Convolutional Neural Nets

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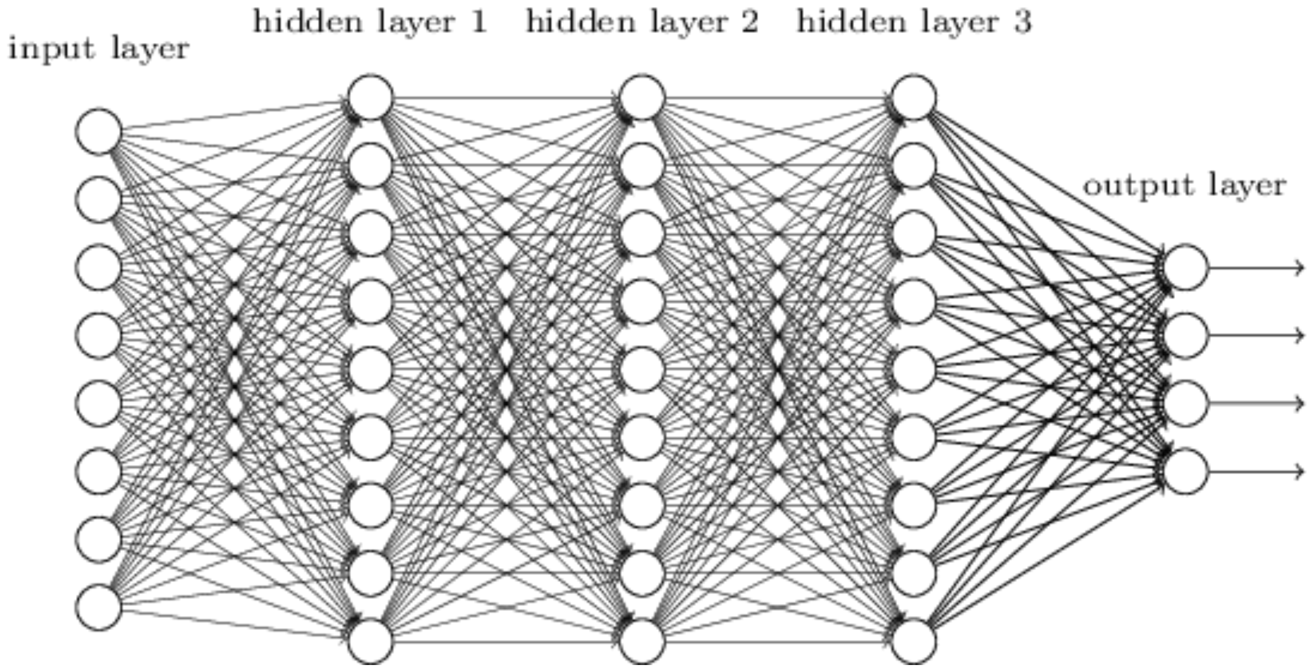
IC-CVLab

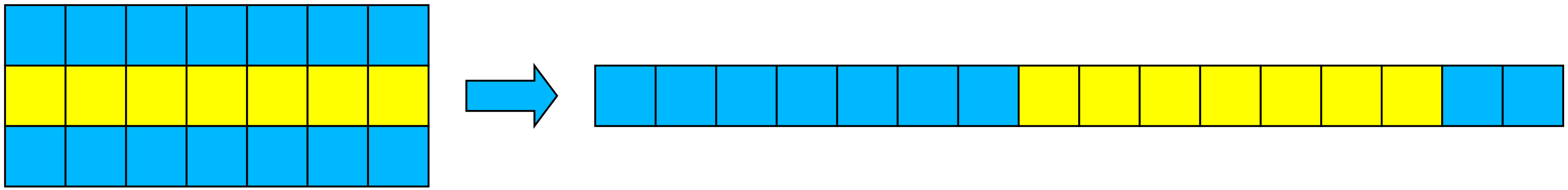
Fully Connected Layers



- The descriptive power of the net increases with the number of layers.
- In the case of a 1D signal, it is roughly proportional to $\prod_n W_n$ where W_n represents the width of a layer.

Processing Digital Images

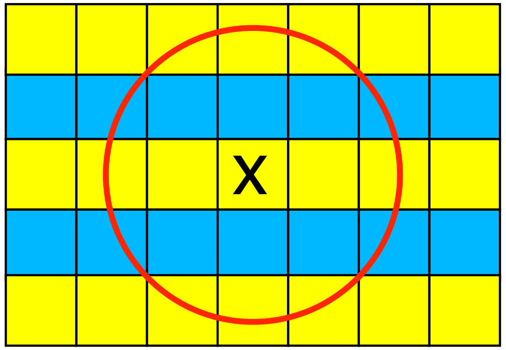
136	134	161	159	163	168	171	173	173	171	166	159	157	155
152	145	136	130	151	149	151	154	158	161	163	163	159	151
145	149	149	145	140	133	145	143	145	145	145	146	148	148
148	143	141	145	145	145	141	136	136	135	135	136	135	133
131	131	129	129	133	136	140	142	142	138	130	128	126	120
115	111	108	106	106	110	120	130	137	142	144	141	129	123
117	109	098	094	094	094	100	110	125	136	141	147	147	145
136	124	116	105	096	096	100	107	116	131	141	147	150	152
152	152	137	124	113	108	105	108	117	129	139	150	157	159
159	157	157	159	135	121	120	120	121	127	136	147	158	163
165	165	163	163	163	166	136	131	135	138	140	145	154	163
166	168	170	168	166	168	170	173	145	143	147	148	152	159
168	173	173	175	173	171	170	173	177	178	151	151	153	156
161	170	176	177	177	179	176	174	174	176	177	179	155	157
161	162	168	176	180	180	180	182	180	175	175	178	180	180



- A MxN image can be represented as an MN vector.
- It can therefore be used an input to an MLP.
- However the neighborhood relationships are then lost.

—> This is not the best approach.

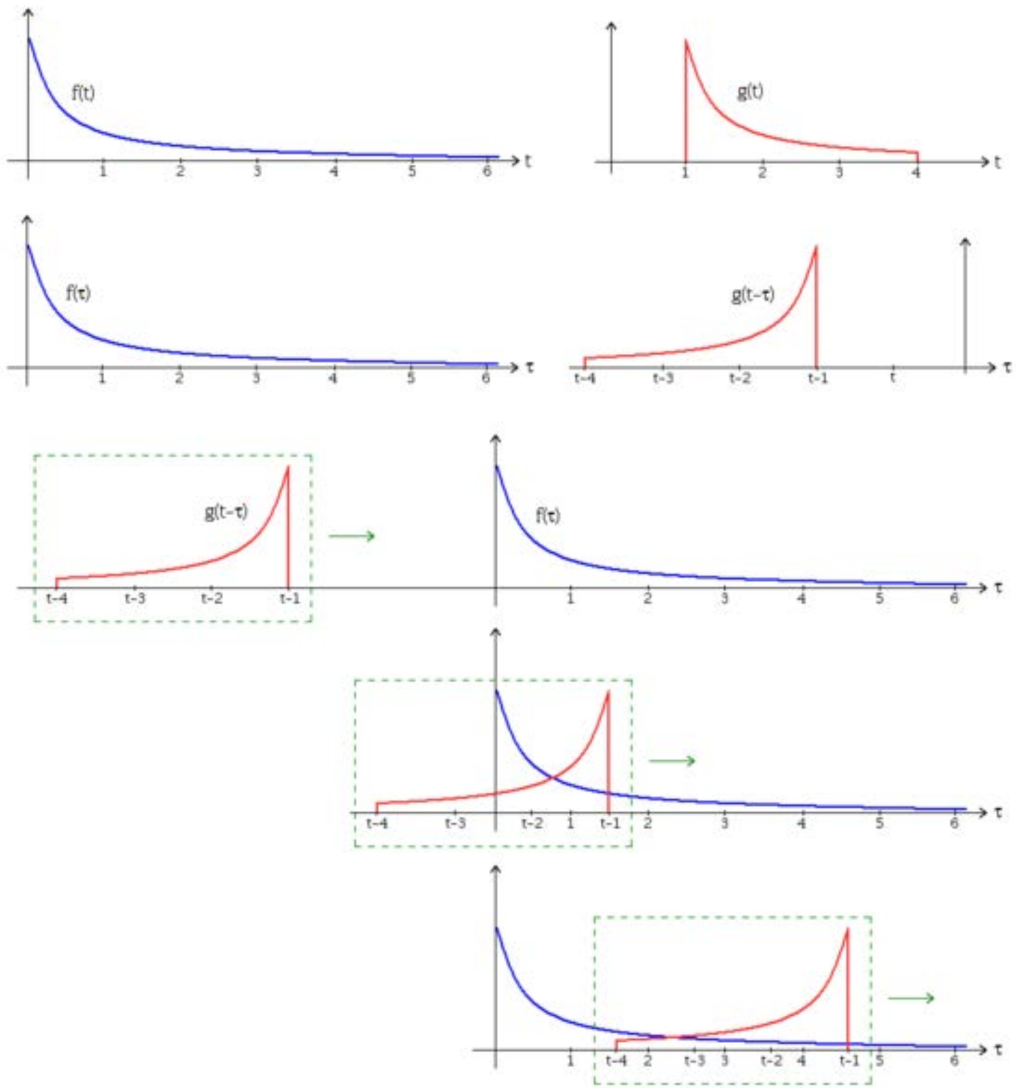
Image Specificities



- In a typical image, the values of neighboring pixels tend to be more highly correlated than those of distant ones.
- An image filter should be translation equivariant.

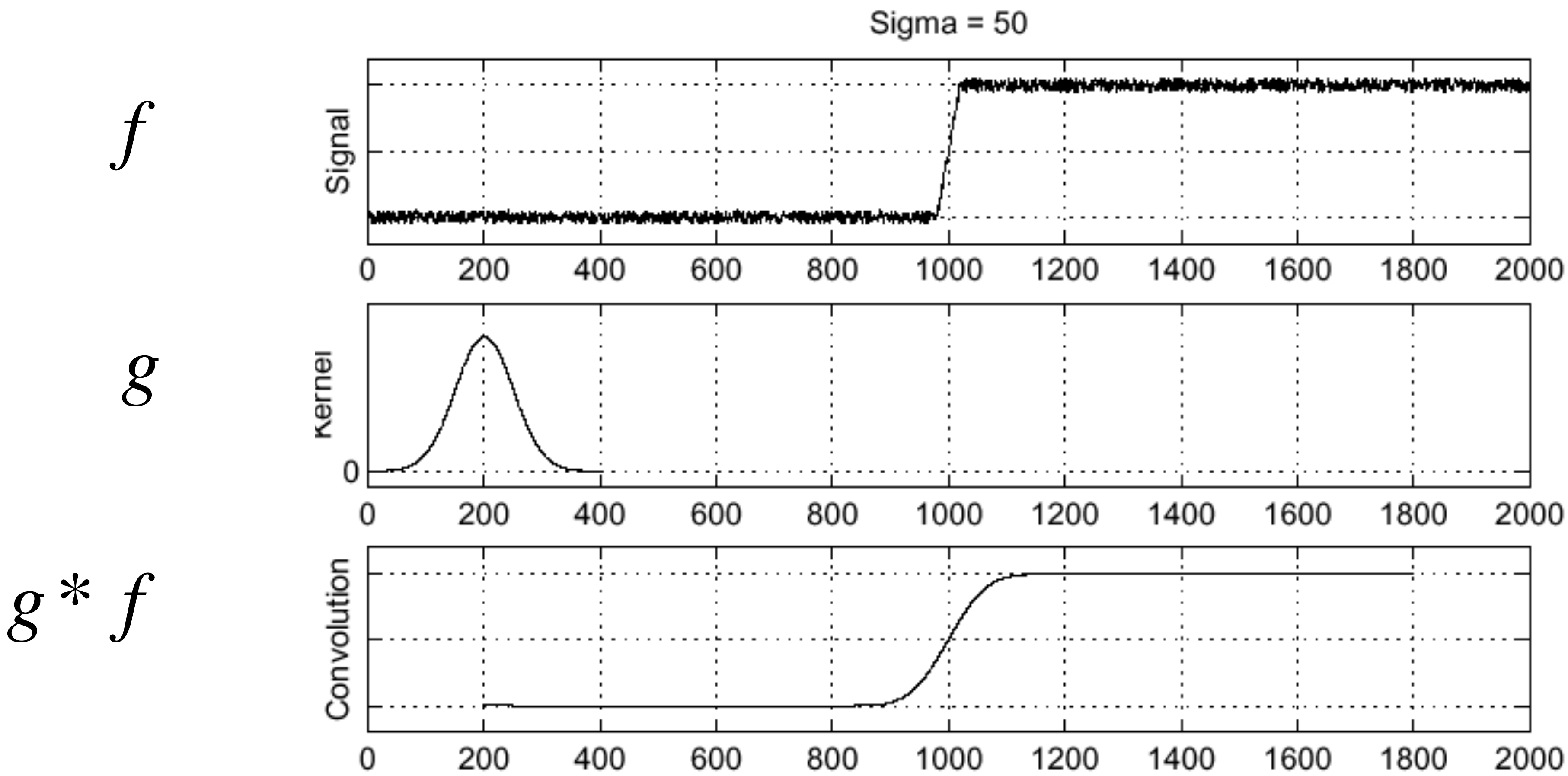
—> These two properties can be exploited to drastically reduce the number of weights required by CNNs using so-called convolutional layers.

1D Convolution in the Continuous Domain



$$g * f(t) = \int_{\tau} g(t-\tau) f(\tau) d\tau$$

Example 1:
Convolution with a Gaussian



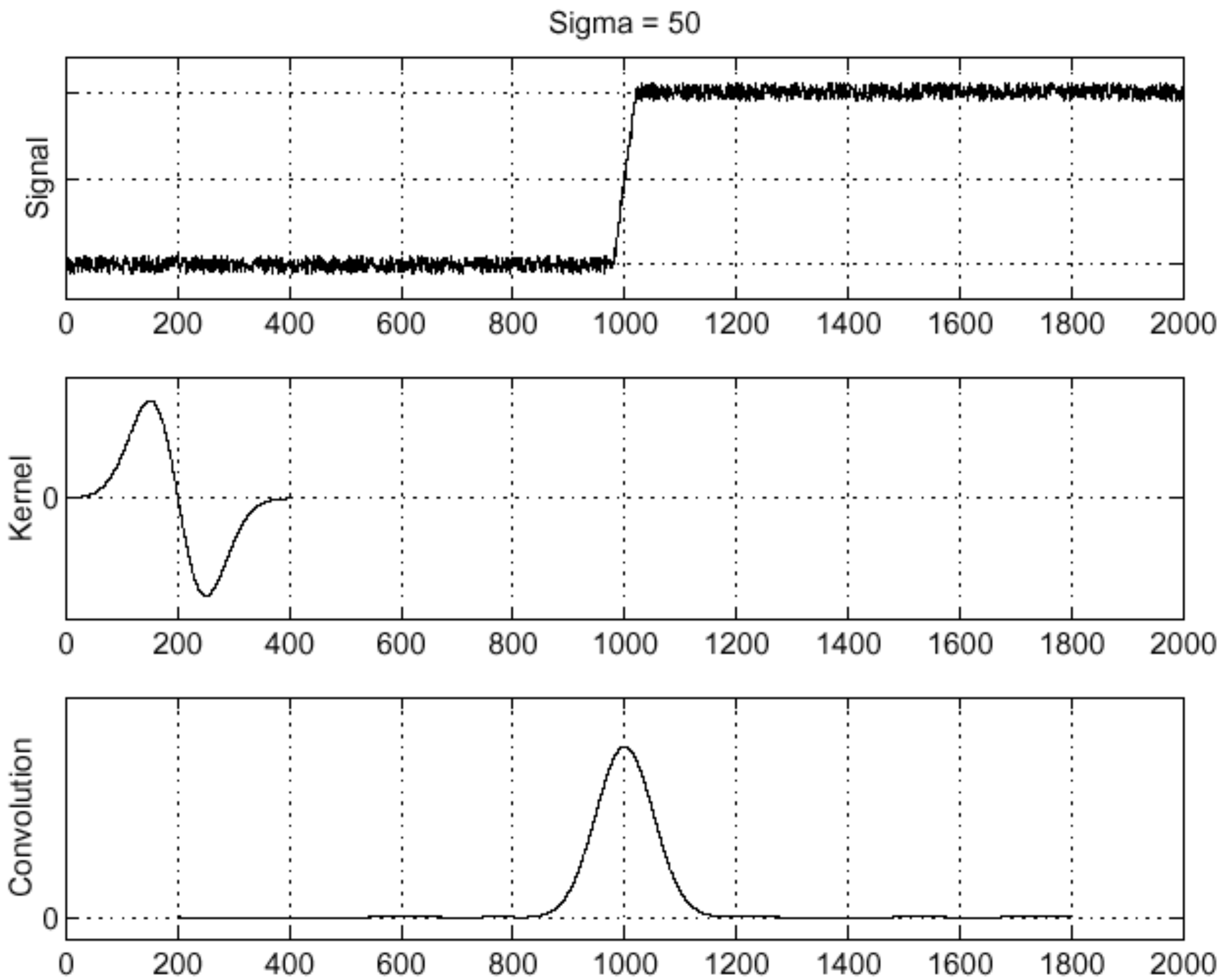
- Each sample is replaces by a weighted average of its neighbors.
- This yields a smoothed version of the original signal.

Example 2: Convolution with the Derivative of a Gaussian

$$f$$

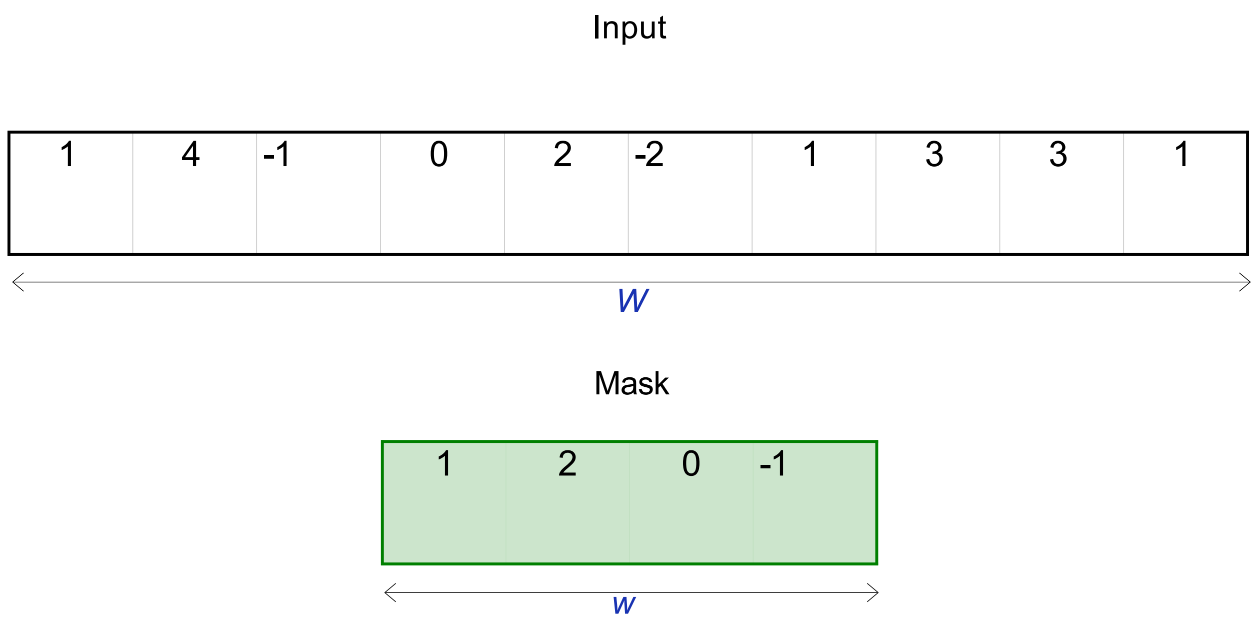
$$\frac{\partial g}{\partial x}$$

$$\frac{\partial}{\partial x}(g * f) = \frac{\partial g}{\partial x} * f$$

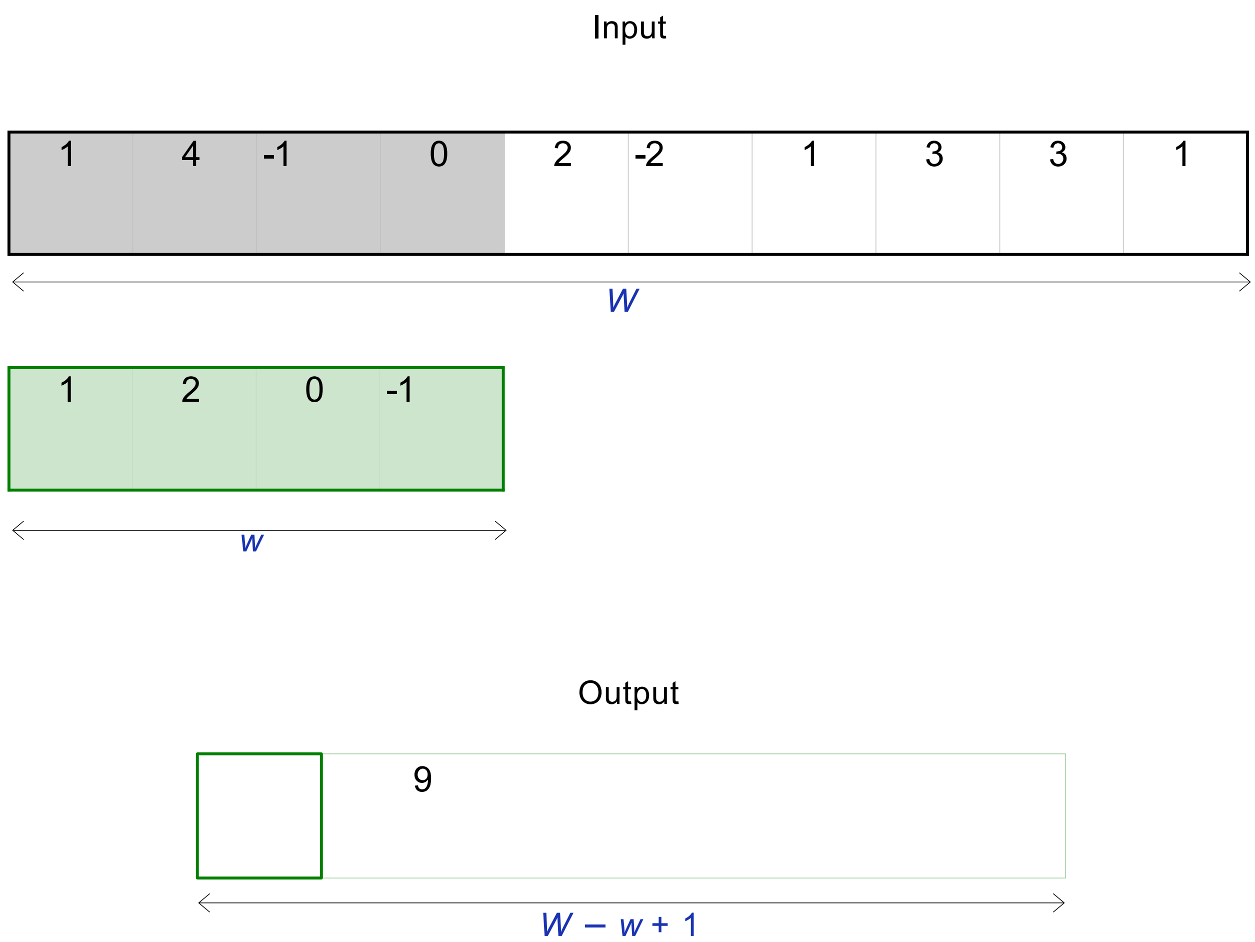


- Convolving with the derivative of a gaussian is the same as smoothing first and then differentiating.

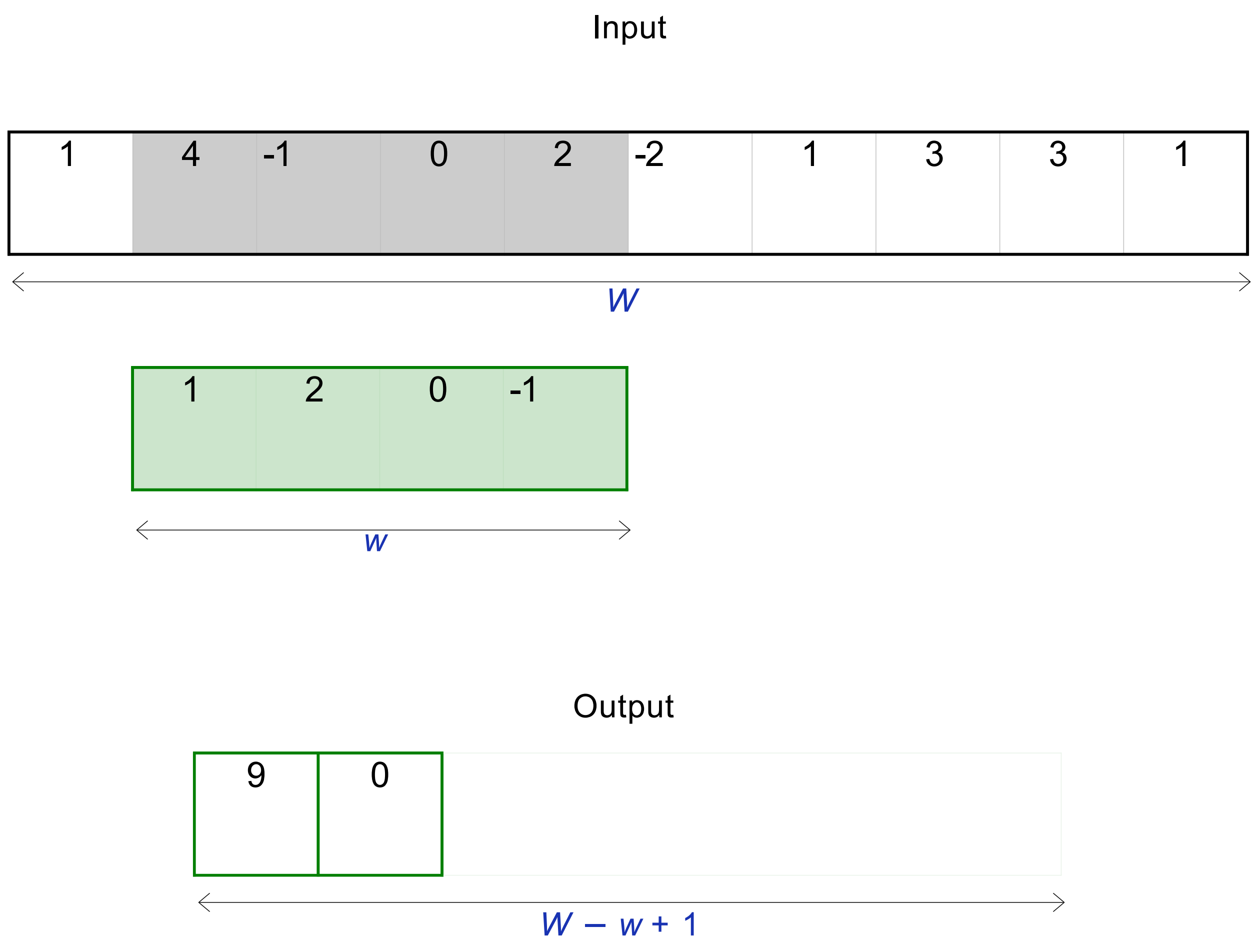
Discrete 1D Convolution



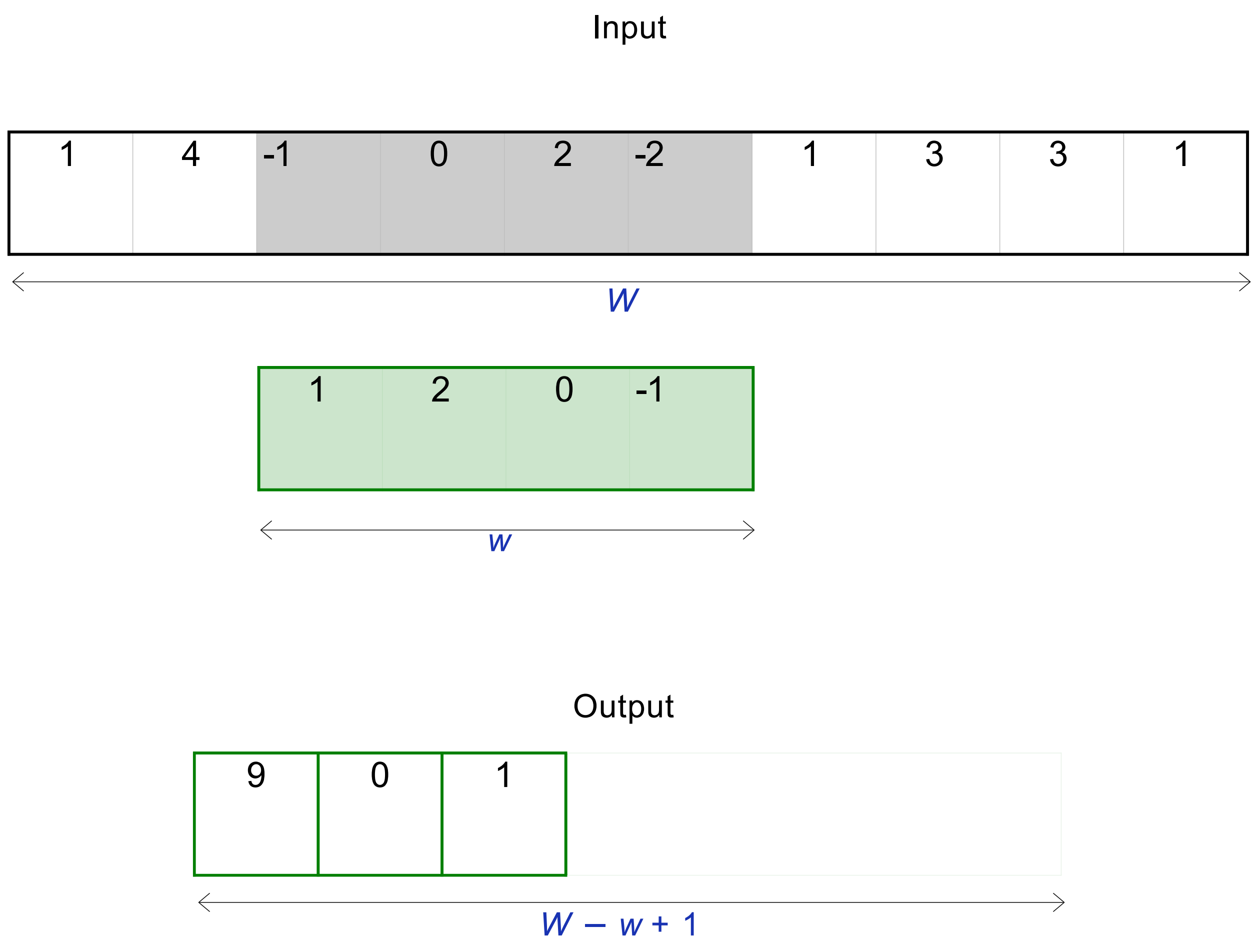
Discrete 1D Convolution



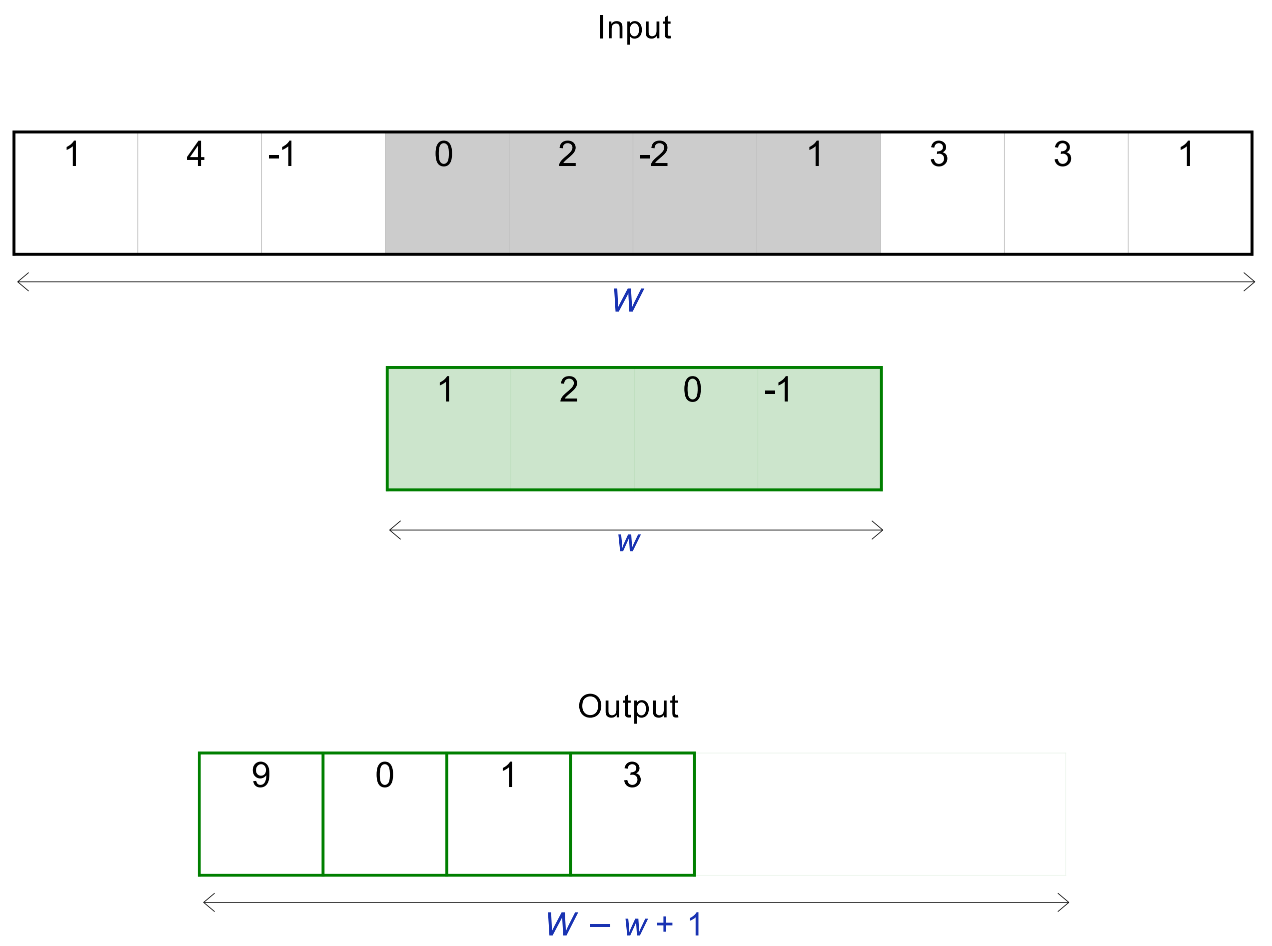
Discrete 1D Convolution



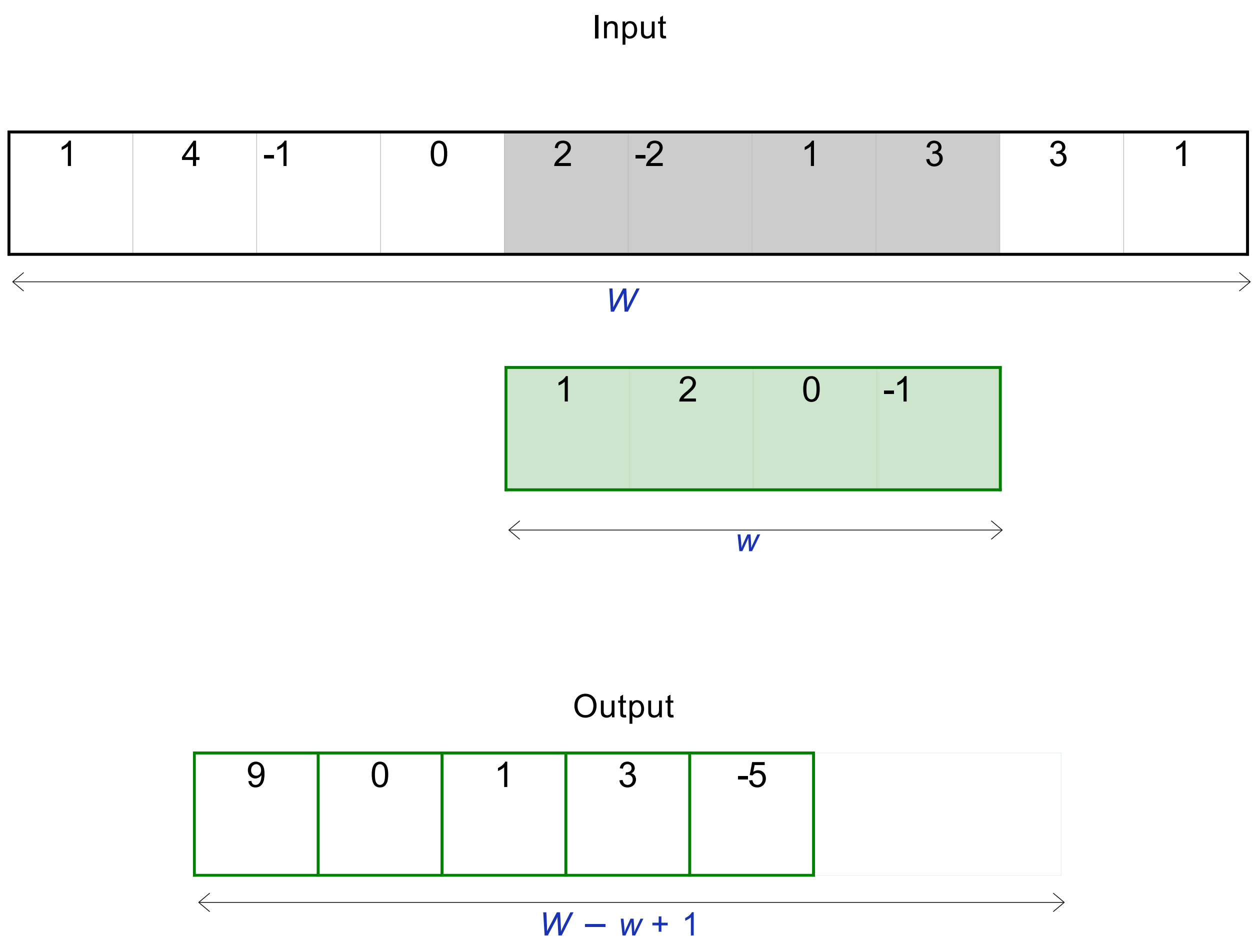
Discrete 1D Convolution



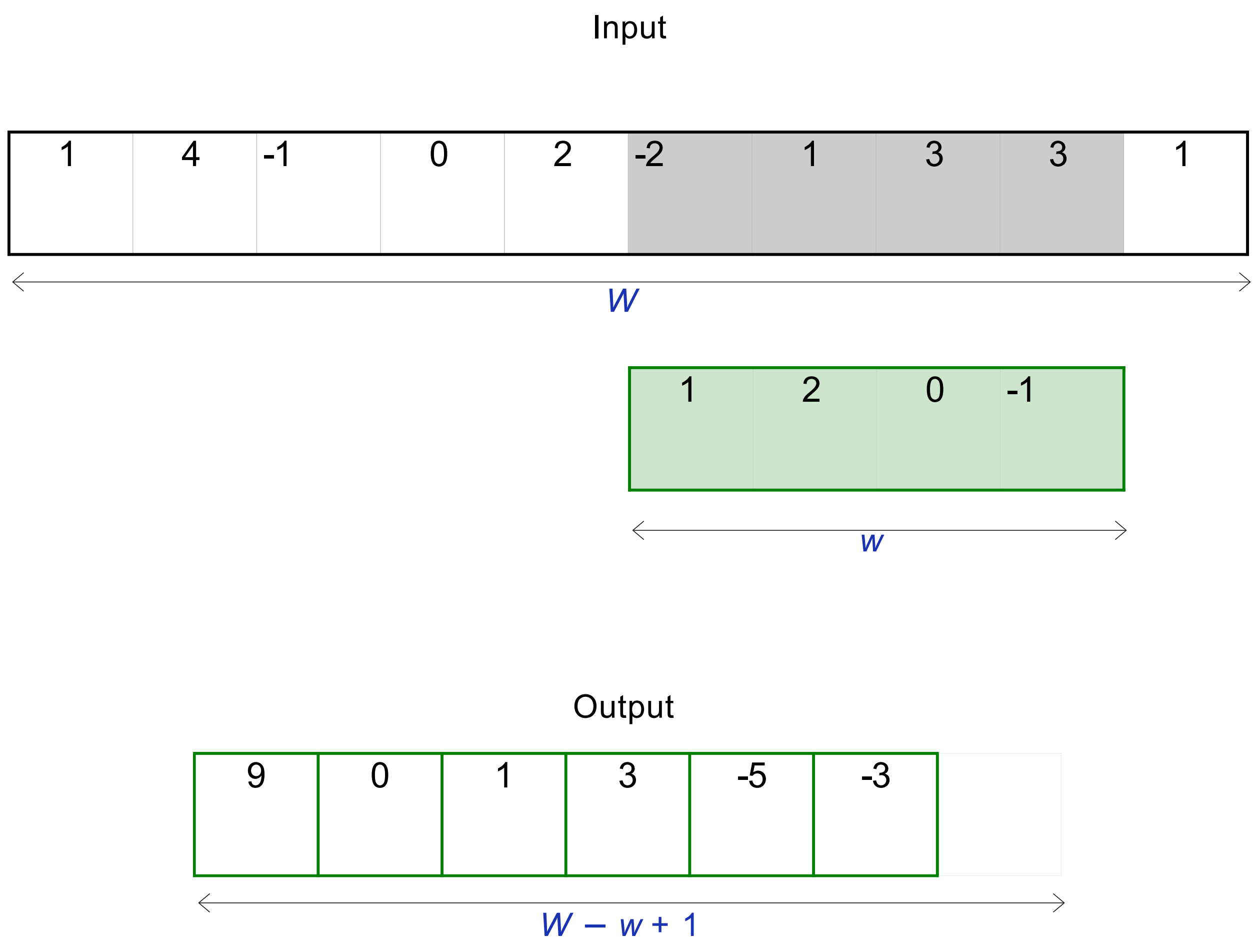
Discrete 1D Convolution



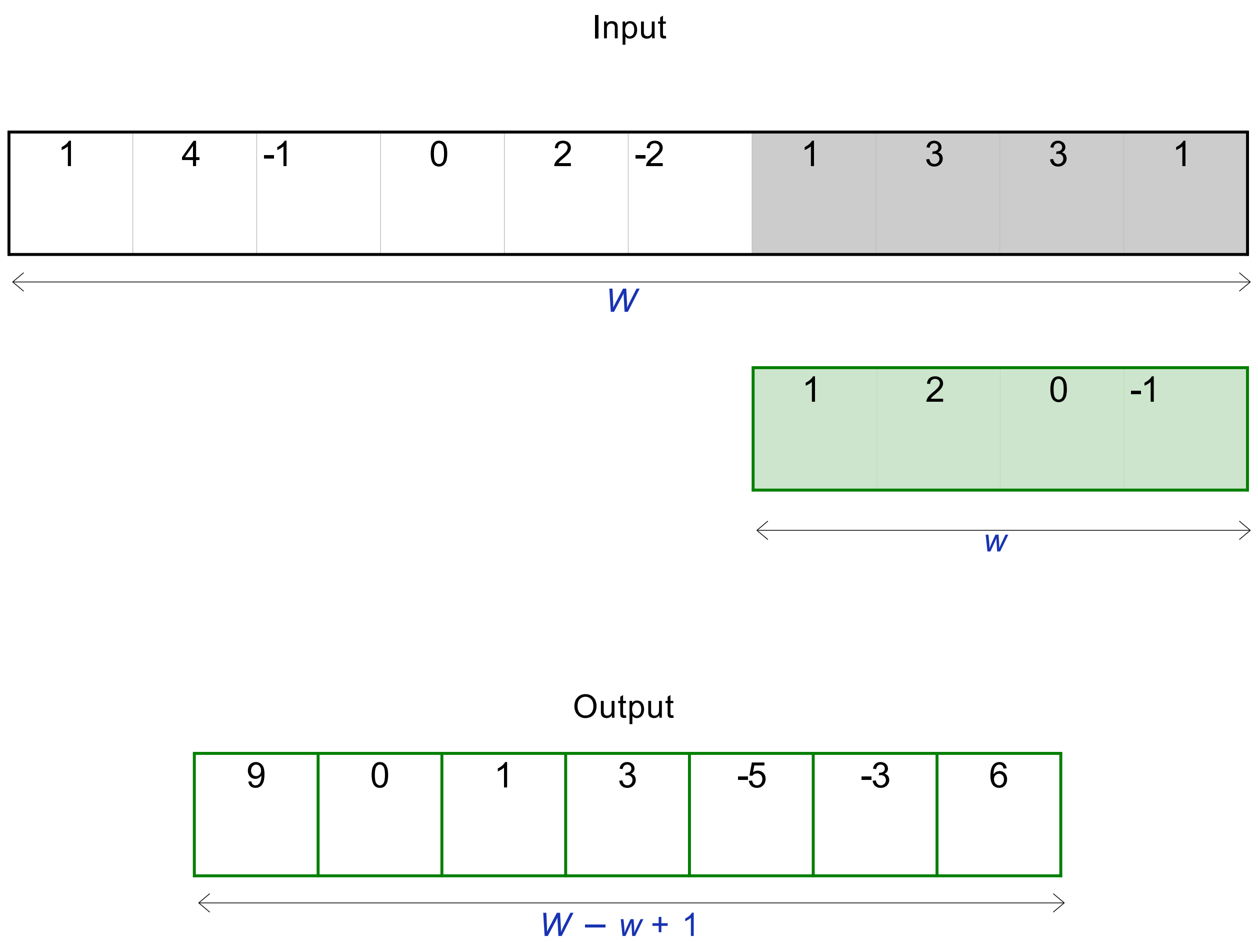
1D Convolution



Discrete 1D Convolution



Discrete 1D Convolution



Discrete 1D Convolution

Input

f

1	4	-1	0	2	-2	1	3	3	1

W

Mask

m

1	2	0	-1

w

Output

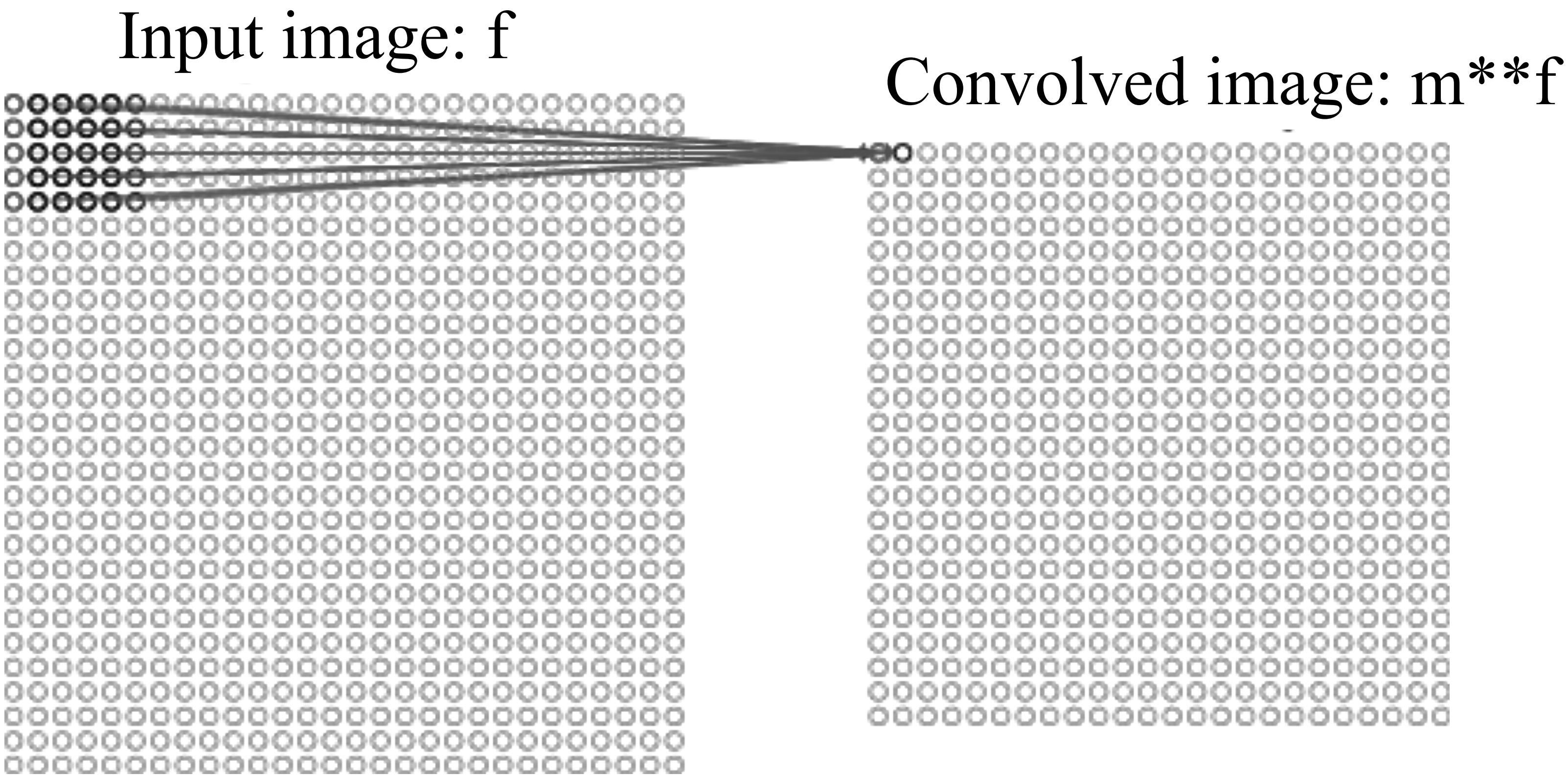
m*f

9	0	1	3	-5	-3	6

$W - w + 1$

$$\boxed{m * f(x) = \sum_{i=0}^{w} m(i) f(x - i)}$$

Discrete 2D Convolution

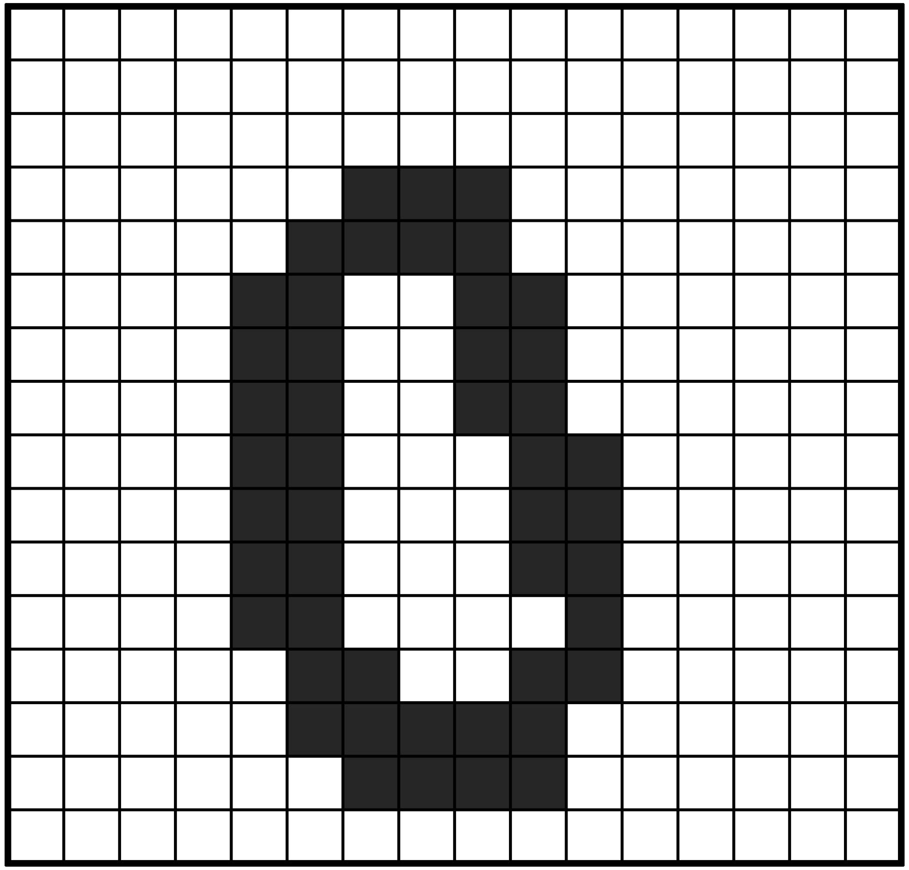


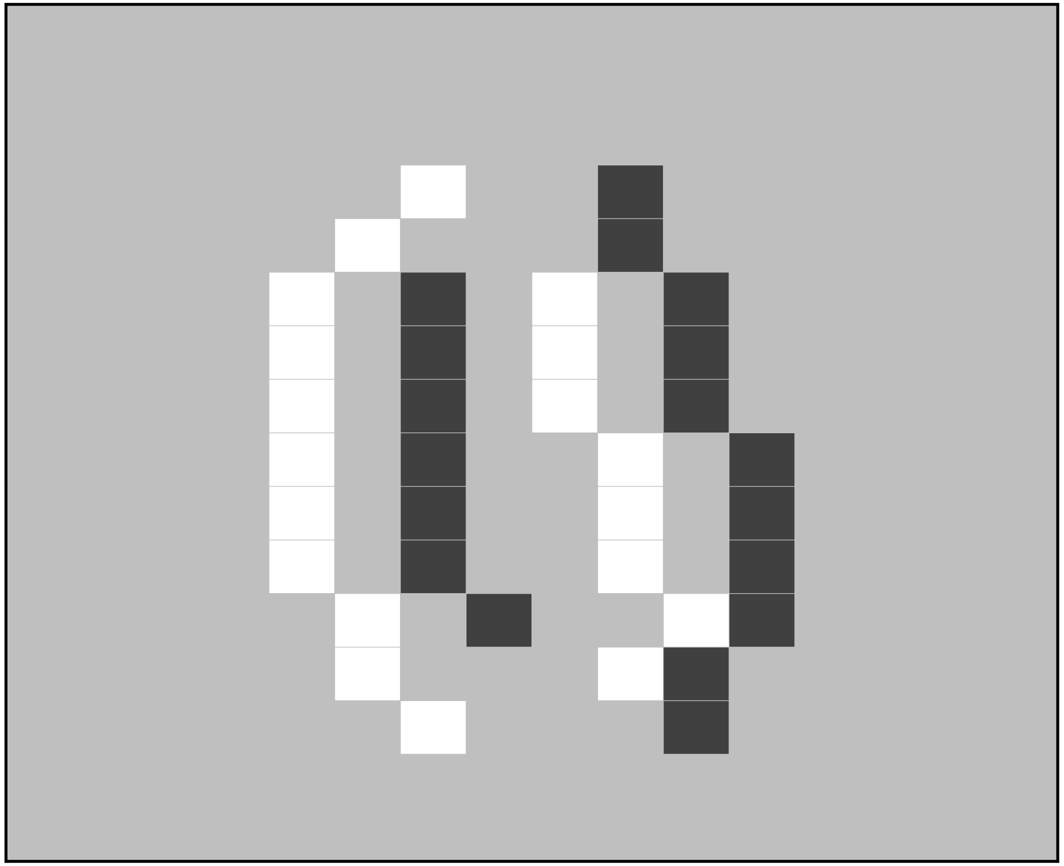
Convolution mask m, also known as a *kernel*.

$$\begin{bmatrix} m_{11} & \dots & m_{1w} \\ \dots & \dots & \dots \\ m_{w1} & \dots & m_{ww} \end{bmatrix}$$

$$\boxed{m**f(x,y) = \sum_{i=0}^{w} \sum_{j=0}^{w} m(i,j) f(x-i, y-j)}$$

Convolution Example



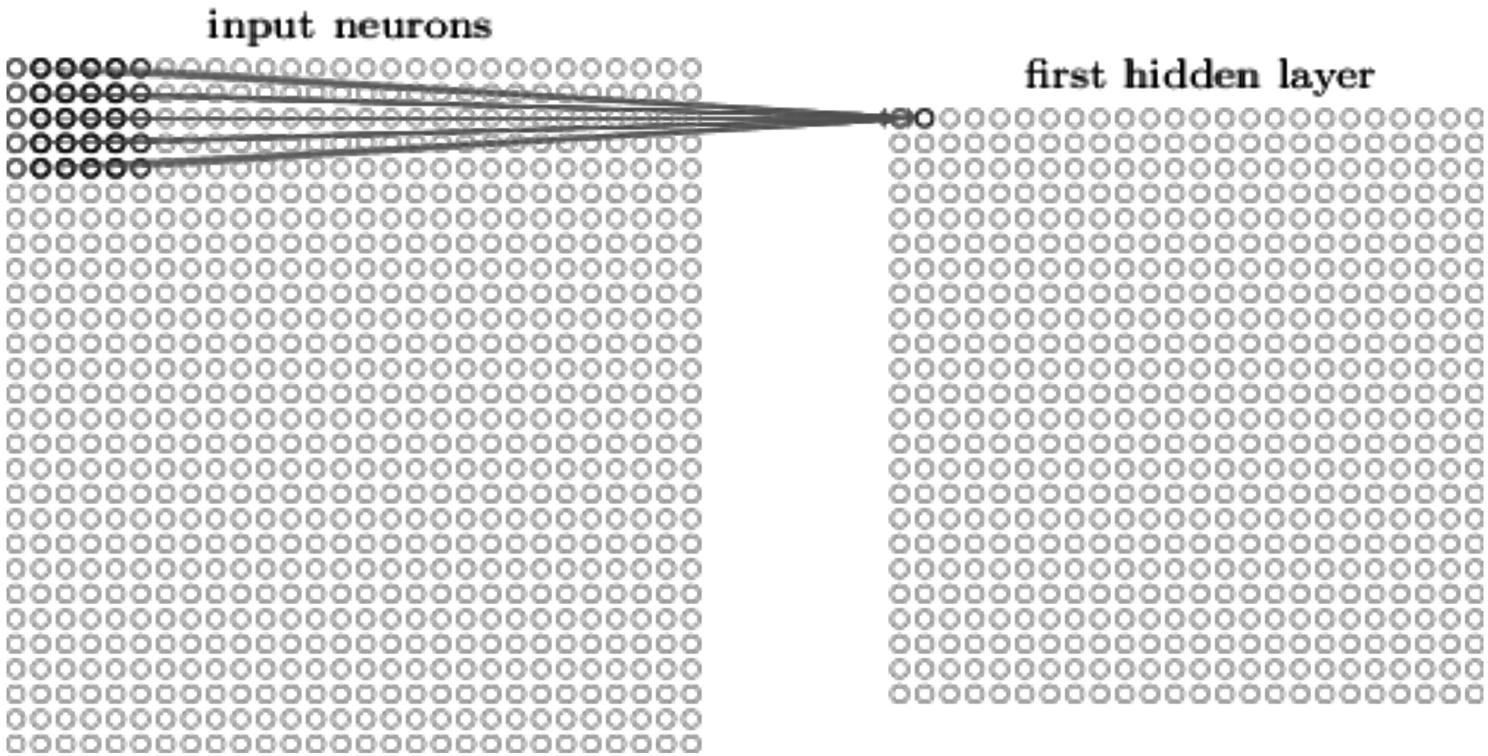


-1	**0**	**+1**
-1	**0**	**+1**
-1	**0**	**+1**

$$\mathbf{h}_{1,1} = \sigma(\mathbf{f}_{1,1} * \mathbf{x} + \mathbf{b}_{1,1})$$

This approximates an x derivative.

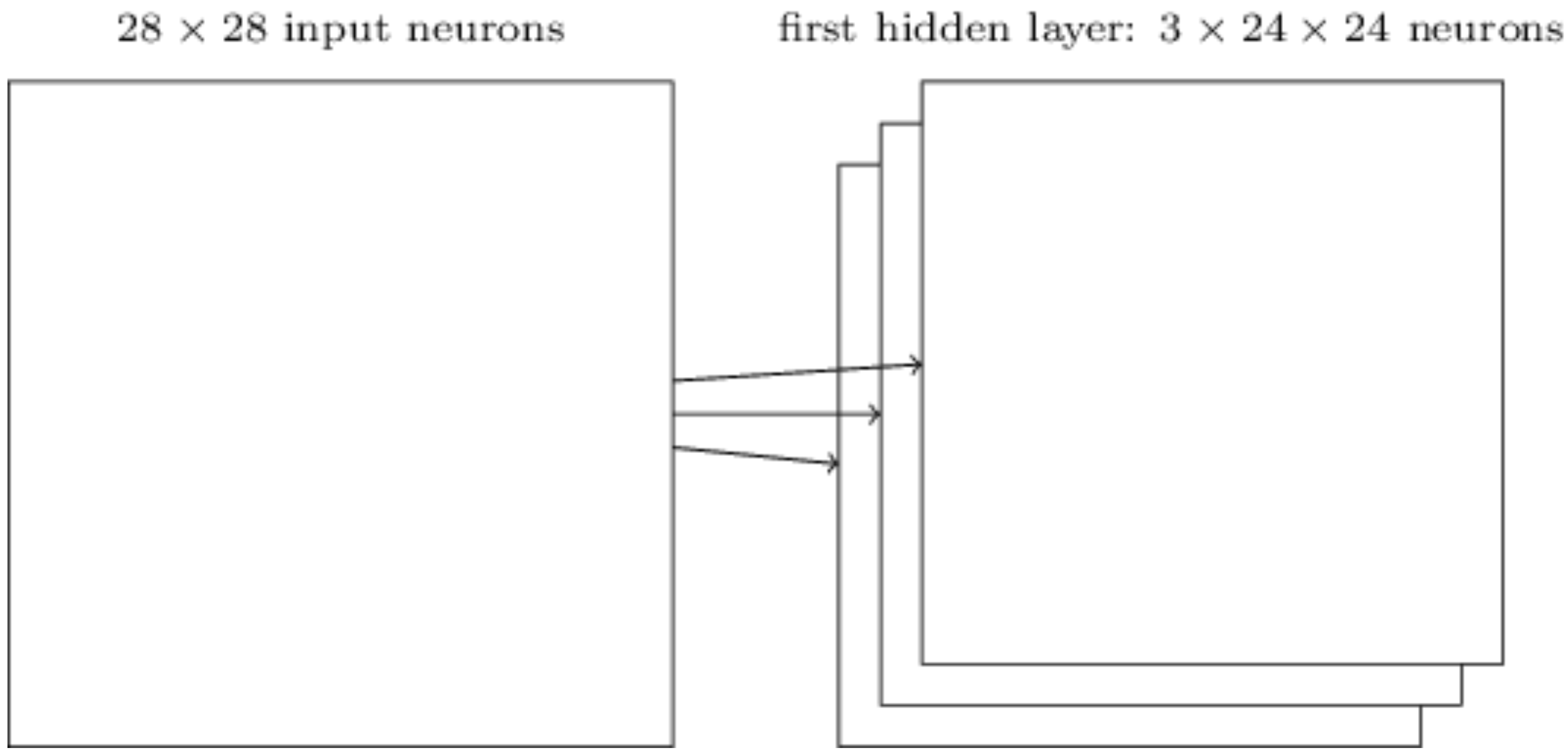
2D Convolutional Layer



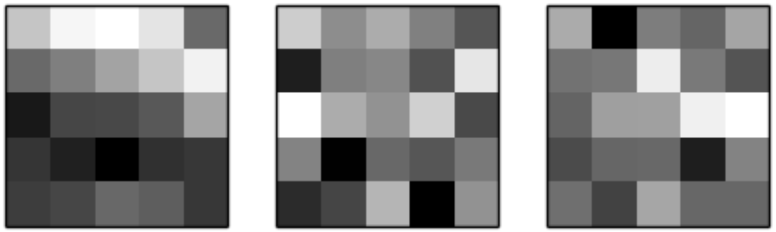
$$a^1_{i,j} = \sigma\left(b + \sum_{x=0}^{n_x} \sum_{y=0}^{n_y} w_{x,y} a^0_{i+x,j+y}\right)$$

- The same weights $w_{x,y}$ are used to compute all the activations.
- There are far fewer weights that in a fully connected layers.
- The neighborhood relationships are explicitly used.

Feature Maps

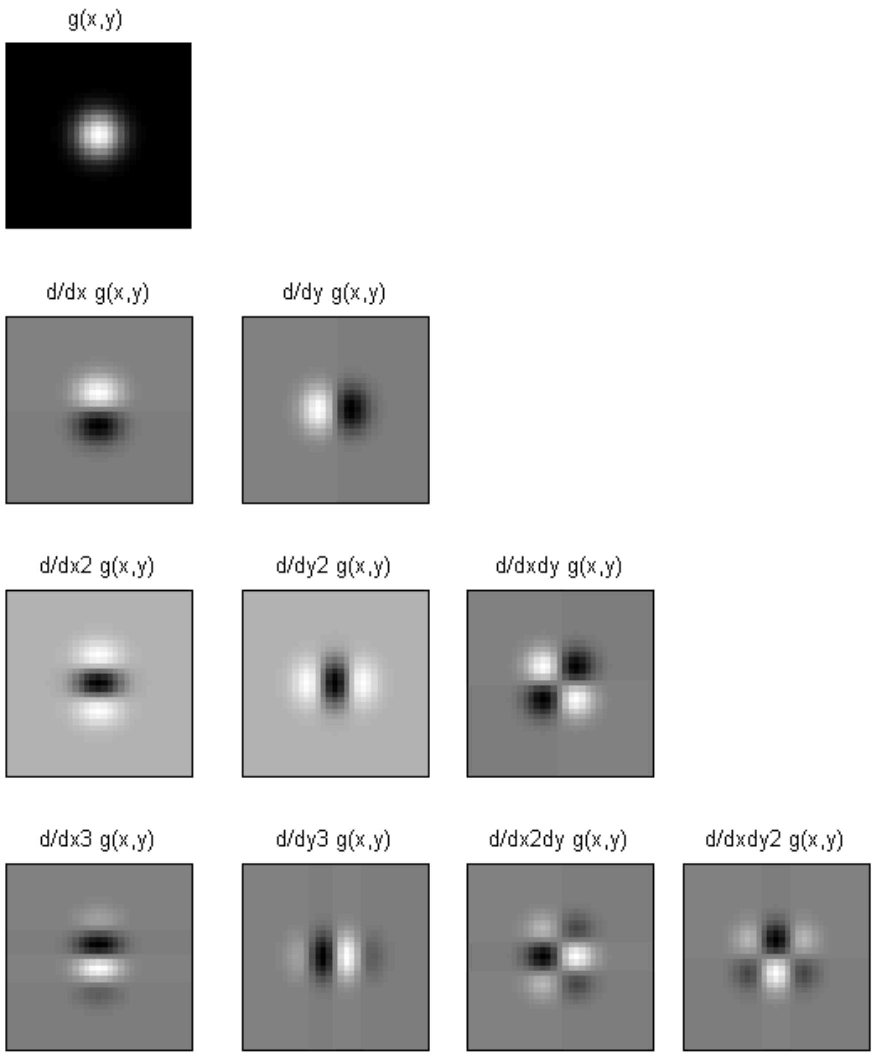


Filters:

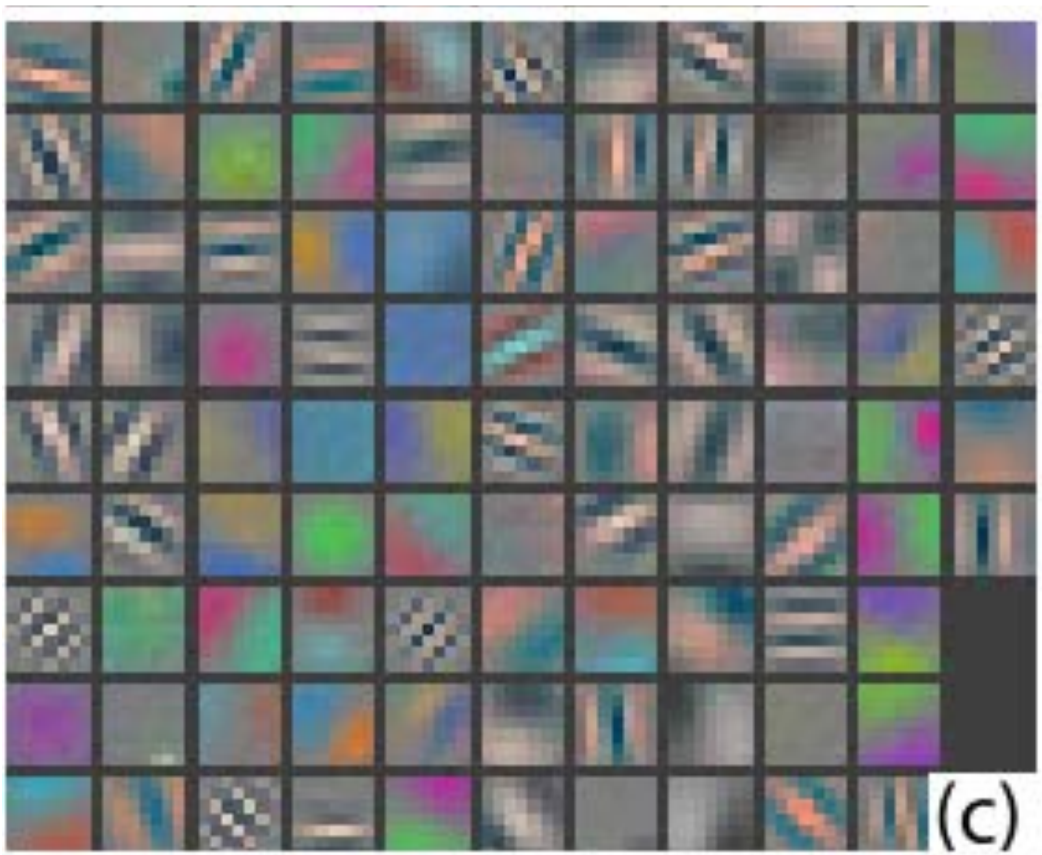


- In practice, one uses several **filters**, that is, sets of weights $w_{x,y,.}$ to compute several convolved versions of the input.
- These are known as **feature maps**.

Derivative Filters

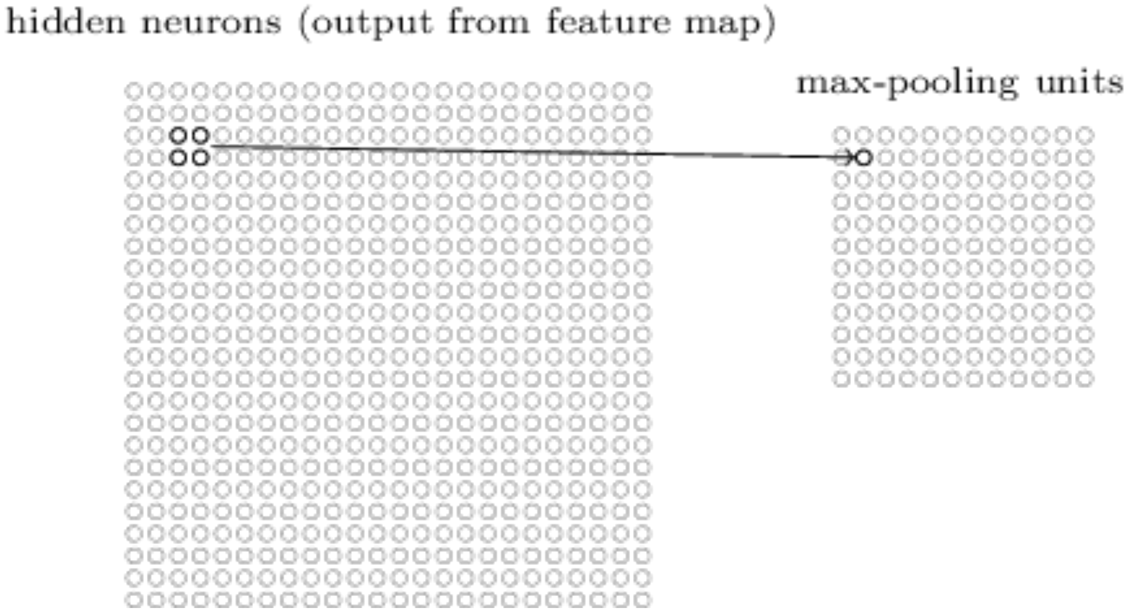


Derivatives



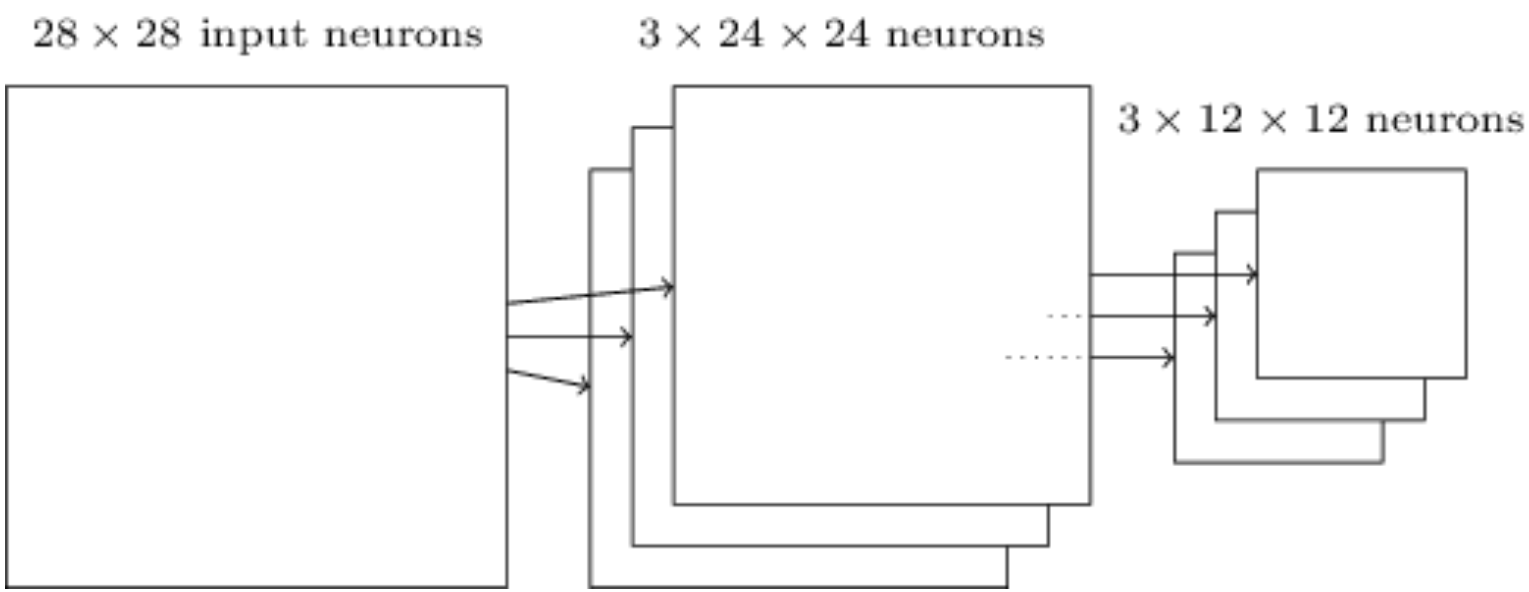
Learned filters

Pooling Layer



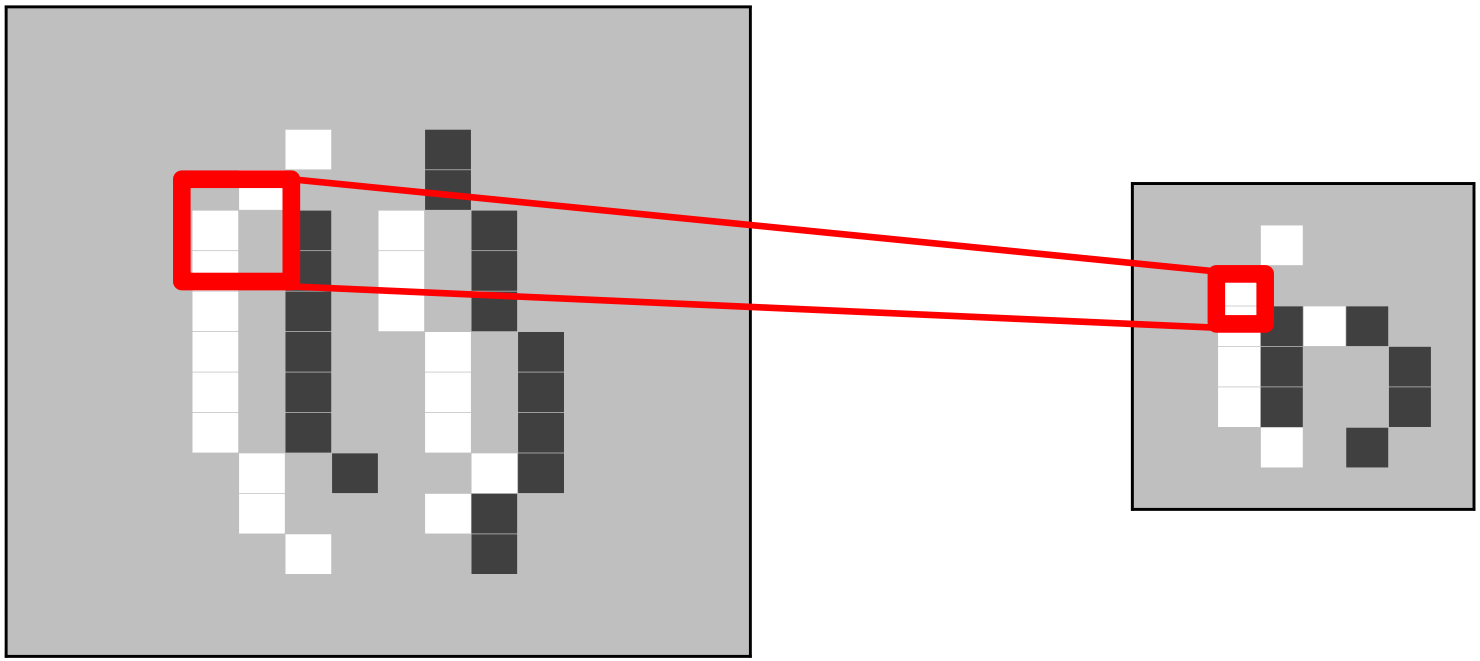
- Reduces the number of inputs by replacing all activations in a neighborhood by a single one.
- Can be thought as asking if a particular feature is present in that neighborhood while ignoring the exact location.

Adding the Pooling Layers



The output size is reduced by the pooling layers.

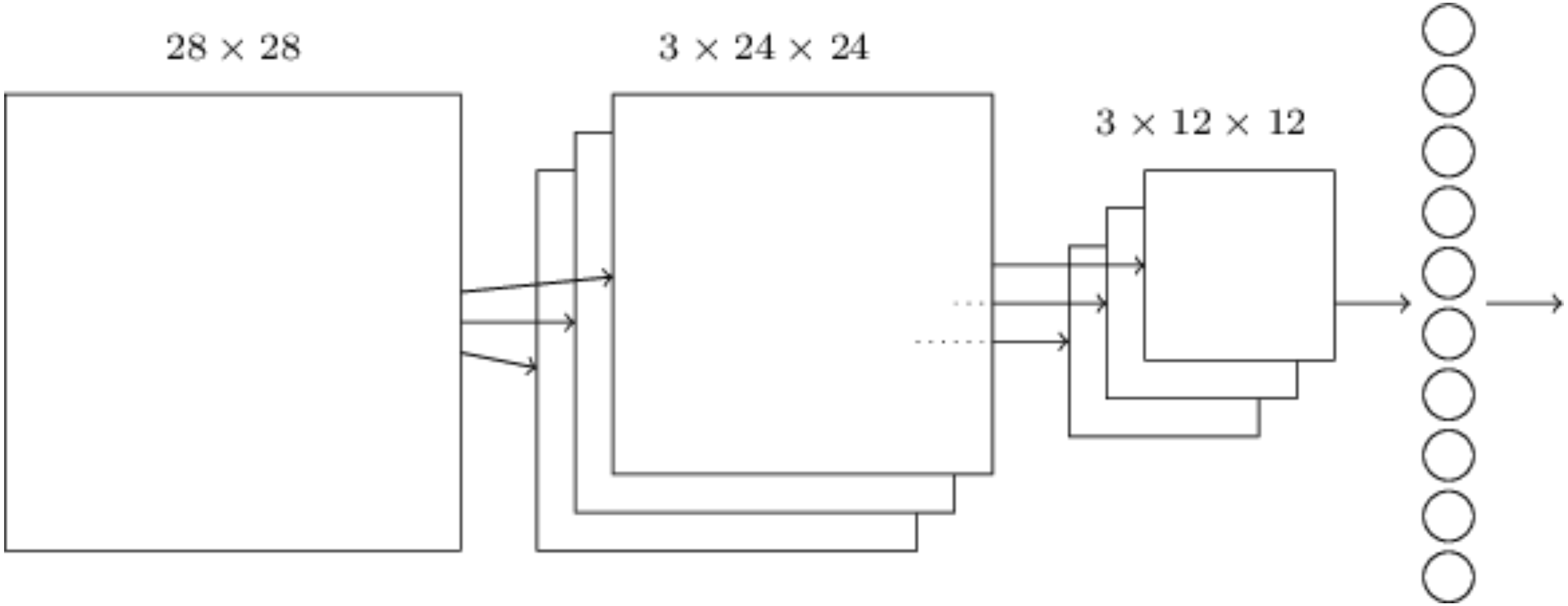
Pooling Example



Max-pooling:

$$\mathbf{h}_i[u,v] = \max\{ \begin{array}{ll} \mathbf{h}_{i-1}[2u, & 2v], \\ \mathbf{h}_{i-1}[2u, & 2v+1], \\ \mathbf{h}_{i-1}[2u+1, & 2v], \\ \mathbf{h}_{i-1}[2u+1, & 2v+1] \end{array} \}$$

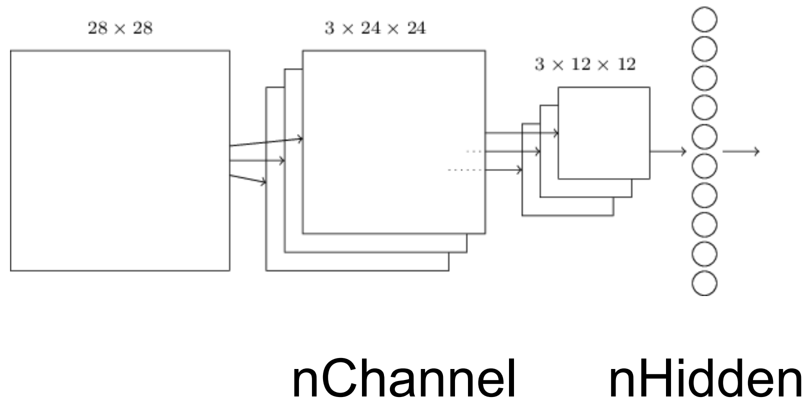
Adding a Fully Connected Layer



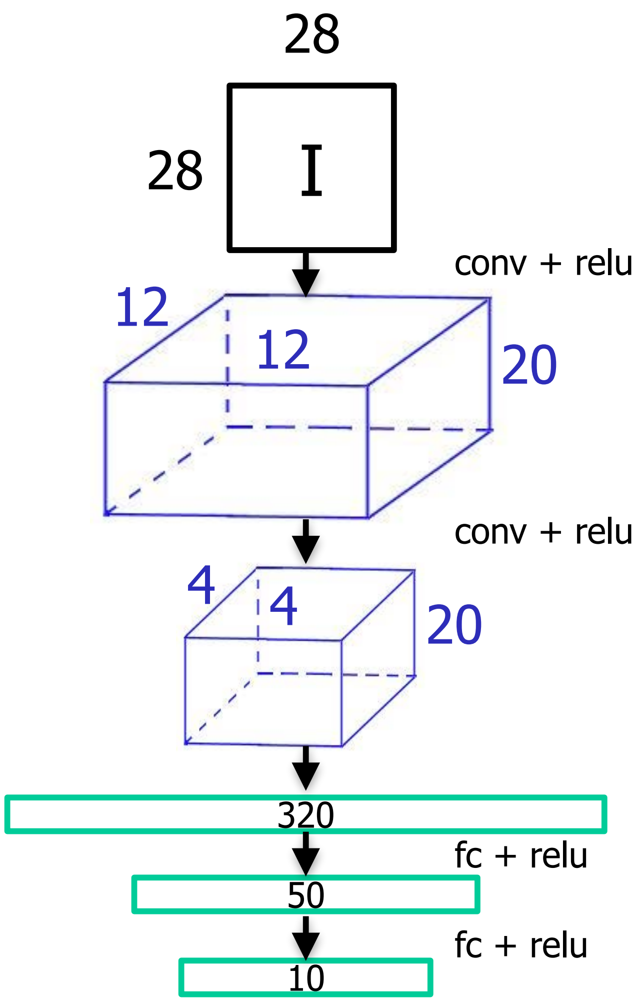
- Each neutron in the final fully connected layer is connected to all neurons in the preceding one.
- Deep architecture with many parameters to learn but still far fewer than an equivalent multilayer perceptron.

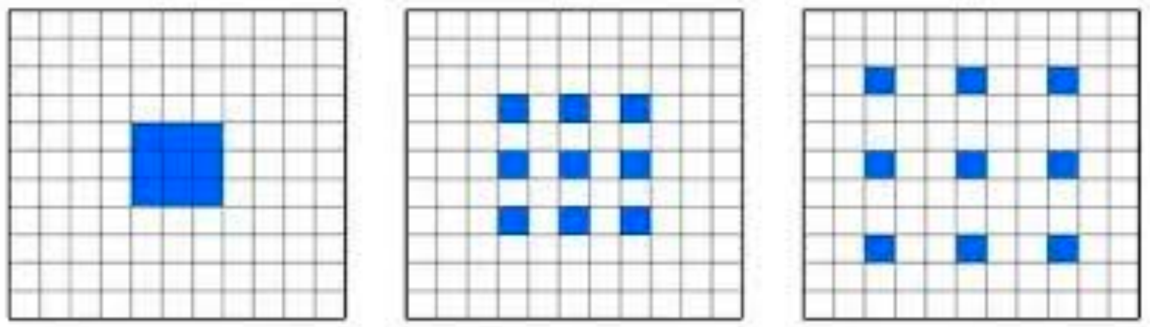
PyTorch Translation (1)

```
class ConvNet(nn.Module):

   def __init__(nChannel=10,nHidden=50):
        self.cv1 = nn.Conv2d(1, nChannel,  kernel_size=5)
        self.cv2 = nn.Conv2d(nChannel, 20, kernel_size=5)
        self.fc1 = nn.Linear(320, nHidden)
        self.fc2 = nn.Linear(nHidden,10)

   def forward(self,x):
        x = F.relu(F.max_pool2d(self.cv1(x), 2))
        x = F.relu(F.max_pool2d(self.cv2(x), 2))
        x = x.view(-1, 320)
        x = F.relu(self.fc1(x))
        x = self.fc2(x)
        return F.log_softmax(x,dim=1)
```



Without Max Pooling





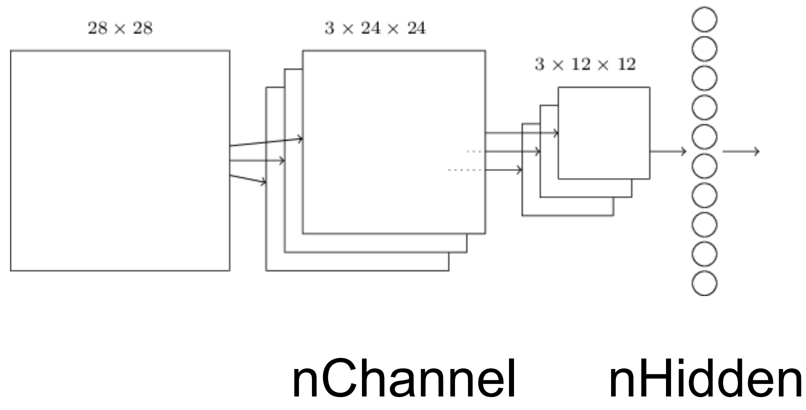
stride=1 stride=2 stride=3

Accuracy	Train	Test
Conv 5x5, stride 1 Max pool 2x3	99.58	98.77
Conv 5x5, stride 2	99.42	98.31
Conv 5x5, stride 1 Conv 3x3, stride 2	99.38	98.57

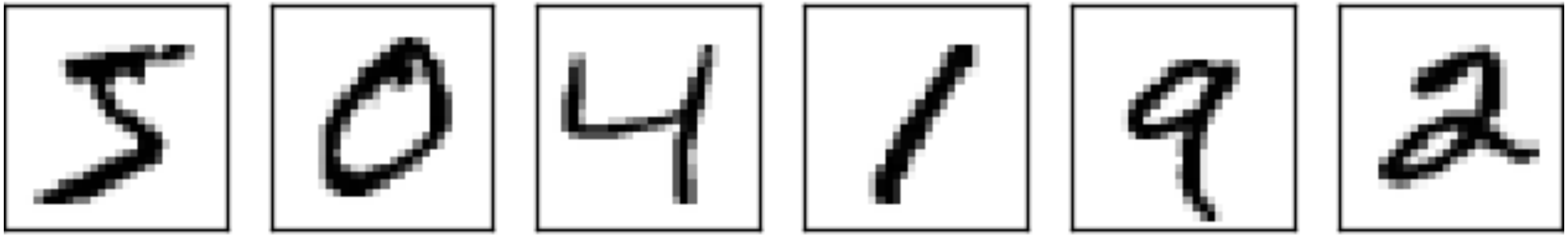
Springenberg et al., ICLR'15

PyTorch Translation (2)

```
class ConvNet(nn.Module):

    def __init__(nChannel=10,nHidden=50):
        self.cv1 = nn.Conv2d(1, nChannel,kernel_size=5,stride=2)
        self.cv2 = nn.Conv2d(nChannel,20,kernel_size=5,stride=2)
        self.fc1 = nn.Linear(320, nHidden)
        self.fc2 = nn.Linear(nHidden,10)

    def forward(self,x):
        x = F.relu(self.cv1(x))
        x = F.relu(self.cv2(x))
        x = x.view(-1, 320)
        x = F.relu(self.fc1(x))
        x = self.fc2(x)
        return F.log_softmax(x,dim=1)
```

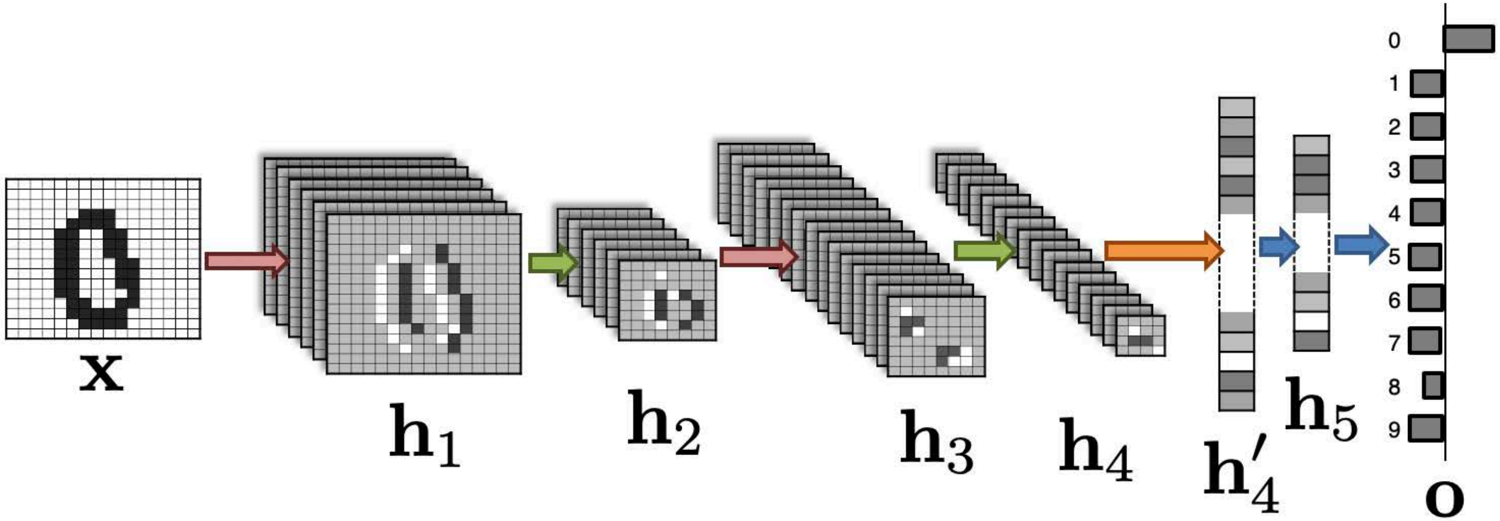


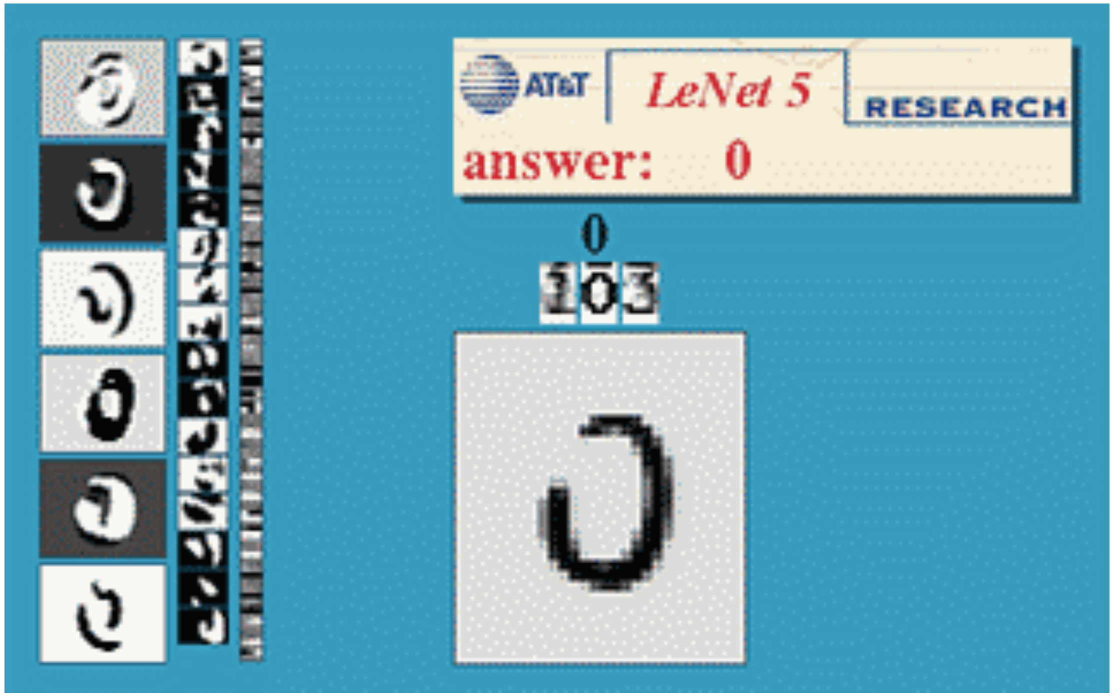
MNIST

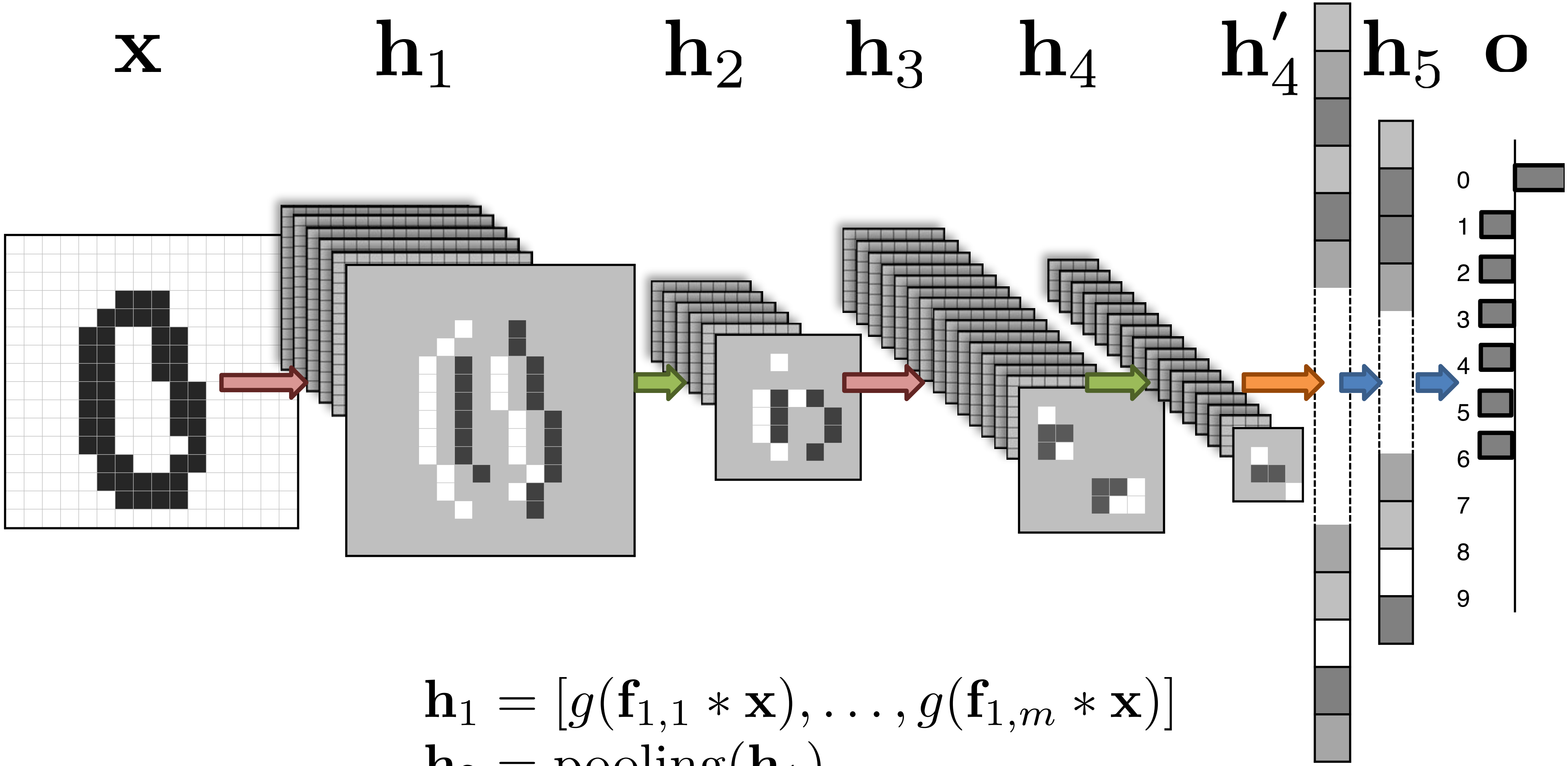


- The network takes as input 28x28 images represented as 784D vectors.
- The output is a 10D vector giving the probability of the image representing any of the 10 digits.
- There are 50’000 training pairs of images and the corresponding label, 10’000 validation pairs, and 5’000 testing pairs.

Lenet (1989-1999)

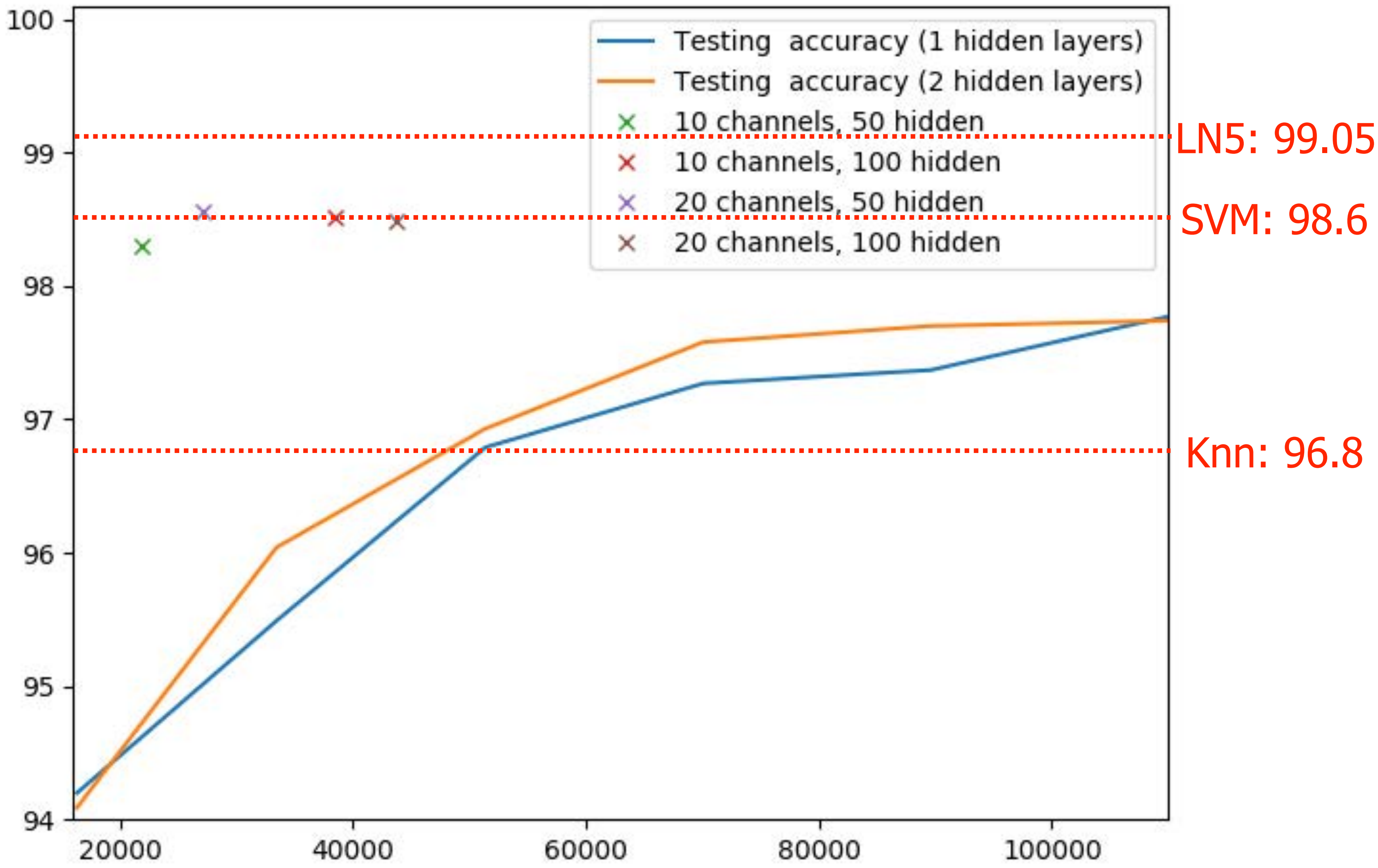






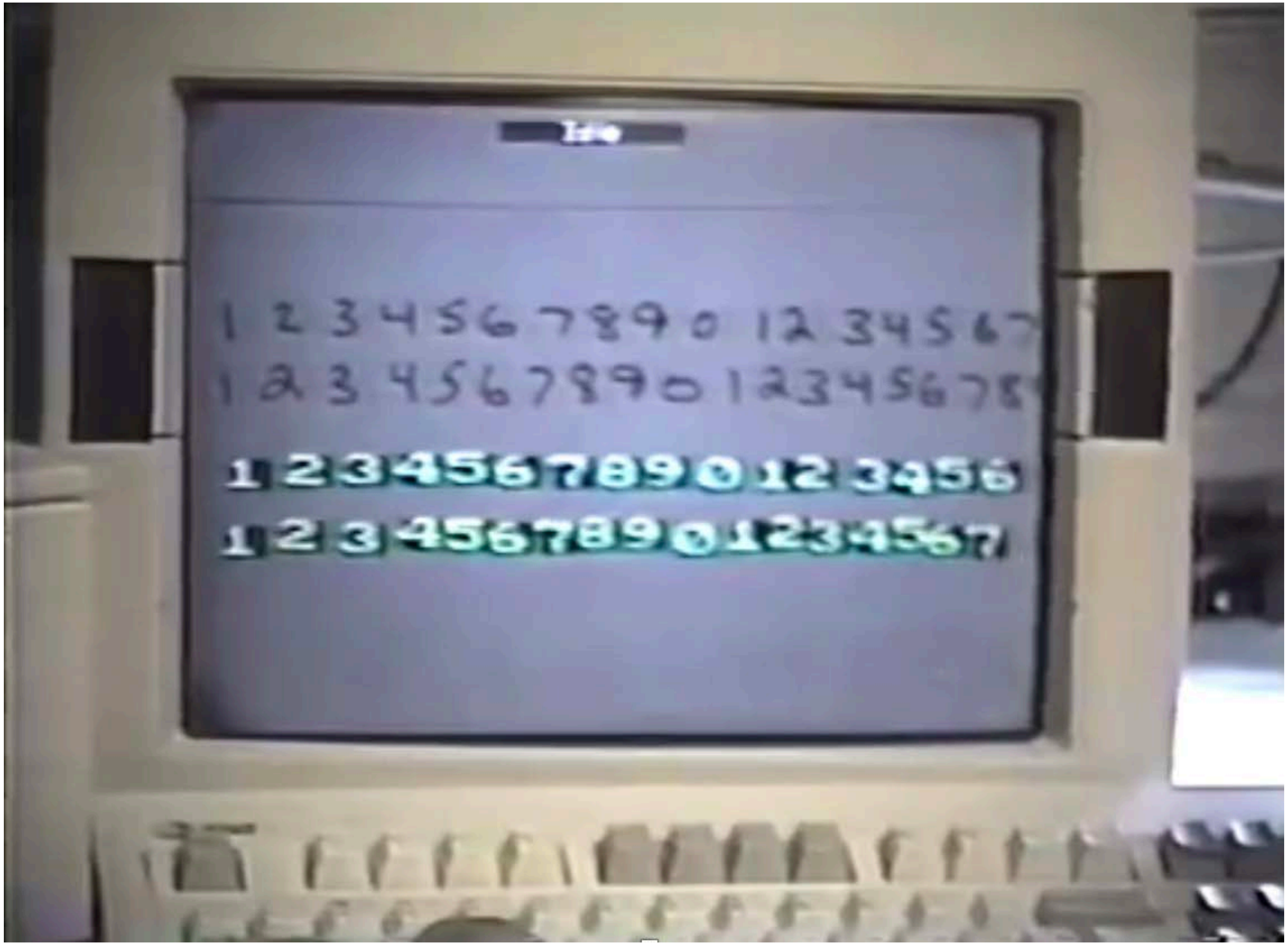
$$
\begin{aligned}
\mathbf{h}_1 &= [g(\mathbf{f}_{1,1} * \mathbf{x}), \ldots, g(\mathbf{f}_{1,m} * \mathbf{x})] \\
\mathbf{h}_2 &= \text{pooling}(\mathbf{h}_1) \\
\mathbf{h}_3 &= [g(\mathbf{f}_{3,1} * \mathbf{h}_2), \ldots, g(\mathbf{f}_{3,n} * \mathbf{h}_2)] \\
\mathbf{h}_4 &= \text{pooling}(\mathbf{h}_3) \\
\mathbf{h}'_4 &= \text{Vec}(\mathbf{h}_4) \\
\mathbf{h}_5 &= g(\mathbf{W}_5 \mathbf{h}'_4 + \mathbf{b}_5) \\
\mathbf{o} &= \mathbf{W}_6 \mathbf{h}_5 + \mathbf{b}_6
\end{aligned}
$$

Lenet Results



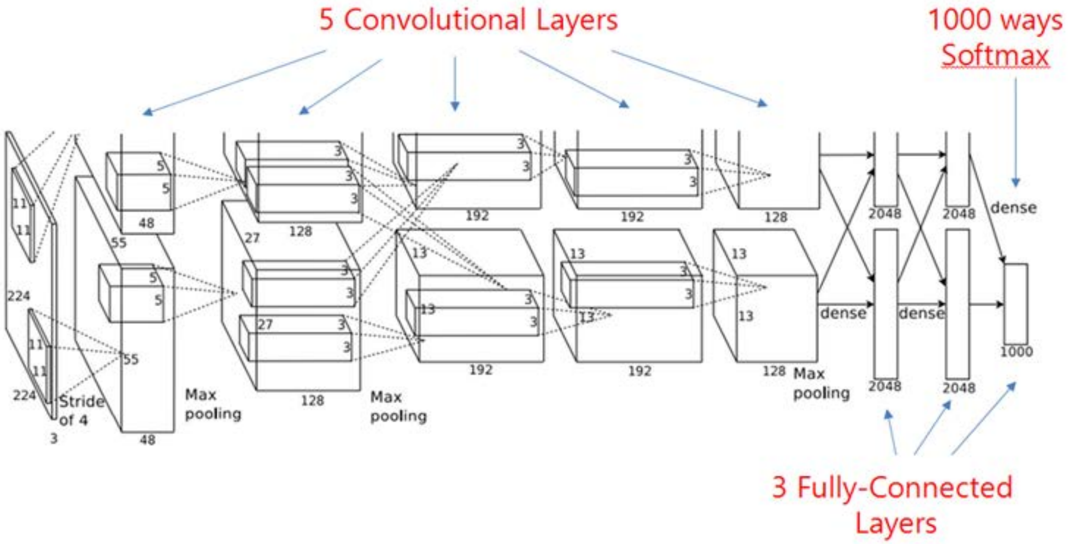
Given the appropriate architecture, the CNN outperforms the other approaches, whereas the MLP did not.

Lenet5 (1992)



- Worked beautifully on MNIST.
- Very few people believed it would scale up.

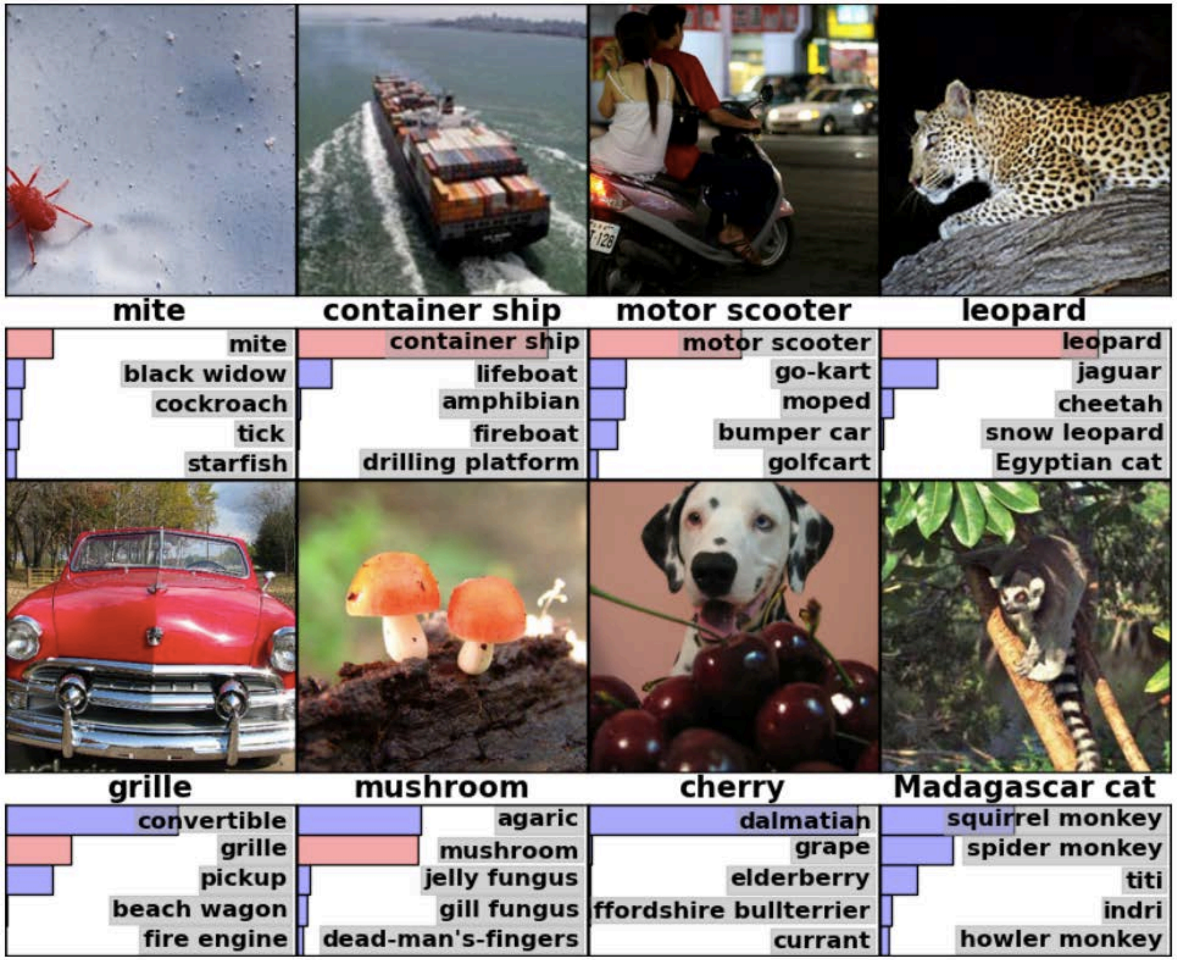
AlexNet (2012)

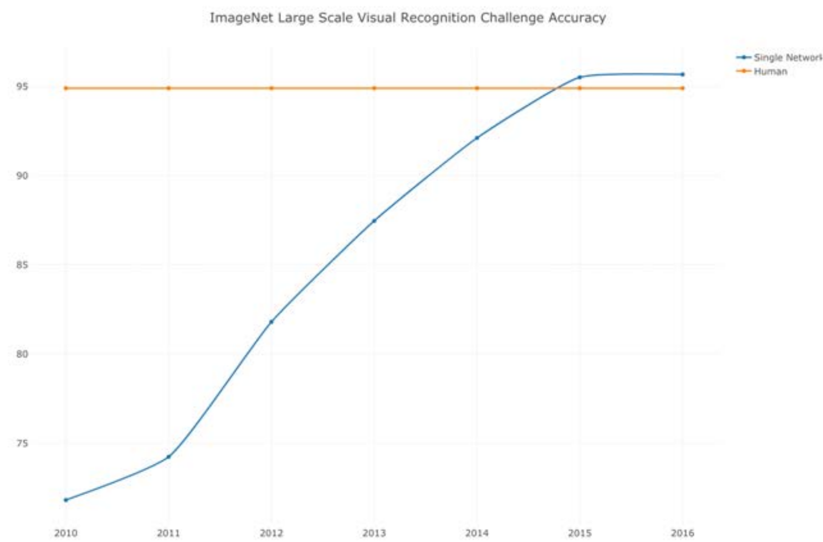


Task: Image classification
Training images: Large Scale Visual Recognition Challenge 2010
Training time: 2 weeks on 2 GPUs

Major Breakthrough: Training large networks has now been shown to be practical!!

AlexNet Results

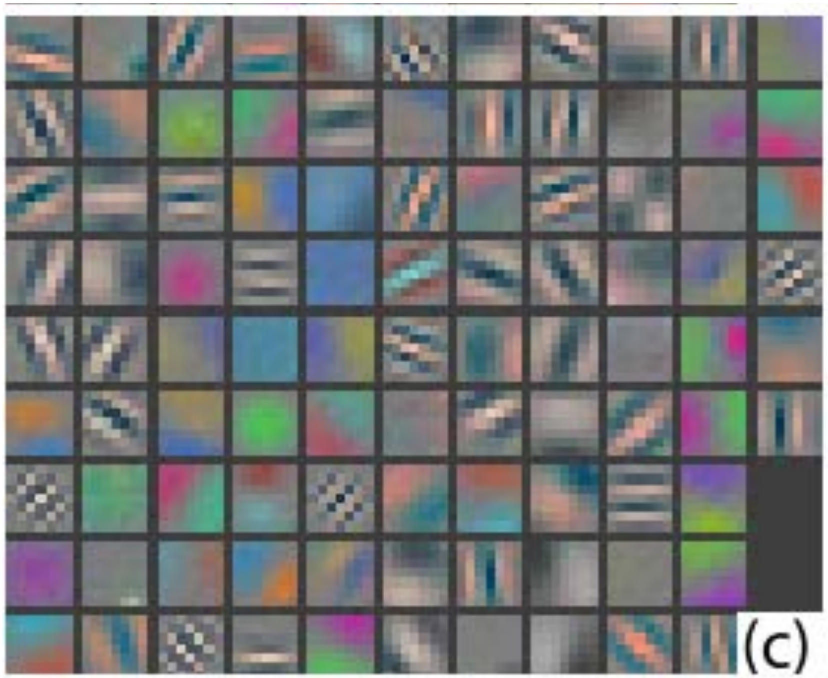




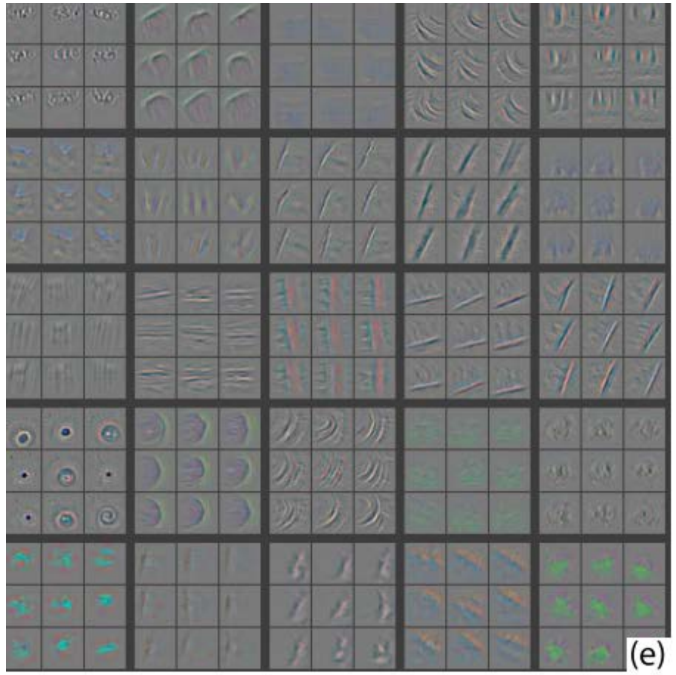
- At the 2012 ImageNet Large Scale Visual Recognition Challenge, AlexNet achieved a top-5 error of 15.3%, more than 10.8% lower than the runner up.
- Since 2015, networks outperform humans on this task.

Krizhevsky, NIPS'12

Feature Maps



First convolutional layer

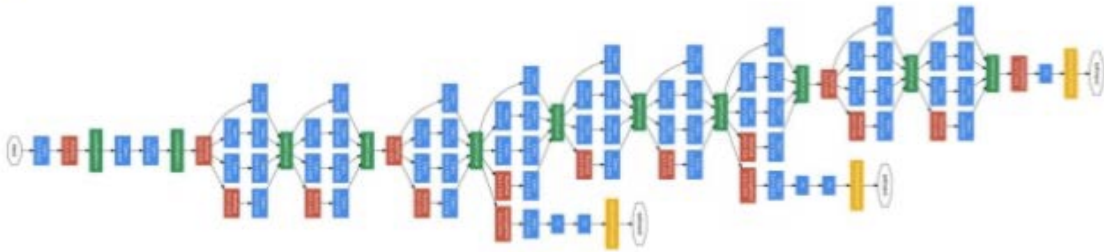


Second convolutional layer

- Some of the convolutional masks are very similar to oriented Gaussian or Gabor filters.
- The trained neural nets compute oriented derivatives, which the brain is also **believed** to do.

Size and Depth Matter

VGG19, 3 weeks of training.

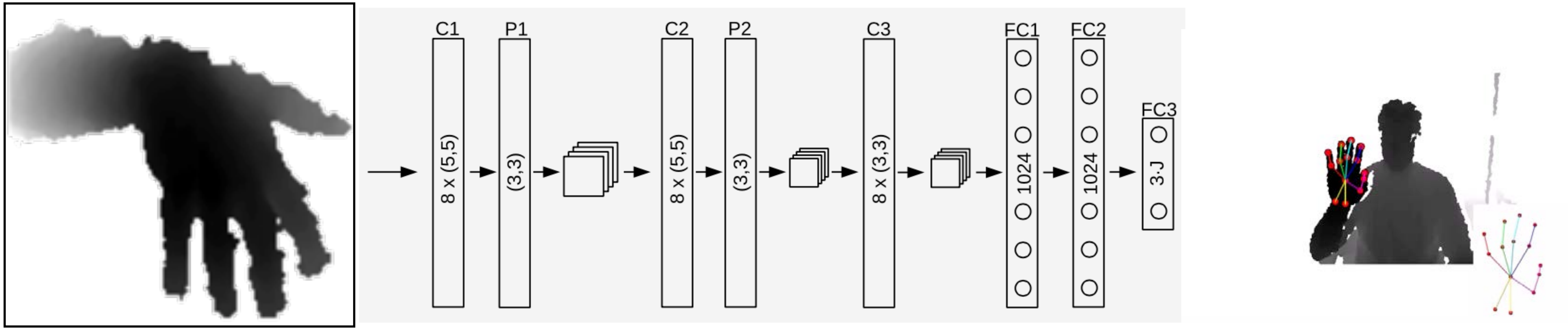


GoogleLeNet.

"It was demonstrated that the representation depth is beneficial for the classification accuracy, and that state-of-the-art performance on the ImageNet challenge dataset can be achieved using a conventional ConvNet architecture."

Simonyan & Zisserman, ICLR'15

Hand Pose Estimation (2015)

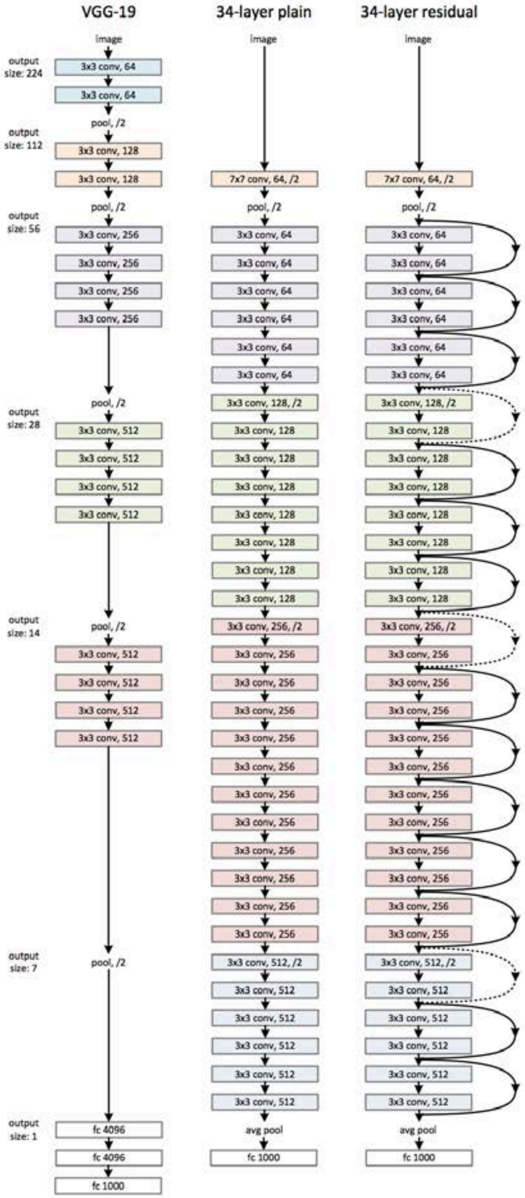


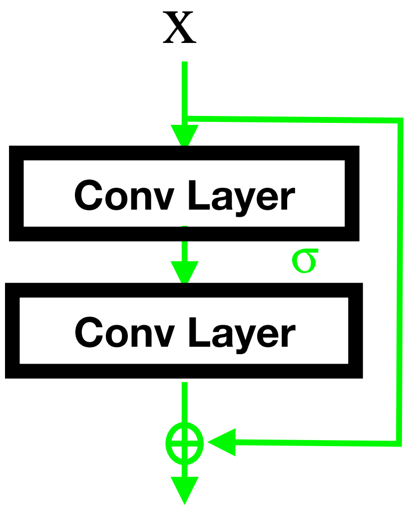
Input: Depth image.

Output: 3D pose vector.

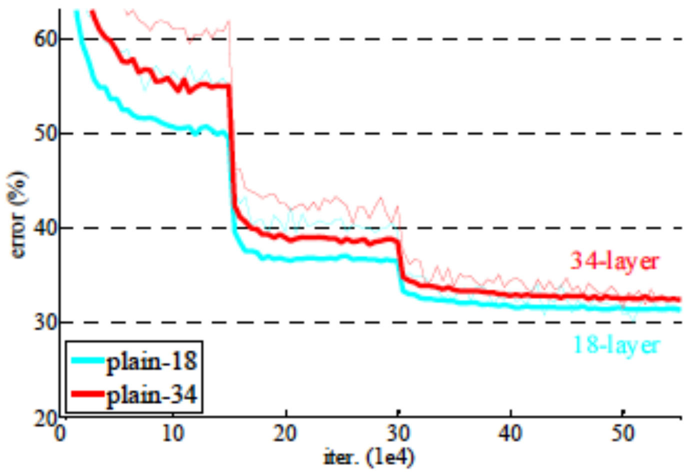
Oberweger et al. , ICCV’15

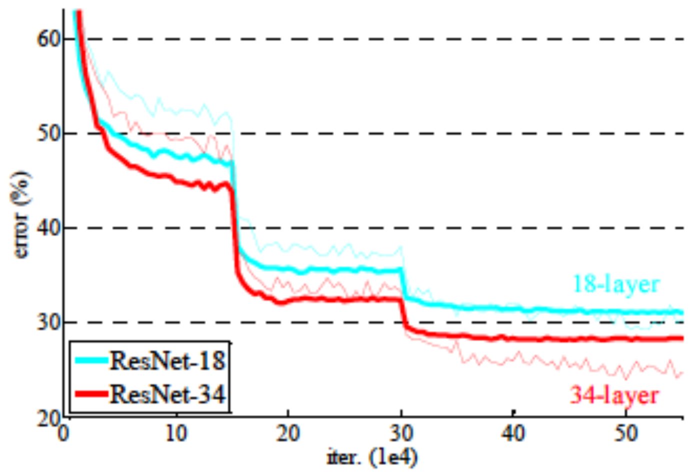
Deeper is Better





$x + l_2(\sigma(l_1(x)))$

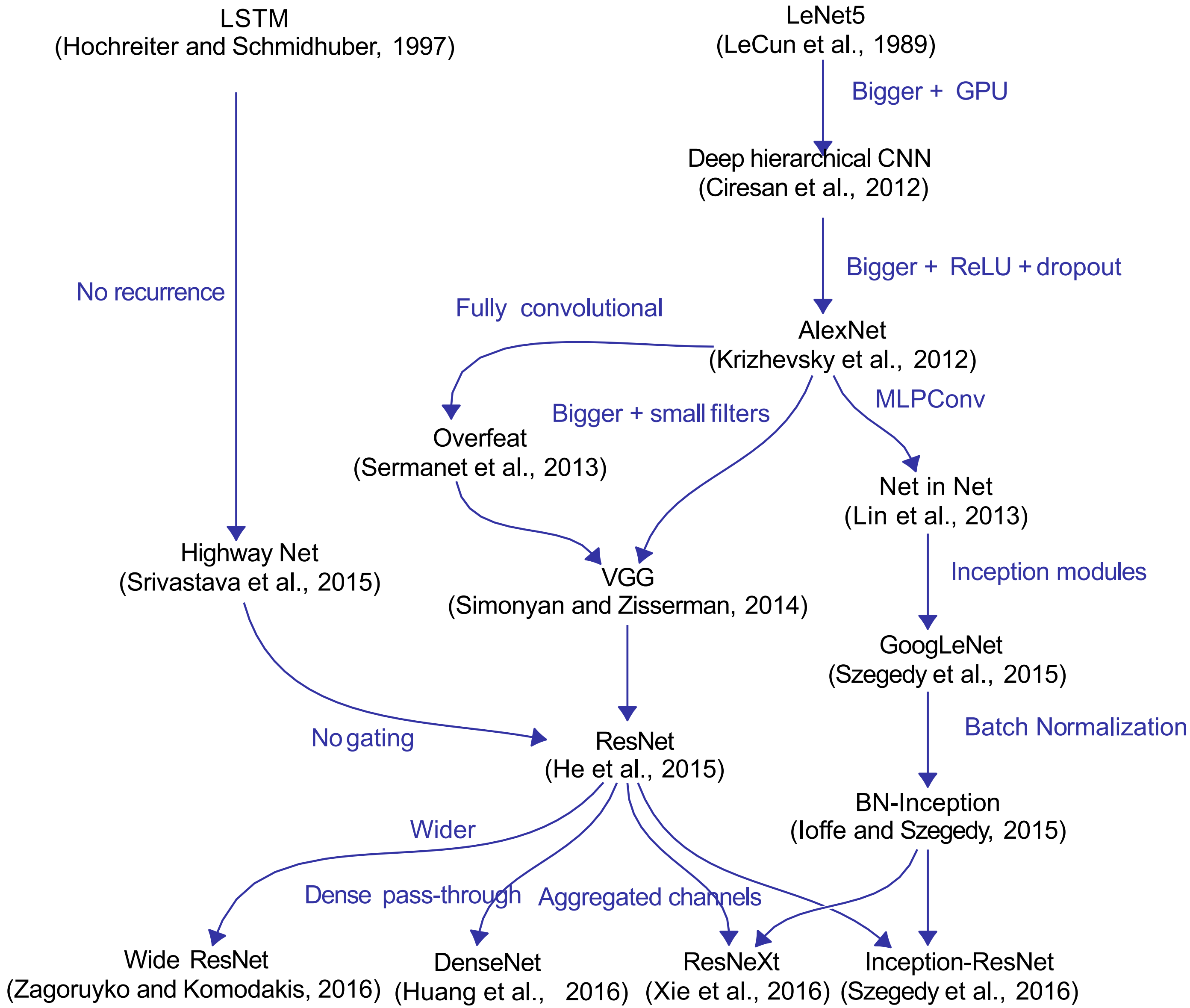




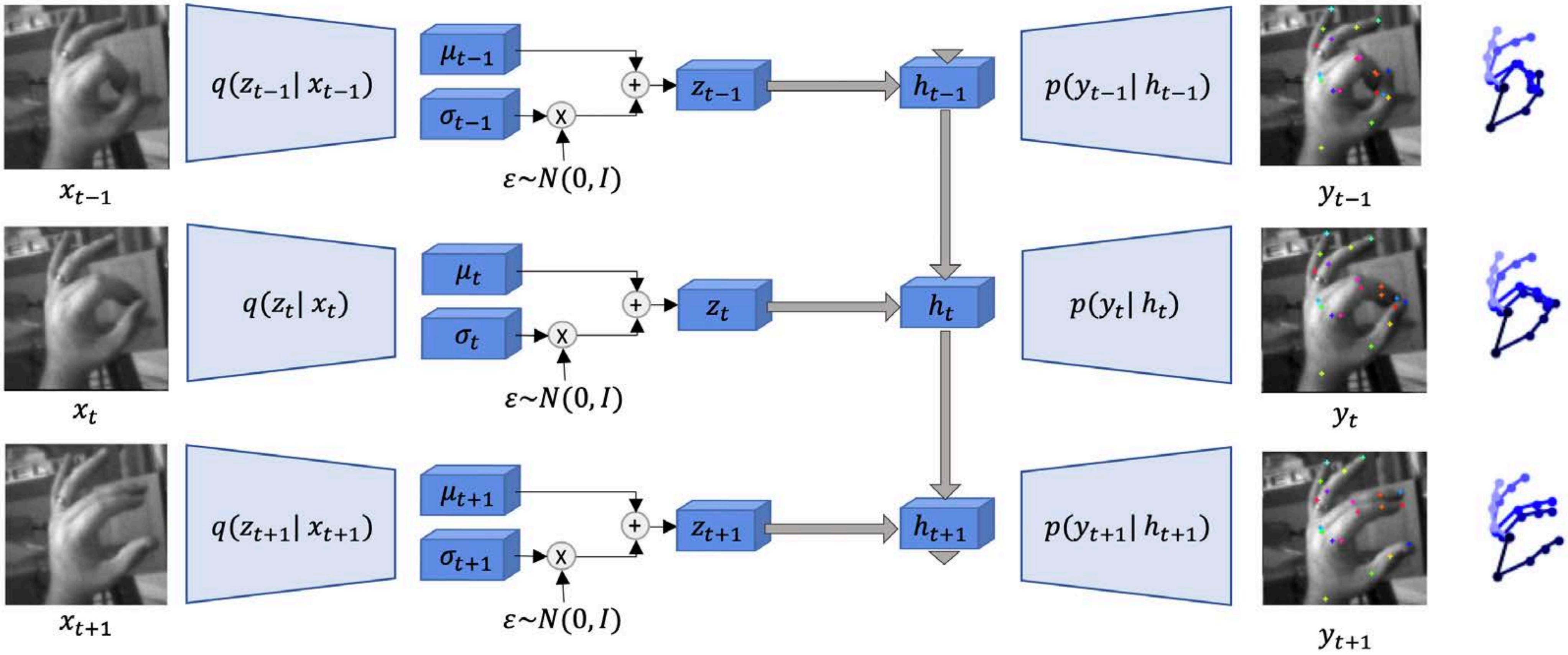
In general, the more ResNet layers, the better the results.

He et al. , CVPR'16

Image Classification Taxonomy

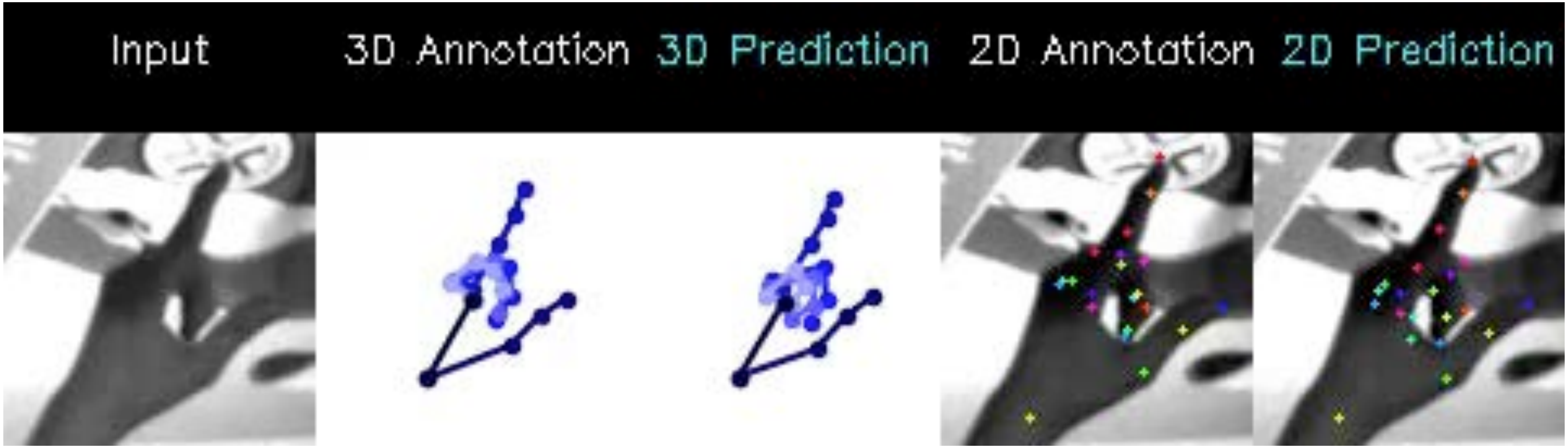


Recurrent Auto Encoder



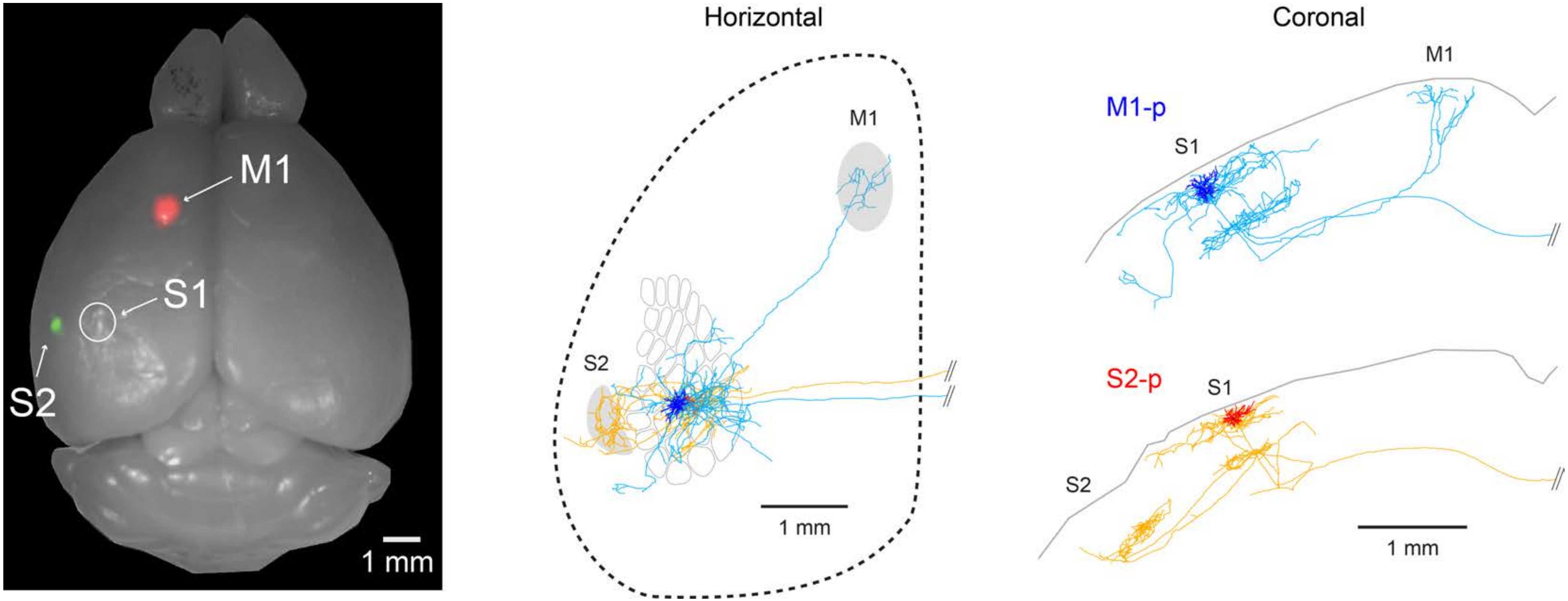
Remelli et al. , ArXiv'19

Hand Pose Estimation from Video (2019)



- This is considerably more difficult than estimating from range images.
- It requires a large training database.

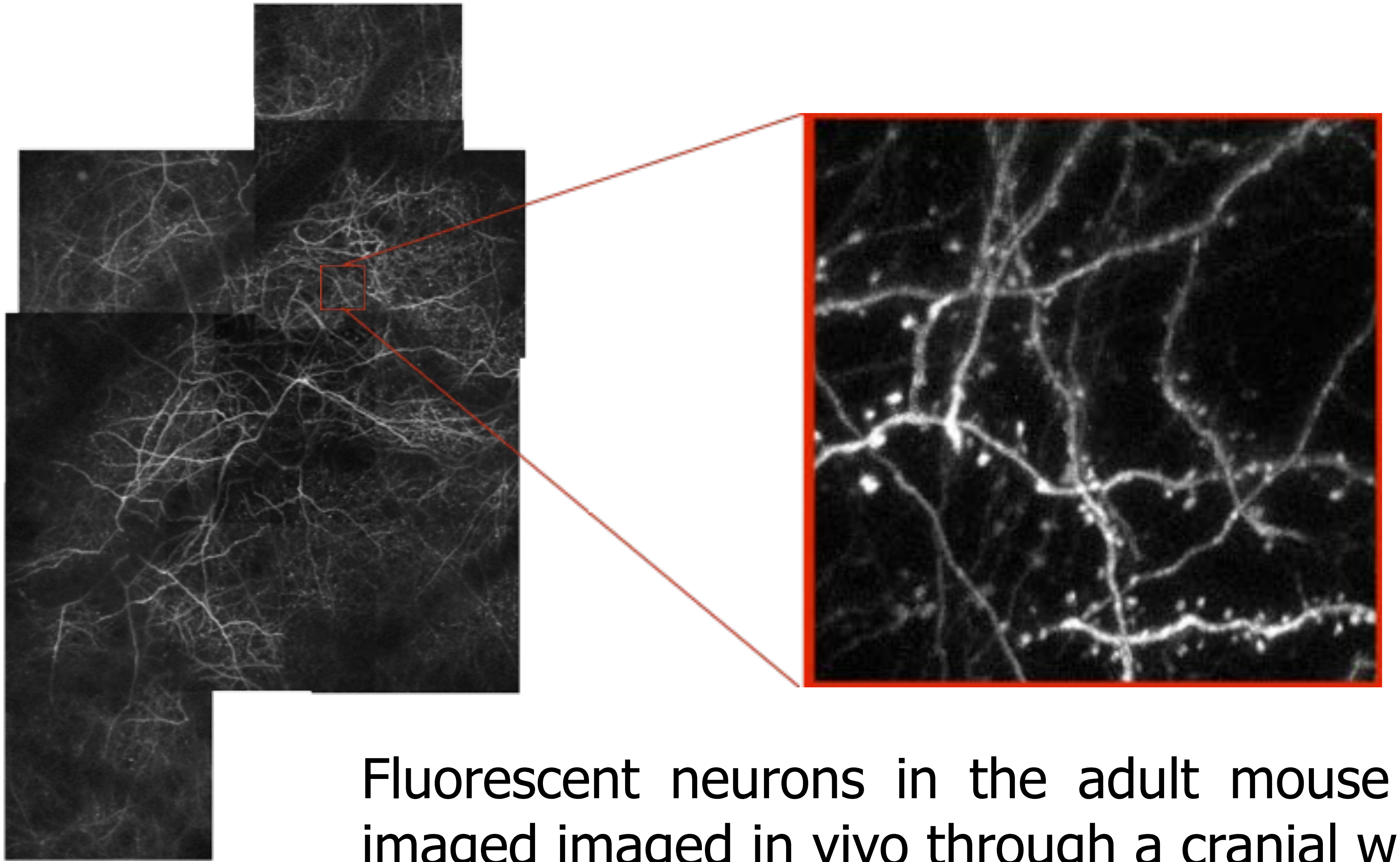
Connectomics



- Building the wiring diagram of the brain.
- Finding long range connections.

—> One step towards understanding how it works.

Dendrites and Axons



Fluorescent neurons in the adult mouse brain imaged imaged in vivo through a cranial window using a 2-photon microscope.

Cartography

The road centerlines are used to plot routes.

Before Machine Learning

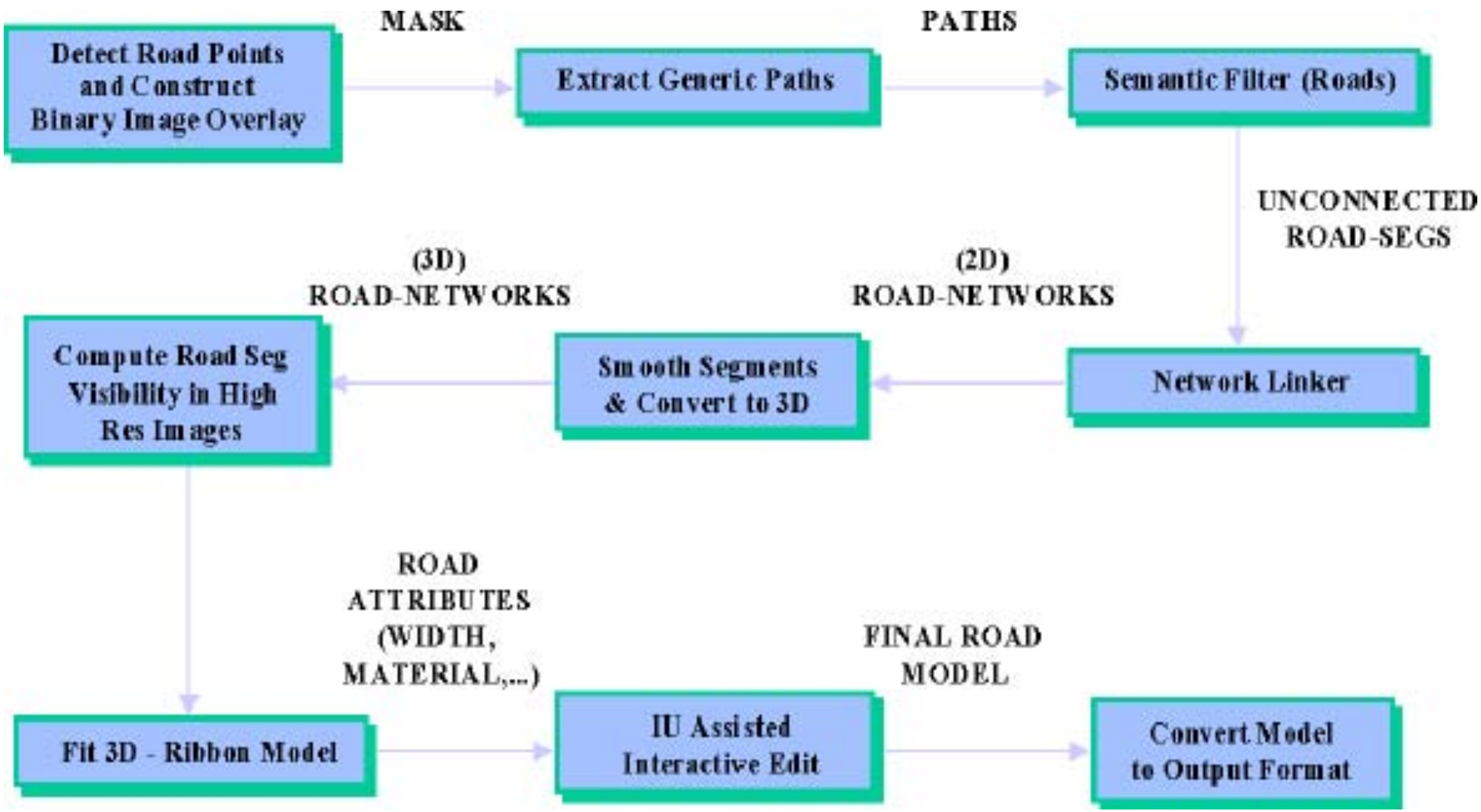
Detect road centerlines

Find generic paths

Apply semantic filter

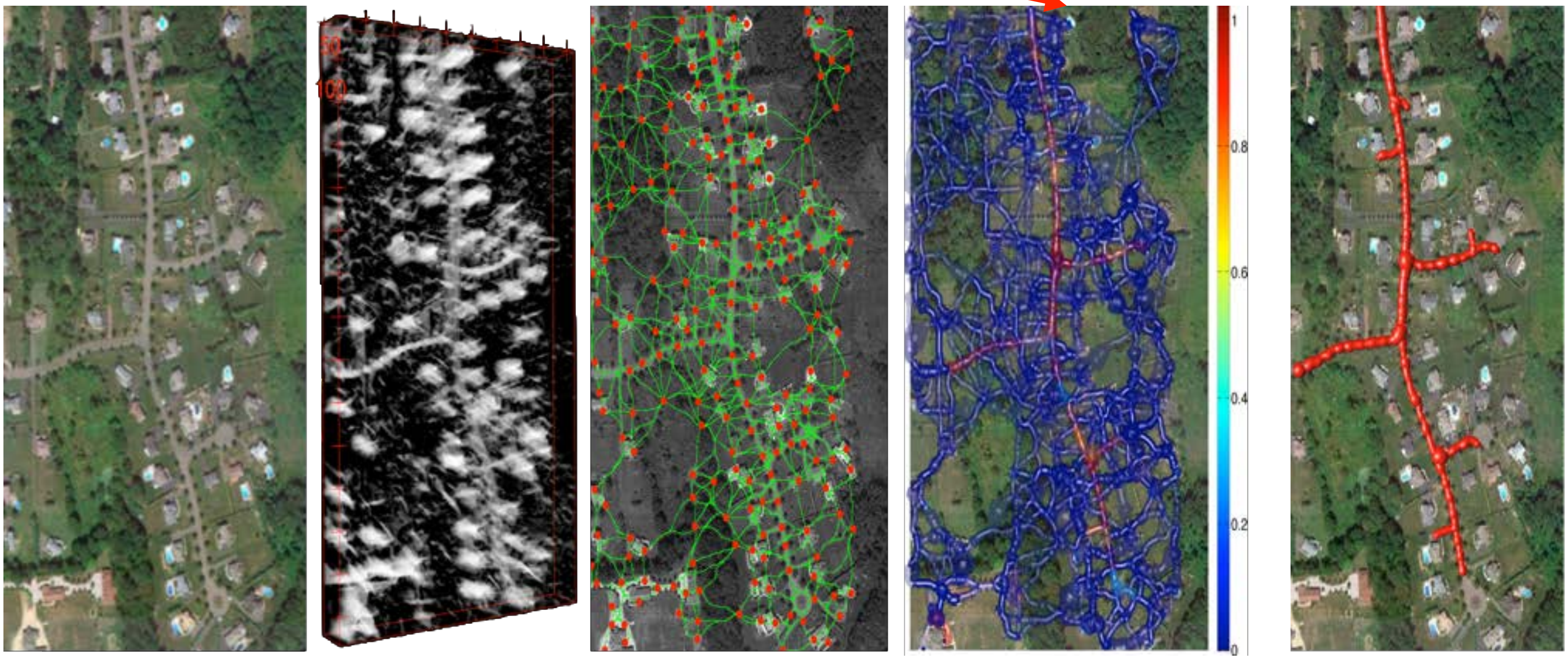
Fischler & Heller, 1998.

Boxology


Detect Road Points and Construct Binary Image Overlay
MASK
Extract Generic Paths
PATHS
Semantic Filter (Roads)
UNCONNECTED ROAD-SEGS
Network Linker
(2D) ROAD-NETWORKS
Smooth Segments & Convert to 3D
(3D) ROAD-NETWORKS
Compute Road Seg Visibility in High Res Images
Fit 3D - Ribbon Model
ROAD ATTRIBUTES (WIDTH, MATERIAL,...)
IU Assisted Interactive Edit
FINAL ROAD MODEL
Convert Model to Output Format

After Machine Learning

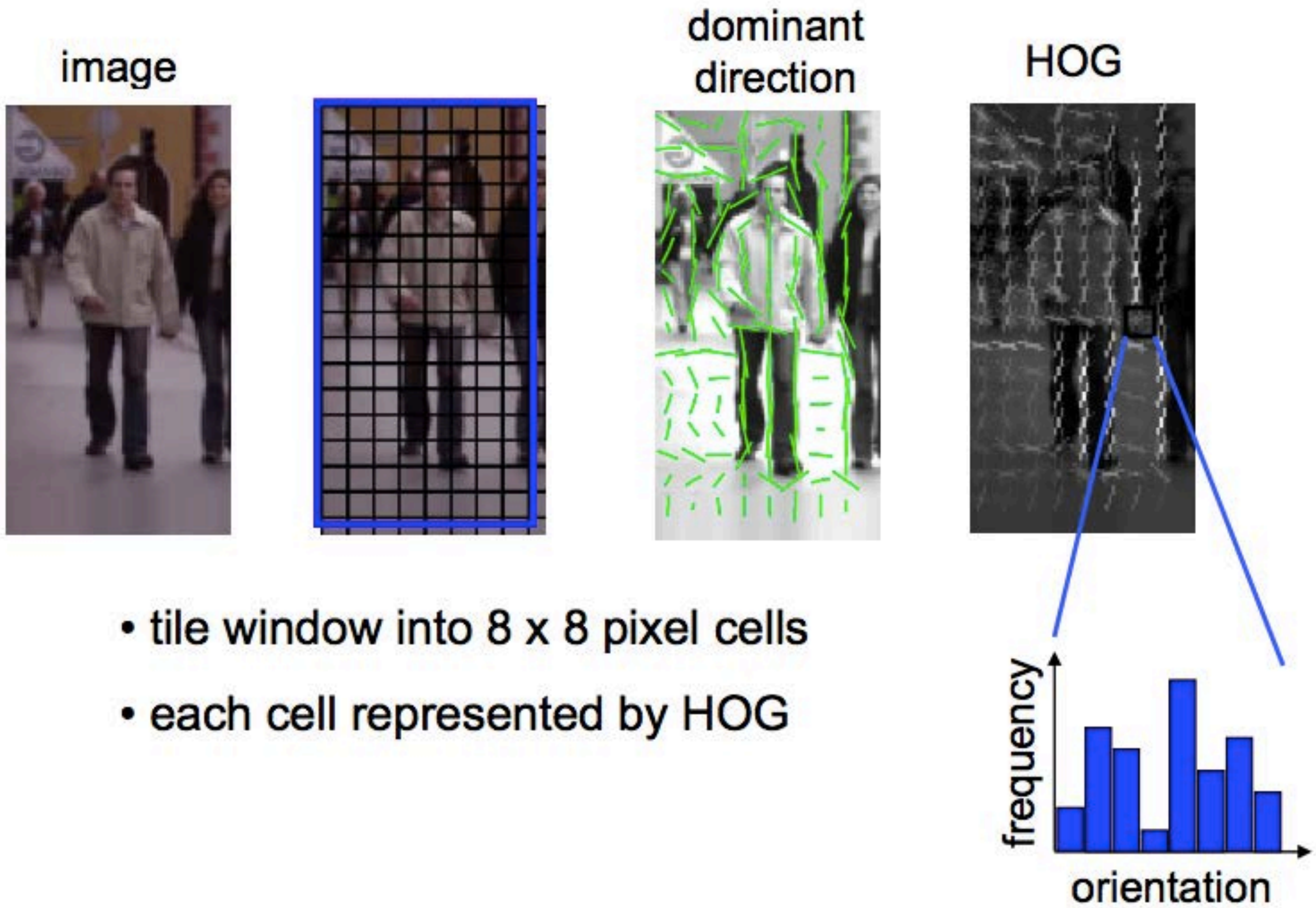
Train a classifier to do this.



To train the classifier, we must associate a feature vector to each path and they all must be of the same dimension.

Turetken et al., PAMI'16.

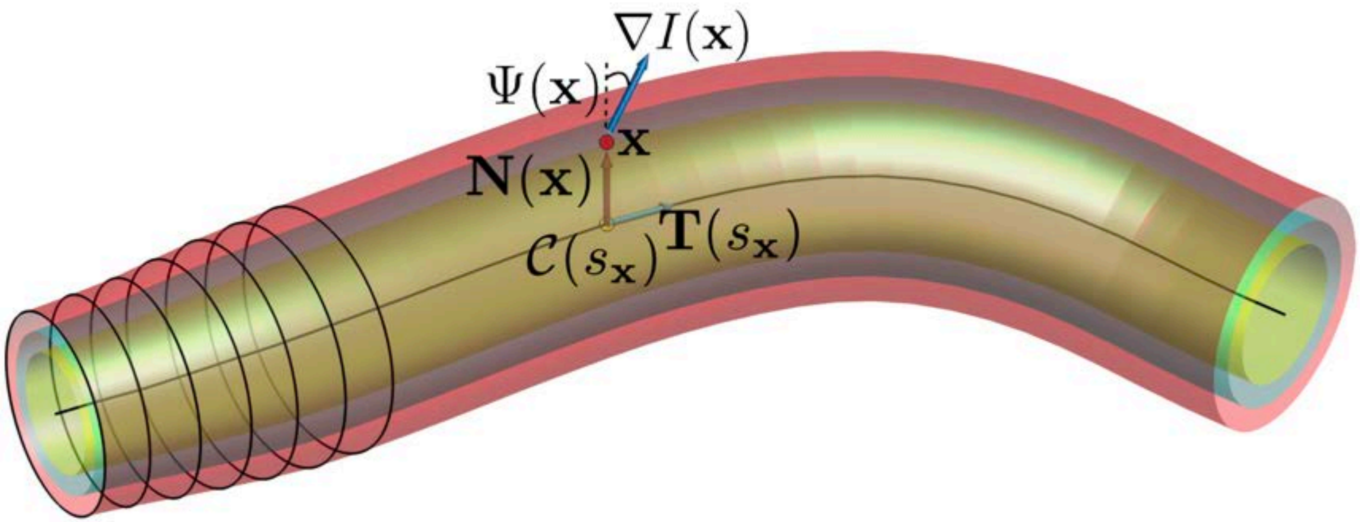
Histogram of Oriented Gradients



- tile window into 8 x 8 pixel cells
- each cell represented by HOG

Feature vector dimension = 16 x 8 (for tiling) x 8 (orientations) = 1024

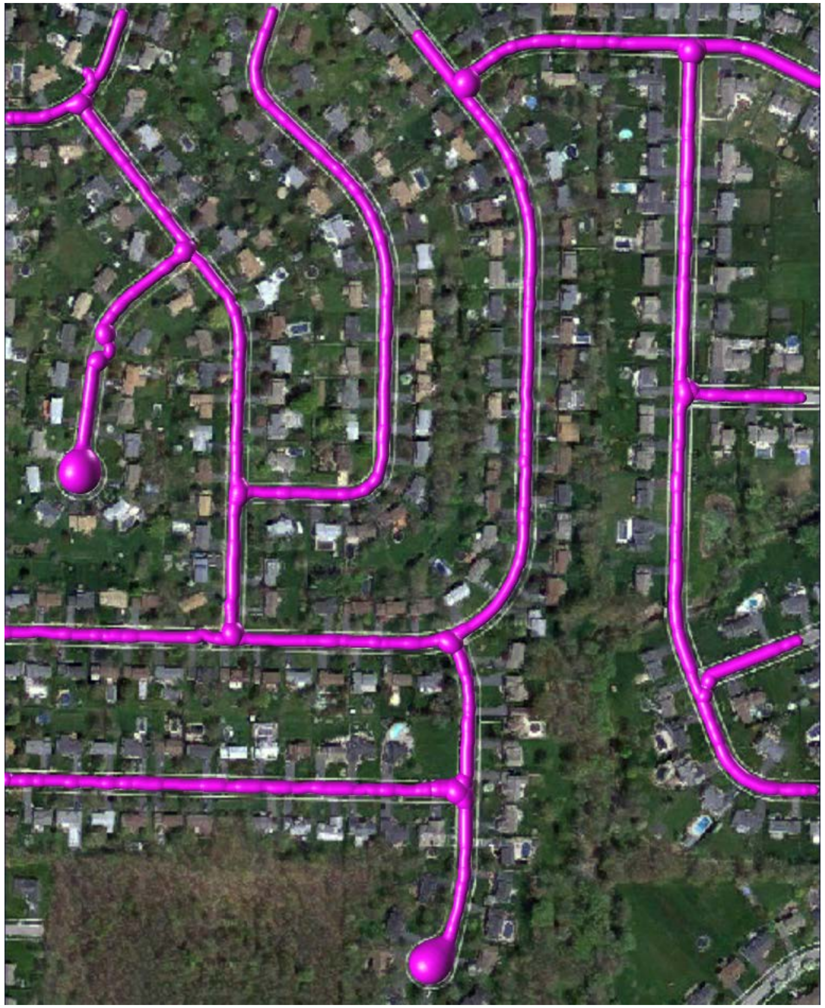
Histogram of Gradient Deviations



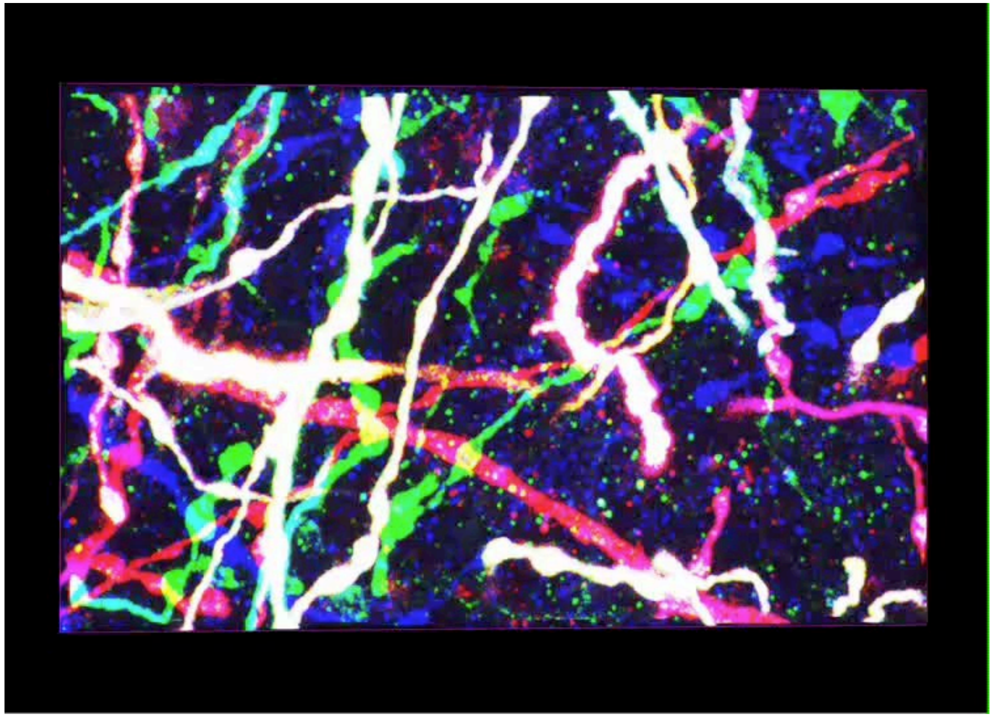
$$\Psi(\mathbf{x}) = \begin{cases} \text{angle}(\nabla I(\mathbf{x}), \mathbf{N}(\mathbf{x})), \text{if } \|\mathbf{x} - \mathcal{C}(s_{\mathbf{x}})\| > \varepsilon \\ \text{angle}(\nabla I(\mathbf{x}), \mathbf{\Pi}(\mathbf{x})), \text{otherwise}, \end{cases}$$

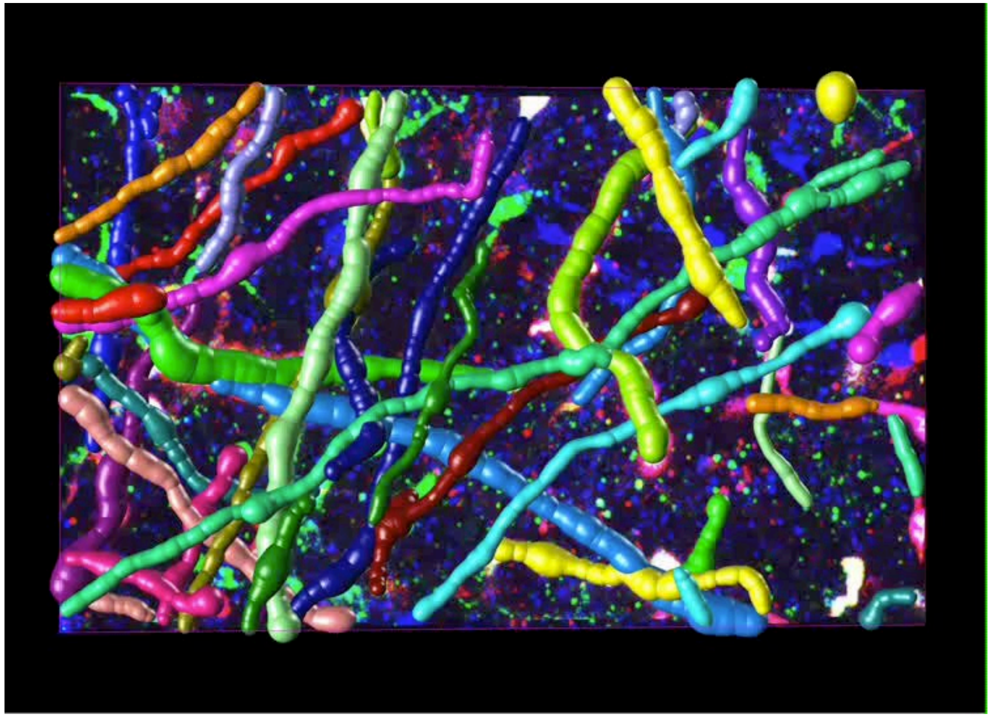
—> One histogram per radius interval plus four geometric features (curvature, tortuosity, ….).

Roads

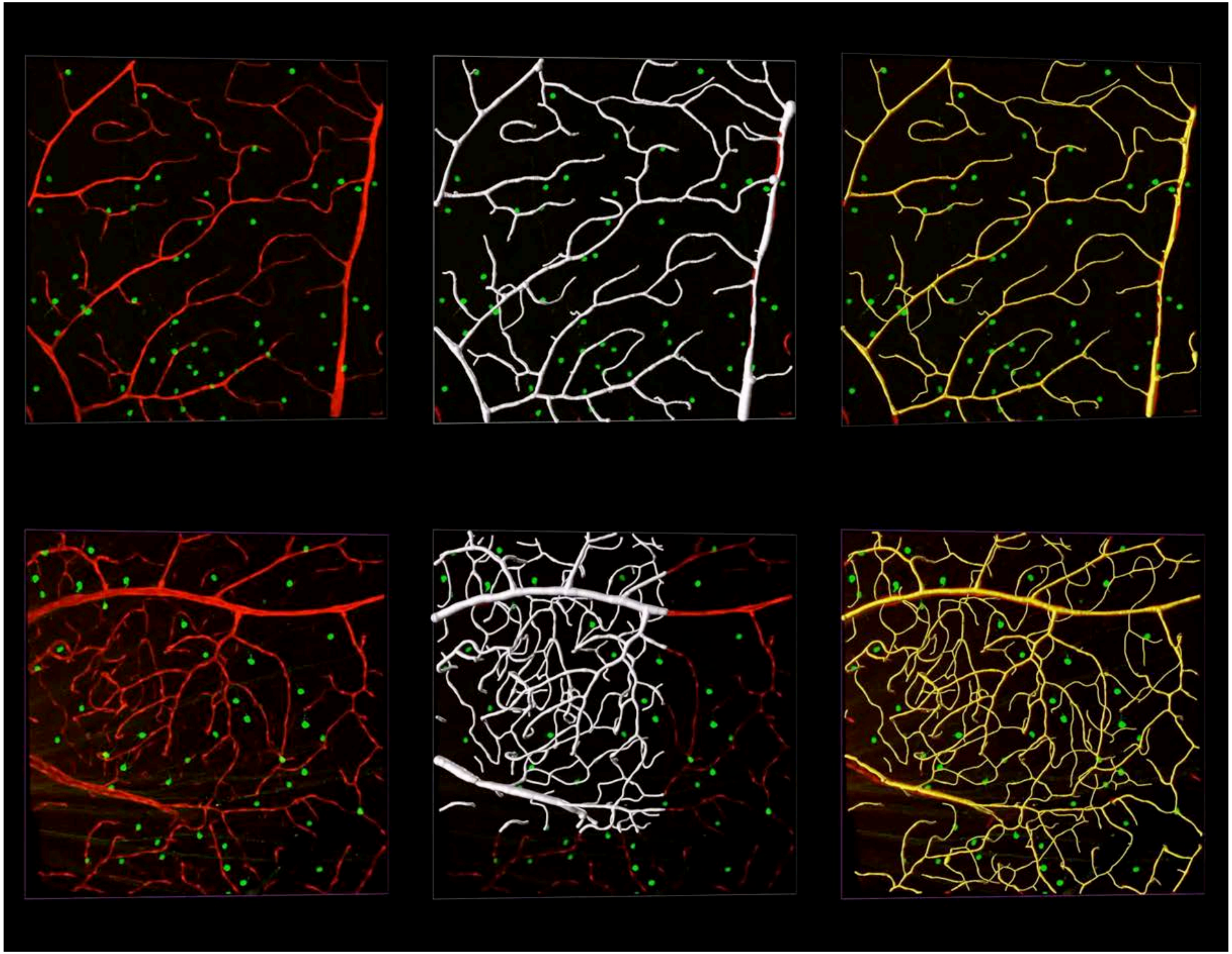


Brainbow Images





Blood Vessels



Deep Learning Tsunami

AlexNet 2012

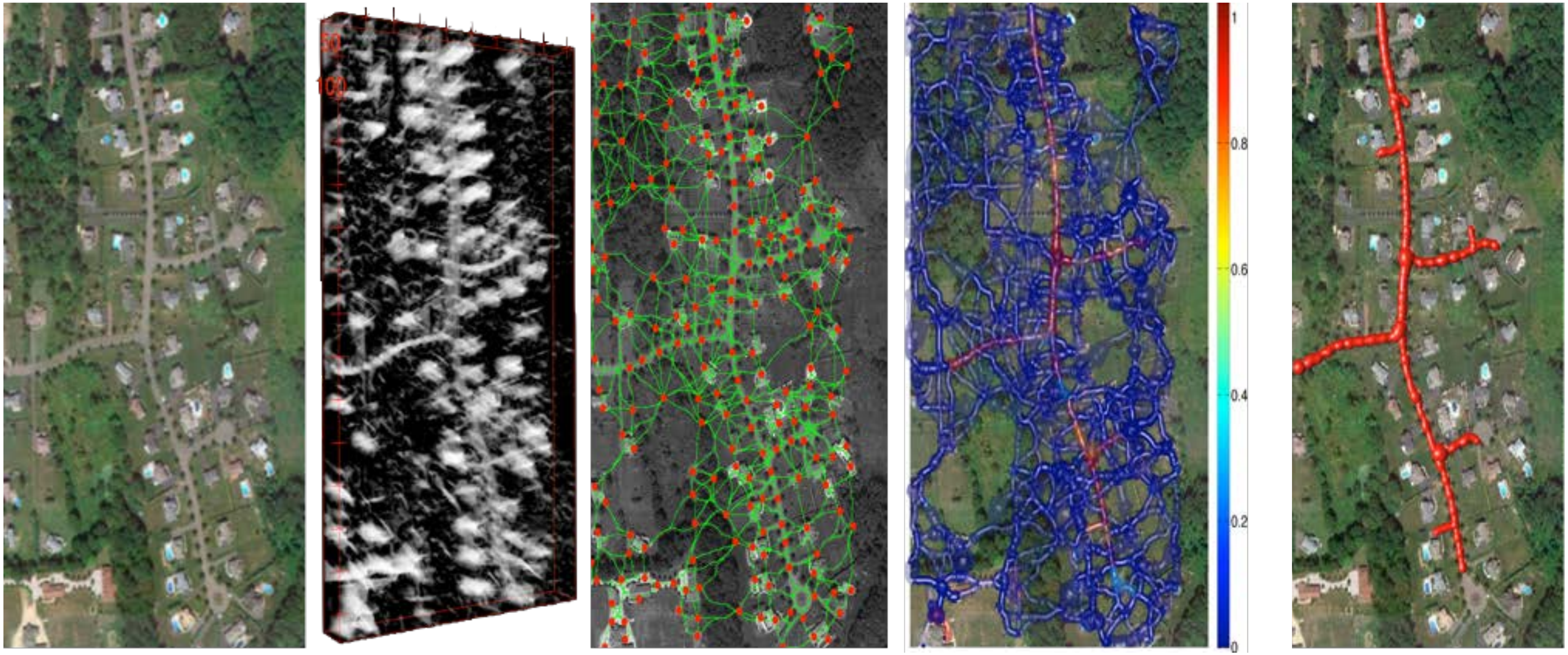
The end of computer science as we know it

or .....

An opportunity to revisit and improve the pipeline:

- Reformulate individual components in terms of CNNs.
- Make them consistent with each other.

Before Deep Learning

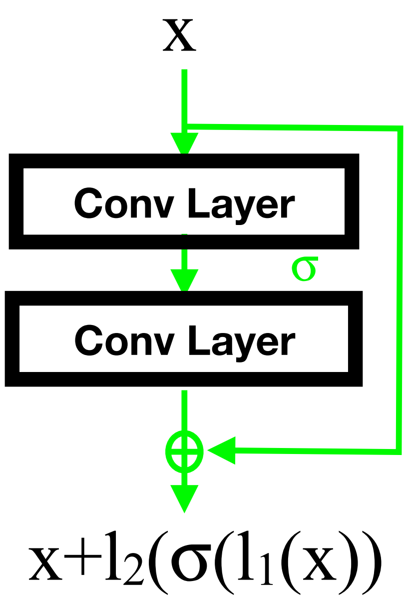


- Machine learning enables the **same** algorithm to work in many different contexts but requires hand-designed features.
- However, computing the tubularity and classifying the paths are closely related tasks. They should not be treated separately.

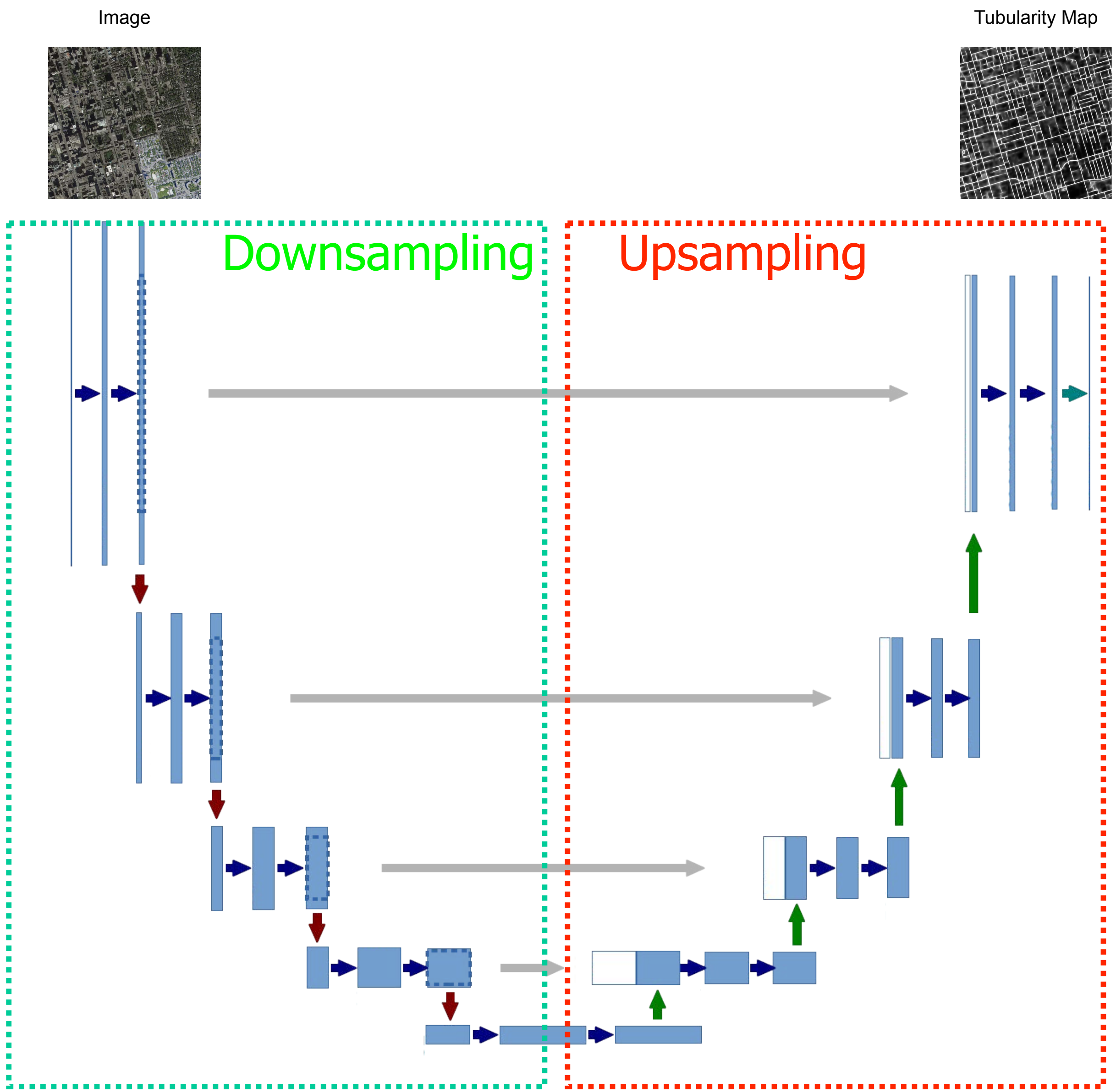
—> Can we use Deep Learning to account for this?

Turetken et al., PAMI'16

ResNet to U-Net

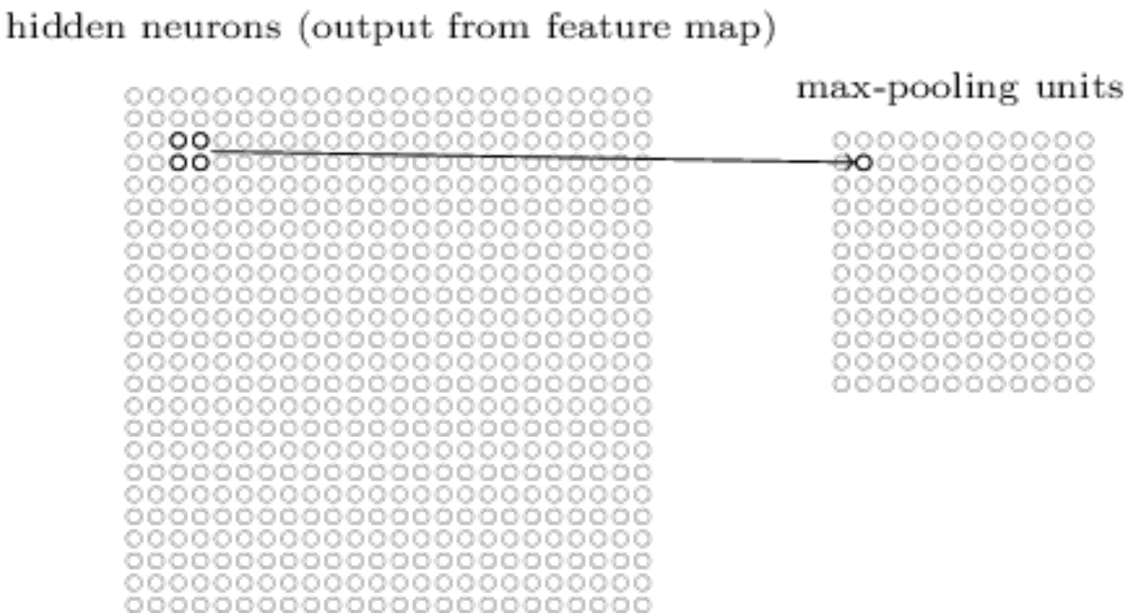


ResNet block



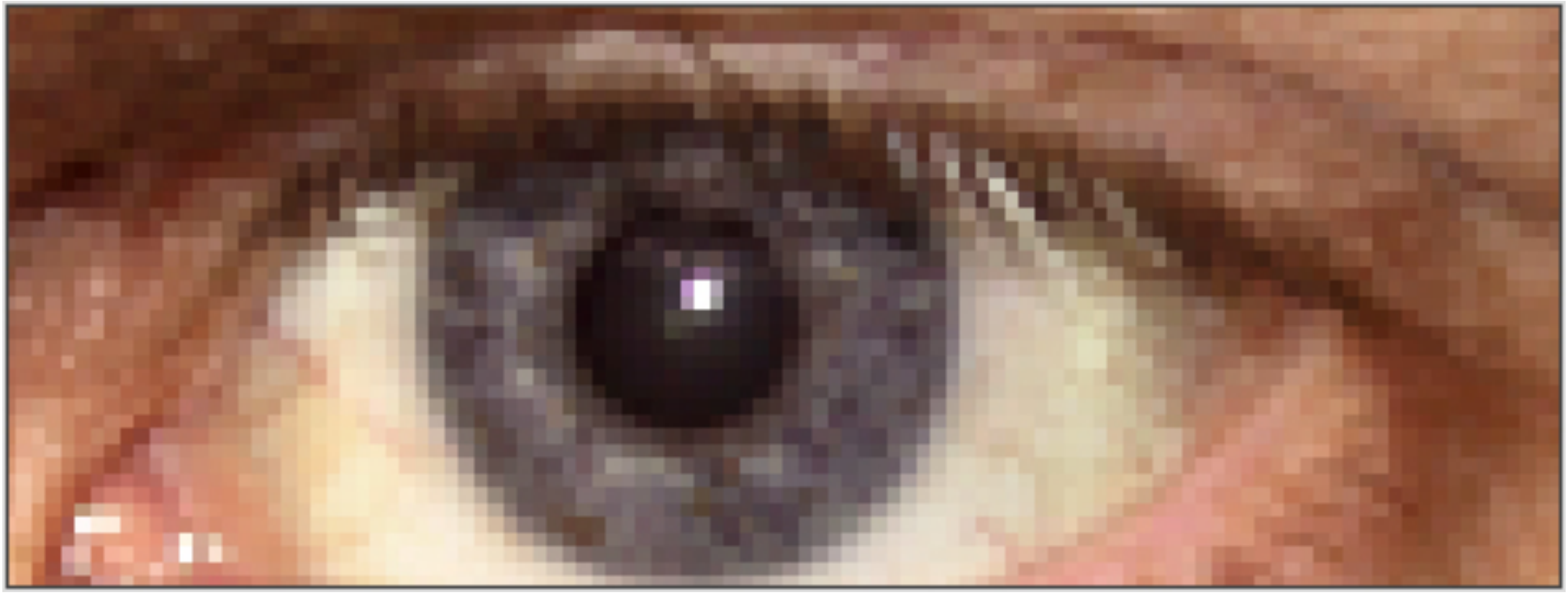
U-Net

Reminder: Downsampling by Pooling



- Reduces the number of inputs by replacing all activations in a neighborhood by a single one.
- Can it be reversed?

Upsampling by Duplication



i1	i2
i3	i4

i1	i1	i2
i1	i1	i2
i3	i3	i4

Upsampling by Interpolation

i1	i2
i3	i4

i1	i5=(i1+i2)/2	i2
i6=(i1+i3)/2	i9=(i1+i2+i3+i4)/4	i7=(i2+i4)/2
i3	i8=(i3+i4)/2	i4

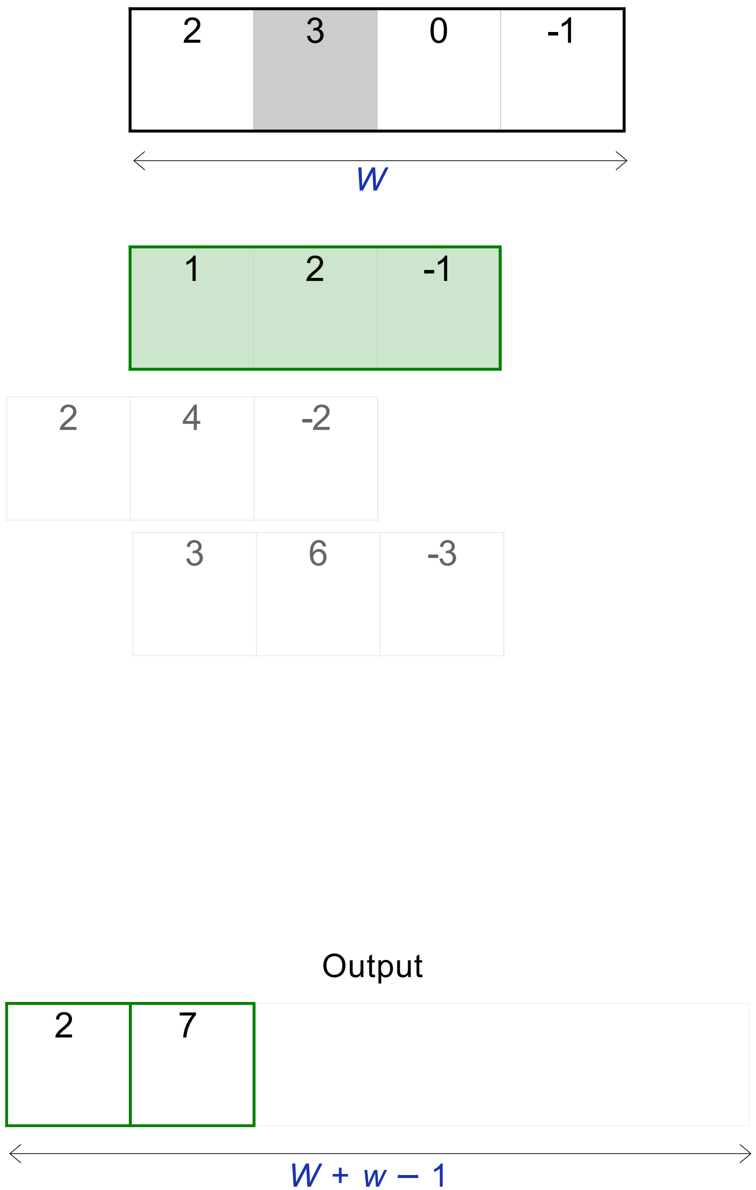
Upsampling by Bilinear Interpolation

i1	i2
i3	i4

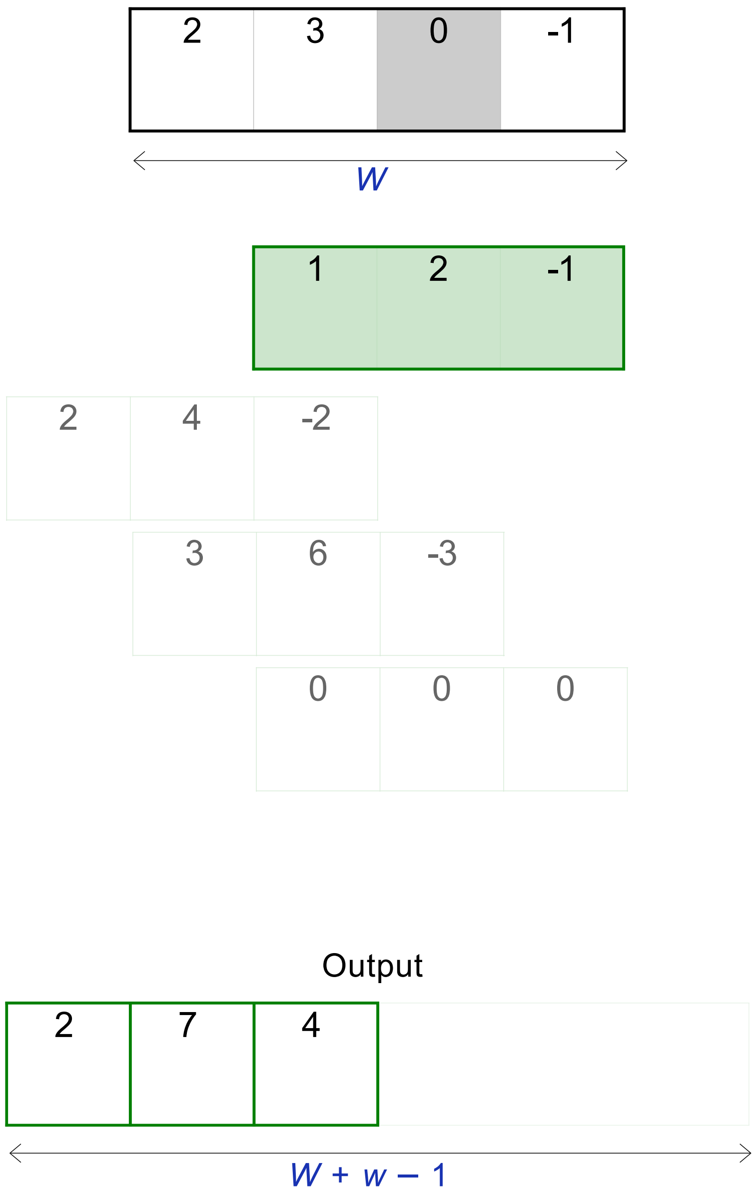
	I10=(i1 + i2 + . + .) / 4	
i1	i11=(i10+i1+i2+i9)/4	i2
	i9=(i1+i2+i3+i4)/4	
i3		i4

—> Each pixel is the average of 4 neighbors.

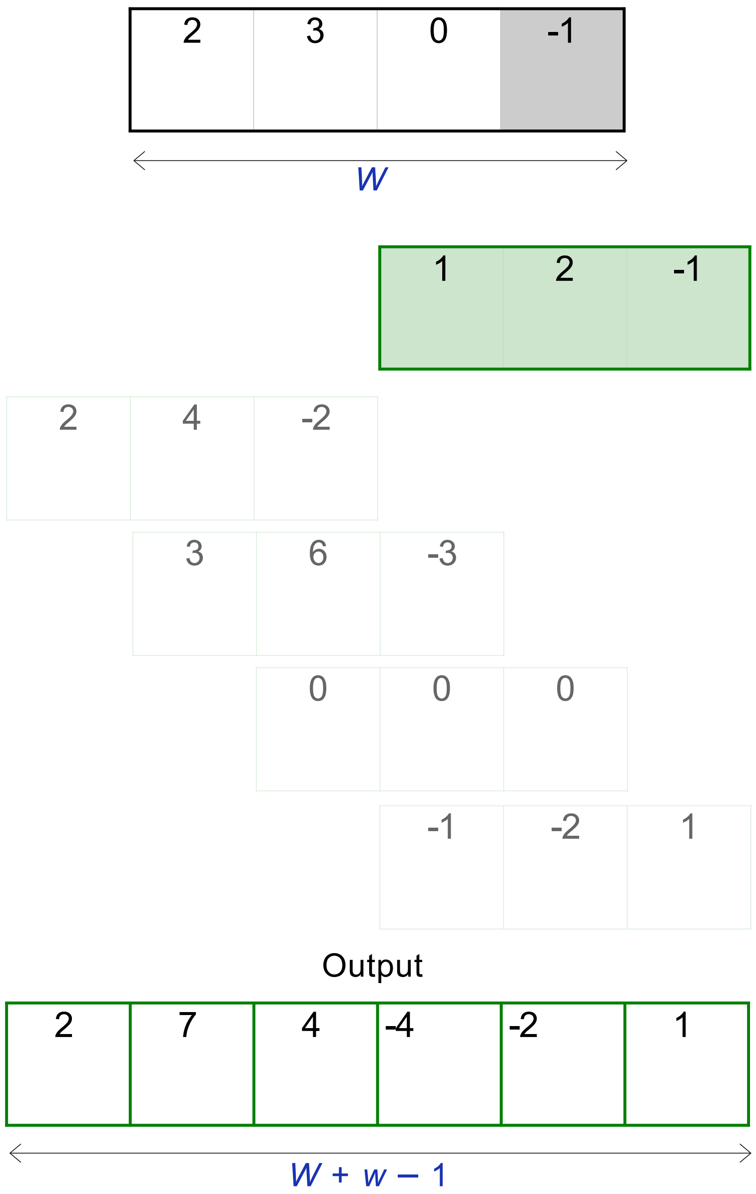
Upsampling by Transposed 1D Convolution



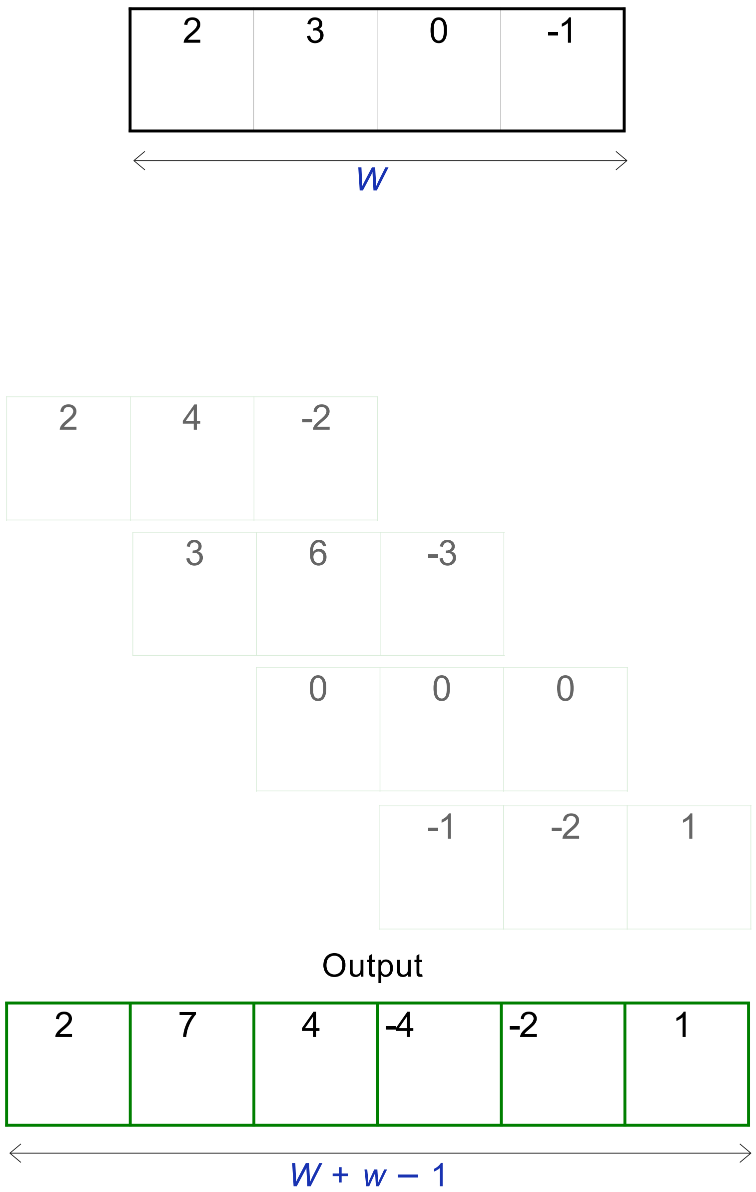
Transposed 1D Convolution



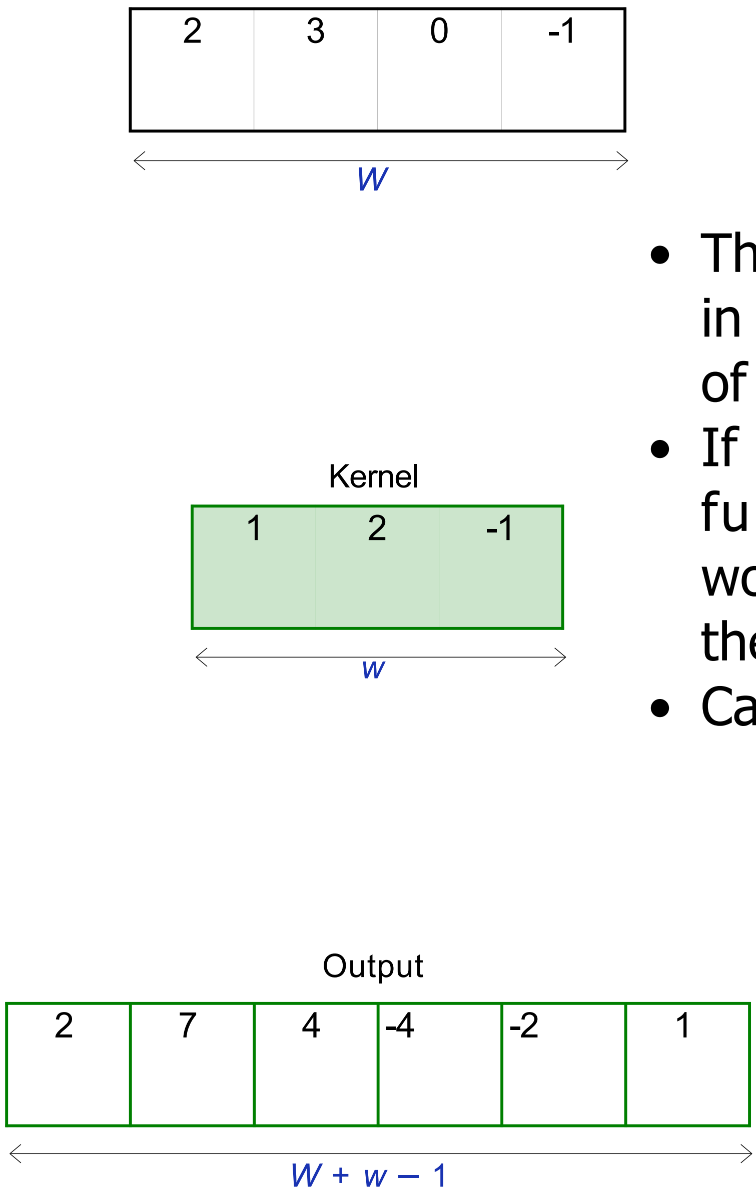
Transposed 1D Convolution



Transpose 1D Convolution

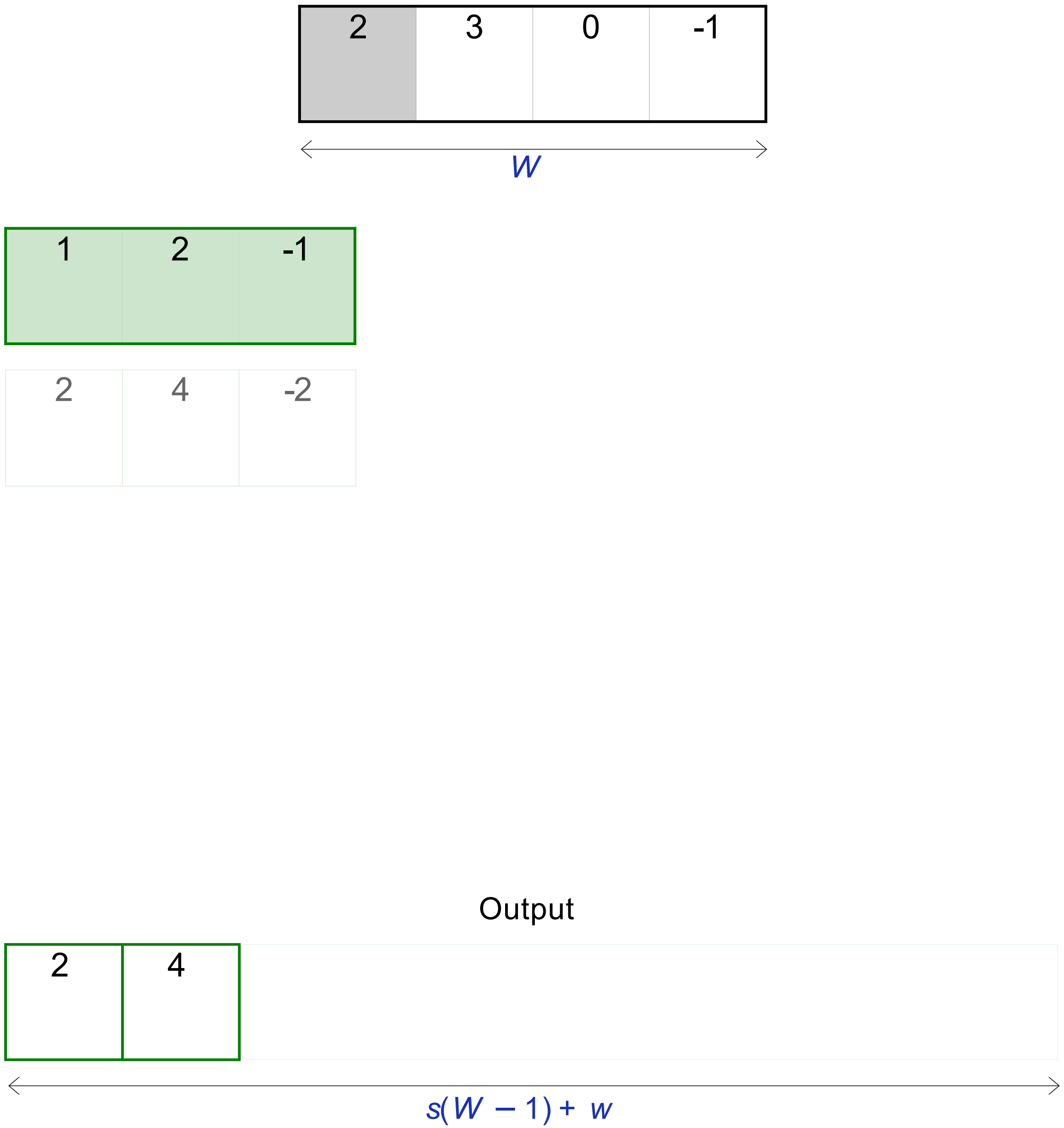


Transposed 1D Convolution

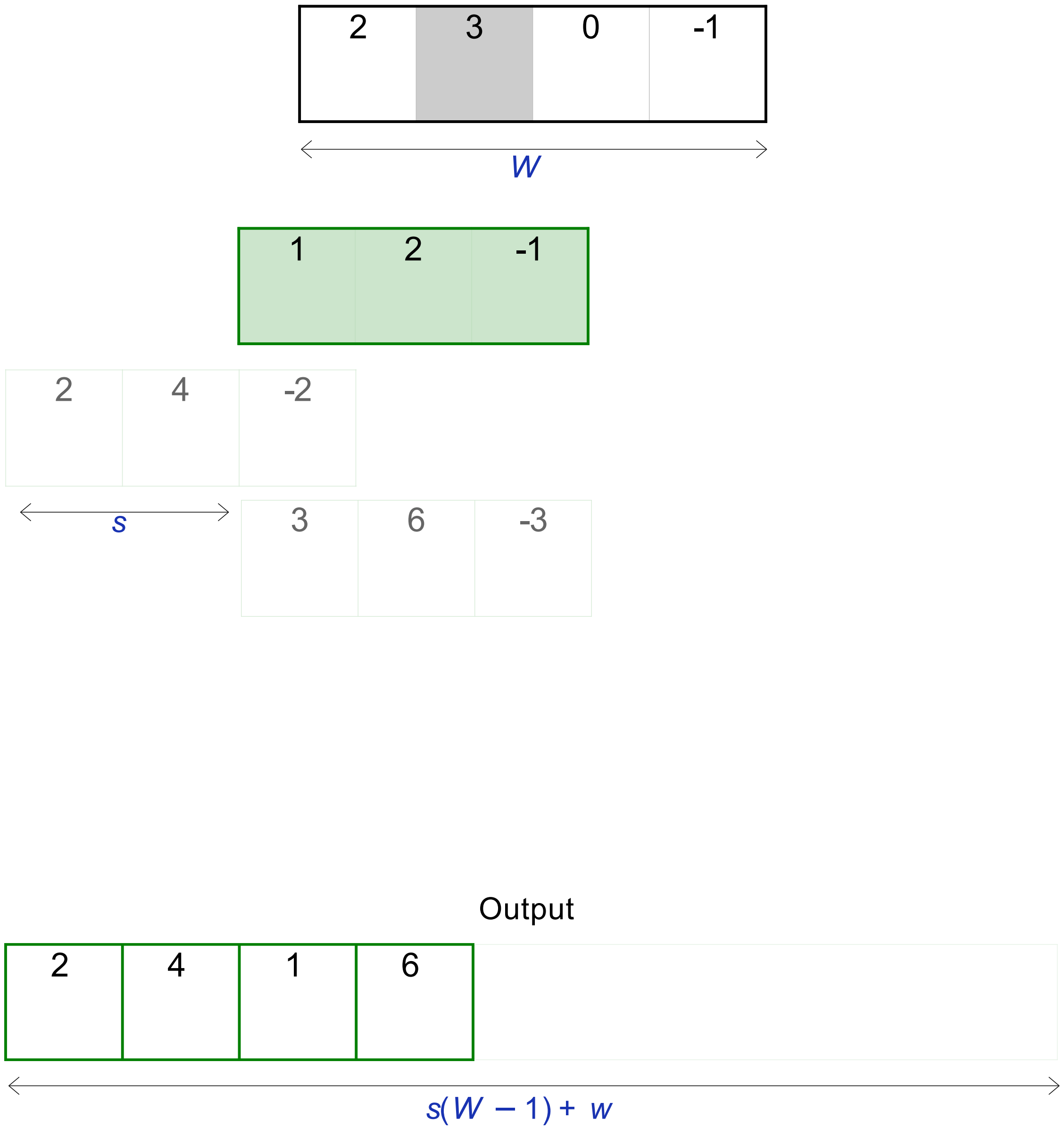


- The summations are performed in the vertical direction instead of the horizontal one.
- If we wrote this in terms of a fully connected layer, this would amount to transposing the weight matrix.
- Can be extended to 2D layers.

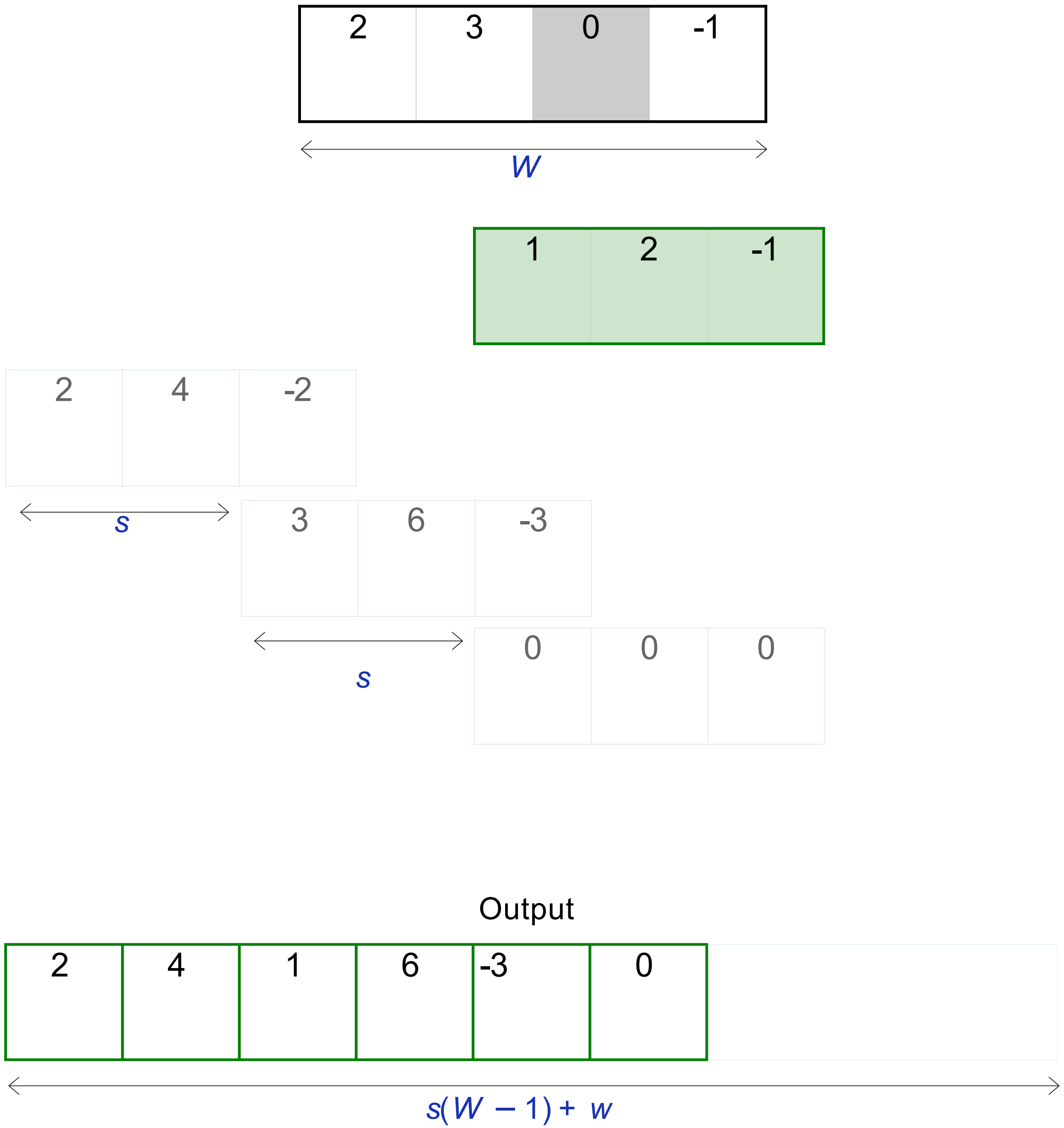
Introducing a Stride Parameter



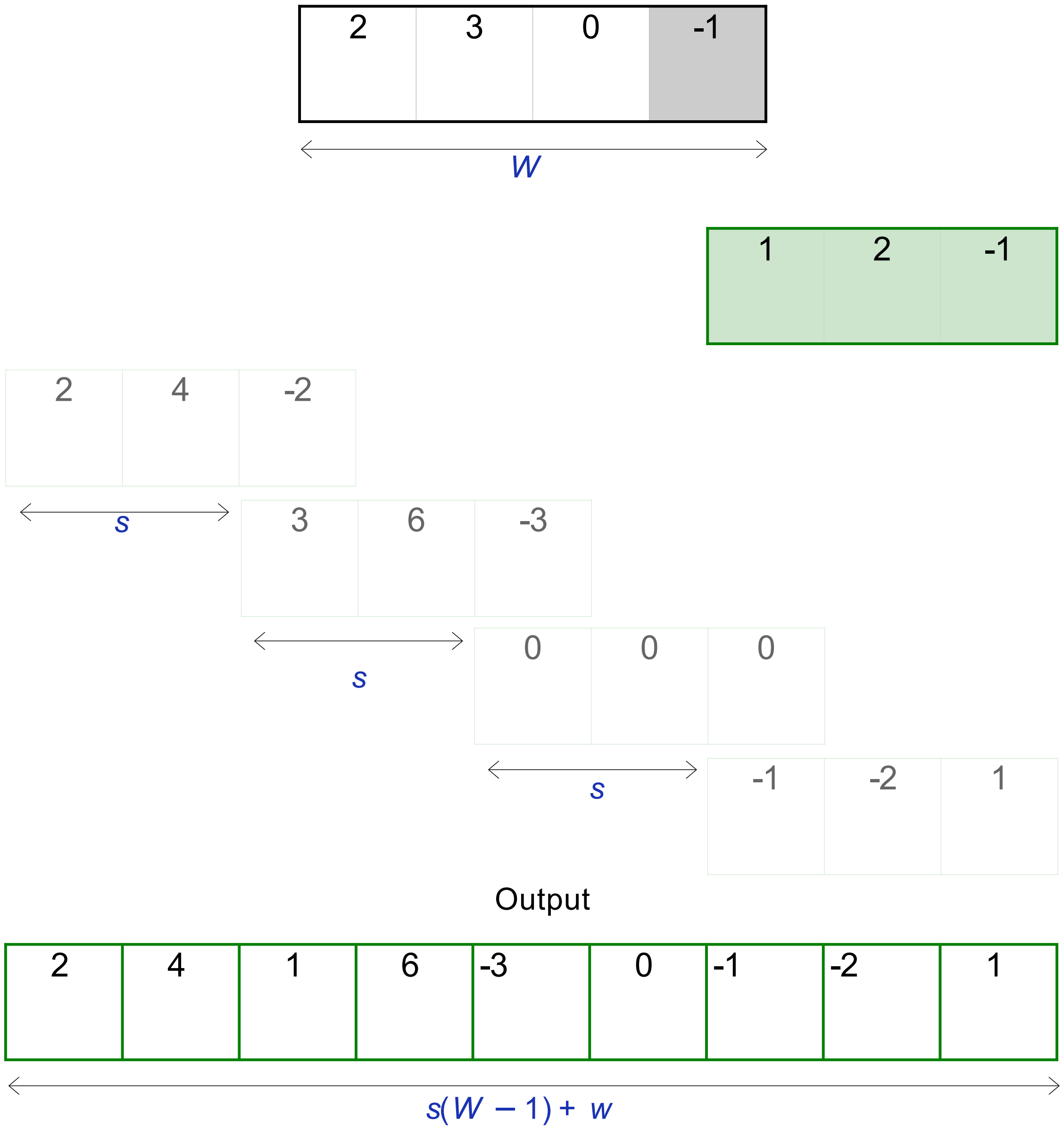
Introducing a Stride Parameter



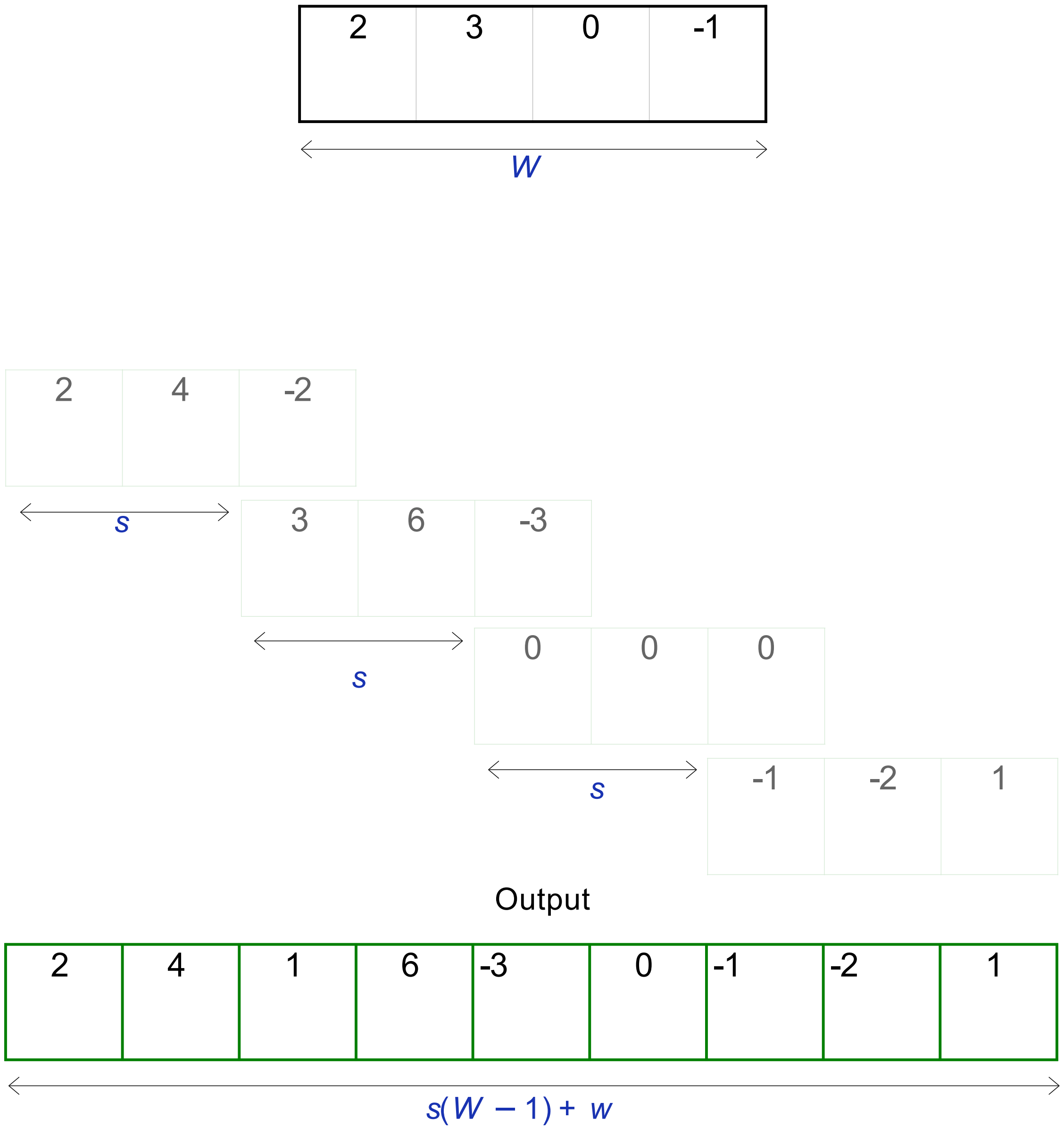
Introducing a Stride Parameter



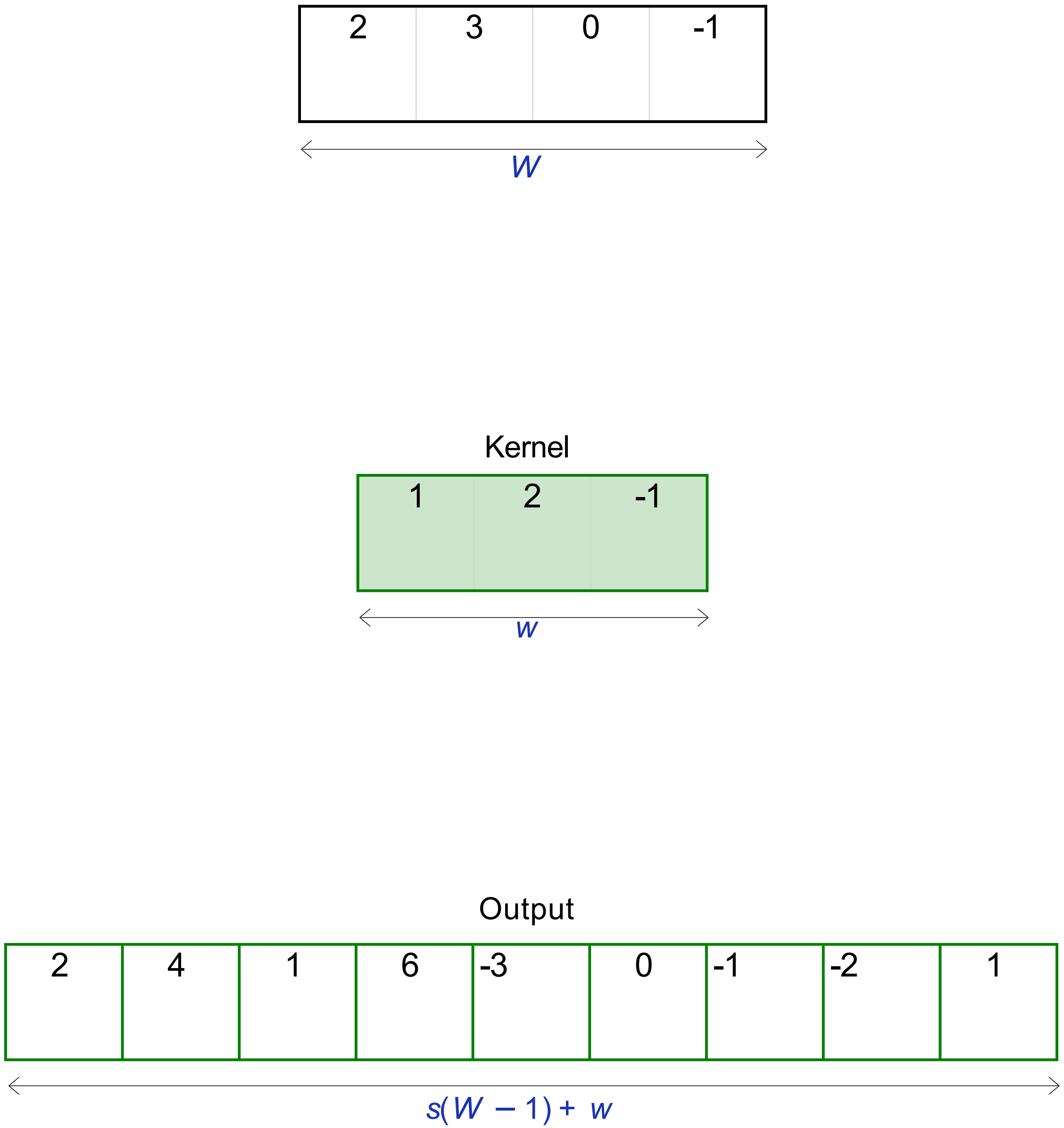
Introducing a Stride Parameter



Introducing a Stride Parameter

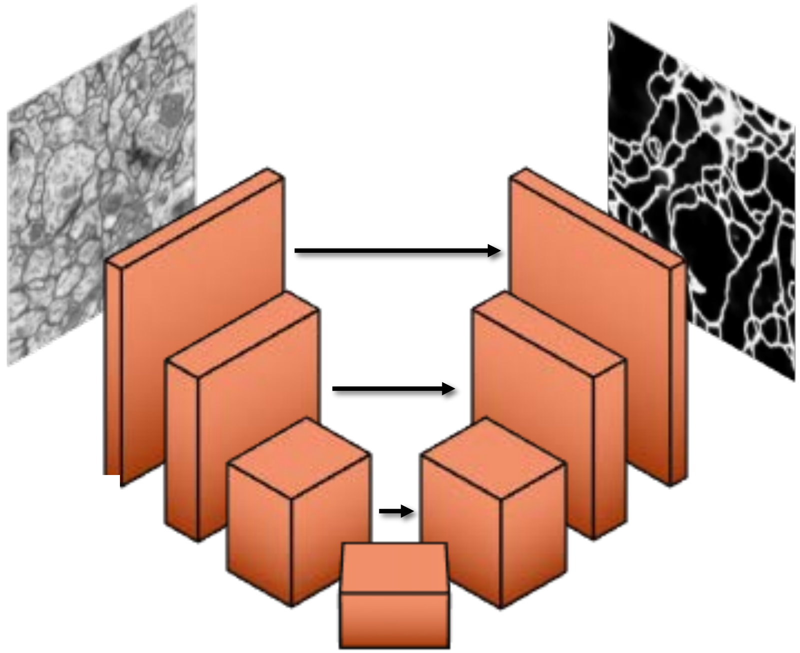


Introducing a Stride Parameter



Estimating the Tubularity

Train Encoder-decoder U-Net architecture using binary cross-entropy



Minimize

$$L_{BCE} = \frac{1}{N}\sum_{i=1}^{N} y_n log(\hat{y}_n) + (1 - y_n) log(\hat{y}_n)$$

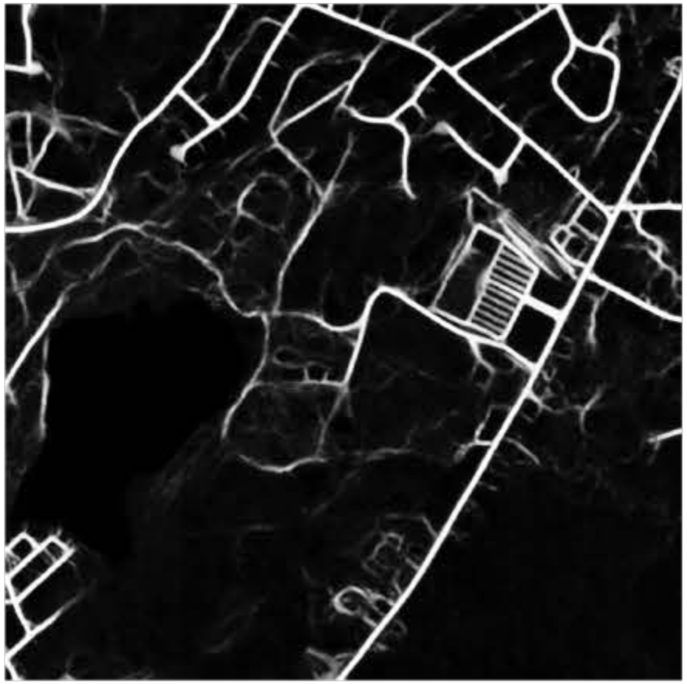
where

- $\hat{\mathbf{y}} = f_{\mathbf{w}}(\mathbf{x})$,
- $\mathbf{x}$ in an input image,
- $\mathbf{y}$ the corresponding ground truth.

Mosinska et al, CVPR'18.

Tubularity Map

Image



BCE Loss

Ground truth

Iterative Refinement

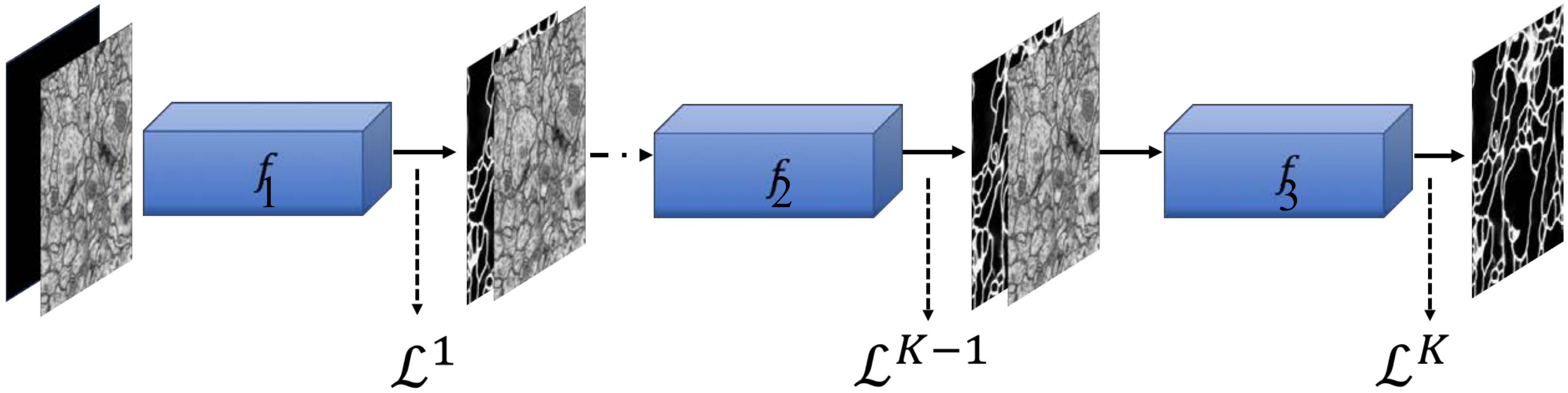
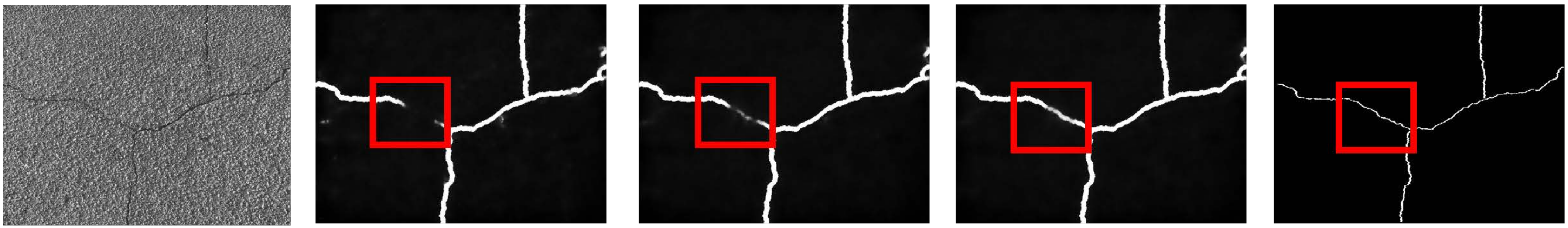


Image　Iter 1　Iter 2　Iter 3　Ground truth

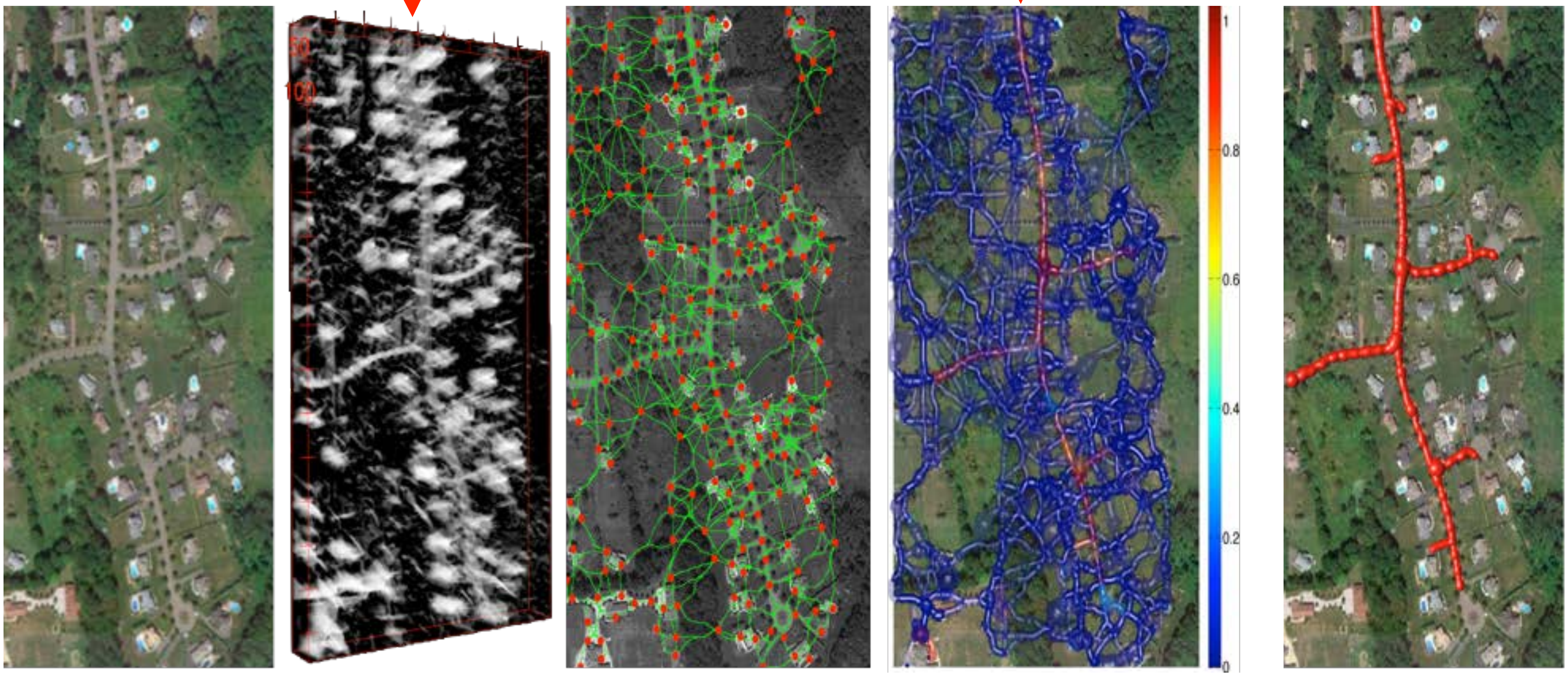


Use <u>the same network</u> to progressively refine the results keeping the number of parameters constant

Before Deep Learning

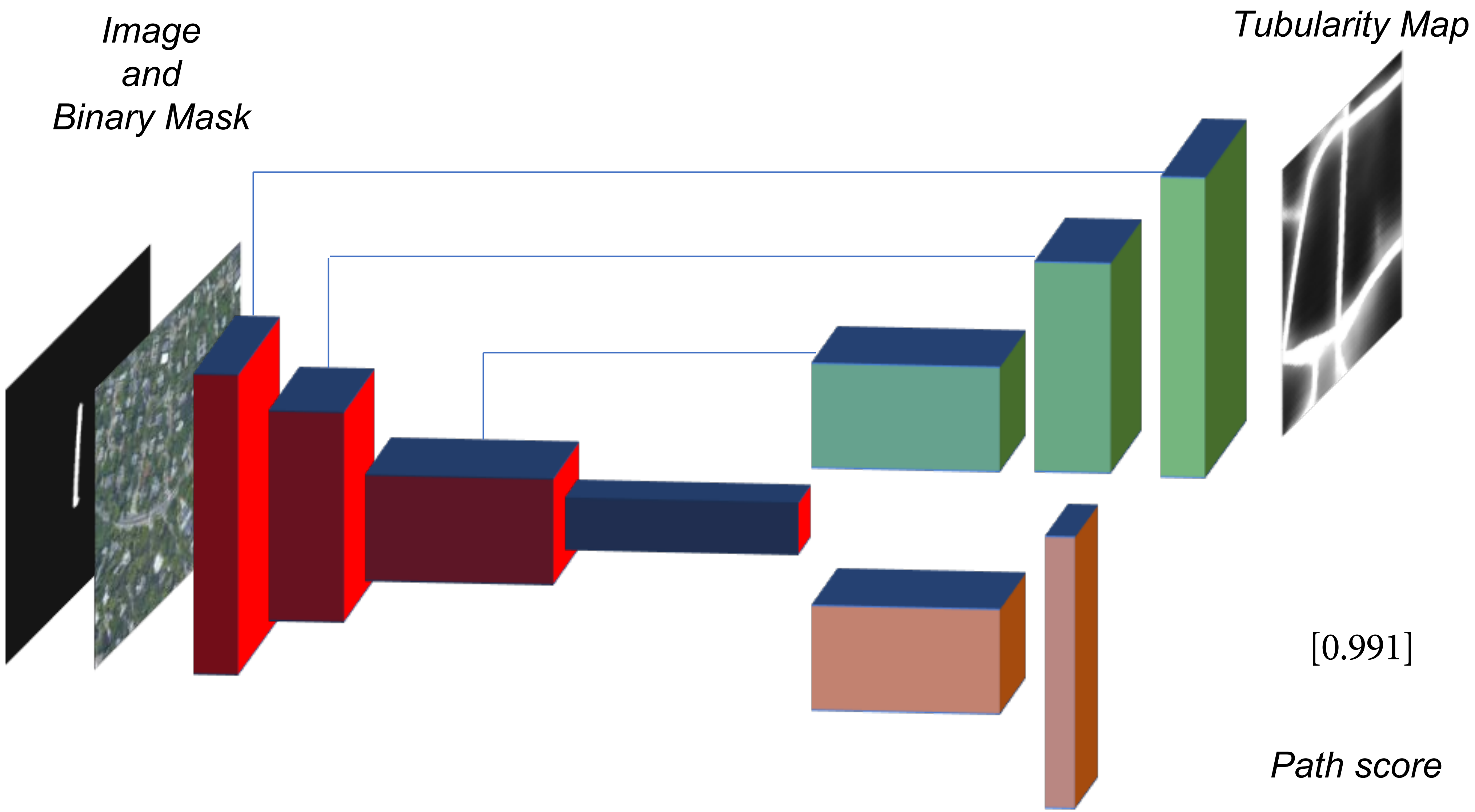
U-Net does this better.

Can it also do this?



Turetken et al., PAMI'16.

Dual Use UNet



After Deep Learning

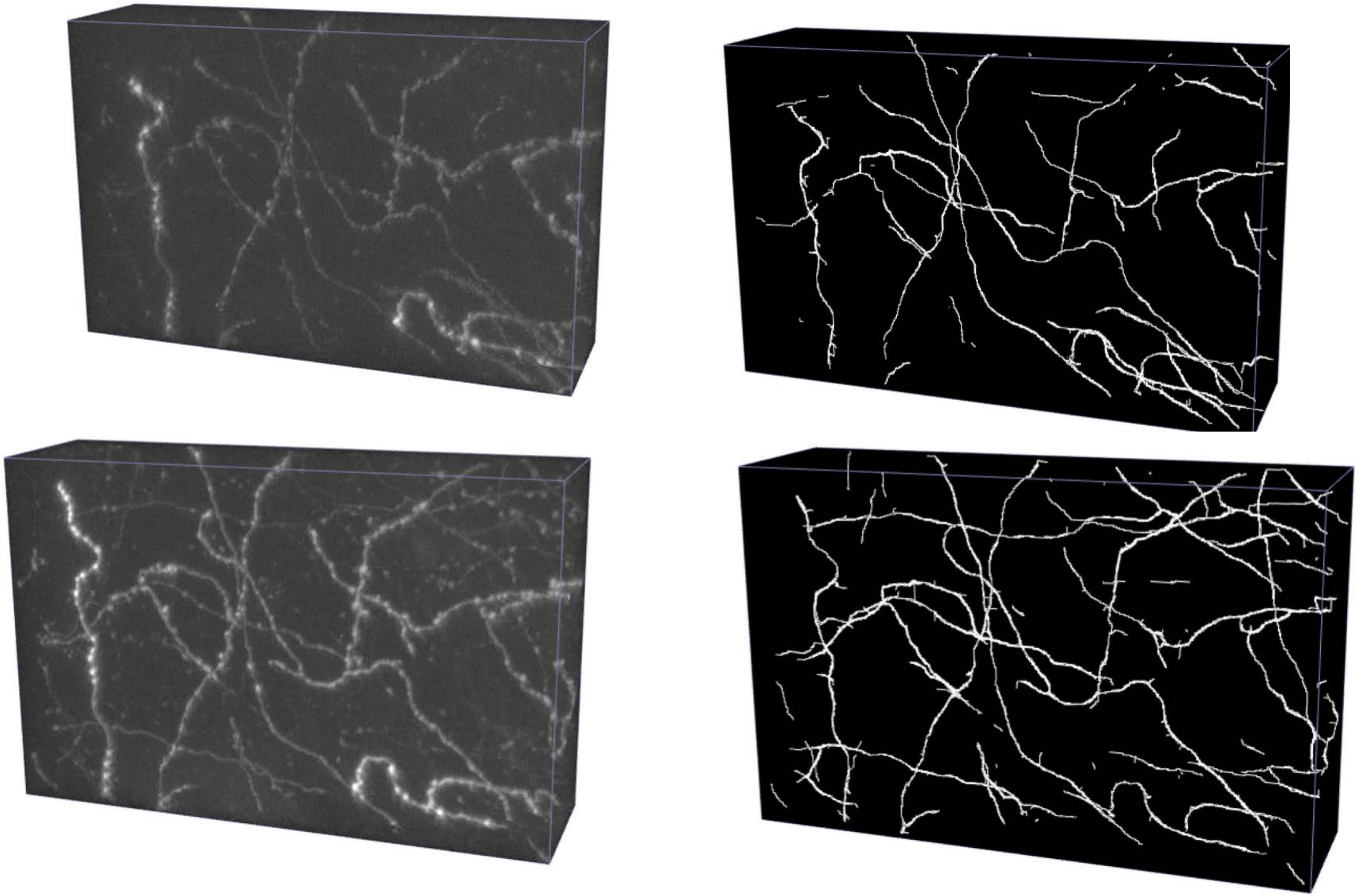
1. Compute a probability map.
2. Sample and connect the samples.
3. Assign a weight to the paths.
4. Retain the best paths.

Streets of Toronto

— False negatives
— False positives

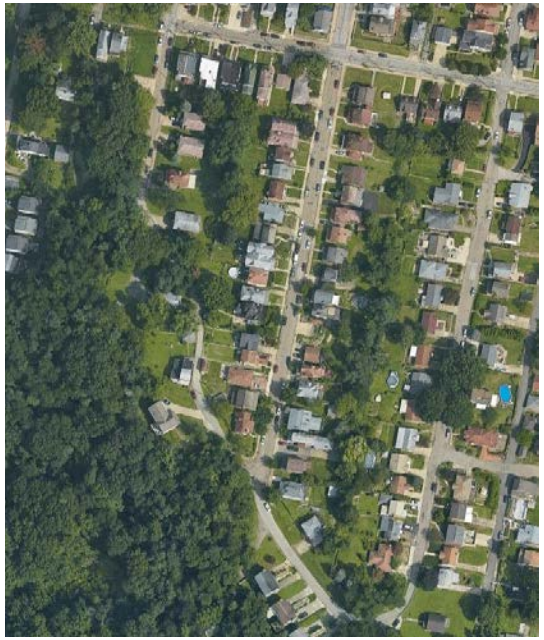
EPFL

Dendrites and Axons

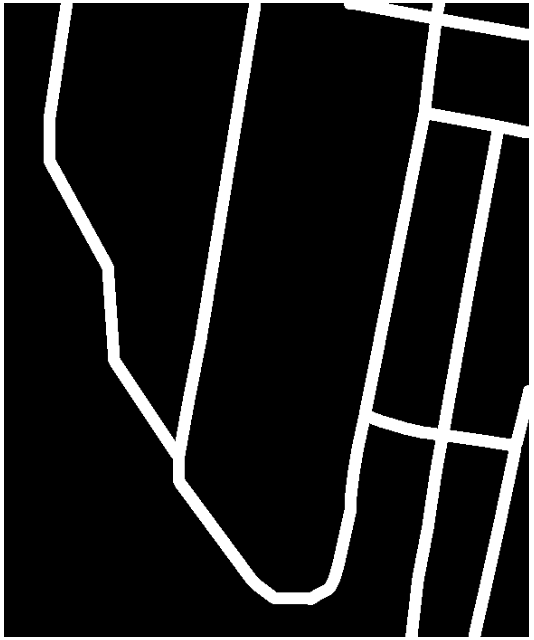


- Deep learning allows the **same** algorithm to work in different contexts.
- The implementation is informed by earlier approaches.

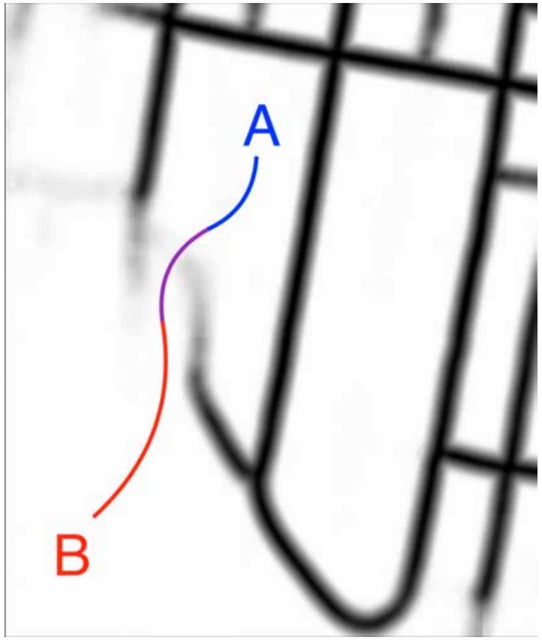
Accounting for Topology



Image

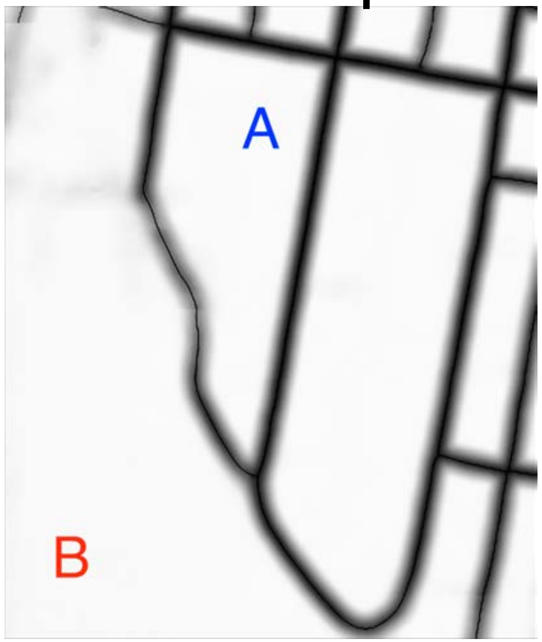


Ground truth



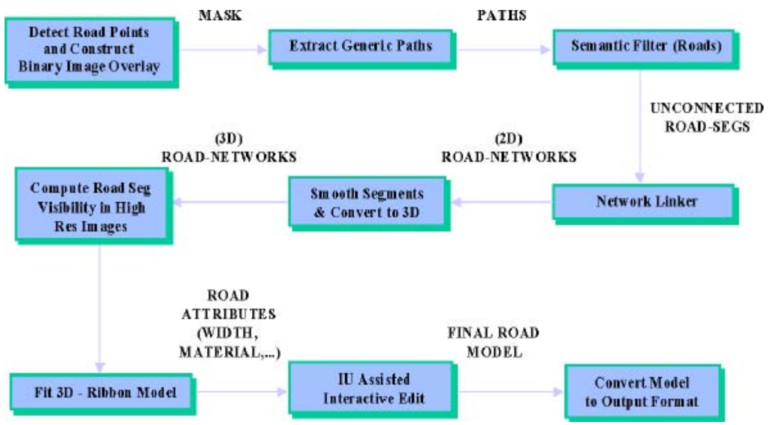
UNet output

—> Add a term in the loss function that penalizes the existence of a path between A and B.

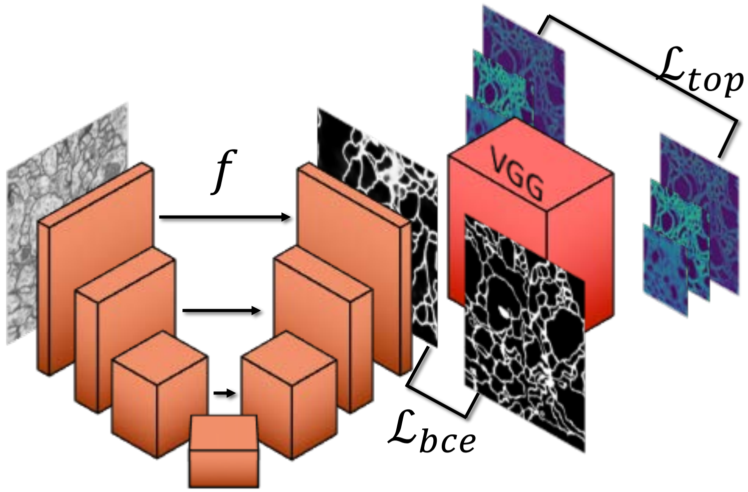


Improved output

1998 - 2038



1998



2018

?

2038

It is difficult to make predictions, especially about the future.
Sometimes attributed to Niels Bohr.

Alpha Go

- Uses Deep Nets to find the most promising locations to focus on.
- Performs Tree based search when possible.
- Relies on reinforcement learning and other ML techniques to train.

—> Beat the world champion in 2017.