t}) - \theta]_+ = \begin{cases} \beta(u(t|\hat{t}) - \theta) & , \text{ if } u(t|\hat{t}) > \theta \\ 0 & , \text{ otherwise} \end{cases}$$

**2.1** Start with the non-leaky integrate-and-fire model by considering the limit of  $\tau_m \rightarrow \infty$ . The membrane potential of the model is then

$$u(t|\hat{t}) = u_r + \frac{1}{C} \int_{\hat{t}}^t I(t') dt'$$

Assume constant input, set  $u_r = 0$  and calculate the hazard and the interval distribution.

**2.2** Consider the leaky integrate-and-fire model with time constant  $\tau_m$  and constant input  $I_0$ . Determine

the membrane potential, the hazard, and the interval distribution.

### Exercise 3: Optimization of a free parameter

Consider a very simple model for the membrane potential at time step  $n$  as a function of a given input:

$$u_n^{model} = RI_n$$

Further, assume you are given measured data  $u_n^{data}$  sampled at the same time steps.

**3.1** Optimize the free scalar parameter  $R$  by minimizing the sum of squared errors

$$E = \sum_n [u_n^{data} - u_n^{model}]^2$$

with respect to this parameter (least squares fit).

**3.2** Calculate the same for constant input  $I_n = I_0$  and interpret the result.

### Exercise 4: Likelihood of a spike train

In an in-vitro experiment, a time-dependent current  $I(t)$  was injected into a neuron for a time  $0 < t < T$  and four spikes were observed at times  $0 < t^{(1)} < t^{(2)} < t^{(3)} < t^{(4)} < T$ .

**4.1** What is the likelihood that this spike train could have been generated by a leaky integrate-and-fire model with linear escape rate defined in exercise 2.

**4.2** Rewrite the likelihood in terms of the interval distribution and hazard of time-dependent renewal theory.