# A note about the analysis of MH for coloring.

- Recall we have a chain in space of proper colorings:

1) Select $v \in V$ at random ; 2) Select $c \in \{1 \ldots q\}$ and recolor $v$ if $c$ is allowed.

- We want to show this chain is <u>irreducible</u> if $q \geqslant \Delta + 2$.
  $\Delta = \underset{v \in V}{Max} \deg(v)$ .

- It should be clear that it is <u>enough</u> to show that any two assignments $\underline{x}$ and $\underline{y}$ can be connected by a path

$$\underline{x} \to \underline{z}_1 \to \underline{z}_2 \to \cdots \to \underline{z}_m \to \underline{y}$$
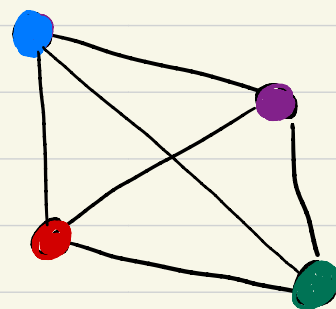
such that <u>two successive assignments differ by only one color</u>:

<span style="color:blue"><u>Question</u> : consider the following working example and find an algorithm assuming $q \geqslant \Delta + 2$ .</span>
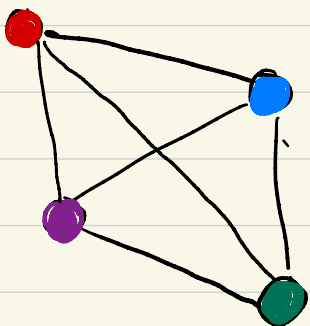
assignment $\underline{x}$                    assignment $\underline{y}$



$\Delta = 4$ ;
$c \in \{ \textcolor{red}{\bullet} \ \textcolor{blue}{\bullet} \ \textcolor{green}{\bullet} \ \textcolor{purple}{\bullet} \ \textcolor{orange}{\bullet} \}$ .

Available colours { 🔴 🔵 🟢 🟣 🟠 }

target: 🏆

initial assignment $\chi$ :
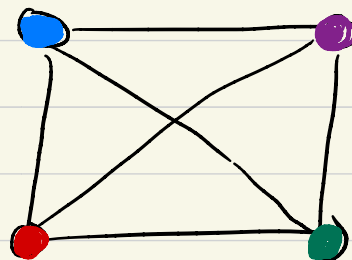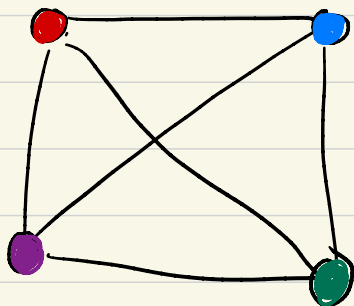


Sequence of recolorings: