A note about the analysis of MH for coloring.

- Recall we have a chain in space of proper colorings:

1) Select $v \in V$ at random ; 2) Select $c \in \{1 \dots q\}$ and recolor v if c is allowed.

LEMMA:

- We want to show this chain is irreducible if $q \geq \Delta + 2$.
$\Delta = \max_{v \in V} \deg(v)$.

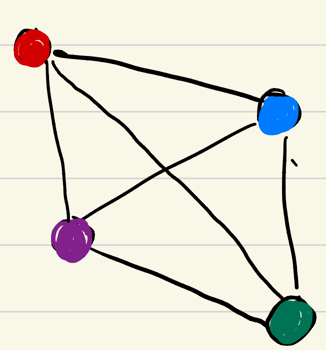
- It should be clear that it is enough to show that any two assignements $\underline{x}$ and $\underline{y}$ can be connected by a path

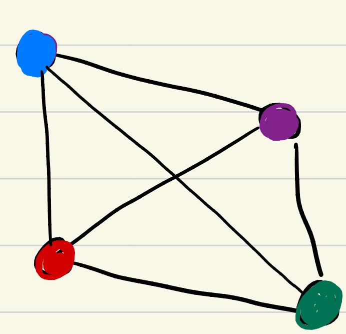
$$\underline{x} \to \underline{z}_1 \to \underline{z}_2 \to \dots \to \underline{z}_m \to \underline{y}$$

such that two successive assignements differ by only one color:

Question : Consider the following working example and find an algorithm assuming $q \geq \Delta + 2$.

assignement $\underline{x}$ assignement $\underline{y}$





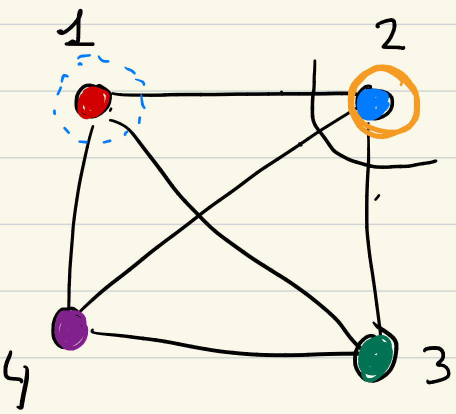
$\Delta = 4$

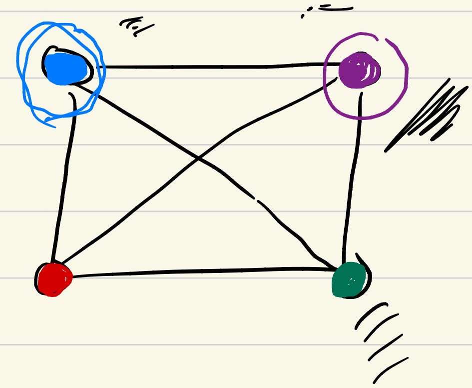
$c \in \{ \text{red}, \text{blue}, \text{green}, \text{purple}, \text{orange} \}$.

$\Delta + 2$ colors.

Available colors { }

initial assignment $\underline{X}$: target: $\underline{Y}$

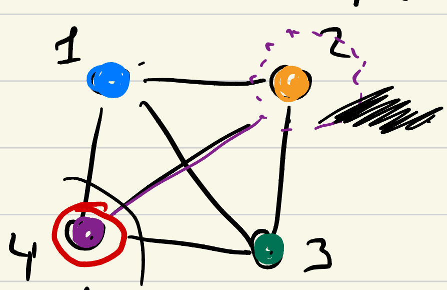




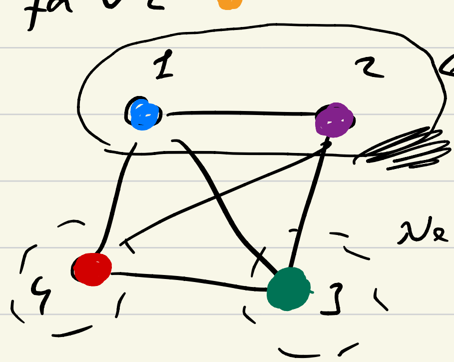
Sequence of recolorings: $\underline{X} \to \underline{Z}_1 \to \underline{Z}_2 \to \dots \to \underline{Y}$.

V1 is recolored in blue? Trial → does not work.

First I take a provisional color for v2



↑ recolor v_4 with provisional color.



part vertices.

Next vertex to recolor

DONE!

Main Idea of Algorithm:

- Order vertices and sequentially take vertices in that order and try to change their color to match those of $\underline{Y}$.
- If you find a conflict (with future vertices) then before coloring current vertex you recolor the conflicting in the future with an available color. $q \geq \Delta + 2$ ensures $\exists$ this available color.