ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 33 Principles of Digital Communications
Homework 13 Dec. 21, 2020

PROBLEM 1. (a) Suppose U and V are binary random variables. The joint distribution induced on (U, V) is given as

$$p_{UV}(u,v) = \begin{cases} 1/3, & (u,v) = (0,0) \\ 1/3, & (u,v) = (1,0) \\ 1/3, & (u,v) = (1,1) \\ 0, & (u,v) = (0,1) \end{cases}.$$

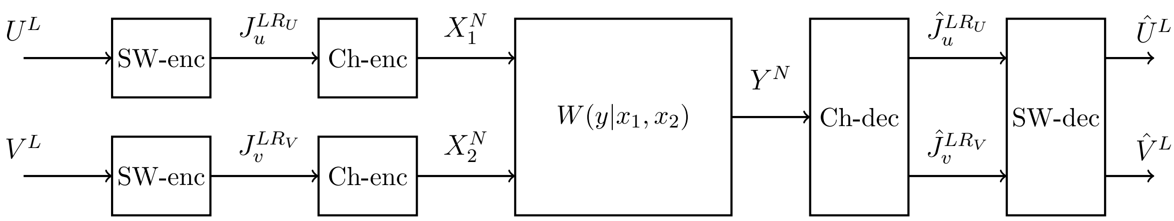
Find the Slepian-Wolf rate region for (U, V) pair.

(b) Now suppose we have a binary additive MAC channel with inputs X_1, X_2 and output Y. The random variables X_1 and X_2 can take values in the set $\{0, 1\}$ and Y can take values in the set $\{0, 1, 2\}$. The relationship between X_1, X_2 and Y is given as

$$Y = X_1 + X_2.$$

Find the capacity region for this MAC.

(c) Now, the aim is to design a communication system that first compresses the source into a bitstream and then employs some channel coding technique to achieve reliable communication. The scheme is given as follows.



Here, SW-enc represents Slepian-Wolf encoder for the source U,V of length L which outputs a bitstream of length LR_U, LR_V respectively. Later, the bitstreams J_u and J_v are encoded by channel encoders (Ch-enc) and then passed through the multiple access channel. As usual, from Y^N; bitstreams $\hat{J}_u$ and $\hat{J}_v$ are estimated by a channel decoder. Finally, the estimated bitstreams are decoded by Slepian-Wolf decoders to obtain U^L and V^L.

For the sources described in part (a) and channel described in part (b), what is the maximum value that L/N can take for a reliable communication?

(d) Consider now an uncoded scheme with the same sources and same channel where $X_1 = U$ and $X_2 = V$. Note that in this scheme, $L = N = 1$. Can (U, V) be recovered from Y? Can the value L/N of this scheme be achieved by schemes as in part (c)?

PROBLEM 2. Consider the multiplicative multiple access channel $Y = X_1X_2$. Find the capacity region when

(a) $X_1 \in \{0, 1\}$, $X_2 \in \{1, 2\}$.

(b) $X_1 \in \{0, 1\}$, $X_2 \in \{1, 2, 3\}$.

(c) $X_1 \in \{1, 2\}$, $X_2 \in \{1, 2\}$.