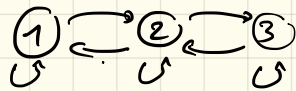


ΠCAA lecture 5: quiz solutions

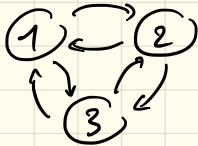
- 1) a) no ($\exists i, j$ with $p_{ij} > 0$ & $p_{ji} = 0$)
b) surely ($P =$ tridiagonal matrix)
c) possibly (depends on the values of the transition probabilities)
d) no ($\exists i, j$ with $p_{ij} > 0$ & $p_{ji} = 0$)
e) surely ($P =$ tridiagonal matrix)
f) surely ($P \neq$ tridiagonal matrix, but still...)
- 2) No! If the chain is reversible and some $p_{ij} > 0$, then $p_{ji} > 0$
so necessarily $p_{ii}^{(2)} \geq p_{ij} \cdot p_{ji} > 0$, i.e., $d \leq 2$

3) a) $N=3$ & $P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/2 & 1/2 \end{pmatrix}$; $\lambda_0=1, \lambda_1=1/2, \lambda_2=0$



$(\gamma = 1/2)$

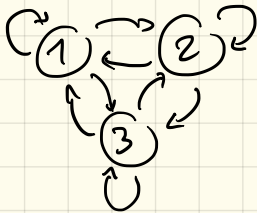
b) $N=3$ & $P = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$; $\lambda_k = \cos\left(\frac{2\pi k}{N}\right)$ $k=0,1,2$



so $\lambda_0=1, \lambda_1=\lambda_2=\cos\left(\frac{2\pi}{3}\right)=-\frac{1}{2}$

$(\gamma = 1/2)$

c) $N=3$ & $P = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$; $\lambda_0=1, \lambda_1=\lambda_2=0$



$(\gamma = 1)$