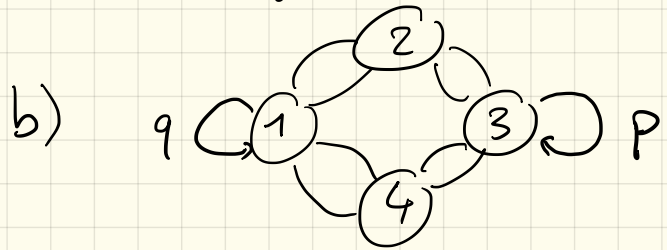
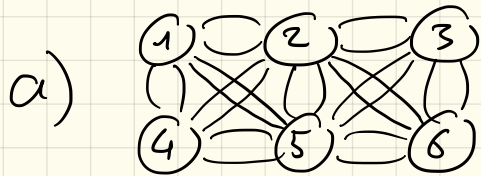
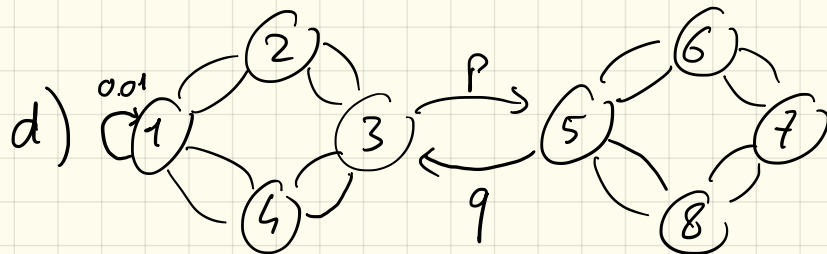
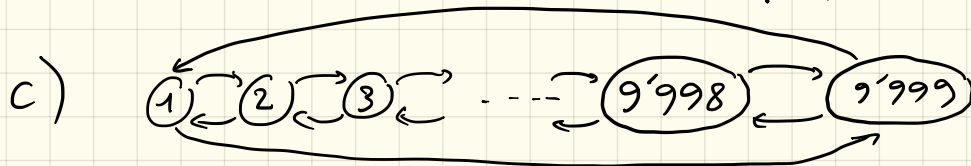


PCA lecture 6: quiz

1) Which of the following chains have a small spectral gap? Why?
 (assuming unmarked arrows have balanced weights)



b1) $p=q=0.01$ b2) $p=q=0.99$



d1) $p=q=0.01$

d2) $p=q=0.99$

2) For which of the previous chains having a small spectral gap does the addition of self-loops (all of the same weight) increase the spectral gap (and therefore the convergence rate)?

3) Let $(p_{ij}, j \in S)$ be probabilities s.t. $p_{ij} > 0 \forall j \in S$ & $\sum_{j \in S} p_{ij} = 1$

a) Show that if $\exists i \in S$ with $0 < c_j < c_i \forall j \in S$ & $\sum_{j \in S} p_{ij} c_j = c_i$
then $c_j = c_i \forall j \in S$.

b) Show that if $d_j \in \{\pm 1\} \forall j \in S$ and $\sum_{j \in S} p_{ij} d_j = d_i$ for some $i \in S$
then $d_j = d_i \forall j \in S$

c) Show that $\nexists d \in \{\pm 1\}^S$ s.t. $\sum_{j \in S} p_{ij} d_j = -d_i$ for some $i \in S$